Phys-ics

HIGH SCHOOL

openstax™

Physics

SENIOR CONTRIBUTING AUTHORS
FATIH GOZUACIK
DENISE PATTISON
CATHERINE TABOR

openstax™

OpenStax
Rice University
6100 Main Street MS-375
Houston, Texas 77005

To learn more about OpenStax, visit https://openstax.org.
Individual print copies and bulk orders can be purchased through our website.

HARDCOVER BOOK ISBN-13	**978-1-975076-51-1**
B&W PAPERBACK BOOK ISBN-13	**978-1-975076-50-4**
DIGITAL VERSION ISBN-13	**978-1-951693-21-3**
ORIGINAL PUBLICATION YEAR	**2020**

10 9 8 7 6 5 4 3 2 1

OpenStax

OpenStax provides free, peer-reviewed, openly licensed textbooks for introductory college and Advanced Placement® courses and low-cost, personalized courseware that helps students learn. A nonprofit ed tech initiative based at Rice University, we're committed to helping students access the tools they need to complete their courses and meet their educational goals.

Rice University

OpenStax, OpenStax CNX, and OpenStax Tutor are initiatives of Rice University. As a leading research university with a distinctive commitment to undergraduate education, Rice University aspires to path-breaking research, unsurpassed teaching, and contributions to the betterment of our world. It seeks to fulfill this mission by cultivating a diverse community of learning and discovery that produces leaders across the spectrum of human endeavor.

Philanthropic Support

OpenStax is grateful for our generous philanthropic partners, who support our vision to improve educational opportunities for all learners.

Laura and John Arnold Foundation

Arthur and Carlyse Ciocca Charitable Foundation

Ann and John Doerr

Bill & Melinda Gates Foundation

Girard Foundation

Google Inc.

The William and Flora Hewlett Foundation

Rusty and John Jaggers

The Calvin K. Kazanjian Economics Foundation

Charles Koch Foundation

Leon Lowenstein Foundation, Inc.

The Maxfield Foundation

Burt and Deedee McMurtry

Michelson 20MM Foundation

National Science Foundation

The Open Society Foundations

Jumee Yhu and David E. Park III

Brian D. Patterson USA-International Foundation

The Bill and Stephanie Sick Fund

Robin and Sandy Stuart Foundation

The Stuart Family Foundation

Tammy and Guillermo Treviño

Contents

PREFACE

Welcome to *Physics*, an OpenStax resource. This textbook was written to increase student access to high-quality learning materials, maintaining highest standards of academic rigor at little to no cost.

About OpenStax

OpenStax is a nonprofit based at Rice University, and it's our mission to improve student access to education. Our first openly licensed college textbook was published in 2012 and our library has since scaled to over 25 books for college and AP® courses used by hundreds of thousands of students. OpenStax Tutor, our low-cost personalized learning tool, is being used in college courses throughout the country. Through our partnerships with philanthropic foundations and our alliance with other educational resource organizations, OpenStax is breaking down the most common barriers to learning and empowering students and instructors to succeed.

About OpenStax resources

Customization

Physics is licensed under a Creative Commons Attribution 4.0 International (CC BY) license, which means that you can distribute, remix, and build upon the content, as long as you provide attribution to OpenStax and its content contributors.

Because our books are openly licensed, you are free to use the entire book or pick and choose the sections that are most relevant to the needs of your course. Feel free to remix the content by assigning your students certain chapters and sections in your syllabus, in the order that you prefer. You can even provide a direct link in your syllabus to the sections in the web view of your book.

Instructors also have the option of creating a customized version of their OpenStax book. The custom version can be made available to students in low-cost print or digital form through their campus bookstore. Visit your book page on OpenStax.org for more information.

Art Attribution in *Physics*

In *Physics*, most art contains attribution to its title, creator or rights holder, host platform, and license within the caption. For art that is openly licensed, anyone may reuse the art as long as they provide the same attribution to its original source. Some art has been provided through permissions and should only be used with the attribution or limitations provided in the credit.

Errata

All OpenStax textbooks undergo a rigorous review process. However, like any professional-grade textbook, errors sometimes occur. The good part is, since our books are web-based, we can make updates periodically. If you have a correction to suggest, submit it through our errata reporting tool. We will review your suggestion and make necessary changes.

Format

You can access this textbook for free in web view or PDF through OpenStax.org, and for a low cost in print.

About *Physics*

This instructional material was initially created through a Texas Education Agency (TEA) initiative to provide high-quality open-source instructional materials to districts free of charge. Funds were allocated by the 84th Texas Legislature (2015) for the creation of state-developed, open-source instructional materials with the request that advanced secondary courses supporting the study of science, technology, engineering, and mathematics should be prioritized.

Physics covers the scope and sequence requirements of a typical one-year physics course. The text provides comprehensive coverage of physical concepts, quantitative examples and skills, and interesting applications. High School Physics has been designed to meet and exceed the requirements of the relevant Texas Essential Knowledge and Skills (TEKS (http://ritter.tea.state.tx.us/rules/tac/chapter112/ch112c.html#112.39)), while allowing significant flexibility for instructors.

Qualified and experienced Texas faculty were involved throughout the development process, and the textbooks were reviewed extensively to ensure effectiveness and usability in each course. Reviewers considered each resource's clarity, accuracy, student support, assessment rigor and appropriateness, alignment to TEKS, and overall quality. Their invaluable suggestions provided the basis for continually improved material and helped to certify that the books are ready for use. The writers and reviewers also considered common course issues, effective teaching strategies, and student engagement to provide instructors and students with useful, supportive content and drive effective learning experiences.

Coverage and scope

Physics presents physical laws, research, concepts, and skills in a logical and engaging progression that should be familiar to most physics faculty. The textbook begins with a general introduction to physics and scientific processes, which is followed by several chapters on motion and Newton's laws. After mechanics, the students will move through thermodynamics, waves and sound, and light and optics. Electricity and magnetism and nuclear physics complete the textbook.

- Chapter 1: What Is Physics?
- Chapter 2: Motion in One Dimension
- Chapter 3: Acceleration
- Chapter 4: Forces and Newton's Laws of Motion
- Chapter 5: Motion in Two Dimensions
- Chapter 6: Circular and Rotational Motion
- Chapter 7: Newton's Law of Gravitation
- Chapter 8: Momentum
- Chapter 9: Work, Energy, and Simple Machines
- Chapter 10: Special Relativity
- Chapter 11: Thermal Energy, Heat, and Work
- Chapter 12: Thermodynamics
- Chapter 13: Waves and Their Properties
- Chapter 14: Sound
- Chapter 15: Light
- Chapter 16: Mirrors and Lenses
- Chapter 17: Diffraction and Interference
- Chapter 18: Static Electricity
- Chapter 19: Electrical Circuits
- Chapter 20: Magnetism
- Chapter 21: The Quantum Nature of Light
- Chapter 22: The Atom
- Chapter 23: Particle Physics

Flexibility

Like any OpenStax content, this textbook can be modified as needed for use by the instructor depending on the needs of the students in the course. Each set of materials created by OpenStax is organized into units and chapters and can be used like a traditional textbook as the entire syllabus for each course. The materials can also be accessed in smaller chunks for more focused use with a single student or an entire class. Instructors are welcome to download and assign the PDF version of the textbook through a learning management system or can use their LMS to link students to specific chapters and sections of the book relevant to the concept being studied. The entire textbook will be available during the fall of 2020 in an editable Google document, and until then instructors are welcome to copy and paste content from the textbook to modify as needed prior to instruction.

Student-centered focus

Physics uses a friendly voice and exciting examples that appeal to a high school audience. The Chapter Openers, for example, include thought-provoking photographs and introductions that connect the content to experiences relevant to student's lives. The writing in our program has been developed with universal design in mind to ensure students of all different backgrounds are reached. Content can be accessed through engaging text, informative visuals, hands-on activities, and online simulations. This diversity of learning media presents a wealth of reinforcement opportunities that allow students to review material in a new and fresh way.

Features

- *Snap Labs*: Give students the opportunity to experience physics through hands-on activities. The labs can be completed quickly and rely primarily on readily available materials so that students can do them at home as they read.
- *Worked Examples*: Promote both analytical and conceptual skills. In each example, the scenario/application is first introduced, followed by a description of the strategy used to solve the problem that emphasizes the concepts involved. These are followed by a fully worked mathematical solution and a discussion of the results.
- *Fun in Physics*: Features physics applications in various entertainment industries.
- *Work in Physics*: Students can explore careers in physics as well as other careers that routinely employ physics.
- *Boundless Physics*: Reveal frontiers in physical knowledge and descriptions of cutting-edge discoveries in physics.
- *Links to Physics*: Highlight connections of physics to other disciplines.
- *Watch Physics*: Support student's understanding of conceptual and computational skills using videos from Khan Academy.
- *Virtual Physics*: Provide inquiry and discovery-based learning by providing a virtual "sandbox" where students can experiment with simulated physics scenarios and equipment using the University of Colorado-developed PhET simulations.
- *Tips for Success*: Offer students advice on how to approach content or problems.

Practice and Assessment

- *Grasp Checks*: Formative assessments that review the comprehension of concepts and skills addressed through reading features, interactive features, and snap labs.
- *Practice Problems*: Challenge students to apply concepts and skills they have seen in a Worked Example to solve a problem.
- *Check Your Understanding*: Conceptual questions that, together with the practice problems, provide formative assessment on key topics in each section.
- *Performance Tasks*: Challenge students to apply the content and skills they have learned to find a solution to a practical situation.
- *Test Prep*: Helps prepare students to successfully respond to the format and rigor of standardized tests. The test prep includes multiple choice, short answer, and extended

response items.

Additional resources

Student and instructor resources

We've compiled additional resources for both students and instructors, including Getting Started Guides, PowerPoint slides, and an instructor answer guide. Instructor resources require a verified instructor account, which you can apply for when you log in or create your account on OpenStax.org. Take advantage of these resources to supplement your OpenStax book.

Partner resources

OpenStax Partners are our allies in the mission to make high-quality learning materials affordable and accessible to students and instructors everywhere. Their tools integrate seamlessly with our OpenStax titles at a low cost. To access the partner resources for your text, visit your book page on OpenStax.org.

About the Authors

Contributing Authors

Fatih Gozuacik (Director of Technology and Education) holds master's degrees in physics and atomic physics from Texas A&M Commerce and Sakarya University in Turkey. In addition to teaching AP® and College Physics, he has been a mentor for the Physics Bowl and the Science Olympiad, and has served as a college counselor for junior and senior students. Fatih was named the 2015 STEM teacher of the year by Educate Texas.

Denise Pattison (East Chambers ISD) was born and raised in Lumberton, Texas. She graduated from Lamar University - Beaumont in 2004 with a degree in science and has taught Physics, Pre AP® Physics, and IPC Physics. She loves teaching because it offers the opportunity to make a difference in a person's life.

Catherine Tabor (Northwest Early High School) holds Bachelors degrees in Mathematics and Physics, a Master's degree in Physics, and is working towards a PhD in Computer Science. She has taught for over twenty years, holding positions at a number of high schools in El Paso and colleges including Evergreen State and UT El Paso. At Northwest Early HS, she teaches Astronomy, AP® Physics, and AP® Computer Science.

Reviewers

- Bryan Callow, Lindenwold High School, Lindenwold, NJ
- John Boehringer, Prosper High School, Prosper, TX
- Wade Green, Stony Point High School, Round Rock, TX
- Susan Vogel, Rockwall-Heath High School, Heath TX
- Stan Hutto, Southwest Christian School, TX
- Richard Lines, Garland High School, Garland, TX
- Penny McCool, San Antonio ISD, Raise Up Texas Coach
- Loren Lykins, Science Department Chairman—Carlisle Independent School District, TX
- Janie Horn, Cleveland High School, Magnolia, TX
- Sean Oshman, Garland Independent School District, TX
- Cort Gillen, Cypress Ranch High School, TX

CHAPTER 1
What is Physics?

Figure 1.1 Galaxies, such as the Andromeda galaxy pictured here, are immense in size. The small blue spots in this photo are also galaxies. The same physical laws apply to objects as large as galaxies or objects as small as atoms. The laws of physics are, therefore, surprisingly few in number. (NASA, JPL-Caltech, P. Barmby, Harvard-Smithsonian Center for Astrophysics).

Chapter Outline

INTRODUCTION Take a look at the image above of the Andromeda Galaxy (Figure 1.1), which contains billions of stars. This galaxy is the nearest one to our own galaxy (the Milky Way) but is still a staggering 2.5 million light years from Earth. (A light year is a measurement of the distance light travels in a year.) Yet, the primary force that affects the movement of stars within Andromeda is the same force that we contend with here on Earth—namely, gravity.

You may soon realize that physics plays a much larger role in your life than you thought. This section introduces you to the realm of physics, and discusses applications of physics in other disciplines of study. It also describes the methods by which science is done, and how scientists communicate their results to each other.

1.1 Physics: Definitions and Applications

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the definition, aims, and branches of physics
- Describe and distinguish classical physics from modern physics and describe the importance of relativity, quantum mechanics, and relativistic quantum mechanics in modern physics
- Describe how aspects of physics are used in other sciences (e.g., biology, chemistry, geology, etc.) as well as in everyday technology

Section Key Terms

atom	classical physics	modern physics
physics	quantum mechanics	theory of relativity

What Physics Is

Think about all of the technological devices that you use on a regular basis. Computers, wireless internet, smart phones, tablets, global positioning system (GPS), MP3 players, and satellite radio might come to mind. Next, think about the most exciting modern technologies that you have heard about in the news, such as trains that levitate above their tracks, *invisibility cloaks* that bend light around them, and microscopic robots that fight diseased cells in our bodies. All of these groundbreaking advancements rely on the principles of **physics**.

Physics is a branch of science. The word *science* comes from a Latin word that means *having knowledge*, and refers the knowledge of how the physical world operates, based on objective evidence determined through observation and experimentation. A key requirement of any scientific explanation of a natural phenomenon is that it must be testable; one must be able to devise and conduct an experimental investigation that either supports or refutes the explanation. It is important to note that some questions fall outside the realm of science precisely because they deal with phenomena that are not scientifically testable. This need for objective evidence helps define the investigative process scientists follow, which will be described later in this chapter.

Physics is the science aimed at describing the fundamental aspects of our universe. This includes what things are in it, what properties of those things are noticeable, and what processes those things or their properties undergo. In simpler terms, physics attempts to describe the basic mechanisms that make our universe behave the way it does. For example, consider a smart phone (Figure 1.2). Physics describes how electric current interacts with the various circuits inside the device. This knowledge helps engineers select the appropriate materials and circuit layout when building the smart phone. Next, consider a GPS. Physics describes the relationship between the speed of an object, the distance over which it travels, and the time it takes to travel that distance. When you use a GPS device in a vehicle, it utilizes these physics relationships to determine the travel time from one location to another.

Figure 1.2 Physics describes the way that electric charge flows through the circuits of this device. Engineers use their knowledge of physics to construct a smartphone with features that consumers will enjoy, such as a GPS function. GPS uses physics equations to determine the driving time between two locations on a map. (@gletham GIS, Social, Mobile Tech Images)

As our technology evolved over the centuries, physics expanded into many branches. Ancient peoples could only study things that they could see with the naked eye or otherwise experience without the aid of scientific equipment. This included the study of kinematics, which is the study of moving objects. For example, ancient people often studied the apparent motion of objects in the sky, such as the sun, moon, and stars. This is evident in the construction of prehistoric astronomical observatories, such as Stonehenge in England (shown in Figure 1.3).

Figure 1.3 Stonehenge is a monument located in England that was built between 3000 and 1000 B.C. It functions as an ancient astronomical observatory, with certain rocks in the monument aligning with the position of the sun during the summer and winter solstices. Other rocks align with the rising and setting of the moon during certain days of the year. (Citypeek, Wikimedia Commons)

Ancient people also studied statics and dynamics, which focus on how objects start moving, stop moving, and change speed and direction in response to forces that push or pull on the objects. This early interest in kinematics and dynamics allowed humans to invent simple machines, such as the lever, the pulley, the ramp, and the wheel. These simple machines were gradually combined and integrated to produce more complicated machines, such as wagons and cranes. Machines allowed humans to gradually do more work more effectively in less time, allowing them to create larger and more complicated buildings and structures, many of which still exist today from ancient times.

As technology advanced, the branches of physics diversified even more. These include branches such as acoustics, the study of sound, and optics, the study of the light. In 1608, the invention of the telescope by a Germany spectacle maker, Hans Lippershey, led to huge breakthroughs in astronomy—the study of objects or phenomena in space. One year later, in 1609, Galileo Galilei began the first studies of the solar system and the universe using a telescope. During the Renaissance era, Isaac Newton used observations made by Galileo to construct his three laws of motion. These laws were the standard for studying kinematics and dynamics even today.

Another major branch of physics is thermodynamics, which includes the study of thermal energy and the transfer of heat. James Prescott Joule, an English physicist, studied the nature of heat and its relationship to work. Joule's work helped lay the foundation for the first of three laws of thermodynamics that describe how energy in our universe is transferred from one object to another or transformed from one form to another. Studies in thermodynamics were motivated by the need to make engines more efficient, keep people safe from the elements, and preserve food.

The 18th and 19th centuries also saw great strides in the study of electricity and magnetism. Electricity involves the study of electric charges and their movements. Magnetism had long ago been noticed as an attractive force between a magnetized object and a metal like iron, or between the opposite poles (North and South) of two magnetized objects. In 1820, Danish physicist Hans Christian Oersted showed that electric currents create magnetic fields. In 1831, English inventor Michael Faraday showed that moving a wire through a magnetic field could induce an electric current. These studies led to the inventions of the electric motor and electric generator, which revolutionized human life by bringing electricity and magnetism into our machines.

The end of the 19th century saw the discovery of radioactive substances by the French scientists Marie and Pierre Curie. Nuclear physics involves studying the nuclei of **atoms**, the source of nuclear radiation. In the 20th century, the study of nuclear physics eventually led to the ability to split the nucleus of an atom, a process called nuclear fission. This process is the basis for nuclear power plants and nuclear weapons. Also, the field of **quantum mechanics**, which involves the mechanics of atoms and molecules, saw great strides during the 20th century as our understanding of atoms and subatomic particles increased (see below).

Early in the 20th century, Albert Einstein revolutionized several branches of physics, especially relativity. Relativity revolutionized our understanding of motion and the universe in general as described further in this chapter. Now, in the 21st century, physicists continue to study these and many other branches of physics.

By studying the most important topics in physics, you will gain analytical abilities that will enable you to apply physics far beyond the scope of what can be included in a single book. These analytical skills will help you to excel academically, and they will also help you to think critically in any career you choose to pursue.

Physics: Past and Present

The word physics is thought to come from the Greek word *phusis*, meaning nature. The study of nature later came to be called *natural philosophy*. From ancient times through the Renaissance, natural philosophy encompassed many fields, including astronomy, biology, chemistry, mathematics, and medicine. Over the last few centuries, the growth of scientific knowledge has resulted in ever-increasing specialization and branching of natural philosophy into separate fields, with physics retaining the most basic facets. Physics, as it developed from the Renaissance to the end of the 19th century, is called **classical physics**. Revolutionary discoveries starting at the beginning of the 20th century transformed physics from classical physics to **modern physics**.

Classical physics is not an exact description of the universe, but it is an excellent approximation under the following conditions: (1) matter must be moving at speeds less than about 1 percent of the speed of light, (2) the objects dealt with must be large enough to be seen with the naked eye, and (3) only weak gravity, such as that generated by Earth, can be involved. Very small objects, such as atoms and molecules, cannot be adequately explained by classical physics. These three conditions apply to almost all of everyday experience. As a result, most aspects of classical physics should make sense on an intuitive level.

Many laws of classical physics have been modified during the 20th century, resulting in revolutionary changes in technology, society, and our view of the universe. As a result, many aspects of modern physics, which occur outside of the range of our everyday experience, may seem bizarre or unbelievable. So why is most of this textbook devoted to classical physics? There are two main reasons. The first is that knowledge of classical physics is necessary to understand modern physics. The second reason is that classical physics still gives an accurate description of the universe under a wide range of everyday circumstances.

Modern physics includes two revolutionary theories: relativity and quantum mechanics. These theories deal with the very fast and the very small, respectively. The **theory of relativity** was developed by Albert Einstein in 1905. By examining how two observers moving relative to each other would see the same phenomena, Einstein devised radical new ideas about time and space. He came to the startling conclusion that the measured length of an object travelling at high speeds (greater than about one percent of the speed of light) is shorter than the same object measured at rest. Perhaps even more bizarre is the idea the time for the same process to occur is different depending on the motion of the observer. Time passes more slowly for an object travelling at high speeds. A trip to the nearest star system, Alpha Centauri, might take an astronaut 4.5 Earth years if the ship travels near the speed of light. However, because time is slowed at higher speeds, the astronaut would age only 0.5 years during the trip. Einstein's ideas of relativity were accepted after they were confirmed by numerous experiments.

Gravity, the force that holds us to Earth, can also affect time and space. For example, time passes more slowly on Earth's surface than for objects farther from the surface, such as a satellite in orbit. The very accurate clocks on global positioning satellites have to correct for this. They slowly keep getting ahead of clocks at Earth's surface. This is called time dilation, and it occurs because gravity, in essence, slows down time.

Large objects, like Earth, have strong enough gravity to distort space. To visualize this idea, think about a bowling ball placed on a trampoline. The bowling ball depresses or curves the surface of the trampoline. If you rolled a marble across the trampoline, it would follow the surface of the trampoline, roll into the depression caused by the bowling ball, and hit the ball. Similarly, the Earth curves space around it in the shape of a funnel. These curves in space due to the Earth cause objects to be attracted to Earth (i.e., gravity).

Because of the way gravity affects space and time, Einstein stated that gravity affects the space-time continuum, as illustrated in Figure 1.4. This is why time proceeds more slowly at Earth's surface than in orbit. In black holes, whose gravity is hundreds of times that of Earth, time passes so slowly that it would appear to a far-away observer to have stopped!

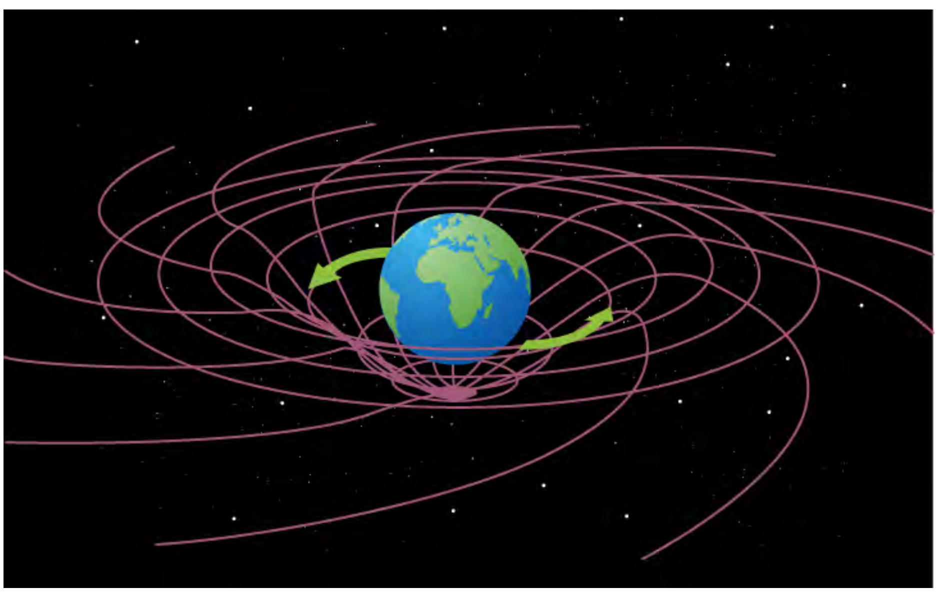


Figure 1.4 Einstein's theory of relativity describes space and time as an interweaved mesh. Large objects, such as a planet, distort space, causing objects to fall in toward the planet due to the action of gravity. Large objects also distort time, causing time to proceed at a slower rate near the surface of Earth compared with the area outside of the distorted region of space-time.

In summary, relativity says that in describing the universe, it is important to realize that time, space and speed are not absolute. Instead, they can appear different to different observers. Einstein's ability to reason out relativity is even more amazing because we cannot see the effects of relativity in our everyday lives.

Quantum mechanics is the second major theory of modern physics. Quantum mechanics deals with the very small, namely, the subatomic particles that make up atoms. Atoms (Figure 1.5) are the smallest units of elements. However, atoms themselves are constructed of even smaller subatomic particles, such as protons, neutrons and electrons. Quantum mechanics strives to describe the properties and behavior of these and other subatomic particles. Often, these particles do not behave in the ways expected by classical physics. One reason for this is that they are small enough to travel at great speeds, near the speed of light.

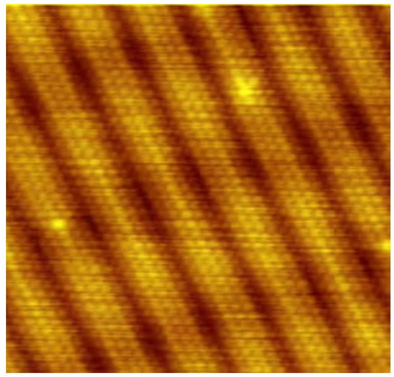


Figure 1.5 Using a scanning tunneling microscope (STM), scientists can see the individual atoms that compose this sheet of gold. (Erwinrossen)

At particle colliders (Figure 1.6), such as the Large Hadron Collider on the France-Swiss border, particle physicists can make subatomic particles travel at very high speeds within a 27 kilometers (17 miles) long superconducting tunnel. They can then study the properties of the particles at high speeds, as well as collide them with each other to see how they exchange energy. This has led to many intriguing discoveries such as the Higgs-Boson particle, which gives matter the property of mass, and antimatter, which causes a huge energy release when it comes in contact with matter.

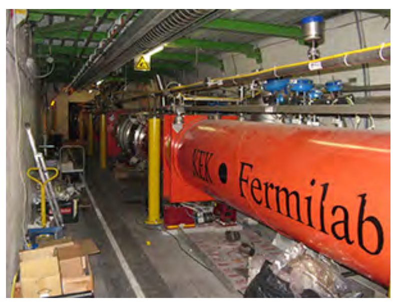


Figure 1.6 Particle colliders such as the Large Hadron Collider in Switzerland or Fermilab in the United States (pictured here), have long tunnels that allows subatomic particles to be accelerated to near light speed. (Andrius.v)

Physicists are currently trying to unify the two theories of modern physics, relativity and quantum mechanics, into a single, comprehensive theory called relativistic quantum mechanics. Relating the behavior of subatomic particles to gravity, time, and space will allow us to explain how the universe works in a much more comprehensive way.

Application of Physics

You need not be a scientist to use physics. On the contrary, knowledge of physics is useful in everyday situations as well as in nonscientific professions. For example, physics can help you understand why you shouldn't put metal in the microwave (Figure 1.7), why a black car radiator helps remove heat in a car engine, and why a white roof helps keep the inside of a house cool. The operation of a car's ignition system, as well as the transmission of electrical signals through our nervous system, are much easier to understand when you think about them in terms of the basic physics of electricity.

Figure 1.7 Why can't you put metal in the microwave? Microwaves are high-energy radiation that increases the movement of electrons in

metal. These moving electrons can create an electrical current, causing sparking that can lead to a fire. (= MoneyBlogNewz)

Physics is the foundation of many important scientific disciplines. For example, chemistry deals with the interactions of atoms and molecules. Not surprisingly, chemistry is rooted in atomic and molecular physics. Most branches of engineering are also applied physics. In architecture, physics is at the heart of determining structural stability, acoustics, heating, lighting, and cooling for buildings. Parts of geology, the study of nonliving parts of Earth, rely heavily on physics; including radioactive dating, earthquake analysis, and heat transfer across Earth's surface. Indeed, some disciplines, such as biophysics and geophysics, are hybrids of physics and other disciplines.

Physics also describes the chemical processes that power the human body. Physics is involved in medical diagnostics, such as x-rays, magnetic resonance imaging (MRI), and ultrasonic blood flow measurements (Figure 1.8). Medical therapy Physics also has many applications in biology, the study of life. For example, physics describes how cells can protect themselves using their cell walls and cell membranes (Figure 1.9). Medical therapy sometimes directly involves physics, such as in using X-rays to diagnose health conditions. Physics can also explain what we perceive with our senses, such as how the ears detect sound or the eye detects color.

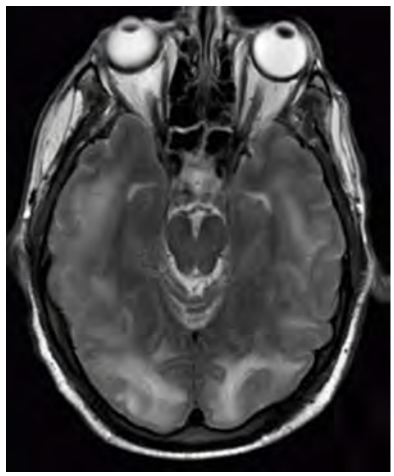


Figure 1.8 Magnetic resonance imaging (MRI) uses electromagnetic waves to yield an image of the brain, which doctors can use to find diseased regions. (Rashmi Chawla, Daniel Smith, and Paul E. Marik)

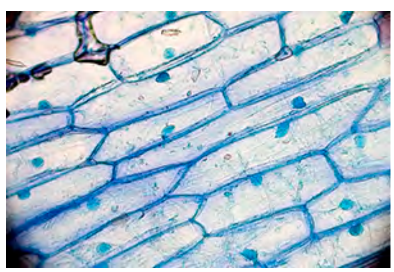


Figure 1.9 Physics, chemistry, and biology help describe the properties of cell walls in plant cells, such as the onion cells seen here. (Umberto Salvagnin)

BOUNDLESS PHYSICS

The Physics of Landing on a Comet

On November 12, 2014, the European Space Agency's Rosetta spacecraft (shown in Figure 1.10) became the first ever to reach and orbit a comet. Shortly after, Rosetta's rover, Philae, landed on the comet, representing the first time humans have ever landed a space probe on a comet.

Figure 1.10 The Rosetta spacecraft, with its large and revolutionary solar panels, carried the Philae lander to a comet. The lander then detached and landed on the comet’s surface. (European Space Agency)

After traveling 6.4 billion kilometers starting from its launch on Earth, Rosetta landed on the comet 67P/Churyumov-Gerasimenko, which is only 4 kilometers wide. Physics was needed to successfully plot the course to reach such a small, distant, and rapidly moving target. Rosetta’s path to the comet was not straight forward. The probe first had to travel to Mars so that Mars’s gravity could accelerate it and divert it in the exact direction of the comet.

This was not the first time humans used gravity to power our spaceships. Voyager 2, a space probe launched in 1977, used the gravity of Saturn to *slingshot* over to Uranus and Neptune (illustrated in Figure 1.11), providing the first pictures ever taken of these planets. Now, almost 40 years after its launch, Voyager 2 is at the very edge of our solar system and is about to enter interstellar space. Its sister ship, Voyager 1 (illustrated in Figure 1.11), which was also launched in 1977, is already there.

To listen to the sounds of interstellar space or see images that have been transmitted back from the Voyager I or to learn more about the Voyager mission, visit the Voyager’s Mission website (https://openstax.org/l/28voyager) .

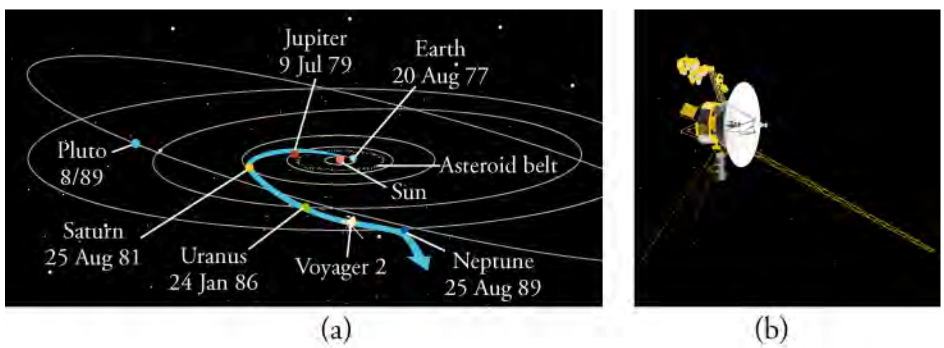


Figure 1.11 a) Voyager 2, launched in 1977, used the gravity of Saturn to slingshot over to Uranus and Neptune. NASA b) A rendering of Voyager 1, the first space probe to ever leave our solar system and enter interstellar space. NASA

Both Voyagers have electrical power generators based on the decay of radioisotopes. These generators have served them for almost 40 years. Rosetta, on the other hand, is solar-powered. In fact, Rosetta became the first space probe to travel beyond the asteroid belt by relying only on solar cells for power generation.

At 800 million kilometers from the sun, Rosetta receives sunlight that is only 4 percent as strong as on Earth. In addition, it is very cold in space. Therefore, a lot of physics went into developing Rosetta’s low-intensity low-temperature solar cells.

In this sense, the Rosetta project nicely shows the huge range of topics encompassed by physics: from modeling the movement of gigantic planets over huge distances within our solar systems, to learning how to generate electric power from low-intensity light. Physics is, by far, the broadest field of science.

GRASP CHECK

What characteristics of the solar system would have to be known or calculated in order to send a probe to a distant planet, such as Jupiter?

a. the effects due to the light from the distant stars
b. the effects due to the air in the solar system
c. the effects due to the gravity from the other planets
d. the effects due to the cosmic microwave background radiation

In summary, physics studies many of the most basic aspects of science. A knowledge of physics is, therefore, necessary to understand all other sciences. This is because physics explains the most basic ways in which our universe works. However, it is not necessary to formally study all applications of physics. A knowledge of the basic laws of physics will be most useful to you, so that you can use them to solve some everyday problems. In this way, the study of physics can improve your problem-solving skills.

Check Your Understanding

1. Which of the following is *not* an essential feature of scientific explanations?
a. They must be subject to testing.
b. They strictly pertain to the physical world.
c. Their validity is judged based on objective observations.
d. Once supported by observation, they can be viewed as a fact.

2. Which of the following does *not* represent a question that can be answered by science?
a. How much energy is released in a given nuclear chain reaction?
b. Can a nuclear chain reaction be controlled?
c. Should uncontrolled nuclear reactions be used for military applications?
d. What is the half-life of a waste product of a nuclear reaction?

3. What are the three conditions under which classical physics provides an excellent description of our universe?
a. 1. Matter is moving at speeds less than about 1 percent of the speed of light
   2. Objects dealt with must be large enough to be seen with the naked eye.
   3. Strong electromagnetic fields are involved.

b. 1. Matter is moving at speeds less than about 1 percent of the speed of light.
   2. Objects dealt with must be large enough to be seen with the naked eye.
   3. Only weak gravitational fields are involved.

c. 1. Matter is moving at great speeds, comparable to the speed of light.
   2. Objects dealt with are large enough to be seen with the naked eye.
   3. Strong gravitational fields are involved.

d. 1. Matter is moving at great speeds, comparable to the speed of light.
   2. Objects are just large enough to be visible through the most powerful telescope.
   3. Only weak gravitational fields are involved.

4. Why is the Greek word for nature appropriate in describing the field of physics?
a. Physics is a natural science that studies life and living organism on habitable planets like Earth.
b. Physics is a natural science that studies the laws and principles of our universe.
c. Physics is a physical science that studies the composition, structure, and changes of matter in our universe.
d. Physics is a social science that studies the social behavior of living beings on habitable planets like Earth.

5. Which aspect of the universe is studied by quantum mechanics?
a. objects at the galactic level
b. objects at the classical level
c. objects at the subatomic level
d. objects at all levels, from subatomic to galactic

1.2 The Scientific Methods

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain how the methods of science are used to make scientific discoveries
- Define a scientific model and describe examples of physical and mathematical models used in physics
- Compare and contrast hypothesis, theory, and law

Section Key Terms

experiment	hypothesis	model	observation	principle
scientific law	scientific methods	theory	universal	

Scientific Methods

Scientists often plan and carry out investigations to answer questions about the universe around us. Such laws are intrinsic to the universe, meaning that humans did not create them and cannot change them. We can only discover and understand them. Their discovery is a very human endeavor, with all the elements of mystery, imagination, struggle, triumph, and disappointment inherent in any creative effort. The cornerstone of discovering natural laws is observation. Science must describe the universe as it is, not as we imagine or wish it to be.

We all are curious to some extent. We look around, make generalizations, and try to understand what we see. For example, we look up and wonder whether one type of cloud signals an oncoming storm. As we become serious about exploring nature, we become more organized and formal in collecting and analyzing data. We attempt greater precision, perform controlled experiments (if we can), and write down ideas about how data may be organized. We then formulate models, theories, and laws based on the data we have collected, and communicate those results with others. This, in a nutshell, describes the **scientific method** that scientists employ to decide scientific issues on the basis of evidence from observation and experiment.

An investigation often begins with a scientist making an **observation**. The scientist observes a pattern or trend within the natural world. Observation may generate questions that the scientist wishes to answer. Next, the scientist may perform some research about the topic and devise a **hypothesis**. A hypothesis is a testable statement that describes how something in the natural world works. In essence, a hypothesis is an educated guess that explains something about an observation.

Scientists may test the hypothesis by performing an **experiment**. During an experiment, the scientist collects data that will help them learn about the phenomenon they are studying. Then the scientists analyze the results of the experiment (that is, the data), often using statistical, mathematical, and/or graphical methods. From the data analysis, they draw conclusions. They may conclude that their experiment either supports or rejects their hypothesis. If the hypothesis is supported, the scientist usually goes on to test another hypothesis related to the first. If their hypothesis is rejected, they will often then test a new and different hypothesis in their effort to learn more about whatever they are studying.

Scientific processes can be applied to many situations. Let's say that you try to turn on your car, but it will not start. You have just made an observation! You ask yourself, "Why won't my car start?" You can now use scientific processes to answer this question. First, you generate a hypothesis such as, "The car won't start because it has no gasoline in the gas tank." To test this hypothesis, you put gasoline in the car and try to start it again. If the car starts, then your hypothesis is supported by the experiment. If the car does not start, then your hypothesis is rejected. You will then need to think up a new hypothesis to test such as, "My car won't start because the fuel pump is broken." Hopefully, your investigations lead you to discover why the car won't start and enable you to fix it.

Modeling

A **model** is a representation of something that is often too difficult (or impossible) to study directly. Models can take the form of physical models, equations, computer programs, or simulations—computer graphics/animations. Models are tools that are especially useful in modern physics because they let us visualize phenomena that we normally cannot observe with our senses, such as very small objects or objects that move at high speeds. For example, we can understand the structure of an atom using models, despite the fact that no one has ever seen an atom with their own eyes. Models are always approximate, so they are simpler to consider than the real situation; the more complete a model is, the more complicated it must be. Models put the intangible or the extremely complex into human terms that we can visualize, discuss, and hypothesize about.

Scientific models are constructed based on the results of previous experiments. Even still, models often only describe a phenomenon partially or in a few limited situations. Some phenomena are so complex that they may be impossible to model them in their entirety, even using computers. An example is the electron cloud model of the atom in which electrons are moving around the atom's center in distinct clouds (Figure 1.12), that represent the likelihood of finding an electron in different places. This model helps us to visualize the structure of an atom. However, it does not show us exactly where an electron will be within its cloud at any one particular time.

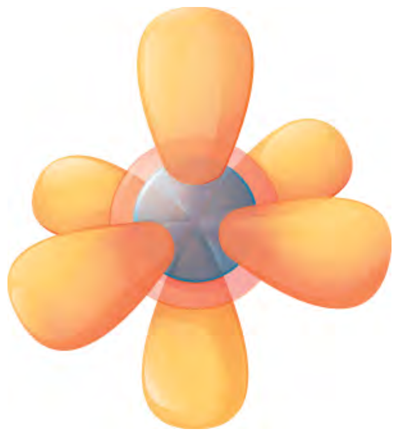


Figure 1.12 The electron cloud model of the atom predicts the geometry and shape of areas where different electrons may be found in an atom. However, it cannot indicate exactly where an electron will be at any one time.

As mentioned previously, physicists use a variety of models including equations, physical models, computer simulations, etc. For example, three-dimensional models are often commonly used in chemistry and physics to model molecules. Properties other than appearance or location are usually modelled using mathematics, where functions are used to show how these properties relate to one another. Processes such as the formation of a star or the planets, can also be modelled using computer simulations. Once a simulation is correctly programmed based on actual experimental data, the simulation can allow us to view processes that happened in the past or happen too quickly or slowly for us to observe directly. In addition, scientists can also run virtual experiments using computer-based models. In a model of planet formation, for example, the scientist could alter the amount or type of rocks present in space and see how it affects planet formation.

Scientists use models and experimental results to construct explanations of observations or design solutions to problems. For example, one way to make a car more fuel efficient is to reduce the friction or drag caused by air flowing around the moving car. This can be done by designing the body shape of the car to be more aerodynamic, such as by using rounded corners instead of sharp ones. Engineers can then construct physical models of the car body, place them in a wind tunnel, and examine the flow of air around the model. This can also be done mathematically in a computer simulation. The air flow pattern can be analyzed for regions smooth air flow and for eddies that indicate drag. The model of the car body may have to be altered slightly to produce the smoothest pattern of air flow (i.e., the least drag). The pattern with the least drag may be the solution to increasing fuel efficiency of the car. This solution might then be incorporated into the car design.

Snap Lab

Using Models and the Scientific Processes

Be sure to secure loose items before opening the window or door.

In this activity, you will learn about scientific models by making a model of how air flows through your classroom or a room in your house.

- One room with at least one window or door that can be opened
- Piece of single-ply tissue paper
  1. Work with a group of four, as directed by your teacher. Close all of the windows and doors in the room you are working in. Your teacher may assign you a specific window or door to study.

2. Before opening any windows or doors, draw a to-scale diagram of your room. First, measure the length and width of your room using the tape measure. Then, transform the measurement using a scale that could fit on your paper, such as 5 centimeters = 1 meter.
3. Your teacher will assign you a specific window or door to study air flow. On your diagram, add arrows showing your hypothesis (before opening any windows or doors) of how air will flow through the room when your assigned window or door is opened. Use pencil so that you can easily make changes to your diagram.
4. On your diagram, mark four locations where you would like to test air flow in your room. To test for airflow, hold a strip of single ply tissue paper between the thumb and index finger. Note the direction that the paper moves when exposed to the airflow. Then, for each location, predict which way the paper will move if your air flow diagram is correct.
5. Now, each member of your group will stand in one of the four selected areas. Each member will test the airflow Agree upon an approximate height at which everyone will hold their papers.
6. When you teacher tells you to, open your assigned window and/or door. Each person should note the direction that their paper points immediately after the window or door was opened. Record your results on your diagram.
7. Did the airflow test data support or refute the hypothetical model of air flow shown in your diagram? Why or why not? Correct your model based on your experimental evidence.
8. With your group, discuss how accurate your model is. What limitations did it have? Write down the limitations that your group agreed upon.

GRASP CHECK

Your diagram is a model, based on experimental evidence, of how air flows through the room. Could you use your model to predict how air would flow through a new window or door placed in a different location in the classroom? Make a new diagram that predicts the room's airflow with the addition of a new window or door. Add a short explanation that describes how.

a. Yes, you could use your model to predict air flow through a new window. The earlier experiment of air flow would help you model the system more accurately.
b. Yes, you could use your model to predict air flow through a new window. The earlier experiment of air flow is not useful for modeling the new system.
c. No, you cannot model a system to predict the air flow through a new window. The earlier experiment of air flow would help you model the system more accurately.
d. No, you cannot model a system to predict the air flow through a new window. The earlier experiment of air flow is not useful for modeling the new system.

Scientific Laws and Theories

A **scientific law** is a description of a pattern in nature that is true in all circumstances that have been studied. That is, physical laws are meant to be **universal**, meaning that they apply throughout the known universe. Laws are often also concise, whereas theories are more complicated. A law can be expressed in the form of a single sentence or mathematical equation. For example, Newton's second law of motion, which relates the motion of an object to the force applied (F), the mass of the object (m), and the object's acceleration (a), is simply stated using the equation

$$F = ma.$$

Scientific ideas and explanations that are true in many, but not all situations in the universe are usually called **principles**. An example is Pascal's principle, which explains properties of liquids, but not solids or gases. However, the distinction between laws and principles is sometimes not carefully made in science.

A **theory** is an explanation for patterns in nature that is supported by much scientific evidence and verified multiple times by multiple researchers. While many people confuse theories with educated guesses or hypotheses, theories have withstood more rigorous testing and verification than hypotheses.

As a closing idea about scientific processes, we want to point out that scientific laws and theories, even those that have been supported by experiments for centuries, can still be changed by new discoveries. This is especially true when new technologies emerge that allow us to observe things that were formerly unobservable. Imagine how viewing previously invisible objects with a microscope or viewing Earth for the first time from space may have instantly changed our scientific theories and laws! What discoveries still await us in the future? The constant retesting and perfecting of our scientific laws and theories allows our knowledge of nature to progress. For this reason, many scientists are reluctant to say that their studies *prove* anything. By saying *support* instead of *prove*, it keeps the door open for future discoveries, even if they won't occur for centuries or even millennia.

Check Your Understanding

6. Explain why scientists sometimes use a model rather than trying to analyze the behavior of the real system.
   a. Models are simpler to analyze.
   b. Models give more accurate results.
   c. Models provide more reliable predictions.
   d. Models do not require any computer calculations.

7. Describe the difference between a question, generated through observation, and a hypothesis.
   a. They are the same.
   b. A hypothesis has been thoroughly tested and found to be true.
   c. A hypothesis is a tentative assumption based on what is already known.
   d. A hypothesis is a broad explanation firmly supported by evidence.

8. What is a scientific model and how is it useful?
   a. A scientific model is a representation of something that can be easily studied directly. It is useful for studying things that can be easily analyzed by humans.
   b. A scientific model is a representation of something that is often too difficult to study directly. It is useful for studying a complex system or systems that humans cannot observe directly.
   c. A scientific model is a representation of scientific equipment. It is useful for studying working principles of scientific equipment.
   d. A scientific model is a representation of a laboratory where experiments are performed. It is useful for studying requirements needed inside the laboratory.

9. Which of the following statements is correct about the hypothesis?
   a. The hypothesis must be validated by scientific experiments.
   b. The hypothesis must not include any physical quantity.
   c. The hypothesis must be a short and concise statement.
   d. The hypothesis must apply to all the situations in the universe.

10. What is a scientific theory?
   a. A scientific theory is an explanation of natural phenomena that is supported by evidence.
   b. A scientific theory is an explanation of natural phenomena without the support of evidence.
   c. A scientific theory is an educated guess about the natural phenomena occurring in nature.
   d. A scientific theory is an uneducated guess about natural phenomena occurring in nature.

11. Compare and contrast a hypothesis and a scientific theory.
   a. A hypothesis is an explanation of the natural world with experimental support, while a scientific theory is an educated guess about a natural phenomenon.
   b. A hypothesis is an educated guess about natural phenomenon, while a scientific theory is an explanation of natural world with experimental support.
   c. A hypothesis is experimental evidence of a natural phenomenon, while a scientific theory is an explanation of the natural world with experimental support.
   d. A hypothesis is an explanation of the natural world with experimental support, while a scientific theory is experimental evidence of a natural phenomenon.

1.3 The Language of Physics: Physical Quantities and Units

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Associate physical quantities with their International System of Units (SI)and perform conversions among SI units using scientific notation
- Relate measurement uncertainty to significant figures and apply the rules for using significant figures in calculations
- Correctly create, label, and identify relationships in graphs using mathematical relationships (e.g., slope, *y*-intercept, inverse, quadratic and logarithmic)

Section Key Terms

accuracy	ampere	constant	conversion factor	dependent variable
derived units	English units	exponential relationship	fundamental physical units	independent variable
inverse relationship	inversely proportional	kilogram	linear relationship	logarithmic (log) scale
log-log plot	meter	method of adding percents	order of magnitude	precision
quadratic relationship	scientific notation	second	semi-log plot	SI units
significant figures	slope	uncertainty	variable	*y*-intercept

The Role of Units

Physicists, like other scientists, make observations and ask basic questions. For example, how big is an object? How much mass does it have? How far did it travel? To answer these questions, they make measurements with various instruments (e.g., meter stick, balance, stopwatch, etc.).

The measurements of physical quantities are expressed in terms of units, which are standardized values. For example, the length of a race, which is a physical quantity, can be expressed in meters (for sprinters) or kilometers (for long distance runners). Without standardized units, it would be extremely difficult for scientists to express and compare measured values in a meaningful way (Figure 1.13).

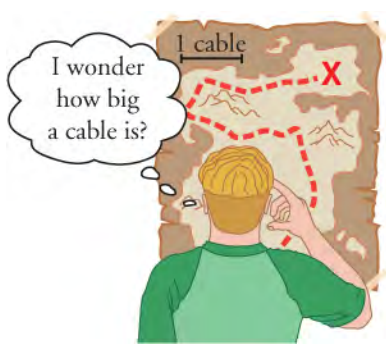


Figure 1.13 Distances given in unknown units are maddeningly useless.

All physical quantities in the International System of Units (SI) are expressed in terms of combinations of seven **fundamental**

physical units, which are units for: length, mass, time, electric current, temperature, amount of a substance, and luminous intensity.

SI Units: Fundamental and Derived Units

There are two major systems of units used in the world: **SI units** (acronym for the French Le Système International d'Unités, also known as the metric system), and **English units** (also known as the imperial system). English units were historically used in nations once ruled by the British Empire. Today, the United States is the only country that still uses English units extensively. Virtually every other country in the world now uses the metric system, which is the standard system agreed upon by scientists and mathematicians.

Some physical quantities are more fundamental than others. In physics, there are seven fundamental physical quantities that are measured in base or physical fundamental units: length, mass, time, electric current temperature, amount of substance, and luminous intensity. Units for other physical quantities (such as force, speed, and electric charge) described by mathematically combining these seven base units. In this course, we will mainly use five of these: length, mass, time, electric current and temperature. The units in which they are measured are the meter, kilogram, second, ampere, kelvin, mole, and candela (Table 1.1). All other units are made by mathematically combining the fundamental units. These are called **derived units**.

Quantity	Name	Symbol
Length	Meter	m
Mass	Kilogram	kg
Time	Second	s
Electric current	Ampere	a
Temperature	Kelvin	k
Amount of substance	Mole	mol
Luminous intensity	Candela	cd

Table 1.1 SI Base Units

The Meter

The SI unit for length is the **meter** (m). The definition of the meter has changed over time to become more accurate and precise. The meter was first defined in 1791 as 1/10,000,000 of the distance from the equator to the North Pole. This measurement was improved in 1889 by redefining the meter to be the distance between two engraved lines on a platinum-iridium bar. (The bar is now housed at the International Bureau of Weights and Meaures, near Paris). By 1960, some distances could be measured more precisely by comparing them to wavelengths of light. The meter was redefined as 1,650,763.73 wavelengths of orange light emitted by krypton atoms. In 1983, the meter was given its present definition as the distance light travels in a vacuum in 1/299,792,458 of a second (Figure 1.14).

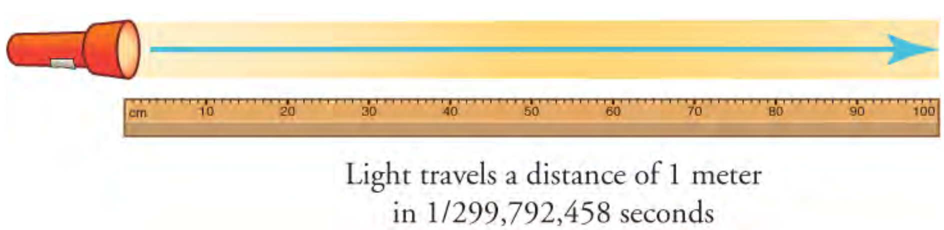


Figure 1.14 The meter is defined to be the distance light travels in 1/299,792,458 of a second through a vacuum. Distance traveled is speed multiplied by time.

The Kilogram

The SI unit for mass is the **kilogram** (kg). It is defined to be the mass of a platinum-iridium cylinder, housed at the International Bureau of Weights and Measures near Paris. Exact replicas of the standard kilogram cylinder are kept in numerous locations throughout the world, such as the National Institute of Standards and Technology in Gaithersburg, Maryland. The determination of all other masses can be done by comparing them with one of these standard kilograms.

The Second

The SI unit for time, the **second** (s) also has a long history. For many years it was defined as 1/86,400 of an average solar day. However, the average solar day is actually very gradually getting longer due to gradual slowing of Earth's rotation. Accuracy in the fundamental units is essential, since all other measurements are derived from them. Therefore, a new standard was adopted to define the second in terms of a non-varying, or constant, physical phenomenon. One constant phenomenon is the very steady vibration of Cesium atoms, which can be observed and counted. This vibration forms the basis of the cesium atomic clock. In 1967, the second was redefined as the time required for 9,192,631,770 Cesium atom vibrations (Figure 1.15).

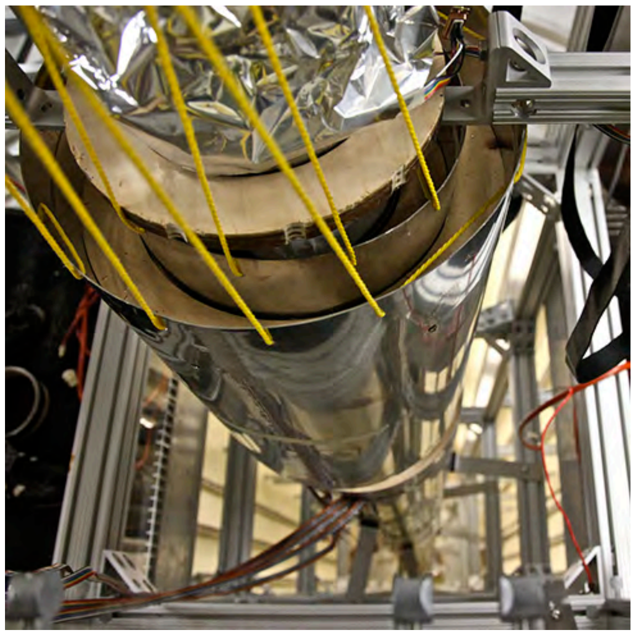


Figure 1.15 An atomic clock such as this one uses the vibrations of cesium atoms to keep time to a precision of one microsecond per year. The fundamental unit of time, the second, is based on such clocks. This image is looking down from the top of an atomic clock. (Steve Jurvetson/Flickr)

The Ampere

Electric current is measured in the **ampere** (A), named after Andre Ampere. You have probably heard of amperes, or *amps*, when people discuss electrical currents or electrical devices. Understanding an ampere requires a basic understanding of electricity and magnetism, something that will be explored in depth in later chapters of this book. Basically, two parallel wires with an electric current running through them will produce an attractive force on each other. One ampere is defined as the amount of electric current that will produce an attractive force of 2.7×10^{-7} newton per meter of separation between the two wires (the newton is the derived unit of force).

Kelvins

The SI unit of temperature is the **kelvin** (or kelvins, but not degrees kelvin). This scale is named after physicist William Thomson, Lord Kelvin, who was the first to call for an absolute temperature scale. The Kelvin scale is based on absolute zero. This is the point at which all thermal energy has been removed from all atoms or molecules in a system. This temperature, 0 K, is equal to –273.15 °C and –459.67 °F. Conveniently, the Kelvin scale actually changes in the same way as the Celsius scale. For example, the freezing point (0 °C) and boiling points of water (100 °C) are 100 degrees apart on the Celsius scale. These two temperatures are also 100 kelvins apart (freezing point = 273.15 K; boiling point = 373.15 K).

Metric Prefixes

Physical objects or phenomena may vary widely. For example, the size of objects varies from something very small (like an atom) to something very large (like a star). Yet the standard metric unit of length is the meter. So, the metric system includes many prefixes that can be attached to a unit. Each prefix is based on factors of 10 (10, 100, 1,000, etc., as well as 0.1, 0.01, 0.001, etc.). Table 1.2 gives the metric prefixes and symbols used to denote the different various factors of 10 in the metric system.

Prefix	Symbol	Value[1]	Example Name	Example Symbol	Example Value	Example Description
exa	E	10^{18}	Exameter	Em	10^{18} m	Distance light travels in a century
peta	P	10^{15}	Petasecond	Ps	10^{15} s	30 million years
tera	T	10^{12}	Terawatt	TW	10^{12} W	Powerful laser output
giga	G	10^{9}	Gigahertz	GHz	10^{9} Hz	A microwave frequency
mega	M	10^{6}	Megacurie	MCi	10^{6} Ci	High radioactivity
kilo	k	10^{3}	Kilometer	km	10^{3} m	About 6/10 mile
hector	h	10^{2}	Hectoliter	hL	10^{2} L	26 gallons
deka	da	10^{1}	Dekagram	dag	10^{1} g	Teaspoon of butter
____	____	10^{0} (=1)				
deci	d	10^{-1}	Deciliter	dL	10^{-1} L	Less than half a soda
centi	c	10^{-2}	Centimeter	Cm	10^{-2} m	Fingertip thickness
milli	m	10^{-3}	Millimeter	Mm	10^{-3} m	Flea at its shoulder
micro	µ	10^{-6}	Micrometer	µm	10^{-6} m	Detail in microscope
nano	n	10^{-9}	Nanogram	Ng	10^{-9} g	Small speck of dust
pico	p	10^{-12}	Picofarad	pF	10^{-12} F	Small capacitor in radio
femto	f	10^{-15}	Femtometer	Fm	10^{-15} m	Size of a proton
atto	a	10^{-18}	Attosecond	as	10^{-18} s	Time light takes to cross an atom

Table 1.2 Metric Prefixes for Powers of 10 and Their Symbols [1]See Appendix A for a discussion of powers of 10. Note—Some examples are approximate.

The metric system is convenient because conversions between metric units can be done simply by moving the decimal place of a number. This is because the metric prefixes are sequential powers of 10. There are 100 centimeters in a meter, 1000 meters in a kilometer, and so on. In nonmetric systems, such as U.S. customary units, the relationships are less simple—there are 12 inches in a foot, 5,280 feet in a mile, 4 quarts in a gallon, and so on. Another advantage of the metric system is that the same unit can be used over extremely large ranges of values simply by switching to the most-appropriate metric prefix. For example, distances in meters are suitable for building construction, but kilometers are used to describe road construction. Therefore, with the metric system, there is no need to invent new units when measuring very small or very large objects—you just have to move the decimal point (and use the appropriate prefix).

Known Ranges of Length, Mass, and Time

Table 1.3 lists known lengths, masses, and time measurements. You can see that scientists use a range of measurement units. This wide range demonstrates the vastness and complexity of the universe, as well as the breadth of phenomena physicists study. As you examine this table, note how the metric system allows us to discuss and compare an enormous range of phenomena, using one system of measurement (Figure 1.16 and Figure 1.17).

Length (m)	Phenomenon Measured	Mass (Kg)	Phenomenon Measured[1]	Time (s)	Phenomenon Measured[1]
10^{-18}	Present experimental limit to smallest observable detail	10^{-30}	Mass of an electron (9.11×10^{-31} kg)	10^{-23}	Time for light to cross a proton
10^{-15}	Diameter of a proton	10^{-27}	Mass of a hydrogen atom (1.67×10^{-27} kg)	10^{-22}	Mean life of an extremely unstable nucleus
10^{14}	Diameter of a uranium nucleus	10^{-15}	Mass of a bacterium	10^{-15}	Time for one oscillation of a visible light
10^{-10}	Diameter of a hydrogen atom	10^{-5}	Mass of a mosquito	10^{-13}	Time for one vibration of an atom in a solid
10^{-8}	Thickness of membranes in cell of living organism	10^{-2}	Mass of a hummingbird	10^{-8}	Time for one oscillation of an FM radio wave
10^{-6}	Wavelength of visible light	1	Mass of a liter of water (about a quart)	10^{-3}	Duration of a nerve impulse
10^{-3}	Size of a grain of sand	10^{2}	Mass of a person	1	Time for one heartbeat
1	Height of a 4-year-old child	10^{3}	Mass of a car	10^{5}	One day (8.64×10^{4} s)
10^{2}	Length of a football field	10^{8}	Mass of a large ship	10^{7}	One year (3.16×10^{7} s)
10^{4}	Greatest ocean depth	10^{12}	Mass of a large iceberg	10^{9}	About half the life expectancy of a human
10^{7}	Diameter of Earth	10^{15}	Mass of the nucleus of a comet	10^{11}	Recorded history
10^{11}	Distance from Earth to the sun	10^{23}	Mass of the moon (7.35×10^{22} kg)	10^{17}	Age of Earth
10^{16}	Distance traveled by light in 1 year (a light year)	10^{25}	Mass of Earth (5.97×10^{24} kg)	10^{18}	Age of the universe
10^{21}	Diameter of the Milky Way Galaxy	10^{30}	Mass of the Sun (1.99×10^{24} kg)		

Table 1.3 Approximate Values of Length, Mass, and Time [1] More precise values are in parentheses.

Length (m)	Phenomenon Measured	Mass (Kg)	Phenomenon Measured[1]	Time (s)	Phenomenon Measured[1]
10^{22}	Distance from Earth to the nearest large galaxy (Andromeda)	10^{42}	Mass of the Milky Way galaxy (current upper limit)		
10^{26}	Distance from Earth to the edges of the known universe	10^{53}	Mass of the known universe (current upper limit)		

Table 1.3 Approximate Values of Length, Mass, and Time [1] More precise values are in parentheses.

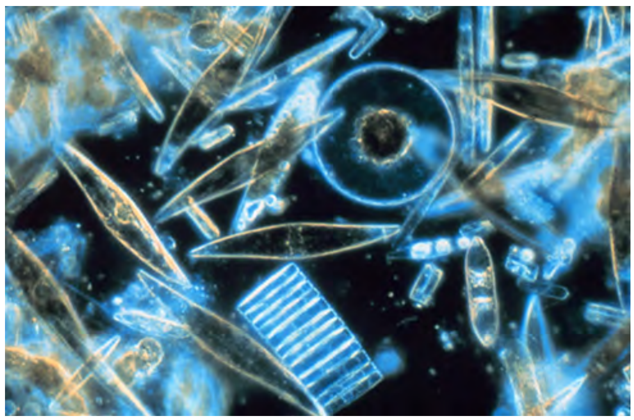


Figure 1.16 Tiny phytoplankton float among crystals of ice in the Antarctic Sea. They range from a few micrometers to as much as 2 millimeters in length. (Prof. Gordon T. Taylor, Stony Brook University; NOAA Corps Collections)

Figure 1.17 Galaxies collide 2.4 billion light years away from Earth. The tremendous range of observable phenomena in nature challenges the imagination. (NASA/CXC/UVic./A. Mahdavi et al. Optical/lensing: CFHT/UVic./H. Hoekstra et al.)

Using Scientific Notation with Physical Measurements

Scientific notation is a way of writing numbers that are too large or small to be conveniently written as a decimal. For example, consider the number 840,000,000,000,000. It's a rather large number to write out. The scientific notation for this number is 8.40×10^{14}. Scientific notation follows this general format

$$x \times 10^y.$$

In this format x is the value of the measurement with all placeholder zeros removed. In the example above, x is 8.4. The x is multiplied by a factor, 10^y, which indicates the number of placeholder zeros in the measurement. Placeholder zeros are those at the end of a number that is 10 or greater, and at the beginning of a decimal number that is less than 1. In the example above, the factor is 10^{14}. This tells you that you should move the decimal point 14 positions to the right, filling in placeholder zeros as you go. In this case, moving the decimal point 14 places creates only 13 placeholder zeros, indicating that the actual measurement value is 840,000,000,000,000.

Numbers that are fractions can be indicated by scientific notation as well. Consider the number 0.0000045. Its scientific notation is 4.5×10^{-6}. Its scientific notation has the same format

$$x \times 10^{y}.$$

Here, x is 4.5. However, the value of y in the 10^{y} factor is negative, which indicates that the measurement is a fraction of 1. Therefore, we move the decimal place to the left, for a negative y. In our example of 4.5×10^{-6}, the decimal point would be moved to the left six times to yield the original number, which would be 0.0000045.

The term **order of magnitude** refers to the power of 10 when numbers are expressed in scientific notation. Quantities that have the same power of 10 when expressed in scientific notation, or come close to it, are said to be of the same order of magnitude. For example, the number 800 can be written as 8×10^{2}, and the number 450 can be written as 4.5×10^{2}. Both numbers have the same value for y. Therefore, 800 and 450 are of the same order of magnitude. Similarly, 101 and 99 would be regarded as the same order of magnitude, 10^{2}. Order of magnitude can be thought of as a ballpark estimate for the scale of a value. The diameter of an atom is on the order of 10^{-9} m, while the diameter of the sun is on the order of 10^{9} m. These two values are 18 orders of magnitude apart.

Scientists make frequent use of scientific notation because of the vast range of physical measurements possible in the universe, such as the distance from Earth to the moon (Figure 1.18), or to the nearest star.

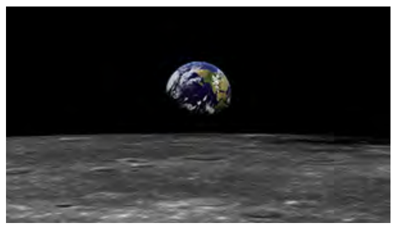


Figure 1.18 The distance from Earth to the moon may seem immense, but it is just a tiny fraction of the distance from Earth to our closest neighboring star. (NASA)

Unit Conversion and Dimensional Analysis

It is often necessary to convert from one type of unit to another. For example, if you are reading a European cookbook in the United States, some quantities may be expressed in liters and you need to convert them to cups. A Canadian tourist driving through the United States might want to convert miles to kilometers, to have a sense of how far away his next destination is. A doctor in the United States might convert a patient's weight in pounds to kilograms.

Let's consider a simple example of how to convert units within the metric system. How can we want to convert 1 hour to seconds?

Next, we need to determine a **conversion factor** relating meters to kilometers. A **conversion factor** is a ratio expressing how many of one unit are equal to another unit. A conversion factor is simply a fraction which equals 1. You can multiply any number by 1 and get the same value. When you multiply a number by a conversion factor, you are simply multiplying it by one. For example, the following are conversion factors: (1 foot)/(12 inches) = 1 to convert inches to feet, (1 meter)/(100 centimeters) = 1 to convert centimeters to meters, (1 minute)/(60 seconds) = 1 to convert seconds to minutes. In this case, we know that there are 1,000 meters in 1 kilometer.

Now we can set up our unit conversion. We will write the units that we have and then multiply them by the conversion factor (1 km/1,000m) = 1, so we are simply multiplying 80m by 1:

$$1\ \cancel{h} \times \frac{60\ \cancel{\text{min}}}{1\ \cancel{h}} \times \frac{60\ \text{s}}{1\ \cancel{\text{min}}} = 3600\ \text{s} = 3.6 \times 10^{2}\ \text{s} \tag{1.1}$$

When there is a unit in the original number, and a unit in the denominator (bottom) of the conversion factor, the units cancel. In this case, hours and minutes cancel and the value in seconds remains.

You can use this method to convert between any types of unit, including between the U.S. customary system and metric system. Notice also that, although you can multiply and divide units algebraically, you cannot add or subtract different units. An expression like *10 km + 5 kg* makes no sense. Even adding two lengths in different units, such as *10 km + 20 m* does not make sense. You express both lengths in the same unit. See Appendix C for a more complete list of conversion factors.

WORKED EXAMPLE

Unit Conversions: A Short Drive Home

Suppose that you drive the 10.0 km from your university to home in 20.0 min. Calculate your average speed (a) in kilometers per hour (km/h) and (b) in meters per second (m/s). (Note—Average speed is distance traveled divided by time of travel.)

Strategy

First we calculate the average speed using the given units. Then we can get the average speed into the desired units by picking the correct conversion factor and multiplying by it. The correct conversion factor is the one that cancels the unwanted unit and leaves the desired unit in its place.

Solution for (a)

1. Calculate average speed. Average speed is distance traveled divided by time of travel. (Take this definition as a given for now—average speed and other motion concepts will be covered in a later module.) In equation form,

$$\text{average speed} = \frac{\text{distance}}{\text{time}}.$$

2. Substitute the given values for distance and time.

$$\text{average speed} = \frac{10.0 \text{ km}}{20.0 \text{ min}} = 0.500 \frac{\text{km}}{\text{min}}$$

3. Convert km/min to km/h: multiply by the conversion factor that will cancel minutes and leave hours. That conversion factor is $60 \text{ min}/1\text{h}$. Thus,

$$\text{average speed} = 0.500 \frac{\text{km}}{\text{min}} \times \frac{60 \text{ min}}{1 \text{ h}} = 30.0 \frac{\text{km}}{\text{h}}.$$

Discussion for (a)

To check your answer, consider the following:

1. Be sure that you have properly cancelled the units in the unit conversion. If you have written the unit conversion factor upside down, the units will not cancel properly in the equation. If you accidentally get the ratio upside down, then the units will not cancel; rather, they will give you the wrong units as follows

$$\frac{\text{km}}{\text{min}} \times \frac{1 \text{ hr}}{60 \text{ min}} = \frac{1}{60} \frac{\text{km}\cdot\text{h}}{\text{min}^2},$$

which are obviously not the desired units of km/h.

2. Check that the units of the final answer are the desired units. The problem asked us to solve for average speed in units of km/h and we have indeed obtained these units.
3. Check the significant figures. Because each of the values given in the problem has three significant figures, the answer should also have three significant figures. The answer 30.0 km/h does indeed have three significant figures, so this is appropriate. Note that the significant figures in the conversion factor are not relevant because an hour is *defined* to be 60 min, so the precision of the conversion factor is perfect.
4. Next, check whether the answer is reasonable. Let us consider some information from the problem—if you travel 10 km in a third of an hour (20 min), you would travel three times that far in an hour. The answer does seem reasonable.

Solution (b)

There are several ways to convert the average speed into meters per second.

1. Start with the answer to (a) and convert km/h to m/s. Two conversion factors are needed—one to convert hours to seconds, and another to convert kilometers to meters.
2. Multiplying by these yields

$$\text{Averagespeed} = 30.0\frac{\text{km}}{\text{h}} \times \frac{1 \text{ h}}{3{,}600 \text{ s}} \times \frac{1{,}000 \text{ m}}{1 \text{ km}}$$

$$\text{Averagespeed} = 8.33\frac{\text{m}}{\text{s}}$$

Discussion for (b)

If we had started with 0.500 km/min, we would have needed different conversion factors, but the answer would have been the same: 8.33 m/s.

You may have noted that the answers in the worked example just covered were given to three digits. Why? When do you need to be concerned about the number of digits in something you calculate? Why not write down all the digits your calculator produces?

WORKED EXAMPLE

Using Physics to Evaluate Promotional Materials

A commemorative coin that is 2" in diameter is advertised to be plated with 15 mg of gold. If the density of gold is 19.3 g/cc, and the amount of gold around the edge of the coin can be ignored, what is the thickness of the gold on the top and bottom faces of the coin?

Strategy

To solve this problem, the volume of the gold needs to be determined using the gold's mass and density. Half of that volume is distributed on each face of the coin, and, for each face, the gold can be represented as a cylinder that is 2" in diameter with a height equal to the thickness. Use the volume formula for a cylinder to determine the thickness.

Solution

The mass of the gold is given by the formula $m = \rho V = 15 \times 10^{-3}\ \text{ g}$, where $\rho = 19.3\ \text{ g/cc}$ and V is the volume. Solving for the volume gives $V = \dfrac{m}{\rho} = \dfrac{15 \times 10^{-3}\ \text{ g}}{19.3\ \text{ g/cc}} \cong 7.8 \times 10^{-4}\ \text{ cc}$.

If t is the thickness, the volume corresponding to half the gold is $\frac{1}{2}\left(7.8 \times 10^{-4}\right) = \pi r^2 t = \pi (2.54)^2 \text{t}$, where the 1" radius has been converted to cm. Solving for the thickness gives $t = \dfrac{\left(3.9 \times 10^{-4}\right)}{\pi (2.54)^2} \cong 1.9 \times 10^{-5}\ \text{ cm} = 0.00019\text{ mm}$.

Discussion

The amount of gold used is stated to be 15 mg, which is equivalent to a thickness of about 0.00019 mm. The mass figure may make the amount of gold sound larger, both because the number is much bigger (15 versus 0.00019), and because people may have a more intuitive feel for how much a millimeter is than for how much a milligram is. A simple analysis of this sort can clarify the significance of claims made by advertisers.

Accuracy, Precision and Significant Figures

Science is based on experimentation that requires good measurements. The validity of a measurement can be described in terms of its accuracy and its precision (see Figure 1.19 and Figure 1.20). **Accuracy** is how close a measurement is to the correct value for that measurement. For example, let us say that you are measuring the length of standard piece of printer paper. The packaging in which you purchased the paper states that it is 11 inches long, and suppose this stated value is correct. You measure the length of the paper three times and obtain the following measurements: 11.1 inches, 11.2 inches, and 10.9 inches. These measurements are quite accurate because they are very close to the correct value of 11.0 inches. In contrast, if you had obtained a measurement of 12 inches, your measurement would not be very accurate. This is why measuring instruments are calibrated based on a known measurement. If the instrument consistently returns the correct value of the known measurement, it is safe for use in finding unknown values.

Figure 1.19 A double-pan mechanical balance is used to compare different masses. Usually an object with unknown mass is placed in one pan and objects of known mass are placed in the other pan. When the bar that connects the two pans is horizontal, then the masses in both pans are equal. The known masses are typically metal cylinders of standard mass such as 1 gram, 10 grams, and 100 grams. (Serge Melki)

Figure 1.20 Whereas a mechanical balance may only read the mass of an object to the nearest tenth of a gram, some digital scales can measure the mass of an object up to the nearest thousandth of a gram. As in other measuring devices, the precision of a scale is limited to the last measured figures. This is the hundredths place in the scale pictured here. (Splarka, Wikimedia Commons)

Precision states how well repeated measurements of something generate the same or similar results. Therefore, the precision of measurements refers to how close together the measurements are when you measure the same thing several times. One way to analyze the precision of measurements would be to determine the range, or difference between the lowest and the highest measured values. In the case of the printer paper measurements, the lowest value was 10.9 inches and the highest value was 11.2 inches. Thus, the measured values deviated from each other by, at most, 0.3 inches. These measurements were reasonably precise because they varied by only a fraction of an inch. However, if the measured values had been 10.9 inches, 11.1 inches, and 11.9 inches, then the measurements would not be very precise because there is a lot of variation from one measurement to another.

The measurements in the paper example are both accurate and precise, but in some cases, measurements are accurate but not precise, or they are precise but not accurate. Let us consider a GPS system that is attempting to locate the position of a restaurant in a city. Think of the restaurant location as existing at the center of a bull's-eye target. Then think of each GPS attempt to locate the restaurant as a black dot on the bull's eye.

In Figure 1.21, you can see that the GPS measurements are spread far apart from each other, but they are all relatively close to the actual location of the restaurant at the center of the target. This indicates a low precision, high accuracy measuring system. However, in Figure 1.22, the GPS measurements are concentrated quite closely to one another, but they are far away from the target location. This indicates a high precision, low accuracy measuring system. Finally, in Figure 1.23, the GPS is both precise and accurate, allowing the restaurant to be located.

Figure 1.21 A GPS system attempts to locate a restaurant at the center of the bull's-eye. The black dots represent each attempt to pinpoint the location of the restaurant. The dots are spread out quite far apart from one another, indicating low precision, but they are each rather close to the actual location of the restaurant, indicating high accuracy. (Dark Evil)

Figure 1.22 In this figure, the dots are concentrated close to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (Dark Evil)

Figure 1.23 In this figure, the dots are concentrated close to one another, indicating high precision, but they are rather far away from the actual location of the restaurant, indicating low accuracy. (Dark Evil)

Uncertainty

The accuracy and precision of a measuring system determine the **uncertainty** of its measurements. Uncertainty is a way to describe how much your measured value deviates from the actual value that the object has. If your measurements are not very accurate or precise, then the uncertainty of your values will be very high. In more general terms, uncertainty can be thought of as a disclaimer for your measured values. For example, if someone asked you to provide the mileage on your car, you might say that it is 45,000 miles, plus or minus 500 miles. The plus or minus amount is the uncertainty in your value. That is, you are indicating that the actual mileage of your car might be as low as 44,500 miles or as high as 45,500 miles, or anywhere in between. All measurements contain some amount of uncertainty. In our example of measuring the length of the paper, we might say that the length of the paper is 11 inches plus or minus 0.2 inches or 11.0 ± 0.2 inches. The uncertainty in a measurement, A, is often denoted as δA ("delta A"),

The factors contributing to uncertainty in a measurement include the following:

1. Limitations of the measuring device
2. The skill of the person making the measurement
3. Irregularities in the object being measured
4. Any other factors that affect the outcome (highly dependent on the situation)

In the printer paper example uncertainty could be caused by: the fact that the smallest division on the ruler is 0.1 inches, the person using the ruler has bad eyesight, or uncertainty caused by the paper cutting machine (e.g., one side of the paper is slightly longer than the other.) It is good practice to carefully consider all possible sources of uncertainty in a measurement and reduce or eliminate them,

Percent Uncertainty

One method of expressing uncertainty is as a percent of the measured value. If a measurement, A, is expressed with uncertainty, δA, the percent uncertainty is

$$\%\text{ uncertainty} = \frac{\delta \text{A}}{\text{A}} \times 100\%. \tag{1.2}$$

WORKED EXAMPLE

Calculating Percent Uncertainty: A Bag of Apples

A grocery store sells 5-lb bags of apples. You purchase four bags over the course of a month and weigh the apples each time. You obtain the following measurements:

- Week 1 weight: $4.\ 8\text{lb}$
- Week 2 weight: $5.3\ \text{lb}$
- Week 3 weight: $4.\ 9\text{lb}$
- Week 4 weight: $5.4\ \text{lb}$

You determine that the weight of the 5 lb bag has an uncertainty of ±0.4 lb. What is the percent uncertainty of the bag's weight?

Strategy

First, observe that the expected value of the bag's weight, A, is 5 lb. The uncertainty in this value, δA, is 0.4 lb. We can use the following equation to determine the percent uncertainty of the weight

$$\%\text{ uncertainty} = \frac{\delta \text{A}}{\text{A}} \times 100\%.$$

Solution

Plug the known values into the equation

$$\%\text{ uncertainty} = \frac{0.4\ \text{lb}}{5\ \text{lb}} \times 100\% = 8\%.$$

Discussion

We can conclude that the weight of the apple bag is 5 lb ± 8 percent. Consider how this percent uncertainty would change if the bag of apples were half as heavy, but the uncertainty in the weight remained the same. Hint for future calculations: when calculating percent uncertainty, always remember that you must multiply the fraction by 100 percent. If you do not do this, you will have a decimal quantity, not a percent value.

Uncertainty in Calculations

There is an uncertainty in anything calculated from measured quantities. For example, the area of a floor calculated from measurements of its length and width has an uncertainty because the both the length and width have uncertainties. How big is the uncertainty in something you calculate by multiplication or division? If the measurements in the calculation have small uncertainties (a few percent or less), then the **method of adding percents** can be used. This method says that the percent uncertainty in a quantity calculated by multiplication or division is the sum of the percent uncertainties in the items used to make the calculation. For example, if a floor has a length of 4.00 m and a width of 3.00 m, with uncertainties of 2 percent and 1 percent, respectively, then the area of the floor is 12.0 m^2 and has an uncertainty of 3 percent (expressed as an area this is 0.36 m^2, which we round to 0.4 m^2 since the area of the floor is given to a tenth of a square meter).

For a quick demonstration of the accuracy, precision, and uncertainty of measurements based upon the units of measurement, try this simulation (http://openstax.org/l/28precision) . You will have the opportunity to measure the length and weight of a desk, using milli- versus centi- units. Which do you think will provide greater accuracy, precision and uncertainty when measuring the desk and the notepad in the simulation? Consider how the nature of the hypothesis or research question might influence how precise of a measuring tool you need to collect data.

Precision of Measuring Tools and Significant Figures

An important factor in the accuracy and precision of measurements is the precision of the measuring tool. In general, a precise measuring tool is one that can measure values in very small increments. For example, consider measuring the thickness of a coin. A standard ruler can measure thickness to the nearest millimeter, while a micrometer can measure the thickness to the nearest 0.005 millimeter. The micrometer is a more precise measuring tool because it can measure extremely small differences in thickness. The more precise the measuring tool, the more precise and accurate the measurements can be.

When we express measured values, we can only list as many digits as we initially measured with our measuring tool (such as the rulers shown in Figure 1.24). For example, if you use a standard ruler to measure the length of a stick, you may measure it with a decimeter ruler as 3.6 cm. You could not express this value as 3.65 cm because your measuring tool was not precise enough to measure a hundredth of a centimeter. It should be noted that the last digit in a measured value has been estimated in some way by the person performing the measurement. For example, the person measuring the length of a stick with a ruler notices that the stick length seems to be somewhere in between 36 mm and 37 mm. He or she must estimate the value of the last digit. The rule is that the last digit written down in a measurement is the first digit with some uncertainty. For example, the last measured value 36.5 mm has three digits, or three significant figures. The number of **significant figures** in a measurement indicates the precision of the measuring tool. The more precise a measuring tool is, the greater the number of significant figures it can report.

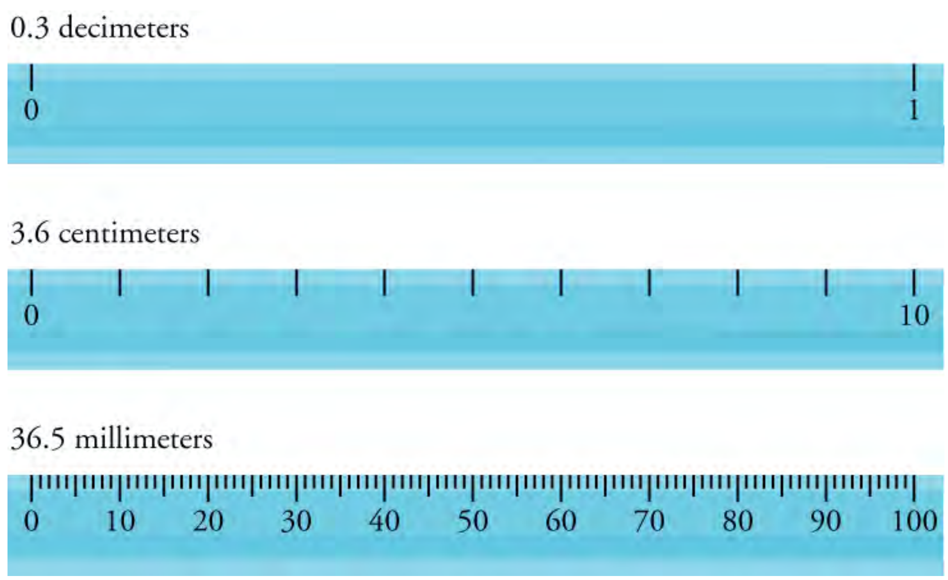


Figure 1.24 Three metric rulers are shown. The first ruler is in decimeters and can measure point three decimeters. The second ruler is in centimeters long and can measure three point six centimeters. The last ruler is in millimeters and can measure thirty-six point five millimeters.

Zeros

Special consideration is given to zeros when counting significant figures. For example, the zeros in 0.053 are not significant because they are only placeholders that locate the decimal point. There are two significant figures in 0.053—the 5 and the 3. However, if the zero occurs between other significant figures, the zeros are significant. For example, both zeros in 10.053 are significant, as these zeros were actually measured. Therefore, the 10.053 placeholder has five significant figures. The zeros in 1300 may or may not be significant, depending on the style of writing numbers. They could mean the number is known to the last zero, or the zeros could be placeholders. So 1300 could have two, three, or four significant figures. To avoid this ambiguity, write 1300 in scientific notation as 1.3×10^3. Only significant figures are given in the x factor for a number in scientific notation (in the form $x \times 10^y$). Therefore, we know that 1 and 3 are the only significant digits in this number. In summary, zeros are significant except when they serve only as placeholders. Table 1.4 provides examples of the number of significant figures in various numbers.

Number	Significant Figures	Rationale
1.657	4	There are no zeros and all non-zero numbers are always significant.
0.4578	4	The first zero is only a placeholder for the decimal point.
0.000458	3	The first four zeros are placeholders needed to report the data to the ten-thoudsandths place.
2000.56	6	The three zeros are significant here because they occur between other significant figures.
45,600	3	With no underlines or scientific notation, we assume that the last two zeros are placeholders and are not significant.
15895<u>00</u>0	7	The two underlined zeros are significant, while the last zero is not, as it is not underlined.
5.457×10^{13}	4	In scientific notation, all numbers reported in front of the multiplication sign are significant
6.520×10^{-23}	4	In scientific notation, all numbers reported in front of the multiplication sign are significant, including zeros.

Table 1.4

Significant Figures in Calculations

When combining measurements with different degrees of accuracy and precision, the number of significant digits in the final answer can be no greater than the number of significant digits in the least precise measured value. There are two different rules, one for multiplication and division and another rule for addition and subtraction, as discussed below.

1. **For multiplication and division:** The answer should have the same number of significant figures as the starting value with the fewest significant figures. For example, the area of a circle can be calculated from its radius using $A = \pi r^2$. Let us see how many significant figures the area will have if the radius has only two significant figures, for example, $r = 2.0$ m. Then, using a calculator that keeps eight significant figures, you would get

$$A = \pi r^2 = (3.1415927...) \times (2.0 \text{ m})^2 = 4.5238934 \text{ m}^2.$$

But because the radius has only two significant figures, the area calculated is meaningful only to two significant figures or

$$A = 4.5 \text{ m}^2$$

even though the value of π is meaningful to at least eight digits.

2. **For addition and subtraction**: The answer should have the same number places (e.g. tens place, ones place, tenths place, etc.) as the least-precise starting value. Suppose that you buy 7.56 kg of potatoes in a grocery store as measured with a scale having a precision of 0.01 kg. Then you drop off 6.052 kg of potatoes at your laboratory as measured by a scale with a precision of 0.001 kg. Finally, you go home and add 13.7 kg of potatoes as measured by a bathroom scale with a precision of 0.1 kg. How many kilograms of potatoes do you now have, and how many significant figures are appropriate in the answer? The mass is found by simple addition and subtraction:

$$\begin{array}{rl} 7.56 & \text{kg} \\ -6.052 & \text{kg} \\ +13.7 & \text{kg} \\ \hline 15.208 & \text{kg} \end{array}$$

The least precise measurement is 13.7 kg. This measurement is expressed to the 0.1 decimal place, so our final answer must also be expressed to the 0.1 decimal place. Thus, the answer should be rounded to the tenths place, giving 15.2 kg. The same is true for non-decimal numbers. For example,

$$6527.23 + 2 = 6528.23 = 6528 \ .$$

We cannot report the decimal places in the answer because *2* has no decimal places that would be significant. Therefore, we can only report to the ones place.

It is a good idea to keep extra significant figures while calculating, and to round off to the correct number of significant figures only in the final answers. The reason is that small errors from rounding while calculating can sometimes produce significant errors in the final answer. As an example, try calculating $5{,}098 - (5.000) \times (1{,}010)$ to obtain a final answer to only two significant figures. Keeping all significant during the calculation gives 48. Rounding to two significant figures in the middle of the calculation changes it to $5{,}100 - (5.000) \times (1{,}000) = 100$, which is way off. You would similarly avoid rounding in the middle of the calculation in counting and in doing accounting, where many small numbers need to be added and subtracted accurately to give possibly much larger final numbers.

Significant Figures in this Text

In this textbook, most numbers are assumed to have three significant figures. Furthermore, consistent numbers of significant figures are used in all worked examples. You will note that an answer given to three digits is based on input good to at least three digits. If the input has fewer significant figures, the answer will also have fewer significant figures. Care is also taken that the number of significant figures is reasonable for the situation posed. In some topics, such as optics, more than three significant figures will be used. Finally, if a number is exact, such as the *2* in the formula, $c = 2\pi r$, it does not affect the number of significant figures in a calculation.

WORKED EXAMPLE

Approximating Vast Numbers: a Trillion Dollars

The U.S. federal deficit in the 2008 fiscal year was a little greater than $10 trillion. Most of us do not have any concept of how much even one trillion actually is. Suppose that you were given a trillion dollars in $100 bills. If you made 100-bill stacks, like that shown in Figure 1.25, and used them to evenly cover a football field (between the end zones), make an approximation of how high the money pile would become. (We will use feet/inches rather than meters here because football fields are measured in yards.) One of your friends says 3 in., while another says 10 ft. What do you think?

Figure 1.25 A bank stack contains one hundred $100 bills, and is worth $10,000. How many bank stacks make up a trillion dollars? (Andrew Magill)

Strategy

When you imagine the situation, you probably envision thousands of small stacks of 100 wrapped $100 bills, such as you might see in movies or at a bank. Since this is an easy-to-approximate quantity, let us start there. We can find the volume of a stack of 100 bills, find out how many stacks make up one trillion dollars, and then set this volume equal to the area of the football field multiplied by the unknown height.

Solution

1. Calculate the volume of a stack of 100 bills. The dimensions of a single bill are approximately 3 in. by 6 in. A stack of 100 of these is about 0.5 in. thick. So the total volume of a stack of 100 bills is

$$\begin{aligned}\text{volume of stack} &= \text{length} \times \text{width} \times \text{height},\\ \text{volume of stack} &= 6 \text{ in.} \times 3 \text{ in.} \times 0.5 \text{ in.},\\ \text{volume of stack} &= 9 \text{ in.}^3.\end{aligned}$$

2. Calculate the number of stacks. Note that a trillion dollars is equal to $\$1 \times 10^{12}$, and a stack of one-hundred $\$100$ bills is equal to $\$10,000$, or $\$1 \times 10^4$. The number of stacks you will have is

$$\$1 \times 10^{12} \text{ (a trillion dollars) / } \$1 \times 10^4 \text{ per stack} = 1 \times 10^8 \text{ stacks.} \tag{1.3}$$

3. Calculate the area of a football field in square inches. The area of a football field is $100 \text{ yd} \times 50 \text{yd}$, which gives $5,000 \text{ yd}^2$. Because we are working in inches, we need to convert square yards to square inches

$$\begin{aligned}\text{Area} &= 5{,}000 \text{ yd}^2 \times \frac{3 \text{ ft}}{1 \text{yd}} \times \frac{3 \text{ ft}}{1 \text{yd}} \times \frac{12 \text{ in.}}{1 \text{ foot}} \times \frac{12 \text{ in.}}{1 \text{ foot}} = 6{,}480{,}000 \text{ in.}^2,\\ \text{Area} &\approx 6 \times 10^6 \text{ in.}^2.\end{aligned}$$

This conversion gives us $6 \times 10^6 \text{ in.}^2$ for the area of the field. (Note that we are using only one significant figure in these calculations.)

4. Calculate the total volume of the bills. The volume of all the $100-bill stacks is
$9 \text{ in.}^3 \text{ / stack} \times 10^8 \text{ stacks} = 9 \times 10^8 \text{ in.}^3$
5. Calculate the height. To determine the height of the bills, use the following equation

$$\begin{aligned} \text{volume of bills} &= \text{area of field} \times \text{height of money} \\ \text{Height of money} &= \frac{\text{volume of bills}}{\text{area of field}} \\ \text{Height of money} &= \frac{9 \times 10^8 \text{ in.}^3}{6 \times 10^6 \text{ in.}^2} = 1.33 \times 10^2 \text{ in.} \\ \text{Height of money} &= 1{\times}10^2 \text{ in.} = 100 \text{ in.} \end{aligned}$$

The height of the money will be about 100 in. high. Converting this value to feet gives

$$100 \text{ in.} \times \frac{1\text{ft}}{12 \text{ in.}} = 8.33 \text{ ft} \approx 8 \text{ ft.}$$

Discussion

The final approximate value is much higher than the early estimate of 3 in., but the other early estimate of 10 ft (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise tell you in terms of rough *guesstimates* versus carefully calculated approximations?

In the example above, the final approximate value is much higher than the first friend's early estimate of 3 in. However, the other friend's early estimate of 10 ft. (120 in.) was roughly correct. How did the approximation measure up to your first guess? What can this exercise suggest about the value of rough *guesstimates* versus carefully calculated approximations?

Graphing in Physics

Most results in science are presented in scientific journal articles using graphs. Graphs present data in a way that is easy to visualize for humans in general, especially someone unfamiliar with what is being studied. They are also useful for presenting large amounts of data or data with complicated trends in an easily-readable way.

One commonly-used graph in physics and other sciences is the line graph, probably because it is the best graph for showing how one quantity changes in response to the other. Let's build a line graph based on the data in Table 1.5, which shows the measured distance that a train travels from its station versus time. Our two **variables**, or things that change along the graph, are time in minutes, and distance from the station, in kilometers. Remember that measured data may not have perfect accuracy.

Time (min)	Distance from Station (km)
0	0
10	24
20	36
30	60
40	84
50	97
60	116
70	140

Table 1.5

1. Draw the two axes. The horizontal axis, or x-axis, shows the **independent variable**, which is the variable that is controlled or manipulated. The vertical axis, or y-axis, shows the **dependent variable**, the non-manipulated variable that changes with (or is dependent on) the value of the independent variable. In the data above, time is the independent variable and should be plotted on the x-axis. Distance from the station is the dependent variable and should be plotted on the y-axis.

2. Label each axes on the graph with the name of each variable, followed by the symbol for its units in parentheses. Be sure to leave room so that you can number each axis. In this example, use *Time (min)* as the label for the *x*-axis.
3. Next, you must determine the best scale to use for numbering each axis. Because the time values on the *x*-axis are taken every 10 minutes, we could easily number the *x*-axis from 0 to 70 minutes with a tick mark every 10 minutes. Likewise, the *y*-axis scale should start low enough and continue high enough to include all of the *distance from station* values. A scale from 0 km to 160 km should suffice, perhaps with a tick mark every 10 km.

   In general, you want to pick a scale for both axes that 1) shows all of your data, and 2) makes it easy to identify trends in your data. If you make your scale too large, it will be harder to see how your data change. Likewise, the smaller and more fine you make your scale, the more space you will need to make the graph. The number of significant figures in the axis values should be coarser than the number of significant figures in the measurements.

4. Now that your axes are ready, you can begin plotting your data. For the first data point, count along the *x*-axis until you find the 10 min tick mark. Then, count up from that point to the 10 km tick mark on the *y*-axis, and approximate where 22 km is along the *y*-axis. Place a dot at this location. Repeat for the other six data points (Figure 1.26).

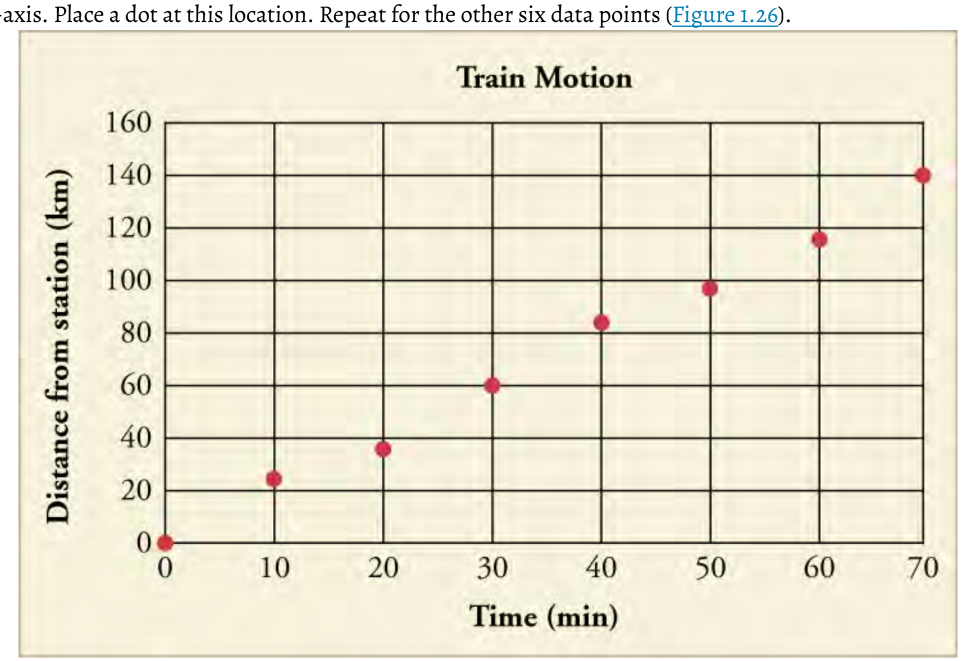


Figure 1.26 The graph of the train's distance from the station versus time from the exercise above.

5. Add a title to the top of the graph to state what the graph is describing, such as the *y*-axis parameter vs. the *x*-axis parameter. In the graph shown here, the title is *train motion*. It could also be titled distance of the train from the station vs. time.
6. Finally, with data points now on the graph, you should draw a trend line (Figure 1.27). The trend line represents the dependence you think the graph represents, so that the person who looks at your graph can see how close it is to the real data. In the present case, since the data points look like they ought to fall on a straight line, you would draw a straight line as the trend line. Draw it to come closest to all the points. Real data may have some inaccuracies, and the plotted points may not all fall on the trend line. In some cases, none of the data points fall exactly on the trend line.

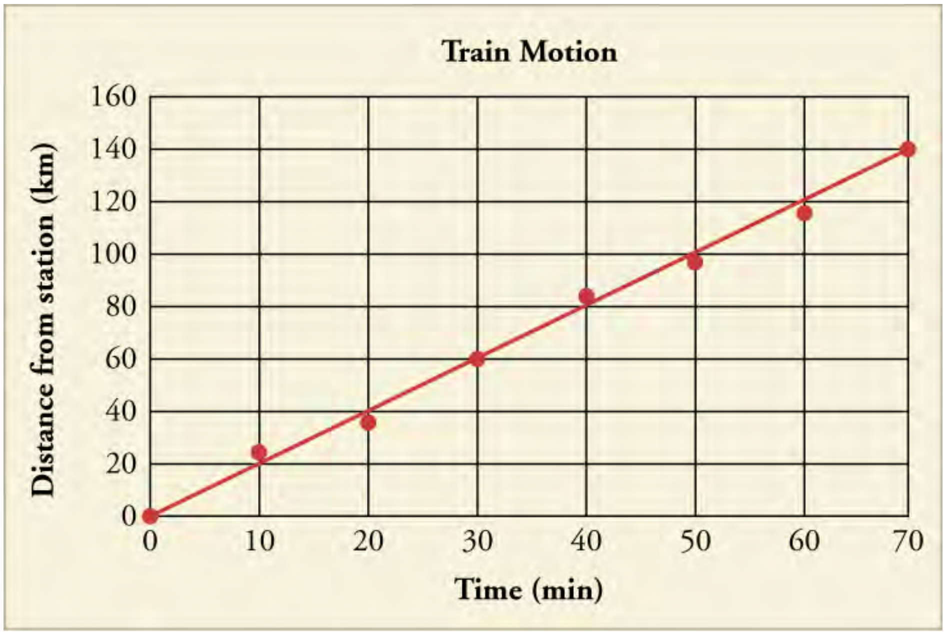


Figure 1.27 The completed graph with the trend line included.

Analyzing a Graph Using Its Equation

One way to get a quick snapshot of a dataset is to look at the equation of its trend line. If the graph produces a straight line, the equation of the trend line takes the form

$$y = mx + b.$$

The b in the equation is the y-intercept while the m in the equation is the **slope**. The y-intercept tells you at what y value the line intersects the y-axis. In the case of the graph above, the y-intercept occurs at 0, at the very beginning of the graph. The y-intercept, therefore, lets you know immediately where on the y-axis the plot line begins.

The m in the equation is the slope. This value describes how much the line on the graph moves up or down on the y-axis along the line's length. The slope is found using the following equation

$$m = \frac{Y_2 - Y_1}{X_2 - X_1}.$$

In order to solve this equation, you need to pick two points on the line (preferably far apart on the line so the slope you calculate describes the line accurately). The quantities Y_2 and Y_1 represent the y-values from the two points on the line (not data points) that you picked, while X_2 and X_1 represent the two x-values of the those points.

What can the slope value tell you about the graph? The slope of a perfectly horizontal line will equal zero, while the slope of a perfectly vertical line will be undefined because you cannot divide by zero. A positive slope indicates that the line moves up the y-axis as the x-value increases while a negative slope means that the line moves down the y-axis. The more negative or positive the slope is, the steeper the line moves up or down, respectively. The slope of our graph in Figure 1.26 is calculated below based on the two endpoints of the line

$$\begin{aligned} m &= \frac{Y_2 - Y_1}{X_2 - X_1} \\ m &= \frac{(80 \text{ km}) - (20 \text{ km})}{(40 \text{ min}) - (10 \text{ min})} \\ m &= \frac{60 \text{ km}}{30 \text{ min}} \\ m &= 2.0 \text{ km/min}. \end{aligned}$$

Equation of line: $y = (2.0 \;\; \text{km/min})\, x + 0$

Because the x axis is time in minutes, we would actually be more likely to use the time t as the independent (x-axis) variable and write the equation as

$$y = (2.0\ \ \text{km/min})\, t + 0. \tag{1.4}$$

The formula $y = mx + b$ only applies to **linear relationships**, or ones that produce a straight line. Another common type of line in physics is the **quadratic relationship**, which occurs when one of the variables is squared. One quadratic relationship in physics is the relation between the speed of an object its centripetal acceleration, which is used to determine the force needed to keep an object moving in a circle. Another common relationship in physics is the **inverse relationship**, in which one variable decreases whenever the other variable increases. An example in physics is Coulomb's law. As the distance between two charged objects increases, the electrical force between the two charged objects decreases. **Inverse proportionality**, such the relation between x and y in the equation

$$y = k/x,$$

for some number k, is one particular kind of inverse relationship. A third commonly-seen relationship is the **exponential relationship**, in which a change in the independent variable produces a proportional change in the dependent variable. As the value of the dependent variable gets larger, its rate of growth also increases. For example, bacteria often reproduce at an exponential rate when grown under ideal conditions. As each generation passes, there are more and more bacteria to reproduce. As a result, the growth rate of the bacterial population increases every generation (Figure 1.28).

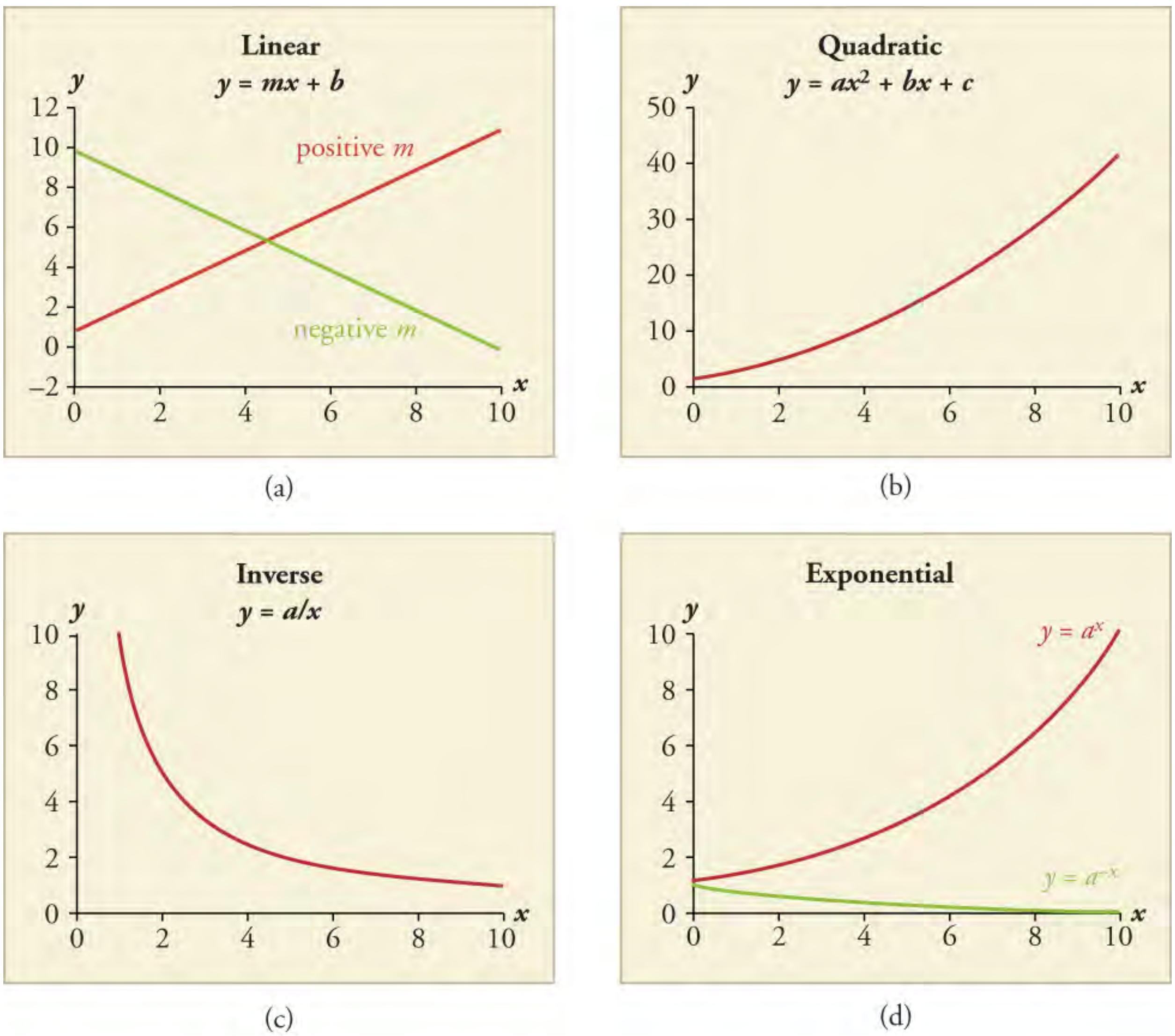


Figure 1.28 Examples of (a) linear, (b) quadratic, (c) inverse, and (d) exponential relationship graphs.

Using Logarithmic Scales in Graphing

Sometimes a variable can have a very large range of values. This presents a problem when you're trying to figure out the best scale to use for your graph's axes. One option is to use a **logarithmic (log) scale**. In a logarithmic scale, the value each mark labels is the previous mark's value multiplied by some constant. For a log base 10 scale, each mark labels a value that is 10 times the value of the mark before it. Therefore, a base 10 logarithmic scale would be numbered: 0, 10, 100, 1,000, etc. You can see how the logarithmic scale covers a much larger range of values than the corresponding linear scale, in which the marks would label the values 0, 10, 20, 30, and so on.

If you use a logarithmic scale on one axis of the graph and a linear scale on the other axis, you are using a **semi-log plot**. The Richter scale, which measures the strength of earthquakes, uses a semi-log plot. The degree of ground movement is plotted on a logarithmic scale against the assigned intensity level of the earthquake, which ranges linearly from 1-10 (Figure 1.29 **(a)**).

If a graph has both axes in a logarithmic scale, then it is referred to as a **log-log plot**. The relationship between the wavelength and frequency of electromagnetic radiation such as light is usually shown as a log-log plot (Figure 1.29 **(b)**). Log-log plots are also commonly used to describe exponential functions, such as radioactive decay.

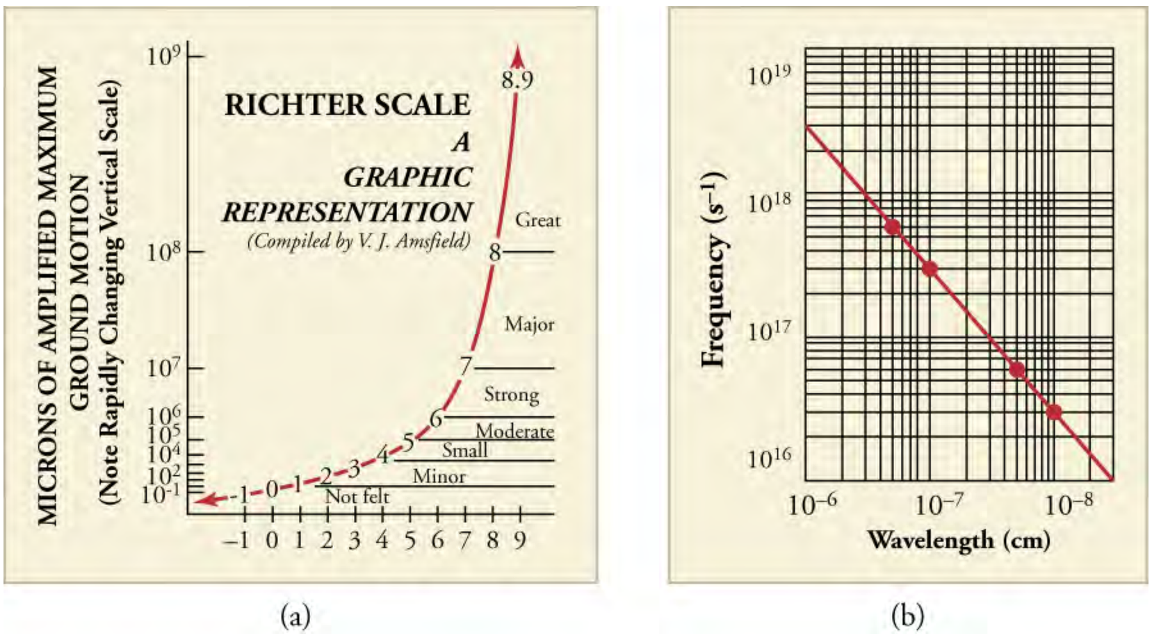


Figure 1.29 (a) The Richter scale uses a log base 10 scale on its *y-axis* (microns of amplified maximum ground motion). (b) The relationship between the frequency and wavelength of electromagnetic radiation can be plotted as a straight line if a log-log plot is used.

Virtual Physics

Graphing Lines

In this simulation you will examine how changing the slope and *y*-intercept of an equation changes the appearance of a plotted line. Select slope-intercept form and drag the blue circles along the line to change the line's characteristics. Then, play the line game and see if you can determine the slope or *y*-intercept of a given line.

Click to view content (https://phet.colorado.edu/sims/html/graphing-lines/latest/graphing-lines_en.html)

GRASP CHECK

How would the following changes affect a line that is neither horizontal nor vertical and has a positive slope?

1. increase the slope but keeping the *y*-intercept constant
2. increase the *y*-intercept but keeping the slope constant
   a. Increasing the slope will cause the line to rotate clockwise around the *y*-intercept. Increasing the *y*-intercept will cause the line to move vertically up on the graph without changing the line's slope.
   b. Increasing the slope will cause the line to rotate counter-clockwise around the *y*-intercept. Increasing the *y*-intercept will cause the line to move vertically up on the graph without changing the line's slope.
   c. Increasing the slope will cause the line to rotate clockwise around the *y*-intercept. Increasing the *y*-intercept will cause the line to move horizontally right on the graph without changing the line's slope.

d. Increasing the slope will cause the line to rotate counter-clockwise around the *y*-intercept. Increasing the *y*-intercept will cause the line to move horizontally right on the graph without changing the line's slope.

Check Your Understanding

12. Identify some advantages of metric units.

a. Conversion between units is easier in metric units.
b. Comparison of physical quantities is easy in metric units.
c. Metric units are more modern than English units.
d. Metric units are based on powers of 2.

13. The length of an American football field is 100 yd, excluding the end zones. How long is the field in meters? Round to the nearest 0.1 m.

a. 10.2 m
b. 91.4 m
c. 109.4 m
d. 328.1 m

14. The speed limit on some interstate highways is roughly 100 km/h. How many miles per hour is this if 1.0 mile is about 1.609 km?

a. 0.1 mi/h
b. 27.8 mi/h
c. 62 mi/h
d. 160 mi/h

15. Briefly describe the target patterns for accuracy and precision and explain the differences between the two.

a. Precision states how much repeated measurements generate the same or closely similar results, while accuracy states how close a measurement is to the true value of the measurement.
b. Precision states how close a measurement is to the true value of the measurement, while accuracy states how much repeated measurements generate the same or closely similar result.
c. Precision and accuracy are the same thing. They state how much repeated measurements generate the same or closely similar results.
d. Precision and accuracy are the same thing. They state how close a measurement is to the true value of the measurement.

KEY TERMS

accuracy how close a measurement is to the correct value for that measurement

ampere the SI unit for electrical current

atom smallest and most basic units of matter

classical physics physics, as it developed from the Renaissance to the end of the nineteenth century

constant a quantity that does not change

conversion factor a ratio expressing how many of one unit are equal to another unit

dependent variable the vertical, or y-axis, variable, which changes with (or is dependent on) the value of the independent variable

derived units units that are derived by combining the fundamental physical units

English units (also known as the customary or imperial system) system of measurement used in the United States; includes units of measurement such as feet, gallons, degrees Fahrenheit, and pounds

experiment process involved with testing a hypothesis

exponential relationship relation between variables in which a constant change in the independent variable is accompanied by change in the dependent variable that is proportional to the value it already had

fundamental physical units the seven fundamental physical units in the SI system of units are length, mass, time, electric current, temperature, amount of a substance, and luminous intensity

hypothesis testable statement that describes how something in the natural world works

independent variable the horizontal, or x-axis, variable, which is not influence by the second variable on the graph, the dependent variable

inverse proportionality a relation between two variables expressible by an equation of the form $y = k/x$ where k stays constant when x and y change; the special form of inverse relationship that satisfies this equation

inverse relationship any relation between variables where one variable decreases as the other variable increases

kilogram the SI unit for mass, abbreviated (kg)

linear relationships relation between variables that produce a straight line when graphed

log-log plot a plot that uses a logarithmic scale in both axes

logarithmic scale a graphing scale in which each tick on an axis is the previous tick multiplied by some value

meter the SI unit for length, abbreviated (m)

method of adding percents calculating the percent uncertainty of a quantity in multiplication or division by adding the percent uncertainties in the quantities being added or divided

model system that is analogous to the real system of interest in essential ways but more easily analyzed

modern physics physics as developed from the twentieth century to the present, involving the theories of relativity and quantum mechanics

observation step where a scientist observes a pattern or trend within the natural world

order of magnitude the size of a quantity in terms of its power of 10 when expressed in scientific notation

physics science aimed at describing the fundamental aspects of our universe—energy, matter, space, motion, and time

precision how well repeated measurements generate the same or closely similar results

principle description of nature that is true in many, but not all situations

quadratic relationship relation between variables that can be expressed in the form $y = ax^2 + bx + c$, which produces a curved line when graphed

quantum mechanics major theory of modern physics which describes the properties and nature of atoms and their subatomic particles

science the study or knowledge of how the physical world operates, based on objective evidence determined through observation and experimentation

scientific law pattern in nature that is true in all circumstances studied thus far

scientific methods techniques and processes used in the constructing and testing of scientific hypotheses, laws, and theories, and in deciding issues on the basis of experiment and observation

scientific notation way of writing numbers that are too large or small to be conveniently written in simple decimal form; the measurement is multiplied by a power of 10, which indicates the number of placeholder zeros in the measurement

second the SI unit for time, abbreviated (s)

semi-log plot A plot that uses a logarithmic scale on one axis of the graph and a linear scale on the other axis.

SI units International System of Units (SI); the international system of units that scientists in most countries have agreed to use; includes units such as meters, liters, and grams; also known as the metric system

significant figures when writing a number, the digits, or number of digits, that express the precision of a measuring tool used to measure the number

slope the ratio of the change of a graph on the y axis to the change along the x-axis, the value of m in the equation of a line, $y = mx + b$

theory explanation of patterns in nature that is supported by much scientific evidence and verified multiple times by various groups of researchers

theory of relativity theory constructed by Albert Einstein which describes how space, time and energy are different for different observers in relative motion

uncertainty a quantitative measure of how much measured values deviate from a standard or expected value

universal applies throughout the known universe

***y*-intercept** the point where a plot line intersects the y-axis

SECTION SUMMARY

1.1 Physics: Definitions and Applications

- Physics is the most fundamental of the sciences, concerning itself with energy, matter, space and time, and their interactions.
- Modern physics involves the theory of relativity, which describes how time, space and gravity are not constant in our universe can be different for different observers, and quantum mechanics, which describes the behavior of subatomic particles.
- Physics is the basis for all other sciences, such as chemistry, biology and geology, because physics describes the fundamental way in which the universe functions.

1.2 The Scientific Methods

- Science seeks to discover and describe the underlying order and simplicity in nature.
- The processes of science include observation, hypothesis, experiment, and conclusion.
- Theories are scientific explanations that are supported by a large body experimental results.
- Scientific laws are concise descriptions of the universe that are universally true.

1.3 The Language of Physics: Physical Quantities and Units

- Physical quantities are a characteristic or property of an object that can be measured or calculated from other measurements.
- The four fundamental units we will use in this textbook are the meter (for length), the kilogram (for mass), the second (for time), and the ampere (for electric current). These units are part of the metric system, which uses powers of 10 to relate quantities over the vast ranges encountered in nature.
- Unit conversions involve changing a value expressed in one type of unit to another type of unit. This is done by using conversion factors, which are ratios relating equal quantities of different units.
- Accuracy of a measured value refers to how close a measurement is to the correct value. The uncertainty in a measurement is an estimate of the amount by which the measurement result may differ from this value.
- Precision of measured values refers to how close the agreement is between repeated measurements.
- Significant figures express the precision of a measuring tool.
- When multiplying or dividing measured values, the final answer can contain only as many significant figures as the least precise value.
- When adding or subtracting measured values, the final answer cannot contain more decimal places than the least precise value.

KEY EQUATIONS

1.3 The Language of Physics: Physical Quantities and Units

slope intercept form	$y = mx + b$
quadratic formula	$y = ax^2 + bx + c$
positive exponential formula	$y = a^x$
negative exponential formula	$y = a^{-x}$

CHAPTER REVIEW

Concept Items

1.1 Physics: Definitions and Applications

1. Which statement best compares and contrasts the aims and topics of natural philosophy had versus physics?
   a. Natural philosophy included all aspects of nature including physics.
   b. Natural philosophy included all aspects of nature excluding physics.
   c. Natural philosophy and physics are different.
   d. Natural philosophy and physics are essentially the

same thing.

2. Which of the following is not an underlying assumption essential to scientific understanding?
   a. Characteristics of the physical universe can be perceived and objectively measured by human beings.
   b. Explanations of natural phenomena can be established with absolute certainty.
   c. Fundamental physical processes dictate how characteristics of the physical universe evolve.
   d. The fundamental processes of nature operate the same way everywhere and at all times.

3. Which of the following questions regarding a strain of genetically modified rice is not one that can be answered by science?
   a. How does the yield of the genetically modified rice compare with that of existing rice?
   b. Is the genetically modified rice more resistant to infestation than existing rice?
   c. How does the nutritional value of the genetically modified rice compare to that of existing rice?
   d. Should the genetically modified rice be grown commercially and sold in the marketplace?

4. What conditions imply that we can use classical physics without considering special relativity or quantum mechanics?
   a. 1. matter is moving at speeds of less than roughly 1 percent the speed of light,
      2. objects are large enough to be seen with the naked eye, and
      3. there is the involvement of a strong gravitational field.
   b. 1. matter is moving at speeds greater than roughly 1 percent the speed of light,
      2. objects are large enough to be seen with the naked eye, and
      3. there is the involvement of a strong gravitational field.
   c. 1. matter is moving at speeds of less than roughly 1 percent the speed of light,
      2. objects are too small to be seen with the naked eye, and
      3. there is the involvement of only a weak gravitational field.
   d. 1. matter is moving at speeds of less than roughly 1 percent the speed of light,
      2. objects are large enough to be seen with the naked eye, and
      3. there is the involvement of a weak gravitational field.

5. How could physics be useful in weather prediction?
   a. Physics helps in predicting how burning fossil fuel releases pollutants.
   b. Physics helps in predicting dynamics and movement of weather phenomena.
   c. Physics helps in predicting the motion of tectonic plates.
   d. Physics helps in predicting how the flowing water affects Earth's surface.

6. How do physical therapists use physics while on the job? Explain.
   a. Physical therapists do not require knowledge of physics because their job is mainly therapy and not physics.
   b. Physical therapists do not require knowledge of physics because their job is more social in nature and unscientific.
   c. Physical therapists require knowledge of physics know about muscle contraction and release of energy.
   d. Physical therapists require knowledge of physics to know about chemical reactions inside the body and make decisions accordingly.

7. What is meant when a physical law is said to be universal?
   a. The law can explain everything in the universe.
   b. The law is applicable to all physical phenomena.
   c. The law applies everywhere in the universe.
   d. The law is the most basic one and all laws are derived from it.

8. What subfield of physics could describe small objects traveling at high speeds or experiencing a strong gravitational field?
   a. general theory of relativity
   b. classical physics
   c. quantum relativity
   d. special theory of relativity

9. Why is Einstein's theory of relativity considered part of modern physics, as opposed to classical physics?
   a. Because it was considered less outstanding than the classics of physics, such as classical mechanics.
   b. Because it was popular physics enjoyed by average people today, instead of physics studied by the elite.
   c. Because the theory deals with very slow-moving objects and weak gravitational fields.
   d. Because it was among the new 19th-century discoveries that changed physics.

1.2 The Scientific Methods

10. Describe the difference between an observation and a hypothesis.

a. An observation is seeing what happens; a hypothesis is a testable, educated guess.
b. An observation is a hypothesis that has been confirmed.
c. Hypotheses and observations are independent of each other.
d. Hypotheses are conclusions based on some observations.

11. Describe how modeling is useful in studying the structure of the atom.
a. Modeling replaces the real system by something similar but easier to examine.
b. Modeling replaces the real system by something more interesting to examine.
c. Modeling replaces the real system by something with more realistic properties.
d. Modeling includes more details than are present in the real system.

12. How strongly is a hypothesis supported by evidence compared to a theory?
a. A theory is supported by little evidence, if any, at first, while a hypothesis is supported by a large amount of available evidence.
b. A hypothesis is supported by little evidence, if any, at first. A theory is supported by a large amount of available evidence.
c. A hypothesis is supported by little evidence, if any, at first. A theory does not need any experiments in support.
d. A theory is supported by little evidence, if any, at first. A hypothesis does not need any experiments in support.

1.3 The Language of Physics: Physical Quantities and Units

13. Which of the following does not contribute to the uncertainty?
a. the limitations of the measuring device
b. the skill of the person making the measurement
c. the regularities in the object being measured
d. other factors that affect the outcome (depending on the situation)

14. How does the independent variable in a graph differ from the dependent variable?
a. The dependent variable varies linearly with the independent variable.
b. The dependent variable depends on the scale of the axis chosen while independent variable does not.
c. The independent variable is directly manipulated or controlled by the person doing the experiment, while dependent variable is the one that changes as a result.
d. The dependent and independent variables are fixed by a convention and hence they are the same.

15. What could you conclude about these two lines?

1. Line A has a slope of -4.7
2. Line B has a slope of 12.0

a. Line A is a decreasing line while line B is an increasing line, with line A being much steeper than line B.
b. Line A is a decreasing line while line B is an increasing line, with line B being much steeper than line A.
c. Line B is a decreasing line while line A is an increasing line, with line A being much steeper than line B.
d. Line B is a decreasing line while line A is an increasing line, with line B being much steeper than line A.

16. Velocity, or speed, is measured using the following formula: $v = \frac{d}{t}$, where v is velocity, d is the distance travelled, and t is the time the object took to travel the distance. If the velocity-time data are plotted on a graph, which variable will be on which axis? Why?
a. Time would be on the x-axis and velocity on the y-axis, because time is an independent variable and velocity is a dependent variable.
b. Velocity would be on the x-axis and time on the y-axis, because time is the independent variable and velocity is the dependent variable.
c. Time would be on the x-axis and velocity on the y-axis, because time is a dependent variable and velocity is a independent variable.
d. Velocity would be on x-axis and time on the y-axis, because time is a dependent variable and velocity is a independent variable.

17. The uncertainty of a triple-beam balance is 0.05 g. What is the percent uncertainty in a measurement of 0.445 kg?
a. 0.011%
b. 0.11%
c. 1.1%
d. 11%

18. What is the definition of uncertainty?
a. Uncertainty is the number of assumptions made prior to the measurement of a physical quantity.
b. Uncertainty is a measure of error in a measurement due to the use of a non-calibrated instrument.
c. Uncertainty is a measure of deviation of the measured value from the standard value.
d. Uncertainty is a measure of error in measurement

due to external factors like air friction and temperature.

Critical Thinking Items

1.1 Physics: Definitions and Applications

19. In what sense does Einstein's theory of relativity illustrate that physics describes fundamental aspects of our universe?
  a. It describes how speed affects different observers' measurements of time and space.
  b. It describes how different parts of the universe are far apart and do not affect each other.
  c. It describes how people think of other people's views from their own frame of reference.
  d. It describes how a frame of reference is necessary to describe position or motion.

20. Can classical physics be used to accurately describe a satellite moving at a speed of 7500 m/s? Explain why or why not.
  a. No, because the satellite is moving at a speed much smaller than the speed of the light and is not in a strong gravitational field.
  b. No, because the satellite is moving at a speed much smaller than the speed of the light and is in a strong gravitational field.
  c. Yes, because the satellite is moving at a speed much smaller than the speed of the light and it is not in a strong gravitational field.
  d. Yes, because the satellite is moving at a speed much smaller than the speed of the light and is in a strong gravitational field.

21. What would be some ways in which physics was involved in building the features of the room you are in right now?
  a. Physics is involved in structural strength, dimensions, etc., of the room.
  b. Physics is involved in the air composition inside the room.
  c. Physics is involved in the desk arrangement inside the room.
  d. Physics is involved in the behavior of living beings inside the room.

22. What theory of modern physics describes the interrelationships between space, time, speed, and gravity?
  a. atomic theory
  b. nuclear physics
  c. quantum mechanics
  d. general relativity

23. According to Einstein's theory of relativity, how could you effectively travel many years into Earth's future, but not age very much yourself?
  a. by traveling at a speed equal to the speed of light
  b. by traveling at a speed faster than the speed of light
  c. by traveling at a speed much slower than the speed of light
  d. by traveling at a speed slightly slower than the speed of light

1.2 The Scientific Methods

24. You notice that the water level flowing in a stream near your house increases when it rains and the water turns brown. Which of these are the best hypothesis to explain why the water turns brown. Assume you have all of the means to test the contents of the stream water.
  a. The water in the stream turns brown because molecular forces between water molecules are stronger than mud molecules
  b. The water in the stream turns brown because of the breakage of a weak chemical bond with the hydrogen atom in the water molecule.
  c. The water in the stream turns brown because it picks up dirt from the bank as the water level increases when it rains.
  d. The water in the stream turns brown because the density of the water increases with increase in water level.

25. Light travels as waves at an approximate speed of 300,000,000 m/s (186,000 mi/s). Designers of devices that use mirrors and lenses model the traveling light by straight lines, or light rays. Describe why it would be useful to model the light as rays of light instead of describing them accurately as electromagnetic waves.
  a. A model can be constructed in such a way that the speed of light decreases.
  b. Studying a model makes it easier to analyze the path that the light follows.
  c. Studying a model will help us to visualize why light travels at such great speed.
  d. Modeling cannot be used to study traveling light as our eyes cannot track the motion of light.

26. A friend says that he doesn't trust scientific explanations because they are just theories, which are basically educated guesses. What could you say to convince him that scientific theories are different from the everyday use of the word theory?
  a. A theory is a scientific explanation that has been repeatedly tested and supported by many experiments.
  b. A theory is a hypothesis that has been tested and

supported by some experiments.
  c. A theory is a set of educated guesses, but at least one of the guesses remain true in each experiment.
  d. A theory is a set of scientific explanations that has at least one experiment in support of it.

27. Give an example of a hypothesis that cannot be tested experimentally.
  a. The structure of any part of the broccoli is similar to the whole structure of the broccoli.
  b. Ghosts are the souls of people who have died.
  c. The average speed of air molecules increases with temperature.
  d. A vegetarian is less likely to be affected by night blindness.

28. Would it be possible to scientifically prove that a supreme being exists or not? Briefly explain your answer.
  a. It can be proved scientifically because it is a testable hypothesis.
  b. It cannot be proved scientifically because it is not a testable hypothesis.
  c. It can be proved scientifically because it is not a testable hypothesis.
  d. It cannot be proved scientifically because it is a testable hypothesis.

1.3 The Language of Physics: Physical Quantities and Units

29. A marathon runner completes a 42.188 km course in 2 h, 30 min, and 12 s. There is an uncertainty of 25 m in the distance traveled and an uncertainty of 1 s in the elapsed time.

1. Calculate the percent uncertainty in the distance.
2. Calculate the uncertainty in the elapsed time.
3. What is the average speed in meters per second?
4. What is the uncertainty in the average speed?

  a. $0.059\ \%, 0.01\ \%, 0.468\text{m/s}, 0.0003\text{m/s}$
  b. $0.059\ \%, 0.01\ \%, 0.468\text{ m/s}, 0.07\text{ m/s}$
  c. $0.59\ \%, 8.33\ \%, 4.681\text{ m/s}, 0.003\text{ m/s}$
  d. $0.059\ \%, 0.01\ \%, 4.681\text{ m/s}, 0.003\text{ m/s}$

30. A car engine moves a piston with a circular cross section of $7.500 \pm 0.002\text{ cm}$ diameter a distance of $3.250 \pm 0.001\text{ cm}$ to compress the gas in the cylinder. By what amount did the gas decrease in volume in cubic centimeters? Find the uncertainty in this volume.
  a. $143.6 \pm 0.002\text{ cm}^3$
  b. $143.6 \pm 0.003\text{ cm}^3$
  c. $143.6 \pm 0.005\text{ cm}^3$
  d. $143.6 \pm 0.1\text{ cm}^3$

31. What would be the slope for a line passing through the two points below?

Point 1: (1, 0.1) Point 2: (7, 26.8)

  a. 2.4
  b. 4.5
  c. 6.2
  d. 6.8

32. The sides of a small rectangular box are measured 1.80 cm and 2.05 cm long and 3.1 cm high. Calculate its volume and uncertainty in cubic centimeters. Assume the measuring device is accurate to $\pm 0.05\text{ cm}$.
  a. $11.4 \pm 0.1\text{ cm}^3$
  b. $11.4 \pm 0.6\text{ cm}^3$
  c. $11.4 \pm 0.8\text{ cm}^3$
  d. $11.4 \pm 0.10\text{ cm}^3$

33. Calculate the approximate number of atoms in a bacterium. Assume that the average mass of an atom in the bacterium is ten times the mass of a hydrogen atom. (Hint—The mass of a hydrogen atom is on the order of 10^{-27} kg and the mass of a bacterium is on the order of 10^{-15} kg .)
  a. 10^{10} atoms
  b. 10^{11} atoms
  c. 10^{12} atoms
  d. 10^{13} atoms

Problems

1.3 The Language of Physics: Physical Quantities and Units

34. A commemorative coin that sells for $40 is advertised to be plated with 15 mg of gold. Suppose gold is worth about $1,300 per ounce. Which of the following best represents the value of the gold in the coin?
  a. $0.33
  b. $0.69
  c. $3.30
  d. $6.90

35. If a marathon runner runs 9.5 miles in one direction, 8.89 miles in another direction and 2.333 miles in a third direction, how much distance did the runner run? Be sure to report your answer using the proper number of significant figures.
  a. 20
  b. 20.7
  c. 20.72

d. 20.732

36. The speed limit on some interstate highways is roughly $80\ \text{km/h}$. What is this in meters per second? How many miles per hour is this?

a. $62\ \text{m/s}$, $27.8\ \text{mi/h}$
b. $22.2\ \text{m/s}$, $49.7\ \text{mi/h}$
c. $62\ \text{m/s}$, $2.78\ \text{mi/h}$
d. $2.78\ \text{m/s}$, $62\ \text{mi/h}$

37. The length and width of a rectangular room are measured to be $3.955 \pm 0.005\ \text{m}$ by $3.050 \pm 0.005\ \text{m}$. Calculate the area of the room and its uncertainty in square meters.

a. $12.06 \pm 0.29\ \text{m}^2$
b. $12.06 \pm 0.01\ \text{m}^2$
c. $12.06 \pm 0.25\ \text{m}^2$
d. $12.06 \pm 0.04\ \text{m}^2$

Performance Task

1.3 The Language of Physics: Physical Quantities and Units

38. a. Create a new system of units to describe something that interests you. Your unit should be described using at least two subunits. For example, you can decide to measure the quality of songs using a new unit called *song awesomeness*. Song awesomeness is measured by: the number of songs downloaded and the number of times the song was used in movies.

b. Create an equation that shows how to calculate your unit. Then, using your equation, create a sample dataset that you could graph. Are your two subunits related linearly, quadratically, or inversely?

TEST PREP

Multiple Choice

1.1 Physics: Definitions and Applications

39. Modern physics could best be described as the combination of which theories?

a. quantum mechanics and Einstein's theory of relativity
b. quantum mechanics and classical physics
c. Newton's laws of motion and classical physics
d. Newton's laws of motion and Einstein's theory of relativity

40. Which of the following could be studied accurately using classical physics?

a. the strength of gravity within a black hole
b. the motion of a plane through the sky
c. the collisions of subatomic particles
d. the effect of gravity on the passage of time

41. Which of the following best describes why knowledge of physics is necessary to understand all other sciences?

a. Physics explains how energy passes from one object to another.
b. Physics explains how gravity works.
c. Physics explains the motion of objects that can be seen with the naked eye.
d. Physics explains the fundamental aspects of the universe.

42. What does radiation therapy, used to treat cancer patients, have to do with physics?

a. Understanding how cells reproduce is mainly about physics.
b. Predictions of the side effects from the radiation therapy are based on physics.
c. The devices used for generating some kinds of radiation are based on principles of physics.
d. Predictions of the life expectancy of patients receiving radiation therapy are based on physics.

1.2 The Scientific Methods

43. The free-electron model of metals explains some of the important behaviors of metals by assuming the metal's electrons move freely through the metal without repelling one another. In what sense is the free-electron theory based on a model?

a. Its use requires constructing replicas of the metal wire in the lab.
b. It involves analyzing an imaginary system simpler than the real wire it resembles.
c. It examines a model, or ideal, behavior that other metals should imitate.
d. It attempts to examine the metal in a very realistic, or model, way.

44. A scientist wishes to study the motion of about 1,000 molecules of gas in a container by modeling them as tiny billiard balls bouncing randomly off one another. Which of the following is needed to calculate and store data on their detailed motion?

a. a group of hypotheses that cannot be practically tested in real life

b. a computer that can store and perform calculations on large data sets
c. a large amount of experimental results on the molecules and their motion
d. a collection of hypotheses that have not yet been tested regarding the molecules

45. When a large body of experimental evidence supports a hypothesis, what may the hypothesis eventually be considered?
a. observation
b. insight
c. conclusion
d. law

46. While watching some ants outside of your house, you notice that the worker ants gather in a specific area on your lawn. Which of the following is a testable hypothesis that attempts to explain why the ants gather in that specific area on the lawn.
a. The worker thought it was a nice location.
b. because ants may have to find a spot for the queen to lay eggs
c. because there may be some food particles lying there
d. because the worker ants are supposed to group together at a place.

1.3 The Language of Physics: Physical Quantities and Units

47. Which of the following would describe a length that is 2.0×10^{-3} of a meter?
a. 2.0 kilometers
b. 2.0 megameters
c. 2.0 millimeters
d. 2.0 micrometers

48. Suppose that a bathroom scale reads a person's mass as 65 kg with a 3 percent uncertainty. What is the uncertainty in their mass in kilograms?
a. a. 2 kg
b. b. 98 kg
c. c. 5 kg
d. d. 0

49. Which of the following best describes a variable?
a. a trend that shows an exponential relationship
b. something whose value can change over multiple measurements
c. a measure of how much a plot line changes along the y-axis
d. something that remains constant over multiple measurements

50. A high school track coach has just purchased a new stopwatch that has an uncertainty of ±0.05 s . Runners on the team regularly clock 100-m sprints in 12.49 s to 15.01 s . At the school's last track meet, the first-place sprinter came in at 12.04 s and the second-place sprinter came in at 12.07 s . Will the coach's new stopwatch be helpful in timing the sprint team? Why or why not?
a. No, the uncertainty in the stopwatch is too large to effectively differentiate between the sprint times.
b. No, the uncertainty in the stopwatch is too small to effectively differentiate between the sprint times.
c. Yes, the uncertainty in the stopwatch is too large to effectively differentiate between the sprint times.
d. Yes, the uncertainty in the stopwatch is too small to effectively differentiate between the sprint times.

Short Answer

1.1 Physics: Definitions and Applications

51. Describe the aims of physics.
a. Physics aims to explain the fundamental aspects of our universe and how these aspects interact with one another.
b. Physics aims to explain the biological aspects of our universe and how these aspects interact with one another.
c. Physics aims to explain the composition, structure and changes in matter occurring in the universe.
d. Physics aims to explain the social behavior of living beings in the universe.

52. Define the fields of magnetism and electricity and state how are they related.
a. Magnetism describes the attractive force between a magnetized object and a metal like iron. Electricity involves the study of electric charges and their movements. Magnetism is not related to the electricity.
b. Magnetism describes the attractive force between a magnetized object and a metal like iron. Electricity involves the study of electric charges and their movements. Magnetism is produced by a flow electrical charges.
c. Magnetism involves the study of electric charges and their movements. Electricity describes the attractive force between a magnetized object and a metal. Magnetism is not related to the electricity.
d. Magnetism involves the study of electric charges and their movements. Electricity describes the attractive force between a magnetized object and a metal. Magnetism is produced by the flow electrical charges.

53. Describe what two topics physicists are trying to unify with relativistic quantum mechanics. How will this unification create a greater understanding of our universe?
   a. Relativistic quantum mechanics unifies quantum mechanics with Einstein's theory of relativity. The unified theory creates a greater understanding of our universe because it can explain objects of all sizes and masses.
   b. Relativistic quantum mechanics unifies classical mechanics with Einstein's theory of relativity. The unified theory creates a greater understanding of our universe because it can explain objects of all sizes and masses.
   c. Relativistic quantum mechanics unifies quantum mechanics with Einstein's theory of relativity. The unified theory creates a greater understanding of our universe because it is unable to explain objects of all sizes and masses.
   d. Relativistic quantum mechanics unifies classical mechanics with the Einstein's theory of relativity. The unified theory creates a greater understanding of our universe because it is unable to explain objects of all sizes and masses.

54. The findings of studies in quantum mechanics have been described as strange or weird compared to those of classical physics. Explain why this would be so.
   a. It is because the phenomena it explains are outside the normal range of human experience which deals with much larger objects.
   b. It is because the phenomena it explains can be perceived easily, namely, ordinary-sized objects.
   c. It is because the phenomena it explains are outside the normal range of human experience, namely, the very large and the very fast objects.
   d. It is because the phenomena it explains can be perceived easily, namely, the very large and the very fast objects.

55. How could knowledge of physics help you find a faster way to drive from your house to your school?
   a. Physics can explain the traffic on a particular street and help us know about the traffic in advance.
   b. Physics can explain about the ongoing construction of roads on a particular street and help us know about delays in the traffic in advance.
   c. Physics can explain distances, speed limits on a particular street and help us categorize faster routes.
   d. Physics can explain the closing of a particular street and help us categorize faster routes.

56. How could knowledge of physics help you build a sound and energy-efficient house?
   a. An understanding of force, pressure, heat, electricity, etc., which all involve physics, will help me design a sound and energy-efficient house.
   b. An understanding of the air composition, chemical composition of matter, etc., which all involves physics, will help me design a sound and energy-efficient house.
   c. An understanding of material cost and economic factors involving physics will help me design a sound and energy-efficient house.
   d. An understanding of geographical location and social environment which involves physics will help me design a sound and energy-efficient house.

57. What aspects of physics would a chemist likely study in trying to discover a new chemical reaction?
   a. Physics is involved in understanding whether the reactants and products dissolve in water.
   b. Physics is involved in understanding the amount of energy released or required in a chemical reaction.
   c. Physics is involved in what the products of the reaction will be.
   d. Physics is involved in understanding the types of ions produced in a chemical reaction.

1.2 The Scientific Methods

58. You notice that it takes more force to get a large box to start sliding across the floor than it takes to get the box sliding faster once it is already moving. Create a testable hypothesis that attempts to explain this observation.
   a. The floor has greater distortions of space-time for moving the sliding box faster than for the box at rest.
   b. The floor has greater distortions of space-time for the box at rest than for the sliding box.
   c. The resistance between the floor and the box is less when the box is sliding then when the box is at rest.
   d. The floor dislikes having objects move across it and therefore holds the box rigidly in place until it cannot resist the force.

59. Design an experiment that will test the following hypothesis: driving on a gravel road causes greater damage to a car than driving on a dirt road.
   a. To test the hypothesis, compare the damage to the car by driving it on a smooth road and a gravel road.
   b. To test the hypothesis, compare the damage to the car by driving it on a smooth road and a dirt road.
   c. To test the hypothesis, compare the damage to the car by driving it on a gravel road and the dirt road.
   d. This is not a testable hypothesis.

60. How is a physical model, such as a spherical mass held in place by springs, used to represent an atom vibrating in a solid, similar to a computer-based model, such as that predicting how gravity affects the orbits of the planets?

a. Both a physical model and a computer-based model should be built around a hypothesis and could be able to test the hypothesis.
b. Both a physical model and a computer-based model should be built around a hypothesis but they cannot be used to test the hypothesis.
c. Both a physical model and a computer-based model should be built around the results of scientific studies and could be used to make predictions about the system under study.
d. Both a physical model and a computer-based model should be built around the results of scientific studies but cannot be used to make predictions about the system under study.

61. Explain the advantages and disadvantages of using a model to predict a life-or-death situation, such as whether or not an asteroid will strike Earth.

a. The advantage of using a model is that it provides predictions quickly, but the disadvantage of using a model is that it could make erroneous predictions.
b. The advantage of using a model is that it provides accurate predictions, but the disadvantage of using a model is that it takes a long time to make predictions.
c. The advantage of using a model is that it provides predictions quickly without any error. There are no disadvantages of using a scientific model.
d. The disadvantage of using models is that it takes longer time to make predictions and the predictions are inaccurate. There are no advantages to using a scientific model.

62. A friend tells you that a scientific law cannot be changed. State whether or not your friend is correct and then briefly explain your answer.

a. Correct, because laws are theories that have been proved true.
b. Correct, because theories are laws that have been proved true.
c. Incorrect, because a law is changed if new evidence contradicts it.
d. Incorrect, because a law is changed when a theory contradicts it.

63. How does a scientific law compare to a local law, such as that governing parking at your school, in terms of whether or not laws can be changed, and how universal a law is?

a. A local law applies only in a specific area, but a scientific law is applicable throughout the universe. Both the local law and the scientific law can change.
b. A local law applies only in a specific area, but a scientific law is applicable throughout the universe. A local law can change, but a scientific law cannot be changed.
c. A local law applies throughout the universe but a scientific law is applicable only in a specific area. Both the local and the scientific law can change.
d. A local law applies throughout the universe, but a scientific law is applicable only in a specific area. A local law can change, but a scientific law cannot be changed.

64. Can the validity of a model be limited, or must it be universally valid? How does this compare to the required validity of a theory or a law?

a. Models, theories and laws must be universally valid.
b. Models, theories, and laws have only limited validity.
c. Models have limited validity while theories and laws are universally valid.
d. Models and theories have limited validity while laws are universally valid.

1.3 The Language of Physics: Physical Quantities and Units

65. The speed of sound is measured at 342 m/s on a certain day. What is this in km/h? Report your answer in scientific notation.

a. 1.23×10^4 km/h
b. 1.23×10^3 km/h
c. 9.5×10^1 km/h
d. 2.05×10^{-1} km/h

66. Describe the main difference between the metric system and the U.S. Customary System.

a. In the metric system, unit changes are based on powers of 10, while in the U.S. customary system, each unit conversion has unrelated conversion factors.
b. In the metric system, each unit conversion has unrelated conversion factors, while in the U.S. customary system, unit changes are based on powers of 10.
c. In the metric system, unit changes are based on powers of 2, while in the U.S. customary system, each unit conversion has unrelated conversion factors.
d. In the metric system, each unit conversion has unrelated conversion factors, while in the U.S. customary system, unit changes are based on

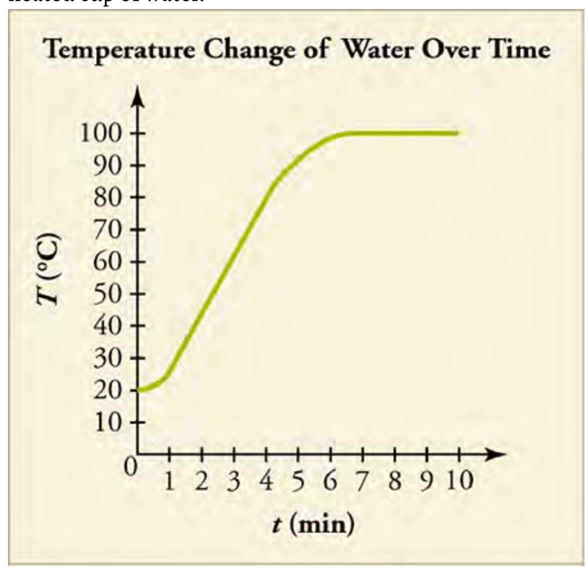
powers of 2.

67. An infant's pulse rate is measured to be 130 ± 5 beats/min. What is the percent uncertainty in this measurement?
a. 2%
b. 3%
c. 4%
d. 5%

68. Explain how the uncertainty of a measurement relates to the accuracy and precision of the measuring device. Include the definitions of accuracy and precision in your answer.
a. A decrease in the precision of a measurement increases the uncertainty of the measurement, while a decrease in accuracy does not.
b. A decrease in either the precision or accuracy of a measurement increases the uncertainty of the measurement.
c. An increase in either the precision or accuracy of a measurement will increase the uncertainty of that measurement.
d. An increase in the accuracy of a measurement will increase the uncertainty of that measurement, while an increase in precision will not.

69. Describe all of the characteristics that can be determined about a straight line with a slope of -3 and a y-intercept of 50 on a graph.
a. Based on the information, the line has a negative slope. Because its y-intercept is 50 and its slope is negative, this line gradually rises on the graph as the x-value increases.
b. Based on the information, the line has a negative slope. Because its y-intercept is 50 and its slope is negative, this line gradually moves downward on the graph as the x-value increases.
c. Based on the information, the line has a positive slope. Because its y-intercept is 50 and its slope is positive, this line gradually rises on the graph as the x-value increases.
d. Based on the information, the line has a positive slope. Because its y-intercept is 50 and its slope is positive, this line gradually moves downward on the graph as the x-value increases.

70. The graph shows the temperature change over time of a heated cup of water.



What is the slope of the graph between the time period 2 min and 5 min?
a. −15 °C/min
b. −0.07 °C/min
c. 0.07 °C/min
d. 15 °C/min

Extended Response

1.2 The Scientific Methods

71. You wish to perform an experiment on the stopping distance of your new car. Create a specific experiment to measure the distance. Be sure to specifically state how you will set up and take data during your experiment.
a. Drive the car at exactly 50 mph and then press harder on the accelerator pedal until the velocity reaches the speed 60 mph and record the distance this takes.
b. Drive the car at exactly 50 mph and then apply the brakes until it stops and record the distance this takes.
c. Drive the car at exactly 50 mph and then apply the brakes until it stops and record the time it takes.
d. Drive the car at exactly 50 mph and then apply the accelerator until it reaches the speed of 60 mph and record the time it takes.

72. You wish to make a model showing how traffic flows around your city or local area. Describe the steps you would take to construct your model as well as some hypotheses that your model could test and the model's limitations in terms of what could not be tested.
a. 1. Testable hypotheses like the gravitational pull on each vehicle while in motion and the average speed of vehicles is 40 mph
   2. Non-testable hypotheses like the average number of vehicles passing is 935 per day and carbon emission from each of the moving vehicle

b. 1. Testable hypotheses like the average number of vehicles passing is 935 per day and the average speed of vehicles is 40 mph
   2. Non-testable hypotheses like the gravitational pull on each vehicle while in motion and the carbon emission from each of the moving vehicle

c. 1. Testable hypotheses like the average number of vehicles passing is 935 per day and the carbon emission from each of the moving vehicle
   2. Non-testable hypotheses like the gravitational pull on each vehicle while in motion and the average speed of the vehicles is 40 mph

d. 1. Testable hypotheses like the average number of vehicles passing is 935 per day and the gravitational pull on each vehicle while in motion
   2. Non-testable hypotheses like the average speed of vehicles is 40 mph and the carbon emission from each of the moving vehicle

73. What would play the most important role in leading to an experiment in the scientific world becoming a scientific law?

a. Further testing would need to show it is a universally followed rule.
b. The observation would have to be described in a published scientific article.
c. The experiment would have to be repeated once or twice.
d. The observer would need to be a well-known scientist whose authority was accepted.

1.3 The Language of Physics: Physical Quantities and Units

74. Tectonic plates are large segments of the Earth's crust that move slowly. Suppose that one such plate has an average speed of 4.0 cm/year. What distance does it move in 1.0 s at this speed? What is its speed in kilometers per million years? Report all of your answers using scientific notation.

a. $1.3 \times 10^{-9}\text{ m};\ 4.0 \times 10^{1}\text{ km/million years}$
b. $1.3 \times 10^{-6}\text{ m};\ 4.0 \times 10^{1}\text{ km/million years}$
c. $1.3 \times 10^{-9}\text{ m};\ 4.0 \times 10^{-11}\text{ km/million years}$
d. $1.3 \times 10^{-6}\text{ m};\ 4.0 \times 10^{-11}\text{ km/million years}$

75. At x = 3, a function f(x) has a positive value, with a positive slope that is decreasing in magnitude with increasing x. Which option could correspond to f(x)?

a. $y = 13x$
b. $y = x^2$
c. $y = 2x + 9$
d. $y = \frac{x}{2} + 9$

CHAPTER 2
Motion in One Dimension

Figure 2.1 Shanghai Maglev. At this rate, a train traveling from Boston to Washington, DC, a distance of 439 miles, could make the trip in under an hour and a half. Presently, the fastest train on this route takes over six hours to cover this distance. (Alex Needham, Public Domain)

Chapter Outline

INTRODUCTION Unless you have flown in an airplane, you have probably never traveled faster than 150 mph. Can you imagine traveling in a train like the one shown in Figure 2.1 that goes over 300 mph? Despite the high speed, the people riding in this train may not notice that they are moving at all unless they look out the window! This is because motion, even motion at 300 mph, is relative to the observer.

In this chapter, you will learn why it is important to identify a reference frame in order to clearly describe motion. For now, the motion you describe will be one-dimensional. Within this context, you will learn the difference between distance and displacement as well as the difference between speed and velocity. Then you will look at some graphing and problem-solving techniques.

2.1 Relative Motion, Distance, and Displacement

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe motion in different reference frames
- Define distance and displacement, and distinguish between the two
- Solve problems involving distance and displacement

Section Key Terms

displacement	distance	kinematics	magnitude
position	reference frame	scalar	vector

Defining Motion

Our study of physics opens with **kinematics**—the study of motion without considering its causes. Objects are in motion everywhere you look. Everything from a tennis game to a space-probe flyby of the planet Neptune involves motion. When you are resting, your heart moves blood through your veins. Even in inanimate objects, atoms are always moving.

How do you know something is moving? The location of an object at any particular time is its **position**. More precisely, you need to specify its position relative to a convenient **reference frame**. Earth is often used as a reference frame, and we often describe the position of an object as it relates to stationary objects in that reference frame. For example, a rocket launch would be described in terms of the position of the rocket with respect to Earth as a whole, while a professor's position could be described in terms of where she is in relation to the nearby white board. In other cases, we use reference frames that are not stationary but are in motion relative to Earth. To describe the position of a person in an airplane, for example, we use the airplane, not Earth, as the reference frame. (See Figure 2.2.) Thus, you can only know how fast and in what direction an object's position is changing against a background of something else that is either not moving or moving with a known speed and direction. The reference frame is the coordinate system from which the positions of objects are described.

Figure 2.2 Are clouds a useful reference frame for airplane passengers? Why or why not? (Paul Brennan, Public Domain)

Your classroom can be used as a reference frame. In the classroom, the walls are not moving. Your motion as you walk to the door, can be measured against the stationary background of the classroom walls. You can also tell if other things in the classroom are moving, such as your classmates entering the classroom or a book falling off a desk. You can also tell in what direction something is moving in the classroom. You might say, "The teacher is moving toward the door." Your reference frame allows you to determine not only that something is moving but also the direction of motion.

You could also serve as a reference frame for others' movement. If you remained seated as your classmates left the room, you would measure their movement away from your stationary location. If you and your classmates left the room together, then your perspective of their motion would be change. You, as the reference frame, would be moving in the same direction as your other moving classmates. As you will learn in the **Snap Lab**, your description of motion can be quite different when viewed from different reference frames.

Snap Lab

Looking at Motion from Two Reference Frames

In this activity you will look at motion from two reference frames. Which reference frame is correct?

- Choose an open location with lots of space to spread out so there is less chance of tripping or falling due to a collision and/or loose basketballs.
- 1 basketball

Procedure

1. Work with a partner. Stand a couple of meters away from your partner. Have your partner turn to the side so that you are looking at your partner's profile. Have your partner begin bouncing the basketball while standing in place. Describe the motion of the ball.
2. Next, have your partner again bounce the ball, but this time your partner should walk forward with the bouncing ball. You will remain stationary. Describe the ball's motion.
3. Again have your partner walk forward with the bouncing ball. This time, you should move alongside your partner while continuing to view your partner's profile. Describe the ball's motion.
4. Switch places with your partner, and repeat Steps 1–3.

GRASP CHECK

How do the different reference frames affect how you describe the motion of the ball?

a. The motion of the ball is independent of the reference frame and is same for different reference frames.
b. The motion of the ball is independent of the reference frame and is different for different reference frames.
c. The motion of the ball is dependent on the reference frame and is same for different reference frames.
d. The motion of the ball is dependent on the reference frames and is different for different reference frames.

LINKS TO PHYSICS

History: Galileo's Ship

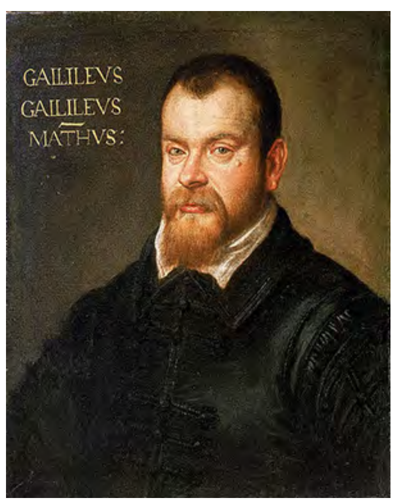


Figure 2.3 Galileo Galilei (1564–1642) studied motion and developed the concept of a reference frame. (Domenico Tintoretto)

The idea that a description of motion depends on the reference frame of the observer has been known for hundreds of years. The 17th-century astronomer Galileo Galilei (Figure 2.3) was one of the first scientists to explore this idea. Galileo suggested the following thought experiment: Imagine a windowless ship moving at a constant speed and direction along a perfectly calm sea. Is there a way that a person inside the ship can determine whether the ship is moving? You can extend this thought experiment by also imagining a person standing on the shore. How can a person on the shore determine whether the ship is moving?

Galileo came to an amazing conclusion. Only by looking at each other can a person in the ship or a person on shore describe the motion of one relative to the other. In addition, their descriptions of motion would be identical. A person inside the ship would describe the person on the land as moving past the ship. The person on shore would describe the ship and the person inside it as moving past. Galileo realized that observers moving at a constant speed and direction relative to each other describe motion in the same way. Galileo had discovered that a description of motion is only meaningful if you specify a reference frame.

GRASP CHECK

Imagine standing on a platform watching a train pass by. According to Galileo's conclusions, how would your description of motion and the description of motion by a person riding on the train compare?

a. I would see the train as moving past me, and a person on the train would see me as stationary.
b. I would see the train as moving past me, and a person on the train would see me as moving past the train.
c. I would see the train as stationary, and a person on the train would see me as moving past the train.
d. I would see the train as stationary, and a person on the train would also see me as stationary.

Distance vs. Displacement

As we study the motion of objects, we must first be able to describe the object's position. Before your parent drives you to school, the car is sitting in your driveway. Your driveway is the starting position for the car. When you reach your high school, the car has changed position. Its new position is your school.

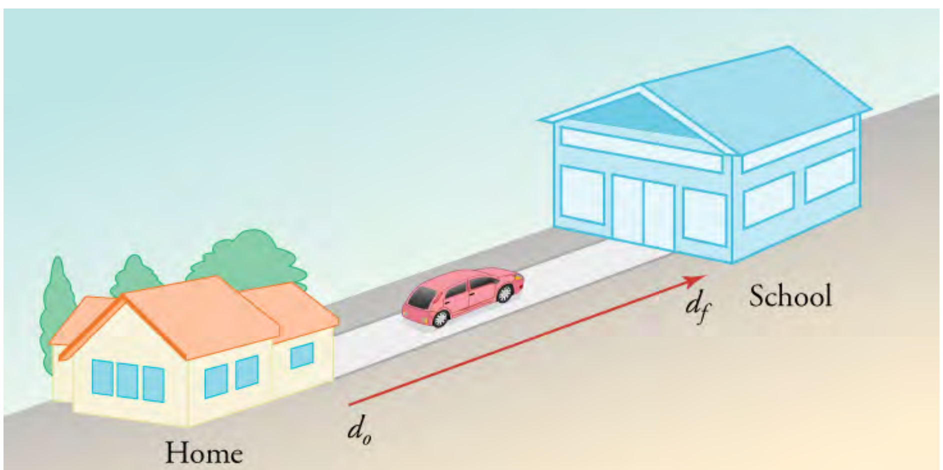


Figure 2.4 Your total change in position is measured from your house to your school.

Physicists use variables to represent terms. We will use **d** to represent car's position. We will use a subscript to differentiate between the initial position, $\mathbf{d}_0$, and the final position, $\mathbf{d}_f$. In addition, vectors, which we will discuss later, will be in bold or will have an arrow above the variable. Scalars will be italicized.

TIPS FOR SUCCESS

In some books, **x** or **s** is used instead of **d** to describe position. In $\mathbf{d}_0$, said *d naught*, the subscript 0 stands for *initial*. When we begin to talk about two-dimensional motion, sometimes other subscripts will be used to describe horizontal position, $\mathbf{d}_x$, or vertical position, $\mathbf{d}_y$. So, you might see references to $\mathbf{d}_{0x}$ and $\mathbf{d}_{fy}$.

Now imagine driving from your house to a friend's house located several kilometers away. How far would you drive? The **distance** an object moves is the length of the path between its initial position and its final position. The distance you drive to your friend's house depends on your path. As shown in Figure 2.5, distance is different from the length of a straight line between two points. The distance you drive to your friend's house is probably longer than the straight line between the two houses.

Figure 2.5 A short line separates the starting and ending points of this motion, but the distance along the path of motion is considerably longer.

We often want to be more precise when we talk about position. The description of an object's motion often includes more than just the distance it moves. For instance, if it is a five kilometer drive to school, the distance traveled is 5 kilometers. After dropping you off at school and driving back home, your parent will have traveled a total distance of 10 kilometers. The car and your parent will end up in the same starting position in space. The net change in position of an object is its **displacement**, or $\Delta \mathbf{d}$. The Greek letter delta, Δ, means *change in*.

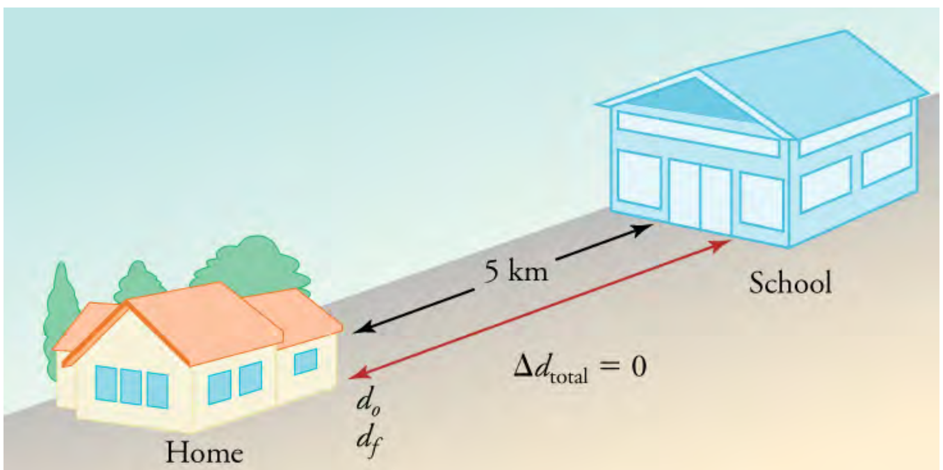


Figure 2.6 The total distance that your car travels is 10 km, but the total displacement is 0.

Snap Lab

Distance vs. Displacement

In this activity you will compare distance and displacement. Which term is more useful when making measurements?

- 1 recorded song available on a portable device
- 1 tape measure
- 3 pieces of masking tape
- A room (like a gym) with a wall that is large and clear enough for all pairs of students to walk back and forth without running into each other.

Procedure

1. One student from each pair should stand with their back to the longest wall in the classroom. Students should stand at least 0.5 meters away from each other. Mark this starting point with a piece of masking tape.
2. The second student from each pair should stand facing their partner, about two to three meters away. Mark this point

with a second piece of masking tape.

3. Student pairs line up at the starting point along the wall.
4. The teacher turns on the music. Each pair walks back and forth from the wall to the second marked point until the music stops playing. Keep count of the number of times you walk across the floor.
5. When the music stops, mark your ending position with the third piece of masking tape.
6. Measure from your starting, initial position to your ending, final position.
7. Measure the length of your path from the starting position to the second marked position. Multiply this measurement by the total number of times you walked across the floor. Then add this number to your measurement from step 6.
8. Compare the two measurements from steps 6 and 7.

GRASP CHECK

1. Which measurement is your total distance traveled?
2. Which measurement is your displacement?
3. When might you want to use one over the other?

a. Measurement of the total length of your path from the starting position to the final position gives the distance traveled, and the measurement from your initial position to your final position is the displacement. Use distance to describe the total path between starting and ending points,and use displacement to describe the shortest path between starting and ending points.
b. Measurement of the total length of your path from the starting position to the final position is distance traveled, and the measurement from your initial position to your final position is displacement. Use distance to describe the shortest path between starting and ending points, and use displacement to describe the total path between starting and ending points.
c. Measurement from your initial position to your final position is distance traveled, and the measurement of the total length of your path from the starting position to the final position is displacement. Use distance to describe the total path between starting and ending points, and use displacement to describe the shortest path between starting and ending points.
d. Measurement from your initial position to your final position is distance traveled, and the measurement of the total length of your path from the starting position to the final position is displacement. Use distance to describe the shortest path between starting and ending points, and use displacement to describe the total path between starting and ending points.

If you are describing only your drive to school, then the distance traveled and the displacement are the same—5 kilometers. When you are describing the entire round trip, distance and displacement are different. When you describe distance, you only include the **magnitude**, the size or amount, of the distance traveled. However, when you describe the displacement, you take into account both the magnitude of the change in position and the direction of movement.

In our previous example, the car travels a total of 10 kilometers, but it drives five of those kilometers forward toward school and five of those kilometers back in the opposite direction. If we ascribe the forward direction a positive (+) and the opposite direction a negative (–), then the two quantities will cancel each other out when added together.

A quantity, such as distance, that has magnitude (i.e., how big or how much) but does not take into account direction is called a **scalar**. A quantity, such as displacement, that has both magnitude and direction is called a **vector**.

WATCH PHYSICS

Vectors & Scalars

This video (http://openstax.org/l/28vectorscalar) introduces and differentiates between vectors and scalars. It also introduces quantities that we will be working with during the study of kinematics.

Click to view content (https://www.khanacademy.org/embed_video?v=ihNZlp7iUHE)

GRASP CHECK

How does this video (https://www.khanacademy.org/science/ap-physics-1/ap-one-dimensional-motion/ap-physics-foundations/v/introduction-to-vectors-and-scalars) help you understand the difference between distance and displacement? Describe the differences between vectors and scalars using physical quantities as examples.

a. It explains that distance is a vector and direction is important, whereas displacement is a scalar and it has no direction attached to it.
b. It explains that distance is a scalar and direction is important, whereas displacement is a vector and it has no direction attached to it.
c. It explains that distance is a scalar and it has no direction attached to it, whereas displacement is a vector and direction is important.
d. It explains that both distance and displacement are scalar and no directions are attached to them.

Displacement Problems

Hopefully you now understand the conceptual difference between distance and displacement. Understanding concepts is half the battle in physics. The other half is math. A stumbling block to new physics students is trying to wade through the math of physics while also trying to understand the associated concepts. This struggle may lead to misconceptions and answers that make no sense. Once the concept is mastered, the math is far less confusing.

So let's review and see if we can make sense of displacement in terms of numbers and equations. You can calculate an object's displacement by subtracting its original position, $\mathbf{d}_\mathbf{o}$, from its final position $\mathbf{d}_\mathbf{f}$. In math terms that means

$$\Delta\mathbf{d} = \mathbf{d}_\mathrm{f} - \mathbf{d}_0 .$$

If the final position is the same as the initial position, then $\Delta\mathbf{d} = 0$.

To assign numbers and/or direction to these quantities, we need to define an axis with a positive and a negative direction. We also need to define an origin, or *O*. In Figure 2.6, the axis is in a straight line with home at zero and school in the positive direction. If we left home and drove the opposite way from school, motion would have been in the negative direction. We would have assigned it a negative value. In the round-trip drive, $\mathbf{d}_\mathrm{f}$ and $\mathbf{d}_\mathrm{o}$ were both at zero kilometers. In the one way trip to school, $\mathbf{d}_\mathrm{f}$ was at 5 kilometers and $\mathbf{d}_\mathrm{o}$ was at zero km. So, $\Delta\mathbf{d}$ was 5 kilometers.

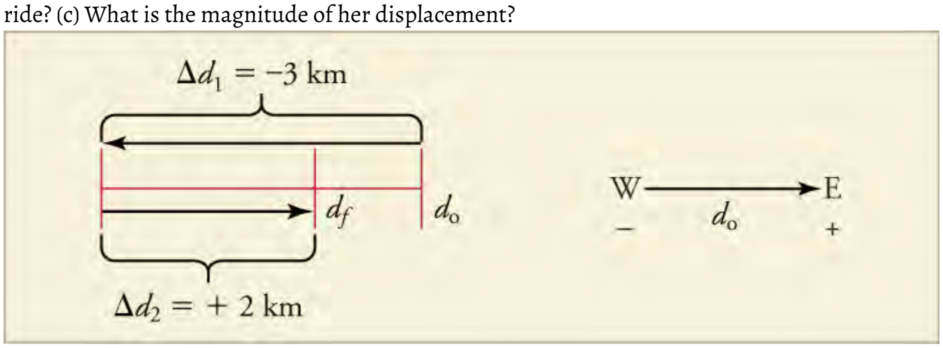
TIPS FOR SUCCESS

You may place your origin wherever you would like. You have to make sure that you calculate all distances consistently from your zero and you define one direction as positive and the other as negative. Therefore, it makes sense to choose the easiest axis, direction, and zero. In the example above, we took home to be zero because it allowed us to avoid having to interpret a solution with a negative sign.

WORKED EXAMPLE

Calculating Distance and Displacement

A cyclist rides 3 km west and then turns around and rides 2 km east. (a) What is her displacement? (b) What distance does she ride? (c) What is the magnitude of her displacement?



Strategy

To solve this problem, we need to find the difference between the final position and the initial position while taking care to note the direction on the axis. The final position is the sum of the two displacements, $\Delta\mathbf{d}_1$ and $\Delta\mathbf{d}_2$.

Solution

a. Displacement: The rider's displacement is $\Delta\mathbf{d} = \mathbf{d}_f - \mathbf{d}_0 = -1\ \text{ km}$.
b. Distance: The distance traveled is 3 km + 2 km = 5 km.
c. The magnitude of the displacement is 1 km.

Discussion

The displacement is negative because we chose east to be positive and west to be negative. We could also have described the displacement as 1 km west. When calculating displacement, the direction mattered, but when calculating distance, the direction did not matter. The problem would work the same way if the problem were in the north–south or *y*-direction.

TIPS FOR SUCCESS

Physicists like to use standard units so it is easier to compare notes. The standard units for calculations are called *SI* units (International System of Units). SI units are based on the metric system. The SI unit for displacement is the meter (m), but sometimes you will see a problem with kilometers, miles, feet, or other units of length. If one unit in a problem is an SI unit and another is not, you will need to convert all of your quantities to the same system before you can carry out the calculation.

Practice Problems

1. On an axis in which moving from right to left is positive, what is the displacement and distance of a student who walks 32 m to the right and then 17 m to the left?
   a. Displacement is -15 m and distance is -49 *m*.
   b. Displacement is -15 m and distance is 49 m.
   c. Displacement is 15 m and distance is -49 m.
   d. Displacement is 15 m and distance is 49 m.

2. Tiana jogs 1.5 km along a straight path and then turns and jogs 2.4 km in the opposite direction. She then turns back and jogs 0.7 km in the original direction. Let Tiana's original direction be the positive direction. What are the displacement and distance she jogged?
   a. Displacement is 4.6 km,and distance is -0.2 km.
   b. Displacement is -0.2 km, and distance is 4.6 km.
   c. Displacement is 4.6 km, and distance is +0.2 km.
   d. Displacement is +0.2 km, and distance is 4.6 km.

WORK IN PHYSICS

Mars Probe Explosion

Figure 2.7 The Mars Climate Orbiter disaster illustrates the importance of using the correct calculations in physics. (NASA)

Physicists make calculations all the time, but they do not always get the right answers. In 1998, NASA, the National Aeronautics and Space Administration, launched the Mars Climate Orbiter, shown in Figure 2.7, a $125-million-dollar satellite designed to monitor the Martian atmosphere. It was supposed to orbit the planet and take readings from a safe distance. The American scientists made calculations in English units (feet, inches, pounds, etc.) and forgot to convert their answers to the standard metric SI units. This was a very costly mistake. Instead of orbiting the planet as planned, the Mars Climate Orbiter ended up flying into the Martian atmosphere. The probe disintegrated. It was one of the biggest embarrassments in NASA's history.

GRASP CHECK

In 1999 the Mars Climate Orbiter crashed because calculation were performed in English units instead of SI units. At one point the orbiter was just 187,000 feet above the surface, which was too close to stay in orbit. What was the height of the orbiter at this time in kilometers? (Assume 1 meter equals 3.281 feet.)

a. 16 km
b. 18 km
c. 57 km
d. 614 km

Check Your Understanding

3. What does it mean when motion is described as relative?
   a. It means that motion of any object is described relative to the motion of Earth.
   b. It means that motion of any object is described relative to the motion of any other object.
   c. It means that motion is independent of the frame of reference.
   d. It means that motion depends on the frame of reference selected.

4. If you and a friend are standing side-by-side watching a soccer game, would you both view the motion from the same reference frame?
   a. Yes, we would both view the motion from the same reference point because both of us are at rest in Earth's frame of reference.
   b. Yes, we would both view the motion from the same reference point because both of us are observing the motion from two points on the same straight line.
   c. No, we would both view the motion from different reference points because motion is viewed from two different points; the reference frames are similar but not the same.
   d. No, we would both view the motion from different reference points because response times may be different; so, the motion observed by both of us would be different.

5. What is the difference between distance and displacement?
   a. Distance has both magnitude and direction, while displacement has magnitude but no direction.
   b. Distance has magnitude but no direction, while displacement has both magnitude and direction.
   c. Distance has magnitude but no direction, while displacement has only direction.
   d. There is no difference. Both distance and displacement have magnitude and direction.

6. Which situation correctly identifies a race car's distance traveled and the magnitude of displacement during a one-lap car race?
   a. The perimeter of the race track is the distance, and the shortest distance between the start line and the finish line is the magnitude of displacement.
   b. The perimeter of the race track is the magnitude of displacement, and the shortest distance between the start and finish line is the distance.
   c. The perimeter of the race track is both the distance and magnitude of displacement.
   d. The shortest distance between the start line and the finish line is both the distance and magnitude of displacement.

7. Why is it important to specify a reference frame when describing motion?
   a. Because Earth is continuously in motion; an object at rest on Earth will be in motion when viewed from outer space.
   b. Because the position of a moving object can be defined only when there is a fixed reference frame.

c. Because motion is a relative term; it appears differently when viewed from different reference frames.
d. Because motion is always described in Earth's frame of reference; if another frame is used, it has to be specified with each situation.

2.2 Speed and Velocity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Calculate the average speed of an object
- Relate displacement and average velocity

Section Key Terms

average speed	average velocity	instantaneous speed
instantaneous velocity	speed	velocity

Speed

There is more to motion than distance and displacement. Questions such as, "How long does a foot race take?" and "What was the runner's speed?" cannot be answered without an understanding of other concepts. In this section we will look at time, speed, and velocity to expand our understanding of motion.

A description of how fast or slow an object moves is its speed. **Speed** is the rate at which an object changes its location. Like distance, speed is a scalar because it has a magnitude but not a direction. Because speed is a rate, it depends on the time interval of motion. You can calculate the elapsed time or the change in time, Δt, of motion as the difference between the ending time and the beginning time

$$\Delta t = t_{\mathrm{f}} - t_0.$$

The SI unit of time is the second (s), and the SI unit of speed is meters per second (m/s), but sometimes kilometers per hour (km/h), miles per hour (mph) or other units of speed are used.

When you describe an object's speed, you often describe the average over a time period. **Average speed**, v_{avg}, is the distance traveled divided by the time during which the motion occurs.

$$v_{\mathrm{avg}} = \frac{\text{distance}}{\text{time}}$$

You can, of course, rearrange the equation to solve for either distance or time

$$\text{time} = \frac{\text{distance}}{v_{\mathrm{avg}}}.$$

$$\text{distance} = v_{\mathrm{avg}} \times \text{time}$$

Suppose, for example, a car travels 150 kilometers in 3.2 hours. Its average speed for the trip is

$$\begin{aligned} v_{\mathrm{avg}} &= \frac{\text{distance}}{\text{time}} \\ &= \frac{150 \text{ km}}{3.2 \text{ h}} \\ &= 47 \text{ km/h}. \end{aligned}$$

A car's speed would likely increase and decrease many times over a 3.2 hour trip. Its speed at a specific instant in time, however, is its **instantaneous speed**. A car's speedometer describes its instantaneous speed.

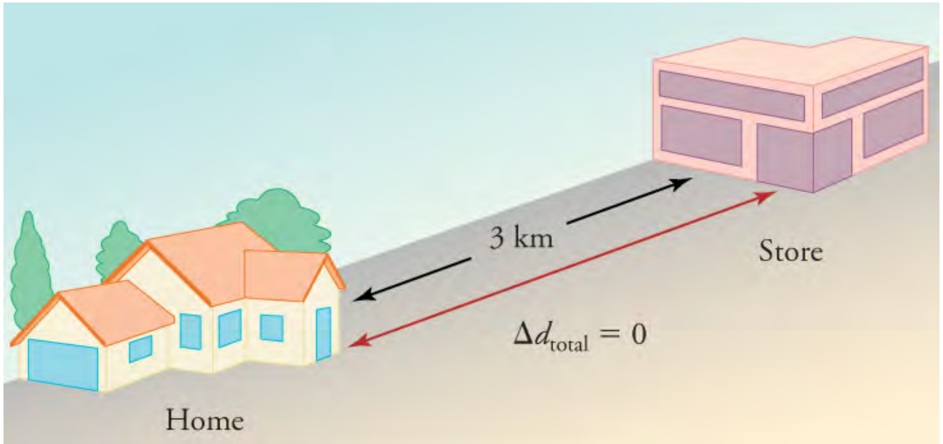


Figure 2.8 During a 30-minute round trip to the store, the total distance traveled is 6 km. The average speed is 12 km/h. The displacement for the round trip is zero, because there was no net change in position.

WORKED EXAMPLE

Calculating Average Speed

A marble rolls 5.2 m in 1.8 s. What was the marble's average speed?

Strategy

We know the distance the marble travels, 5.2 m, and the time interval, 1.8 s. We can use these values in the average speed equation.

Solution

$$v_{avg} = \frac{\text{distance}}{\text{time}} = \frac{5.2 \text{ m}}{1.8 \text{ s}} = 2.9 \text{ m/s}$$

Discussion

Average speed is a scalar, so we do not include direction in the answer. We can check the reasonableness of the answer by estimating: 5 meters divided by 2 seconds is 2.5 m/s. Since 2.5 m/s is close to 2.9 m/s, the answer is reasonable. This is about the speed of a brisk walk, so it also makes sense.

Practice Problems

8. A pitcher throws a baseball from the pitcher's mound to home plate in 0.46 s. The distance is 18.4 m. What was the average speed of the baseball?
   a. 40 m/s
   b. - 40 m/s
   c. 0.03 m/s
   d. 8.5 m/s

9. Cassie walked to her friend's house with an average speed of 1.40 m/s. The distance between the houses is 205 m. How long did the trip take her?
   a. 146 s
   b. 0.01 s
   c. 2.50 min
   d. 287 s

Velocity

The vector version of speed is velocity. **Velocity** describes the speed and direction of an object. As with speed, it is useful to describe either the average velocity over a time period or the velocity at a specific moment. **Average velocity** is displacement divided by the time over which the displacement occurs.

$$v_{avg} = \frac{\text{distance}}{\text{time}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{\mathbf{d}_f - \mathbf{d}_0}{t_f - t_0}$$

Velocity, like speed, has SI units of meters per second (m/s), but because it is a vector, you must also include a direction. Furthermore, the variable **v** for velocity is bold because it is a vector, which is in contrast to the variable *v* for speed which is italicized because it is a scalar quantity.

TIPS FOR SUCCESS

It is important to keep in mind that the average speed is not the same thing as the average velocity without its direction. Like we saw with displacement and distance in the last section, changes in direction over a time interval have a bigger effect on speed and velocity.

Suppose a passenger moved toward the back of a plane with an average velocity of –4 m/s. We cannot tell from the average velocity whether the passenger stopped momentarily or backed up before he got to the back of the plane. To get more details, we must consider smaller segments of the trip over smaller time intervals such as those shown in Figure 2.9. If you consider infinitesimally small intervals, you can define **instantaneous velocity**, which is the velocity at a specific instant in time. Instantaneous velocity and average velocity are the same if the velocity is constant.

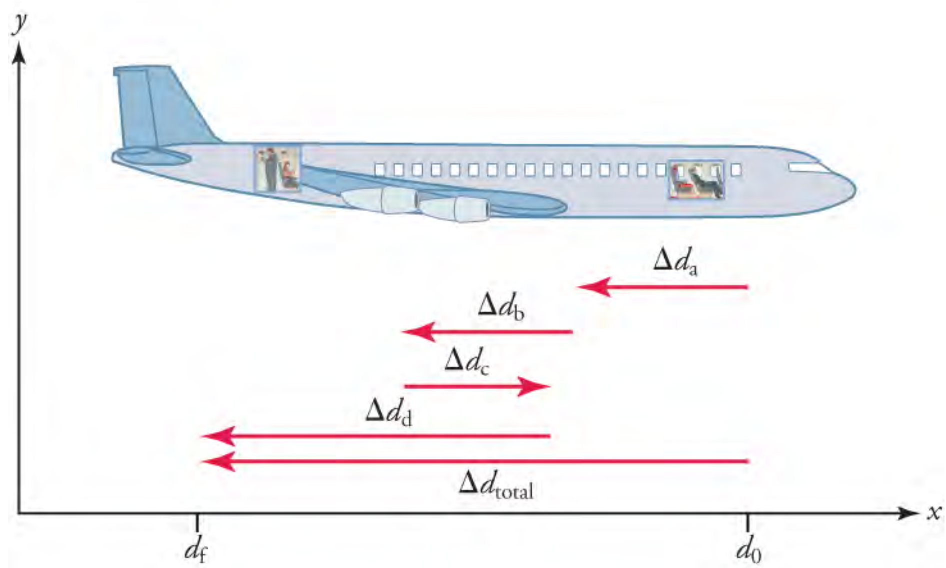


Figure 2.9 The diagram shows a more detailed record of an airplane passenger heading toward the back of the plane, showing smaller segments of his trip.

Earlier, you have read that distance traveled can be different than the magnitude of displacement. In the same way, speed can be different than the magnitude of velocity. For example, you drive to a store and return home in half an hour. If your car's odometer shows the total distance traveled was 6 km, then your average speed was 12 km/h. Your average velocity, however, was zero because your displacement for the round trip is zero.

WATCH PHYSICS

Calculating Average Velocity or Speed

This video (http://openstax.org/l/28avgvelocity) reviews vectors and scalars and describes how to calculate average velocity and average speed when you know displacement and change in time. The video also reviews how to convert km/h to m/s.

Click to view content (https://www.khanacademy.org/embed_video?v=MAS6mBRZZXA)

GRASP CHECK

Which of the following fully describes a vector and a scalar quantity and correctly provides an example of each?

a. A scalar quantity is fully described by its magnitude, while a vector needs both magnitude and direction to fully describe it. Displacement is an example of a scalar quantity and time is an example of a vector quantity.
b. A scalar quantity is fully described by its magnitude, while a vector needs both magnitude and direction to fully describe it. Time is an example of a scalar quantity and displacement is an example of a vector quantity.
c. A scalar quantity is fully described by its magnitude and direction, while a vector needs only magnitude to fully describe it. Displacement is an example of a scalar quantity and time is an example of a vector quantity.
d. A scalar quantity is fully described by its magnitude and direction, while a vector needs only magnitude to fully describe it. Time is an example of a scalar quantity and displacement is an example of a vector quantity.

WORKED EXAMPLE

Calculating Average Velocity

A student has a displacement of 304 m north in 180 s. What was the student's average velocity?

Strategy

We know that the displacement is 304 m north and the time is 180 s. We can use the formula for average velocity to solve the problem.

Solution

$$\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{304\ \text{m}}{180\ \text{s}} = 1.7\ \text{m/s north} \tag{2.1}$$

Discussion

Since average velocity is a vector quantity, you must include direction as well as magnitude in the answer. Notice, however, that the direction can be omitted until the end to avoid cluttering the problem. Pay attention to the significant figures in the problem. The distance 304 m has three significant figures, but the time interval 180 s has only two, so the quotient should have only two significant figures.

TIPS FOR SUCCESS

Note the way scalars and vectors are represented. In this book d represents distance and displacement. Similarly, v represents speed, and v represents velocity. A variable that is not bold indicates a scalar quantity, and a bold variable indicates a vector quantity. Vectors are sometimes represented by small arrows above the variable.

WORKED EXAMPLE

Solving for Displacement when Average Velocity and Time are Known

Layla jogs with an average velocity of 2.4 m/s east. What is her displacement after 46 seconds?

Strategy

We know that Layla's average velocity is 2.4 m/s east, and the time interval is 46 seconds. We can rearrange the average velocity formula to solve for the displacement.

Solution

$$\begin{aligned}\mathbf{v}_{\text{avg}} &= \frac{\Delta \mathbf{d}}{\Delta t} \\ \Delta \mathbf{d} &= \mathbf{v}_{avg}\Delta t \\ &= (2.4\ \text{m/s})(46\ \text{s}) \\ &= 1.1 \times 10^2\ \text{m east}\end{aligned} \tag{2.2}$$

Discussion

The answer is about 110 m east, which is a reasonable displacement for slightly less than a minute of jogging. A calculator shows the answer as 110.4 m. We chose to write the answer using scientific notation because we wanted to make it clear that we only used two significant figures.

TIPS FOR SUCCESS

Dimensional analysis is a good way to determine whether you solved a problem correctly. Write the calculation using only units to be sure they match on opposite sides of the equal mark. In the worked example, you have m = (m/s)(s). Since seconds is in the denominator for the average velocity and in the numerator for the time, the unit cancels out leaving only m and, of course, m = m.

WORKED EXAMPLE

Solving for Time when Displacement and Average Velocity are Known

Phillip walks along a straight path from his house to his school. How long will it take him to get to school if he walks 428 m west with an average velocity of 1.7 m/s west?

Strategy

We know that Phillip's displacement is 428 m west, and his average velocity is 1.7 m/s west. We can calculate the time required for the trip by rearranging the average velocity equation.

Solution

$$\begin{aligned} \mathbf{v}_{\text{avg}} &= \frac{\Delta \mathbf{d}}{\Delta t} \\ \Delta t &= \frac{\Delta \mathbf{d}}{\mathbf{v}_{\text{avg}}} \\ &= \frac{428 \quad \text{m}}{1.7 \quad \text{m/s}} \\ &= 2.5 \times 10^2 \text{ s} \end{aligned} \tag{2.3}$$

Discussion

Here again we had to use scientific notation because the answer could only have two significant figures. Since time is a scalar, the answer includes only a magnitude and not a direction.

Practice Problems

10. A trucker drives along a straight highway for 0.25 h with a displacement of 16 km south. What is the trucker's average velocity?
    a. 4 km/h north
    b. 4 km/h south
    c. 64 km/h north
    d. 64 km/h south

11. A bird flies with an average velocity of 7.5 m/s east from one branch to another in 2.4 s. It then pauses before flying with an average velocity of 6.8 m/s east for 3.5 s to another branch. What is the bird's total displacement from its starting point?
    a. 42 m west
    b. 6 m west
    c. 6 m east
    d. 42 m east

Virtual Physics

The Walking Man

In this simulation you will put your cursor on the man and move him first in one direction and then in the opposite direction. Keep the *Introduction* tab active. You can use the *Charts* tab after you learn about graphing motion later in this chapter. Carefully watch the sign of the numbers in the position and velocity boxes. Ignore the acceleration box for now. See if you can make the man's position positive while the velocity is negative. Then see if you can do the opposite.

Click to view content (https://archive.cnx.org/specials/e2ca52af-8c6b-450e-ac2f-9300b38e8739/moving-man/)

GRASP CHECK

Which situation correctly describes when the moving man's position was negative but his velocity was positive?

a. Man moving toward 0 from left of 0
b. Man moving toward 0 from right of 0
c. Man moving away from 0 from left of 0
d. Man moving away from 0 from right of 0

Check Your Understanding

12. Two runners travel along the same straight path. They start at the same time, and they end at the same time, but at the halfway mark, they have different instantaneous velocities. Is it possible for them to have the same average velocity for the trip?

a. Yes, because average velocity depends on the net or total displacement.
b. Yes, because average velocity depends on the total distance traveled.
c. No, because the velocities of both runners must remain the exactly same throughout the journey.
d. No, because the instantaneous velocities of the runners must remain same midway but can be different elsewhere.

13. If you divide the total distance traveled on a car trip (as determined by the odometer) by the time for the trip, are you calculating the average speed or the magnitude of the average velocity, and under what circumstances are these two quantities the same?

a. Average speed. Both are the same when the car is traveling at a constant speed and changing direction.
b. Average speed. Both are the same when the speed is constant and the car does not change its direction.
c. Magnitude of average velocity. Both are same when the car is traveling at a constant speed.
d. Magnitude of average velocity. Both are same when the car does not change its direction.

14. Is it possible for average velocity to be negative?

a. Yes, in cases when the net displacement is negative.
b. Yes, if the body keeps changing its direction during motion.
c. No, average velocity describes only magnitude and not the direction of motion.
d. No, average velocity describes only the magnitude in the positive direction of motion.

2.3 Position vs. Time Graphs

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the meaning of slope in position vs. time graphs
- Solve problems using position vs. time graphs

Section Key Terms

dependent variable	independent variable	tangent

Graphing Position as a Function of Time

A graph, like a picture, is worth a thousand words. Graphs not only contain numerical information, they also reveal relationships between physical quantities. In this section, we will investigate kinematics by analyzing graphs of position over time.

Graphs in this text have perpendicular axes, one horizontal and the other vertical. When two physical quantities are plotted against each other, the horizontal axis is usually considered the **independent variable**, and the vertical axis is the **dependent variable**. In algebra, you would have referred to the horizontal axis as the x-axis and the vertical axis as the y-axis. As in Figure 2.10, a straight-line graph has the general form $y = mx + b$.

Here m is the slope, defined as the rise divided by the run (as seen in the figure) of the straight line. The letter b is the y-intercept which is the point at which the line crosses the vertical, y-axis. In terms of a physical situation in the real world, these quantities will take on a specific significance, as we will see below. (Figure 2.10.)

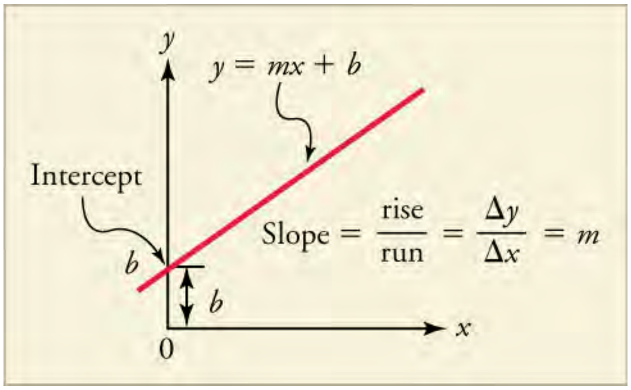


Figure 2.10 The diagram shows a straight-line graph. The equation for the straight line is y equals $mx + b$.

In physics, time is usually the independent variable. Other quantities, such as displacement, are said to depend upon it. A graph of position versus time, therefore, would have position on the vertical axis (dependent variable) and time on the horizontal axis (independent variable). In this case, to what would the slope and y-intercept refer? Let's look back at our original example when studying distance and displacement.

The drive to school was 5 km from home. Let's assume it took 10 minutes to make the drive and that your parent was driving at a constant velocity the whole time. The position versus time graph for this section of the trip would look like that shown in Figure 2.11.

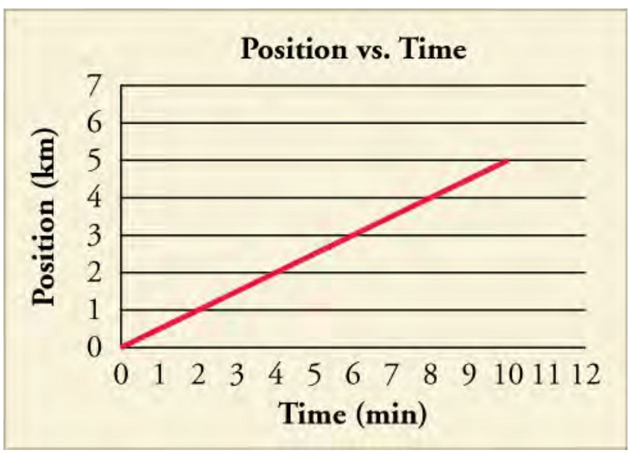


Figure 2.11 A graph of position versus time for the drive to school is shown. What would the graph look like if we added the return trip?

As we said before, $\mathbf{d}_0 = 0$ because we call home our O and start calculating from there. In Figure 2.11, the line starts at $\mathbf{d} = 0$, as well. This is the b in our equation for a straight line. Our initial position in a position versus time graph is always the place where the graph crosses the x-axis at $t = 0$. What is the slope? The *rise* is the change in position, (i.e., displacement) and the *run* is the change in time. This relationship can also be written

$$\frac{\Delta \mathbf{d}}{\Delta t}. \tag{2.4}$$

This relationship was how we defined average velocity. Therefore, the slope in a $\mathbf{d}$ versus t graph, is the average velocity.

TIPS FOR SUCCESS

Sometimes, as is the case where we graph both the trip to school and the return trip, the behavior of the graph looks different during different time intervals. If the graph looks like a series of straight lines, then you can calculate the average velocity for each time interval by looking at the slope. If you then want to calculate the average velocity for the entire trip, you can do a weighted average.

Let's look at another example. Figure 2.12 shows a graph of position versus time for a jet-powered car on a very flat dry lake bed in Nevada.

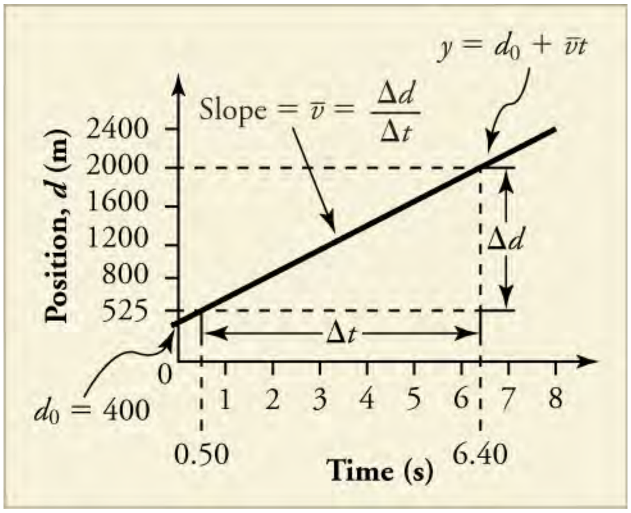


Figure 2.12 The diagram shows a graph of position versus time for a jet-powered car on the Bonneville Salt Flats.

Using the relationship between dependent and independent variables, we see that the slope in the graph in Figure 2.12 is average velocity, $\mathbf{v}_{avg}$ and the intercept is displacement at time zero—that is, $\mathbf{d}_0$. Substituting these symbols into $y = mx + b$ gives

$$\mathbf{d} = \mathbf{v}t + \mathbf{d}_0 \tag{2.5}$$

or

$$\mathbf{d} = \mathbf{d}_0 + \mathbf{v}t. \tag{2.6}$$

Thus a graph of position versus time gives a general relationship among displacement, velocity, and time, as well as giving detailed numerical information about a specific situation. From the figure we can see that the car has a position of 400 m at $t = 0$ s, 650 m at $t = 1.0$ s, and so on. And we can learn about the object's velocity, as well.

Snap Lab

Graphing Motion

In this activity, you will release a ball down a ramp and graph the ball's displacement vs. time.

- Choose an open location with lots of space to spread out so there is less chance for tripping or falling due to rolling balls.

- 1 ball
- 1 board
- 2 or 3 books
- 1 stopwatch
- 1 tape measure
- 6 pieces of masking tape
- 1 piece of graph paper
- 1 pencil

Procedure

1. Build a ramp by placing one end of the board on top of the stack of books. Adjust location, as necessary, until there is no obstacle along the straight line path from the bottom of the ramp until at least the next 3 m.
2. Mark distances of 0.5 m, 1.0 m, 1.5 m, 2.0 m, 2.5 m, and 3.0 m from the bottom of the ramp. Write the distances on the tape.

3. Have one person take the role of the experimenter. This person will release the ball from the top of the ramp. If the ball does not reach the 3.0 m mark, then increase the incline of the ramp by adding another book. Repeat this Step as necessary.
4. Have the experimenter release the ball. Have a second person, the timer, begin timing the trial once the ball reaches the bottom of the ramp and stop the timing once the ball reaches 0.5 m. Have a third person, the recorder, record the time in a data table.
5. Repeat Step 4, stopping the times at the distances of 1.0 m, 1.5 m, 2.0 m, 2.5 m, and 3.0 m from the bottom of the ramp.
6. Use your measurements of time and the displacement to make a position vs. time graph of the ball's motion.
7. Repeat Steps 4 through 6, with different people taking on the roles of experimenter, timer, and recorder. Do you get the same measurement values regardless of who releases the ball, measures the time, or records the result? Discuss possible causes of discrepancies, if any.

GRASP CHECK

True or False: The average speed of the ball will be less than the average velocity of the ball.

a. True
b. False

Solving Problems Using Position vs. Time Graphs

So how do we use graphs to solve for things we want to know like velocity?

WORKED EXAMPLE

Using Position–Time Graph to Calculate Average Velocity: Jet Car

Find the average velocity of the car whose position is graphed in Figure 1.13.

Strategy

The slope of a graph of d vs. t is average velocity, since slope equals rise over run.

$$\text{slope} = \frac{\Delta \mathbf{d}}{\Delta t} = \mathbf{v} \tag{2.7}$$

Since the slope is constant here, any two points on the graph can be used to find the slope.

Solution

1. Choose two points on the line. In this case, we choose the points labeled on the graph: (6.4 s, 2000 m) and (0.50 s, 525 m). (Note, however, that you could choose any two points.)
2. Substitute the **d** and t values of the chosen points into the equation. Remember in calculating change (Δ) we always use final value minus initial value.

$$\begin{aligned} \mathbf{v} &= \frac{\Delta \mathbf{d}}{\Delta t} \\ &= \frac{2000\ \text{m} - 525\ \text{m}}{6.4\ \text{s} - 0.50\ \text{s}}, \\ &= 250\ \text{m/s} \end{aligned} \tag{2.8}$$

Discussion

This is an impressively high land speed (900 km/h, or about 560 mi/h): much greater than the typical highway speed limit of 27 m/s or 96 km/h, but considerably shy of the record of 343 m/s or 1,234 km/h, set in 1997.

But what if the graph of the position is more complicated than a straight line? What if the object speeds up or turns around and goes backward? Can we figure out anything about its velocity from a graph of that kind of motion? Let's take another look at the jet-powered car. The graph in Figure 2.13 shows its motion as it is getting up to speed after starting at rest. Time starts at zero for this motion (as if measured with a stopwatch), and the displacement and velocity are initially 200 m and 15 m/s, respectively.

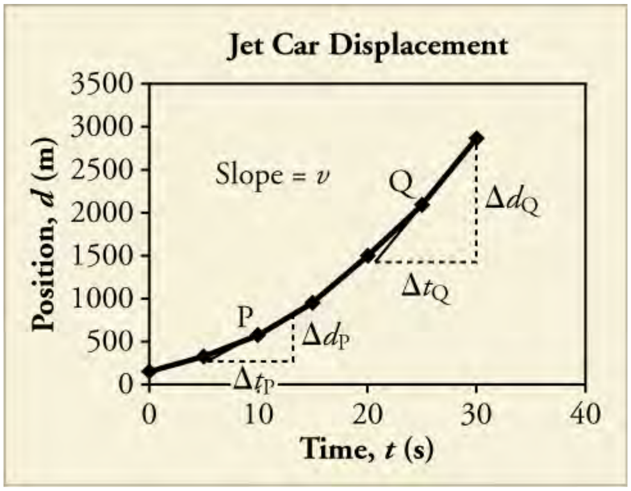


Figure 2.13 The diagram shows a graph of the position of a jet-powered car during the time span when it is speeding up. The slope of a distance versus time graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

Figure 2.14 A U.S. Air Force jet car speeds down a track. (Matt Trostle, Flickr)

The graph of position versus time in Figure 2.13 is a curve rather than a straight line. The slope of the curve becomes steeper as time progresses, showing that the velocity is increasing over time. The slope at any point on a position-versus-time graph is the instantaneous velocity at that point. It is found by drawing a straight line **tangent** to the curve at the point of interest and taking the slope of this straight line. Tangent lines are shown for two points in Figure 2.13. The average velocity is the net displacement divided by the time traveled.

WORKED EXAMPLE

Using Position–Time Graph to Calculate Average Velocity: Jet Car, Take Two

Calculate the instantaneous velocity of the jet car at a time of 25 s by finding the slope of the tangent line at point Q in Figure 2.13.

Strategy

The slope of a curve at a point is equal to the slope of a straight line tangent to the curve at that point.

Solution

1. Find the tangent line to the curve at $t = 25\ \text{s}$.
2. Determine the endpoints of the tangent. These correspond to a position of 1,300 m at time 19 s and a position of 3120 m at time 32 s.

3. Plug these endpoints into the equation to solve for the slope, **v**.

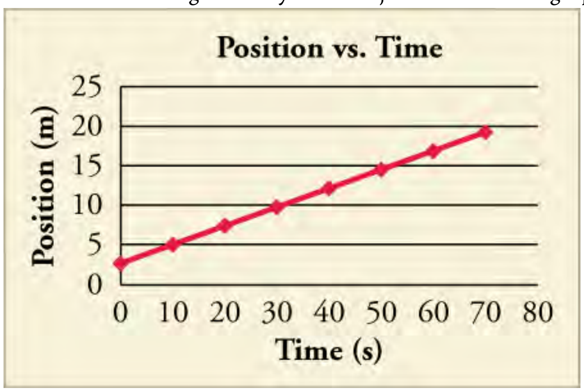
$$\begin{aligned} \text{slope} &= v_Q = \frac{\Delta d_Q}{\Delta t_Q} \\ &= \frac{(3120-1300)\ \text{m}}{(32-19)\ \text{s}} \\ &= \frac{1820\ \ \text{m}}{13\ \ \text{s}} \\ &= 140\ \ \text{m/s} \end{aligned} \tag{2.9}$$

Discussion

The entire graph of **v** versus *t* can be obtained in this fashion.

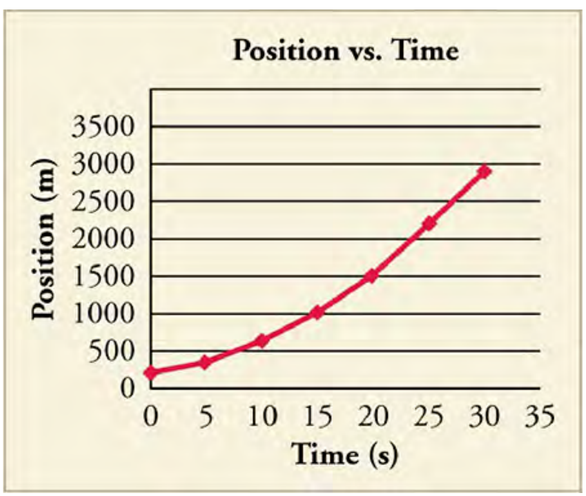
Practice Problems

15. Calculate the average velocity of the object shown in the graph below over the whole time interval.



a. 0.25 m/s
b. 0.31 m/s
c. 3.2 m/s
d. 4.00 m/s

16. True or False: By taking the slope of the curve in the graph you can verify that the velocity of the jet car is 115 m/s at $t = 20$ s.



a. True
b. False

Check Your Understanding

17. Which of the following information about motion can be determined by looking at a position vs. time graph that is a straight line?

a. frame of reference
b. average acceleration
c. velocity
d. direction of force applied

18. True or False: The position vs time graph of an object that is speeding up is a straight line.
a. True
b. False

2.4 Velocity vs. Time Graphs

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the meaning of slope and area in velocity vs. time graphs
- Solve problems using velocity vs. time graphs

Section Key Terms

acceleration

Graphing Velocity as a Function of Time

Earlier, we examined graphs of position versus time. Now, we are going to build on that information as we look at graphs of velocity vs. time. Velocity is the rate of change of displacement. **Acceleration** is the rate of change of velocity; we will discuss acceleration more in another chapter. These concepts are all very interrelated.

Virtual Physics

Maze Game

In this simulation you will use a vector diagram to manipulate a ball into a certain location without hitting a wall. You can manipulate the ball directly with position or by changing its velocity. Explore how these factors change the motion. If you would like, you can put it on the *a* setting, as well. This is acceleration, which measures the rate of change of velocity. We will explore acceleration in more detail later, but it might be interesting to take a look at it here.

Click to view content (https://archive.cnx.org/specials/30e37034-2fbd-11e5-83a2-03be60006ece/maze-game/)

GRASP CHECK

Click to view content (https://archive.cnx.org/specials/30e37034-2fbd-11e5-83a2-03be60006ece/maze-game/#sim-maze-game)

a. The ball can be easily manipulated with displacement because the arena is a position space.
b. The ball can be easily manipulated with velocity because the arena is a position space.
c. The ball can be easily manipulated with displacement because the arena is a velocity space.
d. The ball can be easily manipulated with velocity because the arena is a velocity space.

What can we learn about motion by looking at velocity vs. time graphs? Let’s return to our drive to school, and look at a graph of position versus time as shown in Figure 2.15.

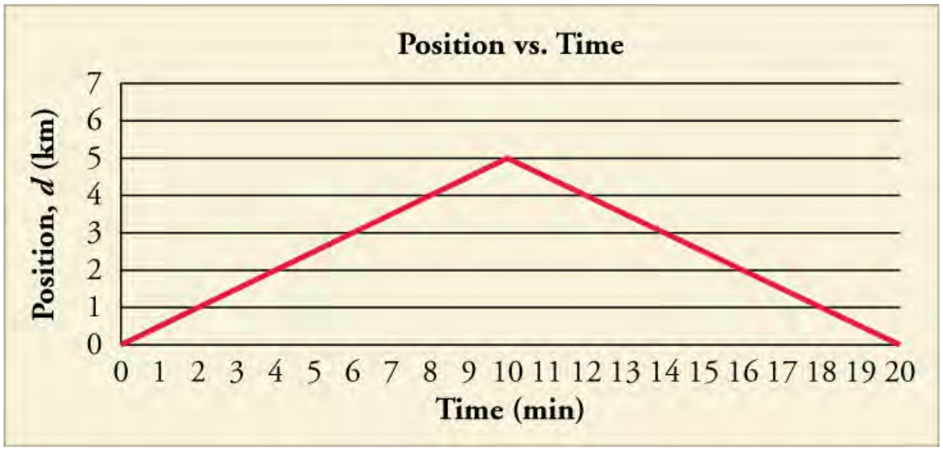


Figure 2.15 A graph of position versus time for the drive to and from school is shown.

We assumed for our original calculation that your parent drove with a constant velocity to and from school. We now know that the car could not have gone from rest to a constant velocity without speeding up. So the actual graph would be curved on either end, but let's make the same approximation as we did then, anyway.

TIPS FOR SUCCESS

It is common in physics, especially at the early learning stages, for certain things to be *neglected*, as we see here. This is because it makes the concept clearer or the calculation easier. Practicing physicists use these kinds of short-cuts, as well. It works out because usually the thing being *neglected* is small enough that it does not significantly affect the answer. In the earlier example, the amount of time it takes the car to speed up and reach its cruising velocity is very small compared to the total time traveled.

Looking at this graph, and given what we learned, we can see that there are two distinct periods to the car's motion—the way to school and the way back. The average velocity for the drive to school is 0.5 km/minute. We can see that the average velocity for the drive back is –0.5 km/minute. If we plot the data showing velocity versus time, we get another graph (Figure 2.16):

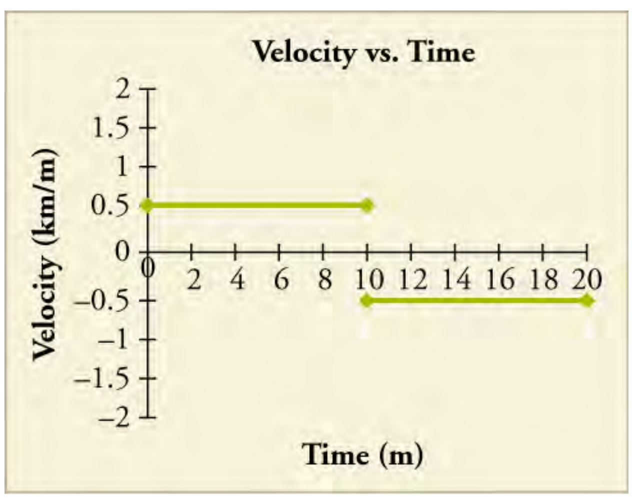


Figure 2.16 Graph of velocity versus time for the drive to and from school.

We can learn a few things. First, we can derive a **v** versus *t* graph from a **d** versus *t* graph. Second, if we have a straight-line position–time graph that is positively or negatively sloped, it will yield a horizontal velocity graph. There are a few other interesting things to note. Just as we could use a position vs. time graph to determine velocity, we can use a velocity vs. time graph to determine position. We know that $\mathbf{v} = \mathbf{d}/t$. If we use a little algebra to re-arrange the equation, we see that $\mathbf{d} = \mathbf{v} \times t$. In Figure 2.16, we have velocity on the *y*-axis and time along the *x*-axis. Let's take just the first half of the motion. We get 0.5 km/minute × 10 minutes. The units for *minutes* cancel each other, and we get 5 km, which is the displacement for the trip to school. If we calculate the same for the return trip, we get –5 km. If we add them together, we see that the net displacement for the whole trip is 0 km, which it should be because we started and ended at the same place.

TIPS FOR SUCCESS

You can treat units just like you treat numbers, so a km/km=1 (or, we say, it cancels out). This is good because it can tell us whether or not we have calculated everything with the correct units. For instance, if we end up with m × s for velocity instead of m/s, we know that something has gone wrong, and we need to check our math. This process is called dimensional analysis, and it is one of the best ways to check if your math makes sense in physics.

The area under a velocity curve represents the displacement. The velocity curve also tells us whether the car is speeding up. In our earlier example, we stated that the velocity was constant. So, the car is not speeding up. Graphically, you can see that the slope of these two lines is 0. This slope tells us that the car is not speeding up, or accelerating. We will do more with this information in a later chapter. For now, just remember that the area under the graph and the slope are the two important parts of the graph. Just like we could define a linear equation for the motion in a position vs. time graph, we can also define one for a velocity vs. time graph. As we said, the slope equals the acceleration, **a**. And in this graph, the *y*-intercept is $\mathbf{v}_0$. Thus, $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$.

But what if the velocity is not constant? Let's look back at our jet-car example. At the beginning of the motion, as the car is speeding up, we saw that its position is a curve, as shown in Figure 2.17.

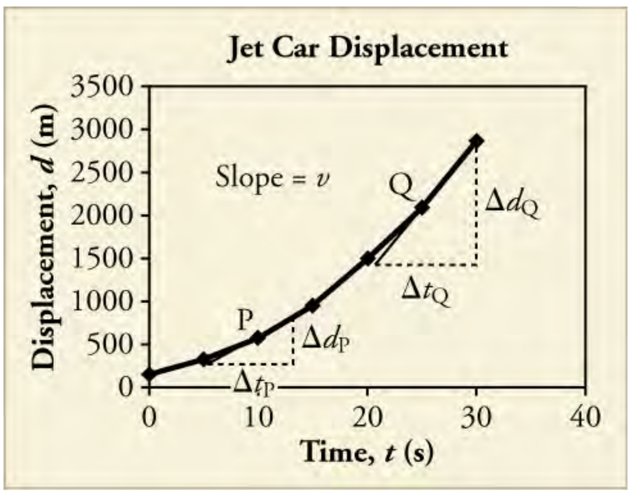


Figure 2.17 A graph is shown of the position of a jet-powered car during the time span when it is speeding up. The slope of a d vs. t graph is velocity. This is shown at two points. Instantaneous velocity at any point is the slope of the tangent at that point.

You do not have to do this, but you could, theoretically, take the instantaneous velocity at each point on this graph. If you did, you would get Figure 2.18, which is just a straight line with a positive slope.

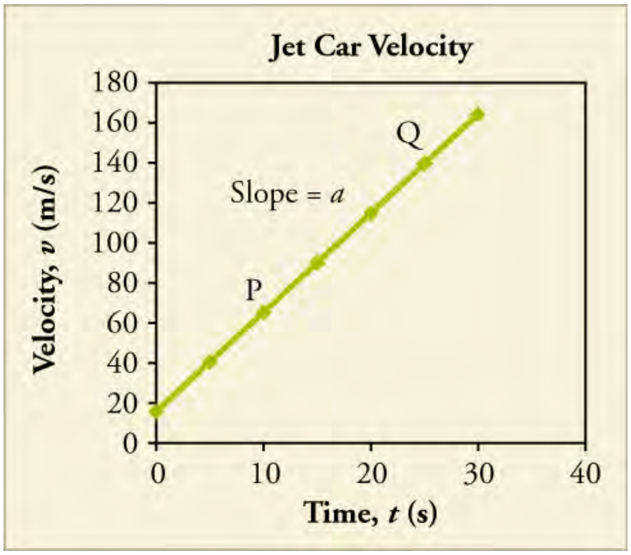


Figure 2.18 The graph shows the velocity of a jet-powered car during the time span when it is speeding up.

Again, if we take the slope of the velocity vs. time graph, we get the acceleration, the rate of change of the velocity. And, if we take the area under the slope, we get back to the displacement.

Solving Problems using Velocity–Time Graphs

Most velocity vs. time graphs will be straight lines. When this is the case, our calculations are fairly simple.

WORKED EXAMPLE

Using Velocity Graph to Calculate Some Stuff: Jet Car

Use this figure to (a) find the displacement of the jet car over the time shown (b) calculate the rate of change (acceleration) of the velocity. (c) give the instantaneous velocity at 5 s, and (d) calculate the average velocity over the interval shown.

Strategy

a. The displacement is given by finding the area under the line in the velocity vs. time graph.
b. The acceleration is given by finding the slope of the velocity graph.
c. The instantaneous velocity can just be read off of the graph.
d. To find the average velocity, recall that $\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{\mathbf{d}_f - \mathbf{d}_0}{t_f - t_0}$

Solution

a. 1. Analyze the shape of the area to be calculated. In this case, the area is made up of a rectangle between 0 and 20 m/s stretching to 30 s. The area of a rectangle is length × width. Therefore, the area of this piece is 600 m.
   2. Above that is a triangle whose base is 30 s and height is 140 m/s. The area of a triangle is 0.5 × length × width. The area of this piece, therefore, is 2,100 m.
   3. Add them together to get a net displacement of 2,700 m.

b. 1. Take two points on the velocity line. Say, $t = 5$ s and $t = 25$ s. At $t = 5$ s, the value of $\mathbf{v} = 40$ m/s. At $t = 25$ s, $\mathbf{v} = 140$ m/s.
   2. Find the slope.
$$\begin{aligned}\mathbf{a} &= \frac{\Delta \mathbf{v}}{\Delta t}\\ &= \frac{100 \text{ m/s}}{20 \text{ s}}\\ &= 5 \text{ m/s}^2\end{aligned}$$

c. The instantaneous velocity at $t = 5$ s, as we found in part (b) is just 40 m/s.

d. 1. Find the net displacement, which we found in part (a) was 2,700 m.
   2. Find the total time which for this case is 30 s.
   3. Divide 2,700 m/30 s = 90 m/s.

Discussion

The average velocity we calculated here makes sense if we look at the graph. 100m/s falls about halfway across the graph and since it is a straight line, we would expect about half the velocity to be above and half below.

TIPS FOR SUCCESS

You can have negative position, velocity, and acceleration on a graph that describes the way the object is moving. You should never see a graph with negative time on an axis. Why?

Most of the velocity vs. time graphs we will look at will be simple to interpret. Occasionally, we will look at curved graphs of velocity vs. time. More often, these curved graphs occur when something is speeding up, often from rest. Let's look back at a more realistic velocity vs. time graph of the jet car's motion that takes this *speeding up* stage into account.

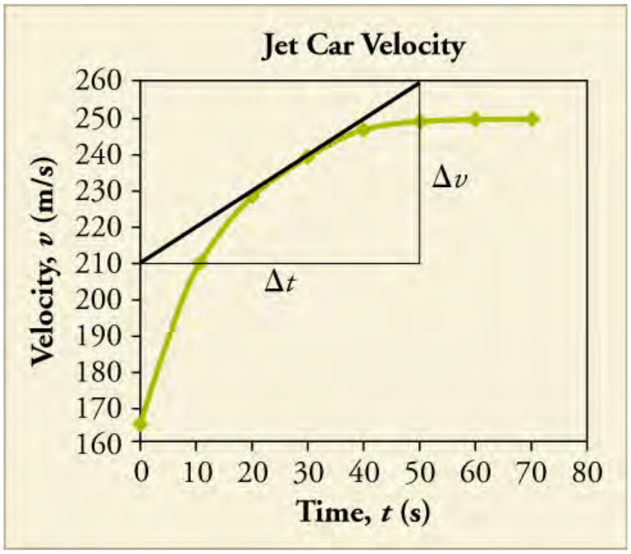


Figure 2.19 The graph shows a more accurate graph of the velocity of a jet-powered car during the time span when it is speeding up.

WORKED EXAMPLE

Using Curvy Velocity Graph to Calculate Some Stuff: jet car, Take Two

Use Figure 2.19 to (a) find the approximate displacement of the jet car over the time shown, (b) calculate the instantaneous acceleration at $t = 30$ s, (c) find the instantaneous velocity at 30 s, and (d) calculate the approximate average velocity over the interval shown.

Strategy

a. Because this graph is an undefined curve, we have to estimate shapes over smaller intervals in order to find the areas.
b. Like when we were working with a curved displacement graph, we will need to take a tangent line at the instant we are interested and use that to calculate the instantaneous acceleration.
c. The instantaneous velocity can still be read off of the graph.
d. We will find the average velocity the same way we did in the previous example.

Solution

a.
1. This problem is more complicated than the last example. To get a good estimate, we should probably break the curve into four sections. $0 \rightarrow 10$ s, $10 \rightarrow 20$ s, $20 \rightarrow 40$ s, and $40 \rightarrow 70$ s.
2. Calculate the bottom rectangle (common to all pieces). 165 m/s $\times$ 70 s = 11,550 m.
3. Estimate a triangle at the top, and calculate the area for each section. Section 1 = 225 m; section 2 = 100 m + 450 m = 550 m; section 3 = 150 m + 1,300 m = 1,450 m; section 4 = 2,550 m.
4. Add them together to get a net displacement of 16,325 m.

b. Using the tangent line given, we find that the slope is 1 m/s^2.

c. The instantaneous velocity at $t = 30$ s, is 240 m/s.

d. 1. Find the net displacement, which we found in part (a), was 16,325 m.
   2. Find the total time, which for this case is 70 s.
   3. Divide $\frac{16{,}325 \text{ m}}{70 \text{ s}} \sim 233 \text{ m/s}$

Discussion

This is a much more complicated process than the first problem. If we were to use these estimates to come up with the average velocity over just the first 30 s we would get about 191 m/s. By approximating that curve with a line, we get an average velocity of 202.5 m/s. Depending on our purposes and how precise an answer we need, sometimes calling a curve a straight line is a worthwhile approximation.

Practice Problems

19.

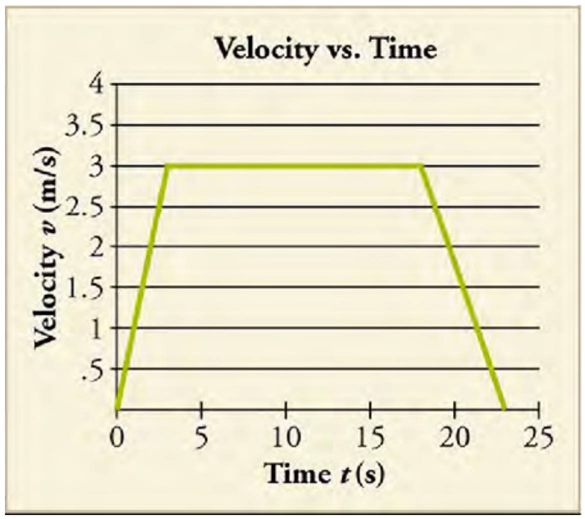


Figure 2.20

Consider the velocity vs. time graph shown below of a person in an elevator. Suppose the elevator is initially at rest. It then speeds up for 3 seconds, maintains that velocity for 15 seconds, then slows down for 5 seconds until it stops. Find the instantaneous velocity at $t = 10$ s and $t = 23$ s.

a. Instantaneous velocity at $t = 10$ s and $t = 23$ s are 0 m/s and 0 m/s.
b. Instantaneous velocity at $t = 10$ s and $t = 23$ s are 0 m/s and 3 m/s.
c. Instantaneous velocity at $t = 10$ s and $t = 23$ s are 3 m/s and 0 m/s.
d. Instantaneous velocity at $t = 10$ s and $t = 23$ s are 3 m/s and 1.5 m/s.

20.

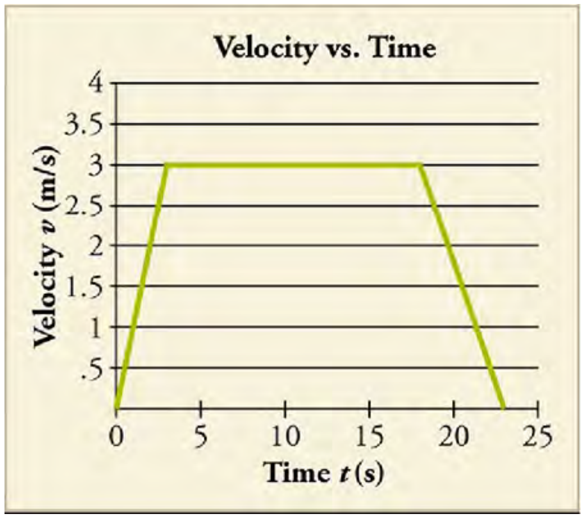


Figure 2.21

Calculate the net displacement and the average velocity of the elevator over the time interval shown.

a. Net displacement is 45 m and average velocity is 2.10 m/s.
b. Net displacement is 45 m and average velocity is 2.28 m/s.
c. Net displacement is 57 m and average velocity is 2.66 m/s.
d. Net displacement is 57 m and average velocity is 2.48 m/s.

Snap Lab

Graphing Motion, Take Two

In this activity, you will graph a moving ball's velocity vs. time.

- your graph from the earlier Graphing Motion Snap Lab!
- 1 piece of graph paper
- 1 pencil

Procedure

1. Take your graph from the earlier Graphing Motion Snap Lab! and use it to create a graph of velocity vs. time.
2. Use your graph to calculate the displacement.

GRASP CHECK

Describe the graph and explain what it means in terms of velocity and acceleration.

a. The graph shows a horizontal line indicating that the ball moved with a constant velocity, that is, it was not accelerating.
b. The graph shows a horizontal line indicating that the ball moved with a constant velocity, that is, it was accelerating.
c. The graph shows a horizontal line indicating that the ball moved with a variable velocity, that is, it was not accelerating.
d. The graph shows a horizontal line indicating that the ball moved with a variable velocity, that is, it was accelerating.

Check Your Understanding

21. What information could you obtain by looking at a velocity vs. time graph?

a. acceleration
b. direction of motion
c. reference frame of the motion

d. shortest path

22. How would you use a position vs. time graph to construct a velocity vs. time graph and vice versa?

a. Slope of position vs. time curve is used to construct velocity vs. time curve, and slope of velocity vs. time curve is used to construct position vs. time curve.
b. Slope of position vs. time curve is used to construct velocity vs. time curve, and area of velocity vs. time curve is used to construct position vs. time curve.
c. Area of position vs. time curve is used to construct velocity vs. time curve, and slope of velocity vs. time curve is used to construct position vs. time curve.
d. Area of position/time curve is used to construct velocity vs. time curve, and area of velocity vs. time curve is used to construct position vs. time curve.

KEY TERMS

acceleration the rate at which velocity changes
average speed distance traveled divided by time during which motion occurs
average velocity displacement divided by time over which displacement occurs
dependent variable the variable that changes as the independent variable changes
displacement the change in position of an object against a fixed axis
distance the length of the path actually traveled between an initial and a final position
independent variable the variable, usually along the horizontal axis of a graph, that does not change based on human or experimental action; in physics this is usually time
instantaneous speed speed at a specific instant in time
instantaneous velocity velocity at a specific instant in time
kinematics the study of motion without considering its causes
magnitude size or amount
position the location of an object at any particular time
reference frame a coordinate system from which the positions of objects are described
scalar a quantity that has magnitude but no direction
speed rate at which an object changes its location
tangent a line that touches another at exactly one point
vector a quantity that has both magnitude and direction
velocity the speed and direction of an object

SECTION SUMMARY

2.1 Relative Motion, Distance, and Displacement

- A description of motion depends on the reference frame from which it is described.
- The distance an object moves is the length of the path along which it moves.
- Displacement is the difference in the initial and final positions of an object.

2.2 Speed and Velocity

- Average speed is a scalar quantity that describes distance traveled divided by the time during which the motion occurs.
- Velocity is a vector quantity that describes the speed and direction of an object.
- Average velocity is displacement over the time period during which the displacement occurs. If the velocity is constant, then average velocity and instantaneous velocity are the same.

2.3 Position vs. Time Graphs

- Graphs can be used to analyze motion.
- The slope of a position vs. time graph is the velocity.
- For a straight line graph of position, the slope is the average velocity.
- To obtain the instantaneous velocity at a given moment for a curved graph, find the tangent line at that point and take its slope.

2.4 Velocity vs. Time Graphs

- The slope of a velocity vs. time graph is the acceleration.
- The area under a velocity vs. time curve is the displacement.
- Average velocity can be found in a velocity vs. time graph by taking the weighted average of all the velocities.

KEY EQUATIONS

2.1 Relative Motion, Distance, and Displacement

Displacement	$\Delta \mathbf{d} = \mathbf{d_f} - \mathbf{d_0}$

2.2 Speed and Velocity

Average speed	$v_{\text{avg}} = \frac{\textbf{distance}}{\textbf{time}}$
Average velocity	$\mathbf{v}_{\text{avg}} = \frac{\Delta \mathbf{d}}{\Delta t} = \frac{\mathbf{d}_{\mathrm{f}} - \mathbf{d}_0}{t_{\mathrm{f}} - t_0}$

2.3 Position vs. Time Graphs

Displacement	$\mathbf{d} = \mathbf{d}_0 + \mathbf{v}t$.

2.4 Velocity vs. Time Graphs

Velocity	$\mathbf{v} = \mathbf{v}_0 + at$
Acceleration	$\mathbf{a} = \frac{\Delta \mathbf{v}}{\Delta t}$

CHAPTER REVIEW

Concept Items

2.1 Relative Motion, Distance, and Displacement

1. Can one-dimensional motion have zero distance but a nonzero displacement? What about zero displacement but a nonzero distance?
   a. One-dimensional motion can have zero distance with a nonzero displacement. Displacement has both magnitude and direction, and it can also have zero displacement with nonzero distance because distance has only magnitude.
   b. One-dimensional motion can have zero distance with a nonzero displacement. Displacement has both magnitude and direction, but it cannot have zero displacement with nonzero distance because distance has only magnitude.
   c. One-dimensional motion cannot have zero distance with a nonzero displacement. Displacement has both magnitude and direction, but it can have zero displacement with nonzero distance because distance has only magnitude and any motion will be the distance it moves.
   d. One-dimensional motion cannot have zero distance with a nonzero displacement. Displacement has both magnitude and direction, and it cannot have zero displacement with nonzero distance because distance has only magnitude.

2. In which example would you be correct in describing an object in motion while your friend would also be correct in describing that same object as being at rest?
   a. You are driving a car toward the east and your friend drives past you in the opposite direction with the same speed. In your frame of reference, you will be in motion. In your friend's frame of reference, you will be at rest.
   b. You are driving a car toward the east and your friend is standing at the bus stop. In your frame of reference, you will be in motion. In your friend's frame of reference, you will be at rest.
   c. You are driving a car toward the east and your friend is standing at the bus stop. In your frame of reference, your friend will be moving toward the west. In your friend's frame of reference, he will be at rest.
   d. You are driving a car toward the east and your friend is standing at the bus stop. In your frame of reference, your friend will be moving toward the east. In your friend's frame of reference, he will be at rest.

3. What does your car's odometer record?
   a. displacement
   b. distance
   c. both distance and displacement
   d. the sum of distance and displacement

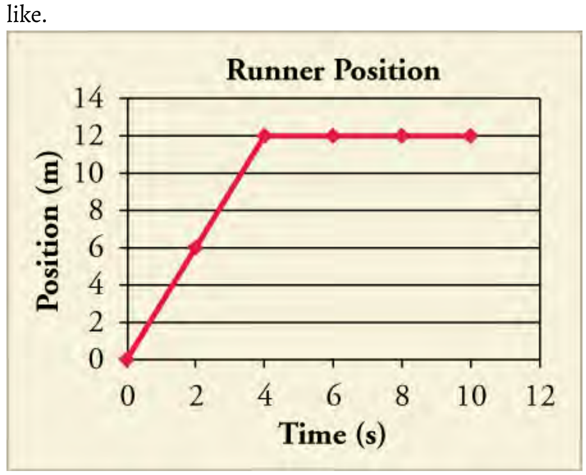
2.2 Speed and Velocity

4. In the definition of velocity, what physical quantity is changing over time?
   a. speed
   b. distance
   c. magnitude of displacement
   d. position vector

5. Which of the following best describes the relationship between instantaneous velocity and instantaneous speed?
   a. Both instantaneous speed and instantaneous velocity are the same, even when there is a change in direction.
   b. Instantaneous speed and instantaneous velocity cannot be the same even if there is no change in direction of motion.
   c. Magnitude of instantaneous velocity is equal to instantaneous speed.
   d. Magnitude of instantaneous velocity is always greater than instantaneous speed.

2.3 Position vs. Time Graphs

6. Use the graph to describe what the runner's motion looks like.



How are average velocity for only the first four seconds and instantaneous velocity related? What is the runner's net displacement over the time shown?
   a. The net displacement is 12 m and the average velocity is equal to the instantaneous velocity.
   b. The net displacement is 12 m and the average velocity

is two times the instantaneous velocity.

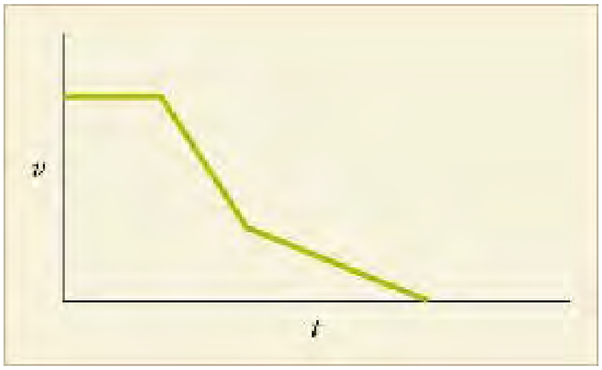
c. The net displacement is 10 + 12 = 22 m and the average velocity is equal to the instantaneous velocity.
d. The net displacement is 10 + 12 = 22 m and the average velocity is two times the instantaneous velocity.

7. A position vs. time graph of a frog swimming across a pond has two distinct straight-line sections. The slope of the first section is $1\ \text{m/s}$. The slope of the second section is $0\ \text{m/s}$. If each section lasts 1 second, then what is the frog's total average velocity?

a. $0\ \text{m/s}$
b. $2\ \text{m/s}$
c. $0.5\ \text{m/s}$
d. $1\ \text{m/s}$

2.4 Velocity vs. Time Graphs

8. A graph of velocity vs. time of a ship coming into a harbor is shown.



Describe the acceleration of the ship based on the graph.

a. The ship is moving in the forward direction at a steady rate. Then it accelerates in the forward direction and then decelerates.
b. The ship is moving in the forward direction at a steady rate. Then it turns around and starts decelerating, while traveling in the reverse direction. It then accelerates, but slowly.
c. The ship is moving in the forward direction at a steady rate. Then it decelerates in the forward direction, and then continues to slow down in the forward direction, but with more deceleration.
d. The ship is moving in the forward direction at a steady rate. Then it decelerates in the forward direction, and then continues to slow down in the forward direction, but with less deceleration.

Critical Thinking Items

2.1 Relative Motion, Distance, and Displacement

9. Boat A and Boat B are traveling at a constant speed in opposite directions when they pass each other. If a person in each boat describes motion based on the boat's own reference frame, will the description by a person in Boat A of Boat B's motion be the same as the description by a person in Boat B of Boat A's motion?

a. Yes, both persons will describe the same motion because motion is independent of the frame of reference.
b. Yes, both persons will describe the same motion because they will perceive the other as moving in the backward direction.
c. No, the motion described by each of them will be different because motion is a relative term.
d. No, the motion described by each of them will be different because the motion perceived by each will be opposite to each other.

10. Passenger A sits inside a moving train and throws a ball vertically upward. How would the motion of the ball be described by a fellow train passenger B and an observer C who is standing on the platform outside the train?

a. Passenger B sees that the ball has vertical, but no horizontal, motion. Observer C sees the ball has vertical as well as horizontal motion.
b. Passenger B sees the ball has vertical as well as horizontal motion. Observer C sees the ball has the vertical, but no horizontal, motion.
c. Passenger B sees the ball has horizontal but no vertical motion. Observer C sees the ball has vertical as well as horizontal motion.
d. Passenger B sees the ball has vertical as well as horizontal motion. Observer C sees the ball has horizontalbut no vertical motion.

2.2 Speed and Velocity

11. Is it possible to determine a car's instantaneous velocity from just the speedometer reading?

a. No, it reflects speed but not the direction.
b. No, it reflects the average speed of the car.
c. Yes, it sometimes reflects instantaneous velocity of the car.
d. Yes, it always reflects the instantaneous velocity of the car.

12. Terri, Aaron, and Jamal all walked along straight paths.

Terri walked 3.95 km north in 48 min. Aaron walked 2.65 km west in 31 min. Jamal walked 6.50 km south in 81 min. Which of the following correctly ranks the three boys in order from lowest to highest average speed?

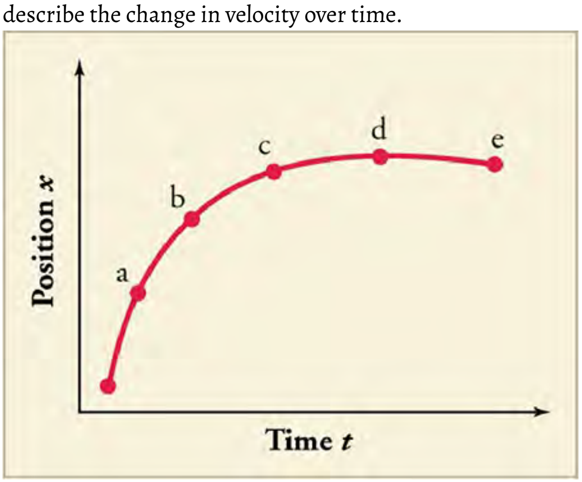
a. Jamal, Terri, Aaron
b. Jamal, Aaron, Terri
c. Terri, Jamal, Aaron
d. Aaron, Terri, Jamal

13. Rhianna and Logan start at the same point and walk due north. Rhianna walks with an average velocity $v_{\text{avg,R}}$. Logan walks three times the distance in twice the time as Rhianna. Which of the following expresses Logan's average velocity in terms of $v_{\text{avg,R}}$?

a. Logan's average velocity = $1.5v_{avg,R}$.
b. Logan's average velocity = $\frac{2}{3}v_{avg,R}$.
c. Logan's average velocity = $3v_{avg,R}$.
d. Logan's average velocity = $\frac{1}{2}v_{avg,R}$.

2.3 Position vs. Time Graphs

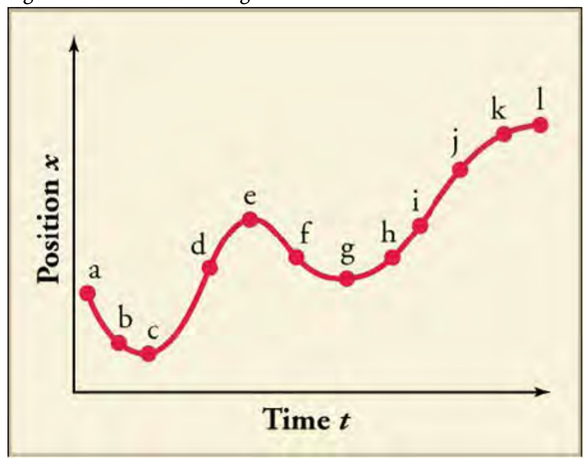
14. Explain how you can use the graph of position vs. time to describe the change in velocity over time.



Identify the time (t_a, t_b, t_c, t_d, or t_e) at which at which the instantaneous velocity is greatest, the time at which it is zero, and the time at which it is negative.

2.4 Velocity vs. Time Graphs

15. Identify the time, or times, at which the instantaneous velocity is greatest, and the time, or times, at which it is negative. A sketch of velocity vs. time derived from the figure will aid in arriving at the correct answers.



a. The instantaneous velocity is greatest at *f*, and it is negative at *d*, *h*, *I*, *j*, and *k*.
b. The instantaneous velocity is greatest at *e*, and it is negative at *a*, *b*, and *f*.
c. The instantaneous velocity is greatest at *f*, and it is negative at *d*, *h*, *I*, *j*, and *k*
d. The instantaneous velocity is greatest at *d*, and it is negative at *a*, *b*, and *f*.

Problems

2.1 Relative Motion, Distance, and Displacement

16. In a coordinate system in which the direction to the right is positive, what are the distance and displacement of a person who walks 35 meters to the left, 18 meters to the right, and then 26 meters to the left?

a. Distance is 79 m and displacement is −43 m.
b. Distance is −79 m and displacement is 43 m.
c. Distance is 43 m and displacement is −79 m.
d. Distance is −43 m and displacement is 79 m.

17. Billy drops a ball from a height of 1 m. The ball bounces back to a height of 0.8 m, then bounces again to a height of 0.5 m, and bounces once more to a height of 0.2 m. Up is the positive direction. What are the total displacement of the ball and the total distance traveled by the ball?

a. The displacement is equal to -4 m and the distance is equal to 4 m.
b. The displacement is equal to 4 m and the distance is equal to 1 m.
c. The displacement is equal to 4 m and the distance is equal to 1 m.
d. The displacement is equal to -1 m and the distance is equal to 4 m.

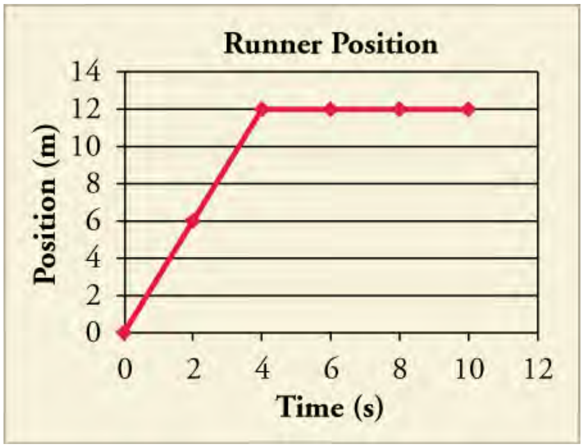
2.2 Speed and Velocity

18. You sit in a car that is moving at an average speed of 86.4 km/h. During the 3.3 s that you glance out the window, how far has the car traveled?

a. 7.27 m
b. 79 m
c. 285 km
d. 1026 m

2.3 Position vs. Time Graphs

19. Using the graph, what is the average velocity for the whole 10 seconds?



a. The total average velocity is 0 m/s.
b. The total average velocity is 1.2 m/s.
c. The total average velocity is 1.5 m/s.
d. The total average velocity is 3.0 m/s.

20. A train starts from rest and speeds up for 15 minutes until it reaches a constant velocity of 100 miles/hour. It stays at this speed for half an hour. Then it slows down for another 15 minutes until it is still. Which of the following correctly describes the position vs time graph of the train's journey?

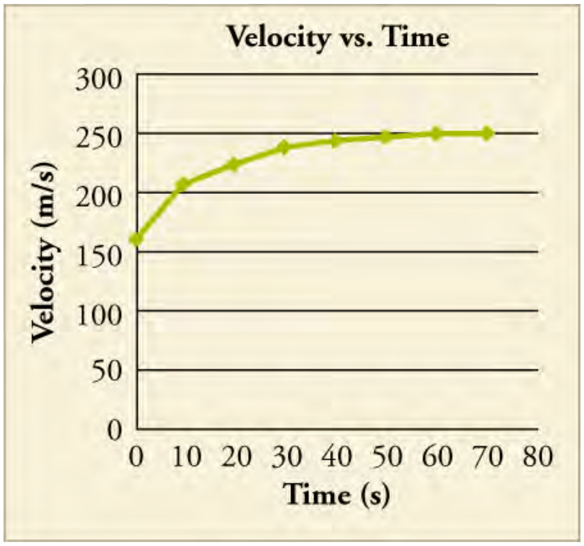
a. The first 15 minutes is a curve that is concave upward, the middle portion is a straight line with slope 100 miles/hour, and the last portion is a concave downward curve.
b. The first 15 minutes is a curve that is concave downward, the middle portion is a straight line with slope 100 miles/hour, and the last portion is a concave upward curve.
c. The first 15 minutes is a curve that is concave upward, the middle portion is a straight line with slope zero, and the last portion is a concave downward curve.
d. The first 15 minutes is a curve that is concave downward, the middle portion is a straight line with slope zero, and the last portion is a concave upward curve.

2.4 Velocity vs. Time Graphs

21. You are characterizing the motion of an object by measuring the location of the object at discrete moments in time. What is the minimum number of data points you would need to estimate the average acceleration of the object?

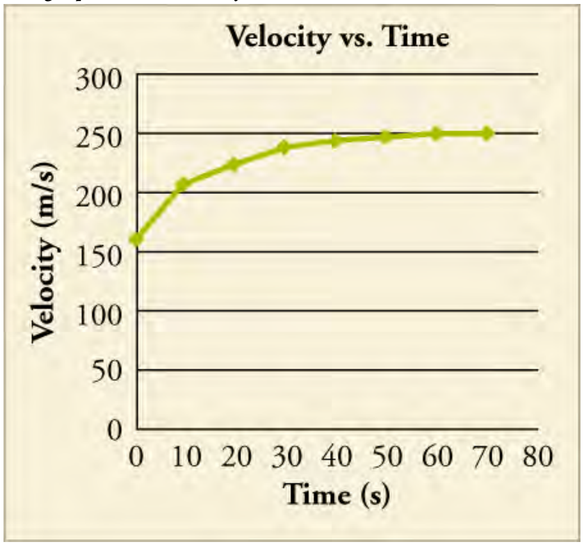
a. 1
b. 2
c. 3
d. 4

22. Which option best describes the average acceleration from 40 to 70 s?



a. It is negative and smaller in magnitude than the initial acceleration.
b. It is negative and larger in magnitude than the initial acceleration.
c. It is positive and smaller in magnitude than the initial acceleration.
d. It is positive and larger in magnitude than the initial acceleration.

23. The graph shows velocity vs. time.



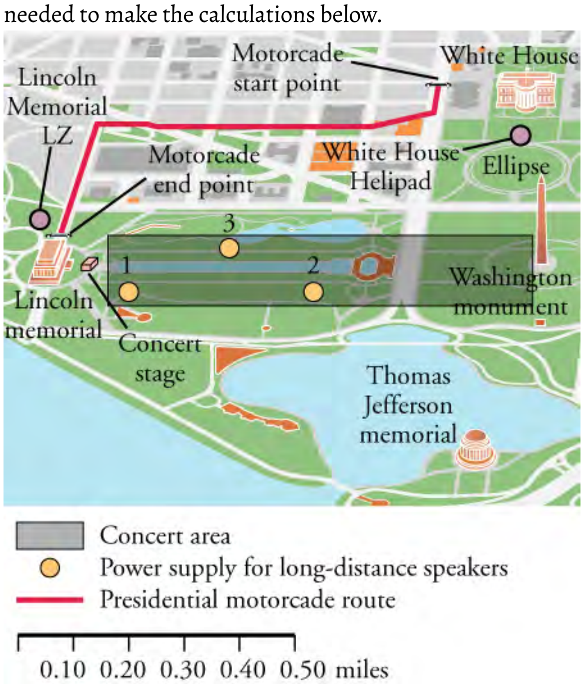
Calculate the net displacement using seven different divisions. Calculate it again using two divisions: 0 → 40 s and 40 → 70 s . Compare. Using both, calculate the average velocity.

a. Displacement and average velocity using seven divisions are 14,312.5 m and 204.5 m/s while with two divisions are 15,500 m and 221.4 m/s respectively.
b. Displacement and average velocity using seven divisions are 15,500 m and 221.4 m/s while with two divisions are 14,312.5 m and 204.5 m/s respectively.
c. Displacement and average velocity using seven divisions are 15,500 m and 204.5 m/s while with two divisions are 14,312.5 m and 221.4 m/s respectively.
d. Displacement and average velocity using seven divisions are 14,312.5 m and 221.4 m/s while with two divisions are 15,500 m and 204.5 m/s respectively.

Performance Task

2.4 Velocity vs. Time Graphs

24. The National Mall in Washington, DC, is a national park containing most of the highly treasured memorials and museums of the United States. However, the National Mall also hosts many events and concerts. The map in shows the area for a benefit concert during which the president will speak. The concert stage is near the Lincoln Memorial. The seats and standing room for the crowd will stretch from the stage east to near the Washington Monument, as shown on the map. You are planning the logistics for the concert. Use the map scale to measure any distances needed to make the calculations below.



The park has three new long-distance speakers. They would like to use these speakers to broadcast the concert audio to other parts of the National Mall. The speakers can project sound up to 0.35 miles away but they must be connected to one of the power supplies within the concert area. What is the minimum amount of wire needed for each speaker, in miles, in order to project the audio to the following areas? Assume that wire cannot be placed over buildings or any memorials.

Part A. The center of the Jefferson Memorial using power supply 1 (This will involve an elevated wire that can travel over water.)

Part B. The center of the Ellipse using power supply 3 (This wire cannot travel over water.)

Part C. The president's motorcade will be traveling to the concert from the White House. To avoid concert traffic, the motorcade travels from the White House west down E Street and then turns south on 23rd Street to reach the Lincoln memorial. What minimum speed, in miles per hour to the nearest tenth, would the motorcade have to travel to make the trip in 5 minutes?

Part D. The president could also simply fly from the White House to the Lincoln Memorial using the presidential helicopter, Marine 1. How long would it take Marine 1, traveling slowly at 30 mph, to travel from directly above the White House landing zone (LZ) to directly above the Lincoln Memorial LZ? Disregard liftoff and landing times and report the travel time in minutes to the nearest minute.

TEST PREP

Multiple Choice

2.1 Relative Motion, Distance, and Displacement

25. Why should you specify a reference frame when describing motion?

a. a description of motion depends on the reference frame
b. motion appears the same in all reference frames
c. reference frames affect the motion of an object
d. you can see motion better from certain reference frames

26. Which of the following is true for the displacement of an object?

a. It is always equal to the distance the object moved

between its initial and final positions.
b. It is both the straight line distance the object moved as well as the direction of its motion.
c. It is the direction the object moved between its initial and final positions.
d. It is the straight line distance the object moved between its initial and final positions.

27. If a biker rides west for 50 miles from his starting position, then turns and bikes back east 80 miles. What is his net displacement?
a. 130 miles
b. 30 miles east
c. 30 miles west
d. Cannot be determined from the information given

28. Suppose a train is moving along a track. Is there a single, correct reference frame from which to describe the train's motion?
a. Yes, there is a single, correct frame of reference because motion is a relative term.
b. Yes, there is a single, correct frame of reference which is in terms of Earth's position.
c. No, there is not a single, correct frame of reference because motion is a relative term.
d. No, there is not a single, correct frame of reference because motion is independent of frame of reference.

29. If a space shuttle orbits Earth once, what is the shuttle's distance traveled and displacement?
a. Distance and displacement both are zero.
b. Distance is circumference of the circular orbit while displacement is zero.
c. Distance is zero while the displacement is circumference of the circular orbit.
d. Distance and displacement both are equal to circumference of the circular orbit.

2.2 Speed and Velocity

30. Four bicyclists travel different distances and times along a straight path. Which cyclist traveled with the greatest average speed?
a. Cyclist 1 travels 95 m in 27 s.
b. Cyclist 2 travels 87 m in 22 s.
c. Cyclist 3 travels 106 m in 26 s.
d. Cyclist 4 travels 108 m in 24 s.

31. A car travels with an average velocity of 23 m/s for 82 s. Which of the following could NOT have been the car's displacement?
a. 1,700 m east
b. 1,900 m west
c. 1,600 m north
d. 1,500 m south

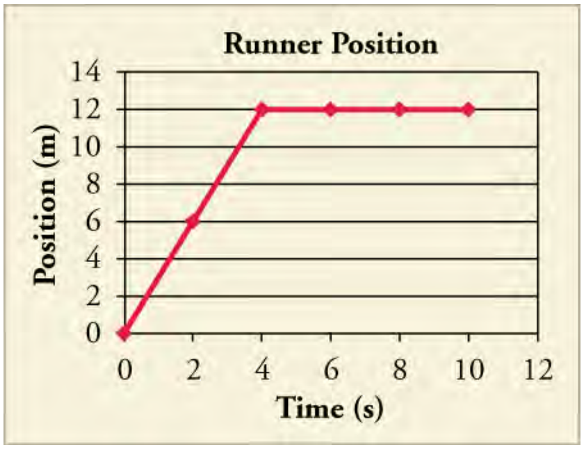
32. A bicyclist covers the first leg of a journey that is d_1 meters long in t_1 seconds, at a speed of v_1 m/s, and the second leg of d_2 meters in t_2 seconds, at a speed of v_2 m/s. If his average speed is equal to the average of v_1 and v_2, then which of the following is true?
a. $t_1 = t_2$
b. $t_1 \neq t_2$
c. $d_1 = d_2$
d. $d_1 \neq d_2$

33. A car is moving on a straight road at a constant speed in a single direction. Which of the following statements is true?
a. Average velocity is zero.
b. The magnitude of average velocity is equal to the average speed.
c. The magnitude of average velocity is greater than the average speed.
d. The magnitude of average velocity is less than the average speed.

2.3 Position vs. Time Graphs

34. What is the slope of a straight line graph of position vs. time?
a. Velocity
b. Displacement
c. Distance
d. Acceleration

35. Using the graph, what is the runner's velocity from 4 to 10 s?



a. −3 m/s
b. 0 m/s
c. 1.2 m/s
d. 3 m/s

2.4 Velocity vs. Time Graphs

36. What does the area under a velocity vs. time graph line represent?
  a. acceleration
  b. displacement
  c. distance
  d. instantaneous velocity

37. An object is moving along a straight path with constant acceleration. A velocity vs. time graph starts at 0 and ends at $10\ \mathrm{m/s}$, stretching over a time-span of $15\ \mathrm{s}$. What is the object's net displacement?
  a. $75\ \mathrm{m}$
  b. $130\ \mathrm{m}$
  c. $150\ \mathrm{m}$
  d. cannot be determined from the information given

Short Answer

2.1 Relative Motion, Distance, and Displacement

38. While standing on a sidewalk facing the road, you see a bicyclist passing by toward your right. In the reference frame of the bicyclist, in which direction are you moving?
  a. in the same direction of motion as the bicyclist
  b. in the direction opposite the motion of the bicyclist
  c. stationary with respect to the bicyclist
  d. in the direction of velocity of the bicyclist

39. Maud sends her bowling ball straight down the center of the lane, getting a strike. The ball is brought back to the holder mechanically. What are the ball's net displacement and distance traveled?
  a. Displacement of the ball is twice the length of the lane, while the distance is zero.
  b. Displacement of the ball is zero, while the distance is twice the length of the lane.
  c. Both the displacement and distance for the ball are equal to zero.
  d. Both the displacement and distance for the ball are twice the length of the lane.

40. A fly buzzes four and a half times around Kit Yan's head. The fly ends up on the opposite side from where it started. If the diameter of his head is $14\ \mathrm{cm}$, what is the total distance the fly travels and its total displacement?
  a. The distance is $63\pi\ \mathrm{cm}$ with a displacement of zero.
  b. The distance is $7\ \mathrm{cm}$ with a displacement of zero.
  c. The distance is $63\pi\ \mathrm{cm}$ with a displacement of $14\ \mathrm{cm}$.
  d. The distance is $7\ \mathrm{cm}$ with a displacement of $63\pi\ \mathrm{cm}$.

2.2 Speed and Velocity

41. Rob drove to the nearest hospital with an average speed of v m/s in t seconds. In terms of t, if he drives home on the same path, but with an average speed of 3v m/s, how long is the return trip home?
  a. t/6
  b. t/3
  c. 3t
  d. 6t

42. What can you infer from the statement, *Velocity of an object is zero*?
  a. Object is in linear motion with constant velocity.
  b. Object is moving at a constant speed.
  c. Object is either at rest or it returns to the initial point.
  d. Object is moving in a straight line without changing its direction.

43. An object has an average speed of 7.4 km/h. Which of the following describes two ways you could increase the average speed of the object to 14.8 km/h?
  a. Reduce the distance that the object travels by half, keeping the time constant, or keep the distance constant and double the time.
  b. Double the distance that the object travels, keeping the time constant, or keep the distance constant and reduce the time by half.
  c. Reduce the distance that the object travels to one-fourth, keeping the time constant, or keep the distance constant and increase the time by fourfold.
  d. Increase the distance by fourfold, keeping the time constant, or keep the distance constant and reduce the time by one-fourth.

44. Swimming one lap in a pool is defined as going across a pool and back again. If a swimmer swims 3 laps in 9 minutes, how can his average velocity be zero?
  a. His average velocity is zero because his total distance is zero.
  b. His average velocity is zero because his total displacement is zero.
  c. His average velocity is zero because the number of laps completed is an odd number.
  d. His average velocity is zero because the velocity of each successive lap is equal and opposite.

2.3 Position vs. Time Graphs

45. A hockey puck is shot down the arena in a straight line. Assume it does not slow until it is stopped by an opposing player who sends it back in the direction it came. The players are 20 m apart and it takes 1 s for the puck to go there and back. Which of the following describes the graph of the displacement over time? Consider the initial direction of the puck to be positive.
   a. The graph is an upward opening V.
   b. The graph is a downward opening V.
   c. The graph is an upward opening U.
   d. The graph is downward opening U.

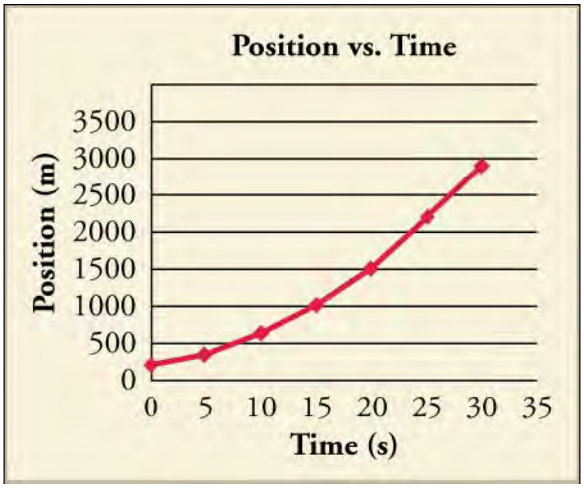
46. A defensive player kicks a soccer ball 20 m back to her own goalie. It stops just as it reaches her. She sends it back to the player. Without knowing the time it takes, draw a rough sketch of the displacement over time. Does this graph look similar to the graph of the hockey puck from the previous question?
   a. Yes, the graph is similar to the graph of the hockey puck.
   b. No, the graph is not similar to the graph of the hockey puck.
   c. The graphs cannot be compared without knowing the time the soccer ball was rolling.

47. What are the net displacement, total distance traveled, and total average velocity in the previous two problems?
   a. net displacement = 0 m, total distance = 20 m, total average velocity = 20 m/s
   b. net displacement = 0 m, total distance = 40 m, total average velocity = 20 m/s
   c. net displacement = 0 m, total distance = 20 m, total average velocity = 0 m/s
   d. net displacement = 0 m, total distance = 40 m, total average velocity = 0 m/s

48. A bee flies straight at someone and then back to its hive along the same path. Assuming it takes no time for the bee to speed up or slow down, except at the moment it changes direction, how would the graph of position vs time look? Consider the initial direction to be positive.
   a. The graph will look like a downward opening V shape.
   b. The graph will look like an upward opening V shape.
   c. The graph will look like a downward opening parabola.
   d. The graph will look like an upward opening parabola.

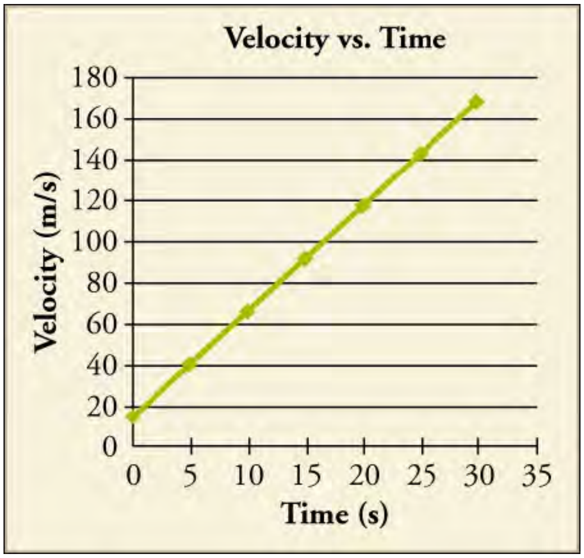
2.4 Velocity vs. Time Graphs

49. What would the velocity vs. time graph of the object whose position is shown in the graph look like?



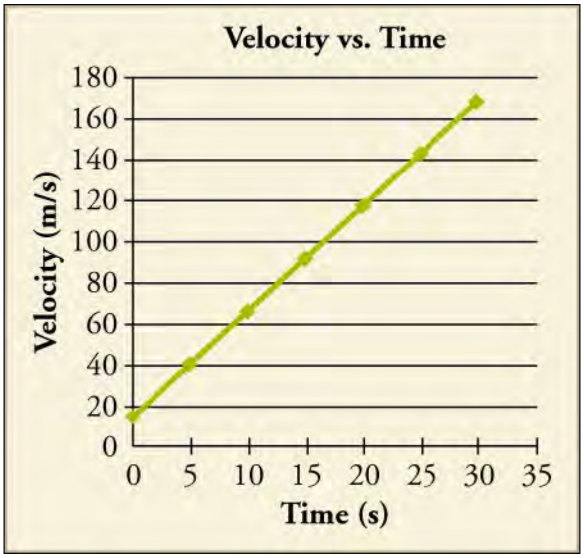
   a. It is a straight line with negative slope.
   b. It is a straight line with positive slope.
   c. It is a horizontal line at some negative value.
   d. It is a horizontal line at some positive value.

50. Which statement correctly describes the object's speed, as well as what a graph of acceleration vs. time would look like?



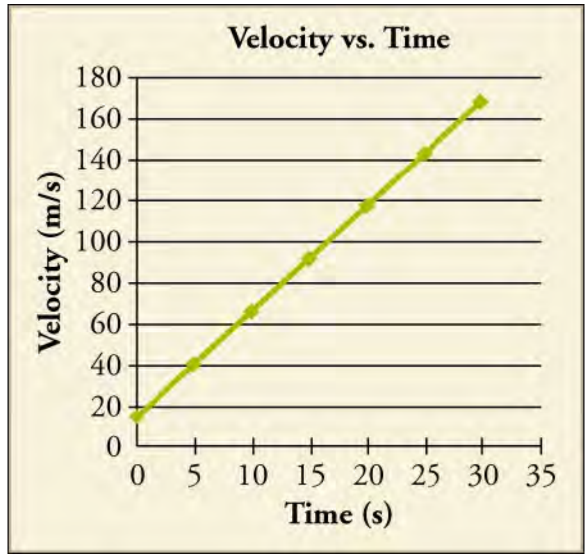
   a. The object is not speeding up, and the acceleration vs. time graph is a horizontal line at some negative value.
   b. The object is not speeding up, and the acceleration vs. time graph is a horizontal line at some positive value.
   c. The object is speeding up, and the acceleration vs. time graph is a horizontal line at some negative value.
   d. The object is speeding up, and the acceleration vs. time graph is a horizontal line at some positive value.

51. Calculate that object's net displacement over the time shown.



a. 540 m
b. 2,520 m
c. 2,790 m
d. 5,040 m

52. What is the object's average velocity?

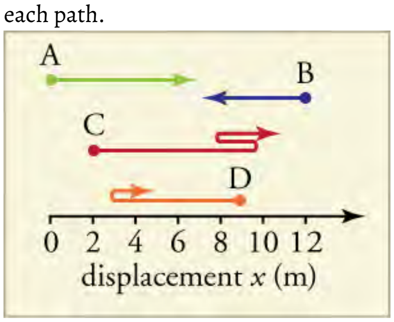


a. 18 m/s
b. 84 m/s
c. 93 m/s
d. 168 m/s

Extended Response

2.1 Relative Motion, Distance, and Displacement

53. Find the distance traveled from the starting point for each path.



Which path has the maximum distance?

a. The distance for Path A is 6 m, Path B is 4 m, Path C is 12 m and for Path D is 7 m. The net displacement for Path A is 7 m, Path B is −4m, Path C is 8 m and for Path D is −5m. Path C has maximum distance and it is equal to 12 meters.
b. The distance for Path A is 6 m, Path B is 4 m, Path C is 8 m and for Path D is 7 m. The net displacement for Path A is 6 m, Path B is −4m, Path C is 12 m and for Path D is −5 m. Path A has maximum distance and it is equal to 6 meters.
c. The distance for Path A is 6 m, Path B is 4 m, Path C is 12 m and for Path D is 7 m. The net displacement for Path A is 6 m, Path B is −4 m, Path C is 8 m and for Path D is −5 m. Path C has maximum distance and it is equal to 12 meters.
d. The distance for Path A is 6 m, Path B is −4 m, Path C is 12 m and for Path D is −5 m. The net displacement for Path A is 7 m, Path B is 4 m, Path C is 8 m and for Path D is 7 m. Path A has maximum distance and it is equal to 6 m.

54. Alan starts from his home and walks 1.3 km east to the library. He walks an additional 0.68 km east to a music store. From there, he walks 1.1 km north to a friend's house and an additional 0.42 km north to a grocery store before he finally returns home along the same path. What is his final displacement and total distance traveled?

a. Displacement is 0 km and distance is 7 km.
b. Displacement is 0 km and distance is 3.5 km.
c. Displacement is 7 km towards west and distance is 7 km.
d. Displacement is 3.5 km towards east and distance is 3.5 km.

2.2 Speed and Velocity

55. Two runners start at the same point and jog at a constant speed along a straight path. Runner A starts at time t = 0 s, and Runner B starts at time t = 2.5 s. The runners both reach a distance 64 m from the starting point at time t = 25 s. If the runners continue at the same speeds, how far from the starting point will each be at time t = 45 s?

a. Runner A will be 72×10^3 m away and Runner B will be 59.5×10^3 m away from the starting point.

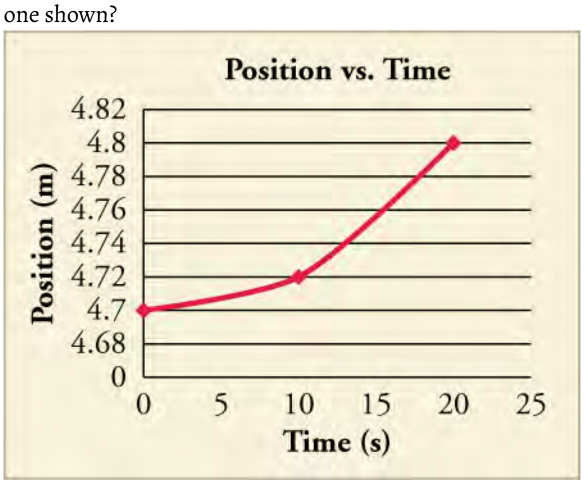
b. Runner A will be 1.2×10^2 m away and runner B will be 1.1×10^2 m away from the starting point.
c. Runner A will be 1.2×10^2 m away and Runner B will be 1.3×10^2 m away from the starting point.
d. Runner A will be 7.2×10^2 m away and Runner B will be 1.3×10^2 m away from the starting point.

56. A father and his daughter go to the bus stop that is located 75 m from their front door. The father walks in a straight line while his daughter runs along a varied path. Despite the different paths, they both end up at the bus stop at the same time. The father's average speed is 2.2 m/s, and his daughter's average speed is 3.5 m/s. (a) How long does it take the father and daughter to reach the bus stop? (b) What was the daughter's total distance traveled? (c) If the daughter maintained her same average speed and traveled in a straight line like her father, how far beyond the bus stop would she have traveled?

a. (a) 21.43 s (b) 75 m (c) 0 m
b. (a) 21.43 s (b) 119 m (c) 44 m
c. (a) 34 s (b) 75 m (c) 0 m
d. (a) 34 s (b) 119 m (c) 44 m

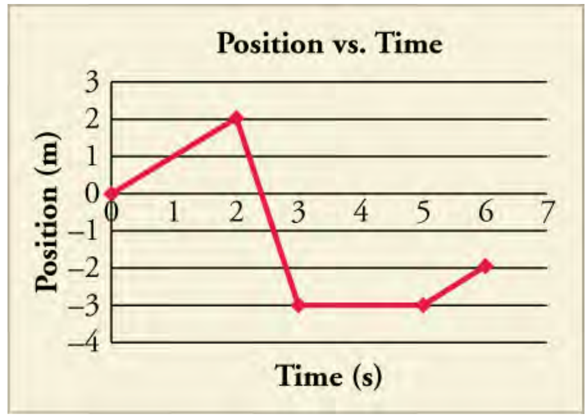
2.3 Position vs. Time Graphs

57. What kind of motion would create a position graph like the one shown?



a. uniform motion
b. any motion that accelerates
c. motion that stops and then starts
d. motion that has constant velocity

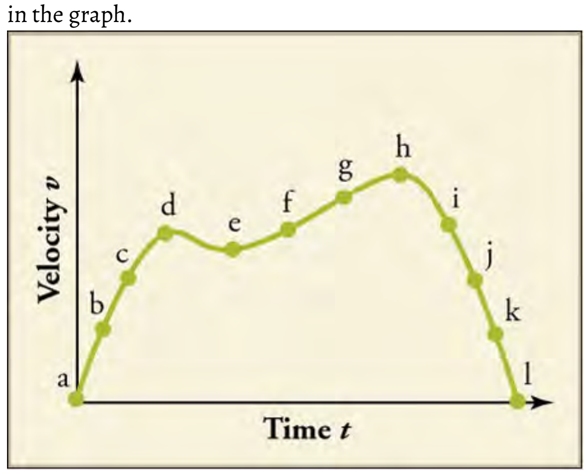
58. What is the average velocity for the whole time period shown in the graph?



a. $-\frac{1}{3}$ m/s
b. $-\frac{3}{4}$ m/s
c. $\frac{1}{3}$ m/s
d. $\frac{3}{4}$ m/s

2.4 Velocity vs. Time Graphs

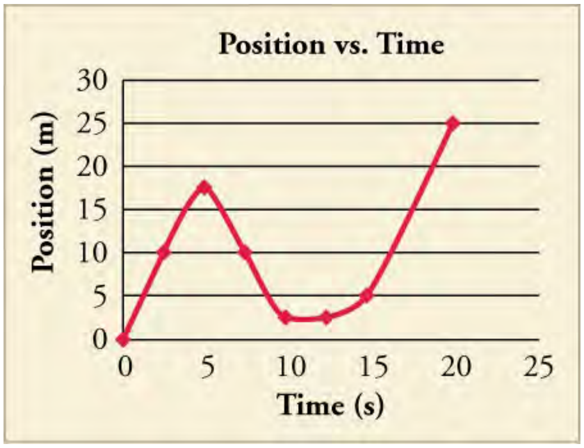
59. Consider the motion of the object whose velocity is charted in the graph.



During which points is the object slowing down and speeding up?

a. It is slowing down between *d* and *e*. It is speeding up between *a* and *d* and *e* and *h*
b. It is slowing down between *a* and *d* and *e* and *h*. It is speeding up between *d* and *e* and then after *i*.
c. It is slowing down between *d* and *e* and then after *h*. It is speeding up between *a* and *d* and *e* and *h*.
d. It is slowing down between *a* and *d* and *e* and *h*. It is speeding up between *d* and *e* and then after *i*.

60. Divide the graph into approximate sections, and use those sections to graph the velocity vs. time of the object.



Then calculate the acceleration during each section, and calculate the approximate average velocity.

a. Acceleration is zero and average velocity is 1.25 m/s.
b. Acceleration is constant with some positive value and average velocity is 1.25 m/s.
c. Acceleration is zero and average velocity is 0.25 m/s.
d. Acceleration is constant with some positive value and average velocity is 0.25 m/s.

CHAPTER 3
Acceleration

Figure 3.1 A plane slows down as it comes in for landing in St. Maarten. Its acceleration is in the opposite direction of its velocity. (Steve Conry, Flickr)

Chapter Outline

INTRODUCTION You may have heard the term *accelerator*, referring to the gas pedal in a car. When the gas pedal is pushed down, the flow of gasoline to the engine increases, which increases the car's velocity. Pushing on the gas pedal results in acceleration because the velocity of the car increases, and acceleration is defined as a change in velocity. You need two quantities to define velocity: a speed and a direction. Changing either of these quantities, or both together, changes the velocity. You may be surprised to learn that pushing on the brake pedal or turning the steering wheel also causes acceleration. The first reduces the *speed* and so changes the velocity, and the second changes the *direction* and also changes the velocity.

In fact, any change in velocity—whether positive, negative, directional, or any combination of these—is called an acceleration in physics. The plane in the picture is said to be accelerating because its velocity is decreasing as it prepares to land. To begin our study of acceleration, we need to have a clear understanding of what acceleration means.

3.1 Acceleration

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain acceleration and determine the direction and magnitude of acceleration in one dimension
- Analyze motion in one dimension using kinematic equations and graphic representations

Section Key Terms

average acceleration instantaneous acceleration negative acceleration

Defining Acceleration

Throughout this chapter we will use the following terms: *time*, *displacement*, *velocity*, and *acceleration*. Recall that each of these terms has a designated variable and SI unit of measurement as follows:

- Time: t, measured in seconds (s)
- Displacement: Δd, measured in meters (m)
- Velocity: v, measured in meters per second (m/s)
- Acceleration: a, measured in meters per second per second (m/s^2, also called meters per second squared)
- Also note the following:
  - Δ means *change in*
  - The subscript 0 refers to an initial value; sometimes subscript i is instead used to refer to initial value.
  - The subscript f refers to final value
  - A bar over a symbol, such as $\overline{a}$, means *average*

Acceleration is the change in velocity divided by a period of time during which the change occurs. The SI units of velocity are m/s and the SI units for time are s, so the SI units for acceleration are m/s^2. **Average acceleration** is given by

$$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_0}.$$

Average acceleration is distinguished from **instantaneous acceleration**, which is acceleration at a specific instant in time. The magnitude of acceleration is often not constant over time. For example, runners in a race accelerate at a greater rate in the first second of a race than during the following seconds. You do not need to know all the instantaneous accelerations at all times to calculate average acceleration. All you need to know is the change in velocity (i.e., the final velocity minus the initial velocity) and the change in time (i.e., the final time minus the initial time), as shown in the formula. Note that the average acceleration can be positive, negative, or zero. A **negative acceleration** is simply an acceleration in the negative direction.

Keep in mind that although acceleration points in the same direction as the *change* in velocity, it is not always in the direction of the velocity itself. When an object slows down, its acceleration is opposite to the direction of its velocity. In everyday language, this is called deceleration; but in physics, it is acceleration—whose direction happens to be opposite that of the velocity. For now, let us assume that motion to the right along the x-axis is *positive* and motion to the left is *negative*.

Figure 3.2 shows a car with positive acceleration in (a) and negative acceleration in (b). The arrows represent vectors showing both direction and magnitude of velocity and acceleration.

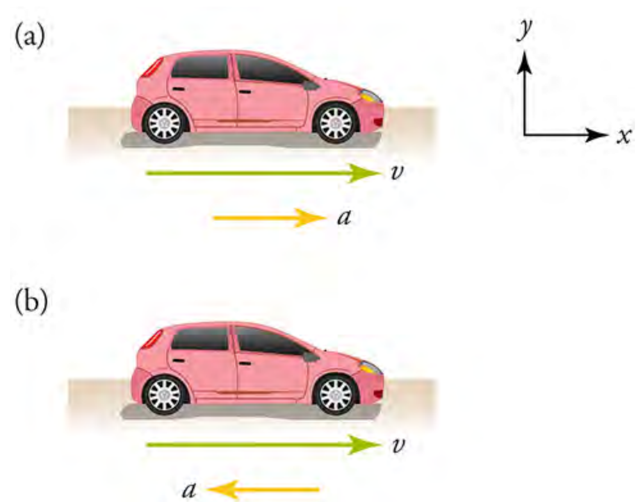


Figure 3.2 The car is speeding up in (a) and slowing down in (b).

Velocity and acceleration are both vector quantities. Recall that vectors have both magnitude and direction. An object traveling at a constant velocity—therefore having no acceleration—does accelerate if it changes direction. So, turning the steering wheel of a moving car makes the car accelerate because the velocity changes direction.

Virtual Physics

The Moving Man

With this animation in , you can produce both variations of acceleration and velocity shown in Figure 3.2, plus a few more variations. Vary the velocity and acceleration by sliding the red and green markers along the scales. Keeping the velocity marker near zero will make the effect of acceleration more obvious. Try changing acceleration from positive to negative while the man is moving. We will come back to this animation and look at the *Charts* view when we study graphical representation of motion.

Click to view content (https://archive.cnx.org/specials/e2ca52af-8c6b-450e-ac2f-9300b38e8739/moving-man/)

GRASP CHECK

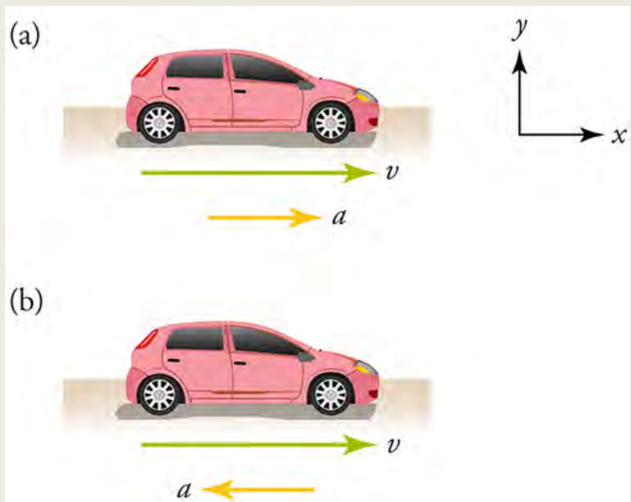


Figure 3.3

Which part, (a) or (b), is represented when the velocity vector is on the positive side of the scale and the acceleration vector is set on the negative side of the scale? What does the car's motion look like for the given scenario?

a. Part (a). The car is slowing down because the acceleration and the velocity vectors are acting in the opposite direction.
b. Part (a). The car is speeding up because the acceleration and the velocity vectors are acting in the same direction.
c. Part (b). The car is slowing down because the acceleration and velocity vectors are acting in the opposite directions.
d. Part (b). The car is speeding up because the acceleration and the velocity vectors are acting in the same direction.

Calculating Average Acceleration

Look back at the equation for average acceleration. You can see that the calculation of average acceleration involves three values: change in time, (Δt); change in velocity, (Δv); and acceleration (a).

Change in time is often stated as a time interval, and change in velocity can often be calculated by subtracting the initial velocity from the final velocity. Average acceleration is then simply change in velocity divided by change in time. Before you begin calculating, be sure that all distances and times have been converted to meters and seconds. Look at these examples of acceleration of a subway train.

WORKED EXAMPLE

An Accelerating Subway Train

A subway train accelerates from rest to 30.0 km/h in 20.0 s. What is the average acceleration during that time interval?

Strategy

Start by making a simple sketch.

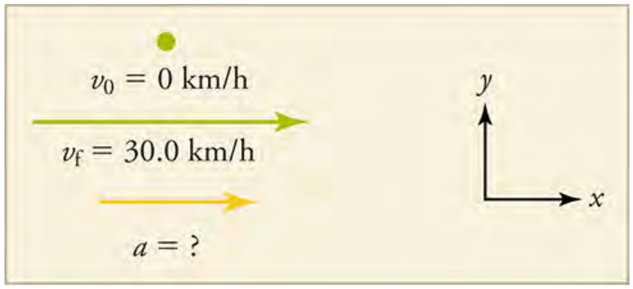


Figure 3.4

This problem involves four steps:

1. Convert to units of meters and seconds.
2. Determine the change in velocity.
3. Determine the change in time.
4. Use these values to calculate the average acceleration.

Solution

1. Identify the knowns. Be sure to read the problem for given information, which may not *look* like numbers. When the problem states that the train starts from rest, you can write down that the initial velocity is 0 m/s. Therefore, $v_0 = 0$; $v_f = 30.0$ km/h; and $\Delta t = 20.0$ s.
2. Convert the units.

$$\frac{30.0 \text{ km}}{\text{h}} \times \frac{10^3 \text{m}}{1 \text{ km}} \times \frac{1 \text{ h}}{3600 \text{ s}} = 8.333 \frac{\text{m}}{\text{s}} \tag{3.1}$$

3. Calculate change in velocity, $\Delta v = v_f - v_0 = 8.333 \text{ m/s} - 0 = +8.333 \text{ m/s}$, where the plus sign means the change in velocity is to the right.
4. We know Δt, so all we have to do is insert the known values into the formula for average acceleration.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{8.333 \text{ m/s}}{20.00 \text{ s}} = +0.417 \frac{\text{m}}{\text{s}^2} \tag{3.2}$$

Discussion

The plus sign in the answer means that acceleration is to the right. This is a reasonable conclusion because the train starts from rest and ends up with a velocity directed to the right (i.e., positive). So, acceleration is in the same direction as the *change* in velocity, as it should be.

WORKED EXAMPLE

An Accelerating Subway Train

Now, suppose that at the end of its trip, the train slows to a stop in 8.00 s from a speed of 30.0 km/h. What is its average acceleration during this time?

Strategy

Again, make a simple sketch.

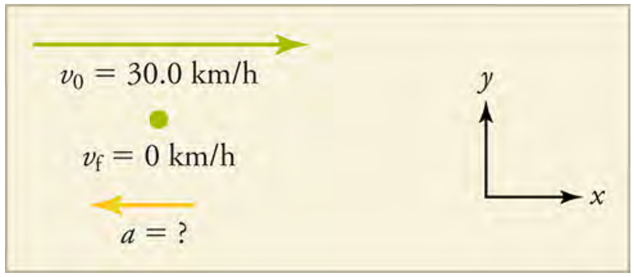


Figure 3.5

In this case, the train is decelerating and its acceleration is negative because it is pointing to the left. As in the previous example, we must find the change in velocity and change in time, then solve for acceleration.

Solution

1. Identify the knowns: $v_0 = 30.0$ km/h; $v_f = 0$; and $\Delta t = 8.00$ s.
2. Convert the units. From the first problem, we know that 30.0 km/h = 8.333 m/s.
3. Calculate change in velocity, $\Delta v = v_f - v_0 = 0 - 8.333 \text{ m/s} = -8.333 \text{ m/s}$, where the minus sign means that the change in velocity points to the left.
4. We know $\Delta t = 8.00$ s, so all we have to do is insert the known values into the equation for average acceleration.

$$\bar{a} = \frac{\Delta v}{\Delta t} = \frac{-8.333 \text{ m/s}}{8.00 \text{ s}} = -1.04 \frac{\text{m}}{\text{s}^2} \quad (3.3)$$

Discussion

The minus sign indicates that acceleration is to the left. This is reasonable because the train initially has a positive velocity in this problem, and a negative acceleration would reduce the velocity. Again, acceleration is in the same direction as the *change* in velocity, which is negative in this case. This acceleration can be called a deceleration because it has a direction opposite to the velocity.

TIPS FOR SUCCESS

- It is easier to get plus and minus signs correct if you always assume that motion is away from zero and toward positive values on the *x*-axis. This way *v* always starts off being positive and points to the right. If speed is increasing, then acceleration is positive and also points to the right. If speed is decreasing, then acceleration is negative and points to the left.
- It is a good idea to carry two extra significant figures from step-to-step when making calculations. Do not round off with each step. When you arrive at the final answer, apply the rules of significant figures for the operations you carried out and round to the correct number of digits. Sometimes this will make your answer slightly more accurate.

Practice Problems

1. A cheetah can accelerate from rest to a speed of 30.0 m/s in 7.00 s. What is its acceleration?

a. -0.23 m/s^2
b. -4.29 m/s^2
c. 0.23 m/s^2
d. 4.29 m/s^2

2. A women backs her car out of her garage with an acceleration of 1.40 m/s^2. How long does it take her to reach a speed of 2.00 m/s?

a. 0.70 s
b. 1.43 s
c. 2.80 s
d. 3.40 s

WATCH PHYSICS

Acceleration

This video shows the basic calculation of acceleration and some useful unit conversions.

Click to view content (https://www.khanacademy.org/embed_video?v=FOkQszg1-j8)

GRASP CHECK

Why is acceleration a vector quantity?

a. It is a vector quantity because it has magnitude as well as direction.
b. It is a vector quantity because it has magnitude but no direction.
c. It is a vector quantity because it is calculated from distance and time.
d. It is a vector quantity because it is calculated from speed and time.

GRASP CHECK

What will be the change in velocity each second if acceleration is 10 m/s/s?

a. An acceleration of 10 m/s/s means that every second, the velocity increases by 10 m/s.
b. An acceleration of 10 m/s/s means that every second, the velocity decreases by 10 m/s.
c. An acceleration of 10 m/s/s means that every 10 seconds, the velocity increases by 10 m/s.
d. An acceleration of 10 m/s/s means that every 10 seconds, the velocity decreases by 10 m/s.

Snap Lab

Measure the Acceleration of a Bicycle on a Slope

In this lab you will take measurements to determine if the acceleration of a moving bicycle is constant. If the acceleration is constant, then the following relationships hold: $\overline{v} = \frac{\Delta d}{\Delta t} = \frac{v_0 + v_f}{2}$ If $v_0 = 0$, then $v_f = 2\overline{v}$ and $\overline{a} = \frac{v_f}{\Delta t}$

You will work in pairs to measure and record data for a bicycle coasting down an incline on a smooth, gentle slope. The data will consist of distances traveled and elapsed times.

- Find an open area to minimize the risk of injury during this lab.

- stopwatch
- measuring tape
- bicycle

1. Find a gentle, paved slope, such as an incline on a bike path. The more gentle the slope, the more accurate your data will likely be.
2. Mark uniform distances along the slope, such as 5 m, 10 m, etc.
3. Determine the following roles: the bike rider, the timer, and the recorder. The recorder should create a data table to collect the distance and time data.
4. Have the rider at the starting point at rest on the bike. When the timer calls *Start*, the timer starts the stopwatch and the rider begins coasting down the slope on the bike without pedaling.
5. Have the timer call out the elapsed times as the bike passes each marked point. The recorder should record the times in the data table. It may be necessary to repeat the process to practice roles and make necessary adjustments.
6. Once acceptable data has been recorded, switch roles. Repeat Steps 3–5 to collect a second set of data.
7. Switch roles again to collect a third set of data.
8. Calculate average acceleration for each set of distance-time data. If your result for $\overline{a}$ is not the same for different pairs of Δv and Δt, then acceleration is not constant.
9. Interpret your results.

GRASP CHECK

If you graph the average velocity (y-axis) vs. the elapsed time (x-axis), what would the graph look like if acceleration is uniform?

a. a horizontal line on the graph
b. a diagonal line on the graph
c. an upward-facing parabola on the graph
d. a downward-facing parabola on the graph

Check Your Understanding

3. What are three ways an object can accelerate?
    a. By speeding up, maintaining constant velocity, or changing direction
    b. By speeding up, slowing down, or changing direction
    c. By maintaining constant velocity, slowing down, or changing direction
    d. By speeding up, slowing down, or maintaining constant velocity

4. What is the difference between average acceleration and instantaneous acceleration?
    a. Average acceleration is the change in displacement divided by the elapsed time; instantaneous acceleration is the acceleration at a given point in time.
    b. Average acceleration is acceleration at a given point in time; instantaneous acceleration is the change in displacement divided by the elapsed time.
    c. Average acceleration is the change in velocity divided by the elapsed time; instantaneous acceleration is acceleration at a given point in time.
    d. Average acceleration is acceleration at a given point in time; instantaneous acceleration is the change in velocity divided by the elapsed time.

5. What is the rate of change of velocity called?
    a. Time
    b. Displacement
    c. Velocity
    d. Acceleration

3.2 Representing Acceleration with Equations and Graphs

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the kinematic equations related to acceleration and illustrate them with graphs
- Apply the kinematic equations and related graphs to problems involving acceleration

Section Key Terms

acceleration due to gravity	kinematic equations	uniform acceleration

How the Kinematic Equations are Related to Acceleration

We are studying concepts related to motion: time, displacement, velocity, and especially acceleration. We are only concerned with motion in one dimension. The **kinematic equations** apply to conditions of constant acceleration and show how these concepts are related. **Constant acceleration** is acceleration that does not change over time. The first kinematic equation relates displacement d, average velocity $\bar{v}$, and time t.

$$d = d_0 + \bar{v}t \tag{3.4}$$

The initial displacement d_0 is often 0, in which case the equation can be written as $\bar{v} = \frac{d}{t}$

This equation says that average velocity is displacement per unit time. We will express velocity in meters per second. If we graph displacement versus time, as in Figure 3.6, the slope will be the velocity. Whenever a rate, such as velocity, is represented graphically, time is usually taken to be the independent variable and is plotted along the x axis.

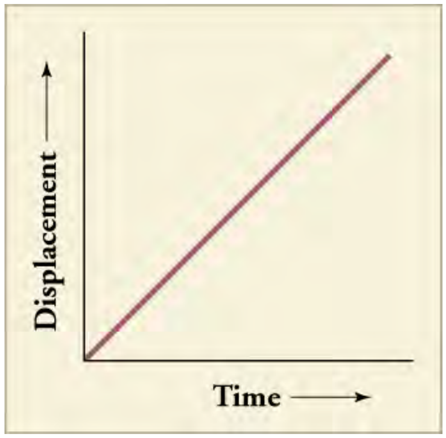


Figure 3.6 The slope of displacement versus time is velocity.

The second kinematic equation, another expression for average velocity $\overline{v}$, is simply the initial velocity plus the final velocity divided by two.

$$\overline{v} = \frac{v_0 + v_f}{2} \tag{3.5}$$

Now we come to our main focus of this chapter; namely, the kinematic equations that describe motion with constant acceleration. In the third kinematic equation, acceleration is the rate at which velocity increases, so velocity at any point equals initial velocity plus acceleration multiplied by time

$$v = v_0 + at \text{ Also, if we start from rest } (v_0 = 0\text{), we can write } a = \frac{v}{t} \tag{3.6}$$

Note that this third kinematic equation does not have displacement in it. Therefore, if you do not know the displacement and are not trying to solve for a displacement, this equation might be a good one to use.

The third kinematic equation is also represented by the graph in Figure 3.7.

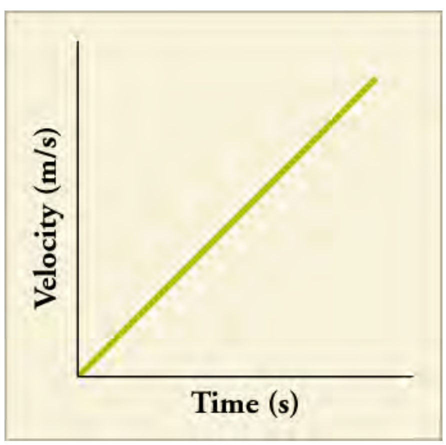


Figure 3.7 The slope of velocity versus time is acceleration.

The fourth kinematic equation shows how displacement is related to acceleration

$$d = d_0 + v_0 t + \frac{1}{2}at^2. \tag{3.7}$$

When starting at the origin, $d_0 = 0$ and, when starting from rest, $v_0 = 0$, in which case the equation can be written as

$$a = \frac{2d}{t^2}.$$

This equation tells us that, for constant acceleration, the slope of a plot of $2d$ versus t^2 is acceleration, as shown in Figure 3.8.

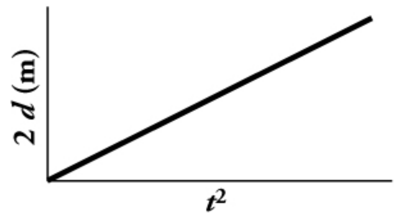


Figure 3.8 When acceleration is constant, the slope of $2d$ versus t^2 gives the acceleration.

The fifth kinematic equation relates velocity, acceleration, and displacement

$$v^2 = v_0^2 + 2a\left(d - d_0\right). \tag{3.8}$$

This equation is useful for when we do not know, or do not need to know, the time.

When starting from rest, the fifth equation simplifies to

$$a = \frac{v^2}{2d}.$$

According to this equation, a graph of velocity squared versus twice the displacement will have a slope equal to acceleration.

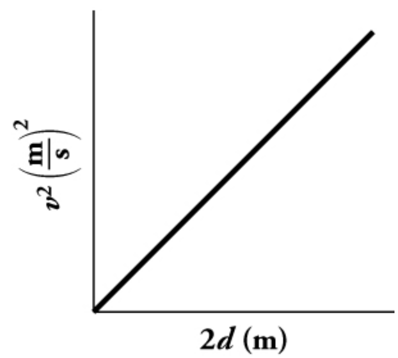


Figure 3.9

Note that, in reality, knowns and unknowns will vary. Sometimes you will want to rearrange a kinematic equation so that the knowns are the values on the axes and the unknown is the slope. Sometimes the intercept will not be at the origin (0,0). This will happen when v_0 or d_0 is not zero. This will be the case when the object of interest is already in motion, or the motion begins at some point other than at the origin of the coordinate system.

Virtual Physics

The Moving Man (Part 2)

Look at the Moving Man simulation again and this time use the *Charts* view. Again, vary the velocity and acceleration by sliding the red and green markers along the scales. Keeping the velocity marker near zero will make the effect of acceleration more obvious. Observe how the graphs of position, velocity, and acceleration vary with time. Note which are linear plots and which are not.

Click to view content (https://archive.cnx.org/specials/e2ca52af-8c6b-450e-ac2f-9300b38e8739/moving-man/)

GRASP CHECK

On a velocity versus time plot, what does the slope represent?

a. Acceleration

b. Displacement
c. Distance covered
d. Instantaneous velocity

GRASP CHECK

On a position versus time plot, what does the slope represent?

a. Acceleration
b. Displacement
c. Distance covered
d. Instantaneous velocity

The kinematic equations are applicable when you have constant acceleration.

1. $d = d_0 + \bar{v}t$, or $\bar{v} = \frac{d}{t}$ when $d_0 = 0$
2. $\bar{v} = \frac{v_0 + v_f}{2}$
3. $v = v_0 + at$, or $a = \frac{v}{t}$ when $v_0 = 0$
4. $d = d_0 + v_0 t + \frac{1}{2}at^2$, or $a = \frac{2d}{t^2}$ when $d_0 = 0$ and $v_0 = 0$
5. $v^2 = v_0^2 + 2a(d - d_0)$, or $a = \frac{2d}{t^2}$ when $d_0 = 0$ and $v_0 = 0$

Applying Kinematic Equations to Situations of Constant Acceleration

Problem-solving skills are essential to success in a science and life in general. The ability to apply broad physical principles, which are often represented by equations, to specific situations is a very powerful form of knowledge. It is much more powerful than memorizing a list of facts. Analytical skills and problem-solving abilities can be applied to new situations, whereas a list of facts cannot be made long enough to contain every possible circumstance. Essential analytical skills will be developed by solving problems in this text and will be useful for understanding physics and science in general throughout your life.

Problem-Solving Steps

While no single step-by-step method works for every problem, the following general procedures facilitate problem solving and make the answers more meaningful. A certain amount of creativity and insight are required as well.

1. *Examine the situation to determine which physical principles are involved. It is vital to draw a simple sketch at the outset.* Decide which direction is positive and note that on your sketch.
2. *Identify the knowns: Make a list of what information is given or can be inferred from the problem statement.* Remember, not all given information will be in the form of a number with units in the problem. If something starts *from rest*, we know the initial velocity is zero. If something *stops*, we know the final velocity is zero.
3. *Identify the unknowns: Decide exactly what needs to be determined in the problem.*
4. *Find an equation or set of equations that can help you solve the problem.* Your list of knowns and unknowns can help here. For example, if time is not needed or not given, then the fifth kinematic equation, which does not include time, could be useful.
5. *Insert the knowns along with their units into the appropriate equation and obtain numerical solutions complete with units.* This step produces the numerical answer; it also provides a check on units that can help you find errors. If the units of the answer are incorrect, then an error has been made.
6. *Check the answer to see if it is reasonable: Does it make sense?* This final step is extremely important because the goal of physics is to accurately describe nature. To see if the answer is reasonable, check its magnitude, its sign, and its units. Are the significant figures correct?

Summary of Problem Solving

- Determine the knowns and unknowns.
- Find an equation that expresses the unknown in terms of the knowns. More than one unknown means more than one equation is needed.
- Solve the equation or equations.

- Be sure units and significant figures are correct.
- Check whether the answer is reasonable.

FUN IN PHYSICS

Drag Racing

Figure 3.10 Smoke rises from the tires of a dragster at the beginning of a drag race. (Lt. Col. William Thurmond. Photo courtesy of U.S. Army.)

The object of the sport of drag racing is acceleration. Period! The races take place from a standing start on a straight one-quarter-mile (402 m) track. Usually two cars race side by side, and the winner is the driver who gets the car past the quarter-mile point first. At the finish line, the cars may be going more than 300 miles per hour (134 m/s). The driver then deploys a parachute to bring the car to a stop because it is unsafe to brake at such high speeds. The cars, called dragsters, are capable of accelerating at 26 m/s^2. By comparison, a typical sports car that is available to the general public can accelerate at about 5 m/s^2.

Several measurements are taken during each drag race:

- Reaction time is the time between the starting signal and when the front of the car crosses the starting line.
- Elapsed time is the time from when the vehicle crosses the starting line to when it crosses the finish line. The record is a little over 3 s.
- Speed is the average speed during the last 20 m before the finish line. The record is a little under 400 mph.

The video shows a race between two dragsters powered by jet engines. The actual race lasts about four seconds and is near the end of the video (https://openstax.org/l/28dragsters) .

GRASP CHECK

A dragster crosses the finish line with a velocity of $140\ \text{m/s}$. Assuming the vehicle maintained a constant acceleration from start to finish, what was its average velocity for the race?

a. $0\ \text{m/s}$
b. $35\ \text{m/s}$
c. $70\ \text{m/s}$
d. $140\ \text{m/s}$

WORKED EXAMPLE

Acceleration of a Dragster

The time it takes for a dragster to cross the finish line is unknown. The dragster accelerates from rest at 26 m/s^2 for a quarter mile (0.250 mi). What is the final velocity of the dragster?

Strategy

The equation $v^2 = v_0^2 + 2a\,(d - d_0)$ is ideally suited to this task because it gives the velocity from acceleration and displacement, without involving the time.

Solution

1. Convert miles to meters.

$$(0.250 \text{ mi}) \times \frac{1609 \text{ m}}{1 \text{ mi}} = 402 \text{ m} \tag{3.9}$$

2. Identify the known values. We know that $v_0 = 0$ since the dragster starts from rest, and we know that the distance traveled, $d - d_0$ is 402 m. Finally, the acceleration is constant at $a = 26.0 \text{ m/s}^2$.
3. Insert the knowns into the equation $v^2 = v_0^2 + 2a(d - d_0)$ and solve for v.

$$v^2 = 0 + 2\left(26.0 \frac{\text{m}}{\text{s}^2}\right)(402 \text{ m}) = 2.09 \times 10^4 \frac{\text{m}^2}{\text{s}^2} \tag{3.10}$$

Taking the square root gives us $v = \sqrt{2.09 \times 10^4 \frac{\text{m}^2}{\text{s}^2}} = 145 \frac{\text{m}}{\text{s}}$.

Discussion

145 m/s is about 522 km/hour or about 324 mi/h, but even this breakneck speed is short of the record for the quarter mile. Also, note that a square root has two values. We took the positive value because we know that the velocity must be in the same direction as the acceleration for the answer to make physical sense.

An examination of the equation $v^2 = v_0^2 + 2a(d - d_0)$ can produce further insights into the general relationships among physical quantities:

- The final velocity depends on the magnitude of the acceleration and the distance over which it applies.
- For a given acceleration, a car that is going twice as fast does not stop in twice the distance—it goes much further before it stops. This is why, for example, we have reduced speed zones near schools.

Practice Problems

6. Dragsters can reach a top speed of 145 m/s in only 4.45 s. Calculate the average acceleration for such a dragster.
   a. -32.6 m/s^2
   b. 0 m/s^2
   c. 32.6 m/s^2
   d. 145 m/s^2

7. An Olympic-class sprinter starts a race with an acceleration of 4.50 m/s^2. Assuming she can maintain that acceleration, what is her speed 2.40 s later?
   a. 4.50 m/s
   b. 10.8 m/s
   c. 19.6 m/s
   d. 44.1 m/s

Constant Acceleration

In many cases, acceleration is not uniform because the force acting on the accelerating object is not constant over time. A situation that gives constant acceleration is the acceleration of falling objects. When air resistance is not a factor, all objects near Earth's surface fall with an acceleration of about 9.80 m/s^2. Although this value decreases slightly with increasing altitude, it may be assumed to be essentially constant. The value of 9.80 m/s^2 is labeled g and is referred to as **acceleration due to gravity**. Gravity is the force that causes nonsupported objects to accelerate downward—or, more precisely, toward the center of Earth. The magnitude of this force is called the weight of the object and is given by mg where m is the mass of the object (in kg). In places other than on Earth, such as the Moon or on other planets, g is not 9.80 m/s^2, but takes on other values. When using g for the acceleration a in a kinematic equation, it is usually given a negative sign because the acceleration due to gravity is downward.

WORK IN PHYSICS

Effects of Rapid Acceleration

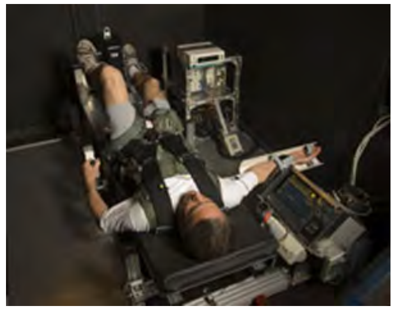


Figure 3.11 Astronauts train using G Force Simulators. (NASA)

When in a vehicle that accelerates rapidly, you experience a force on your entire body that accelerates your body. You feel this force in automobiles and slightly more on amusement park rides. For example, when you ride in a car that turns, the car applies a force on your body to make you accelerate in the direction in which the car is turning. If enough force is applied, you will accelerate at 9.80 m/s^2. This is the same as the acceleration due to gravity, so this force is called one G.

One G is the force required to accelerate an object at the acceleration due to gravity at Earth's surface. Thus, one G for a paper cup is much less than one G for an elephant, because the elephant is much more massive and requires a greater force to make it accelerate at 9.80 m/s^2. For a person, a G of about 4 is so strong that his or her face will distort as the bones accelerate forward through the loose flesh. Other symptoms at extremely high Gs include changes in vision, loss of consciousness, and even death. The space shuttle produces about 3 Gs during takeoff and reentry. Some roller coasters and dragsters produce forces of around 4 Gs for their occupants. A fighter jet can produce up to 12 Gs during a sharp turn.

Astronauts and fighter pilots must undergo G-force training in simulators. The video (https://www.youtube.com/watch?v=n-8QHOUWECU) shows the experience of several people undergoing this training.

People, such as astronauts, who work with G forces must also be trained to experience zero G—also called free fall or weightlessness—which can cause queasiness. NASA has an aircraft that allows it occupants to experience about 25 s of free fall. The aircraft is nicknamed the *Vomit Comet*.

GRASP CHECK

A common way to describe acceleration is to express it in multiples of g, Earth's gravitational acceleration. If a dragster accelerates at a rate of 39.2 m/s^2, how many g's does the driver experience?

a. 1.5 g
b. 4.0 g
c. 10.5 g
d. 24.5 g

WORKED EXAMPLE

Falling Objects

A person standing on the edge of a high cliff throws a rock straight up with an initial velocity v_0 of 13 m/s.

(a) Calculate the position and velocity of the rock at 1.00, 2.00, and 3.00 seconds after it is thrown. Ignore the effect of air resistance.

Strategy

Sketch the initial velocity and acceleration vectors and the axes.

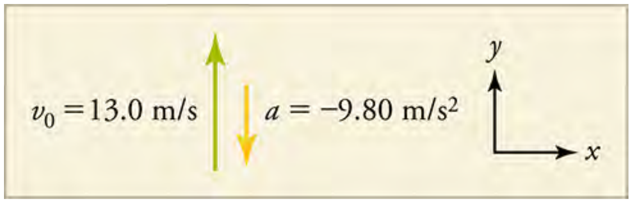


Figure 3.12 Initial conditions for rock thrown straight up.

List the knowns: time $t = 1.00$ s, 2.00 s, and 3.00 s; initial velocity $v_0 = 13$ m/s; acceleration $a = g = -9.80$ m/s^2; and position $y_0 = 0$ m

List the unknowns: y_1, y_2, and y_3; v_1, v_2, and v_3—where 1, 2, 3 refer to times 1.00 s, 2.00 s, and 3.00 s

Choose the equations.

$$d = d_0 + v_0 t + \frac{1}{2}at^2 \text{ becomes } y = y_0 + v_0 t - \frac{1}{2}gt^2 \tag{3.11}$$

$$v = v_0 + at \text{ becomes } v = v_0 + -gt \tag{3.12}$$

These equations describe the unknowns in terms of knowns only.

Solution

$$y_1 = 0 + (13.0 \text{ m/s})\,(1.00 \text{ s}) + \frac{(-9.80\text{m/s}^2)(1.00 \text{ s})^2}{2} = 8.10 \text{ m}$$
$$y_2 = 0 + (13.0 \text{ m/s})\,(2.00 \text{ s}) + \frac{(-9.80\text{m/s}^2)(2.00 \text{ s})^2}{2} = 6.40 \text{ m}$$
$$y_3 = 0 + (13.0 \text{ m/s})\,(3.00 \text{ s}) + \frac{(-9.80\text{m/s}^2)(3.00 \text{ s})^2}{2} = -5.10 \text{ m}$$
$$v_1 = 13.0 \text{ m/s} + \left(-9.80\text{m/s}^2\right)(1.00 \text{ s}) = 3.20 \text{ m/s}$$
$$v_2 = 13.0 \text{ m/s} + \left(-9.80\text{m/s}^2\right)(2.00 \text{ s}) = -6.60 \text{ m/s}$$
$$v_3 = 13.0 \text{ m/s} + \left(-9.80\text{m/s}^2\right)(3.00 \text{ s}) = -16.4 \text{ m/s}$$

Discussion

The first two positive values for y show that the rock is still above the edge of the cliff, and the third negative y value shows that it has passed the starting point and is below the cliff. Remember that we set *up* to be positive. Any position with a positive value is above the cliff, and any velocity with a positive value is an upward velocity. The first value for v is positive, so the rock is still on the way up. The second and third values for v are negative, so the rock is on its way down.

(b) Make graphs of position versus time, velocity versus time, and acceleration versus time. Use increments of 0.5 s in your graphs.

Strategy

Time is customarily plotted on the *x*-axis because it is the independent variable. Position versus time will not be linear, so calculate points for 0.50 s, 1.50 s, and 2.50 s. This will give a curve closer to the true, smooth shape.

Solution

The three graphs are shown in Figure 3.13.

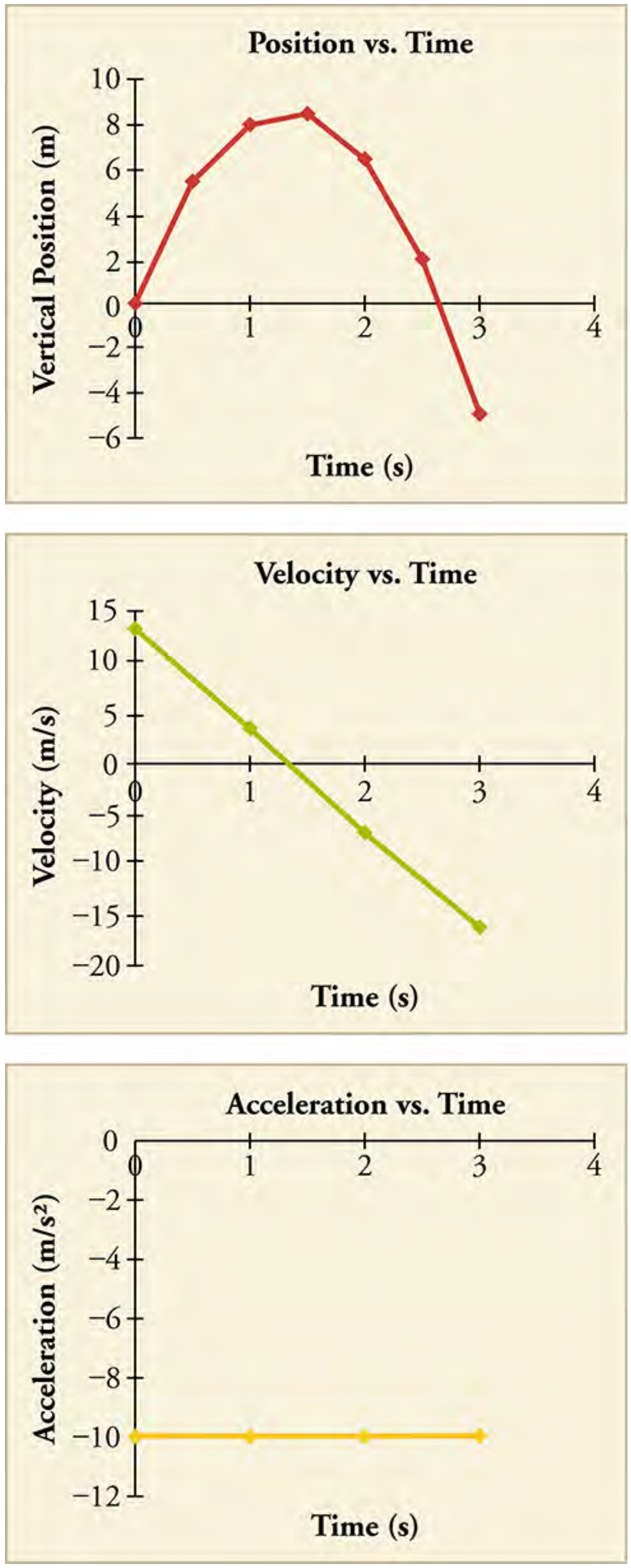


Figure 3.13

Discussion

- y vs. t does *not* represent the two-dimensional parabolic path of a trajectory. The path of the rock is straight up and straight down. The slope of a line tangent to the curve at any point on the curve equals the velocity at that point—i.e., the instantaneous velocity.

- Note that the v vs. t line crosses the vertical axis at the initial velocity and crosses the horizontal axis at the time when the rock changes direction and begins to fall back to Earth. This plot is linear because acceleration is constant.
- The a vs. t plot also shows that acceleration is constant; that is, it does not change with time.

Practice Problems

8. A cliff diver pushes off horizontally from a cliff and lands in the ocean 2.00 s later. How fast was he going when he entered the water?
  a. 0 m/s
  b. 19.0 m/s
  c. 19.6 m/s
  d. 20.0 m/s

9. A girl drops a pebble from a high cliff into a lake far below. She sees the splash of the pebble hitting the water 2.00 s later. How fast was the pebble going when it hit the water?
  a. 9.80 m/s
  b. 10.0 m/s
  c. 19.6 m/s
  d. 20.0 m/s

Check Your Understanding

10. Identify the four variables found in the kinematic equations.
  a. Displacement, Force, Mass, and Time
  b. Acceleration, Displacement, Time, and Velocity
  c. Final Velocity, Force, Initial Velocity, and Mass
  d. Acceleration, Final Velocity, Force, and Initial Velocity

11. Which of the following steps is always required to solve a kinematics problem?
  a. Find the force acting on the body.
  b. Find the acceleration of a body.
  c. Find the initial velocity of a body.
  d. Find a suitable kinematic equation and then solve for the unknown quantity.

12. Which of the following provides a correct answer for a problem that can be solved using the kinematic equations?
  a. A body starts from rest and accelerates at 4 m/s^2 for 2 s. The body's final velocity is 8 m/s.
  b. A body starts from rest and accelerates at 4 m/s^2 for 2 s. The body's final velocity is 16 m/s.
  c. A body with a mass of 2 kg is acted upon by a force of 4 N. The acceleration of the body is 2 m/s^2.
  d. A body with a mass of 2 kg is acted upon by a force of 4 N. The acceleration of the body is 0.5 m/s^2.

KEY TERMS

acceleration due to gravity acceleration of an object that is subject only to the force of gravity; near Earth's surface this acceleration is 9.80 m/s^2

average acceleration change in velocity divided by the time interval over which it changed

constant acceleration acceleration that does not change with respect to time

instantaneous acceleration rate of change of velocity at a specific instant in time

kinematic equations the five equations that describe motion in terms of time, displacement, velocity, and acceleration

negative acceleration acceleration in the negative direction

SECTION SUMMARY

3.1 Acceleration

- Acceleration is the rate of change of velocity and may be negative or positive.
- Average acceleration is expressed in m/s^2 and, in one dimension, can be calculated using $\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_o}$.

3.2 Representing Acceleration with Equations and Graphs

- The kinematic equations show how time, displacement, velocity, and acceleration are related for objects in motion.
- In general, kinematic problems can be solved by identifying the kinematic equation that expresses the unknown in terms of the knowns.
- Displacement, velocity, and acceleration may be displayed graphically versus time.

KEY EQUATIONS

3.1 Acceleration

Average acceleration	$\overline{a} = \frac{\Delta v}{\Delta t} = \frac{v_f - v_0}{t_f - t_o}$

3.2 Representing Acceleration with Equations and Graphs

Average velocity	$d = d_0 + \overline{v}t$, or $\overline{v} = \frac{d}{t}$ when $d_0 = 0$
Average velocity	$\overline{v} = \frac{v_0 + v_f}{2}$
Velocity	$v = v_0 + at$, or when $v_0 = 0$
Displacement	$d = d_0 + v_0 t + \frac{1}{2}at^2$, or $a = \frac{2d}{t^2}$ when $d_0 = 0$ and $v_0 = 0$
Acceleration	$v^2 = v_0^2 + 2a(d - d_0)$, or $a = \frac{v^2}{2d}$ when $d_0 = 0$ and $v_0 = 0$

CHAPTER REVIEW

Concept Items

3.1 Acceleration

1. How can you use the definition of acceleration to explain the units in which acceleration is measured?
   a. Acceleration is the rate of change of velocity. Therefore, its unit is m/s^2.
   b. Acceleration is the rate of change of displacement. Therefore, its unit is m/s.
   c. Acceleration is the rate of change of velocity. Therefore, its unit is m^2/s.
   d. Acceleration is the rate of change of displacement. Therefore, its unit is m^2/s.

2. What are the SI units of acceleration?
   a. m^2/s
   b. cm^2/s
   c. m/s^2
   d. cm/s^2

3. Which of the following statements explains why a racecar going around a curve is accelerating, even if the speed is constant?
   a. The car is accelerating because the magnitude as well as the direction of velocity is changing.
   b. The car is accelerating because the magnitude of velocity is changing.
   c. The car is accelerating because the direction of velocity is changing.

d. The car is accelerating because neither the magnitude nor the direction of velocity is changing.

3.2 Representing Acceleration with Equations and Graphs

4. A student calculated the final velocity of a train that decelerated from 30.5 m/s and got an answer of −43.34 m/s. Which of the following might indicate that he made a mistake in his calculation?
   a. The sign of the final velocity is wrong.
   b. The magnitude of the answer is too small.
   c. There are too few significant digits in the answer.
   d. The units in the initial velocity are incorrect.

5. Create your own kinematics problem. Then, create a flow chart showing the steps someone would need to take to solve the problem.
   a. Acceleration
   b. Distance
   c. Displacement
   d. Force

6. Which kinematic equation would you use to find the velocity of a skydiver 2.0 s after she jumps from a plane and before she opens her parachute? Assume the positive direction is downward.
   a. $v = v_0 + at$
   b. $v = v_0 - at$
   c. $v^2 = {v_0}^2 + at$
   d. $v^2 = {v_0}^2 - at$

Critical Thinking Items

3.1 Acceleration

7. Imagine that a car is traveling from your left to your right at a constant velocity. Which two actions could the driver take that may be represented as (a) a velocity vector and an acceleration vector both pointing to the right and then (b) changing so the velocity vector points to the right and the acceleration vector points to the left?
   a. (a) Push down on the accelerator and then (b) push down again on the accelerator a second time.
   b. (a) Push down on the accelerator and then (b) push down on the brakes.
   c. (a) Push down on the brakes and then (b) push down on the brakes a second time.
   d. (a) Push down on the brakes and then (b) push down on the accelerator.

8. A motorcycle moving at a constant velocity suddenly accelerates at a rate of $4.0\ \text{m/s}^2$ to a speed of 35 m/s in 5.0 s. What was the initial speed of the motorcycle?
   a. -34 m/s
   b. -15 m/s
   c. 15 m/s
   d. 34 m/s

3.2 Representing Acceleration with Equations and Graphs

9. A student is asked to solve a problem:
   An object falls from a height for 2.0 s, at which point it is still 60 m above the ground. What will be the velocity of the object when it hits the ground?
   Which of the following provides the correct order of kinematic equations that can be used to solve the problem?
   a. First use $v^2 = {v_0}^2 + 2a(d - d_0)$, then use $v = v_0 + at$.
   b. First use $v = v_0 + at$, then use $v^2 = {v_0}^2 + 2a(d - d_0)$.
   c. First use $d = d_0 + v_0 t + \frac{1}{2}at^2$, then use $v = v_0 + at$.
   d. First use $v = v_0 + at$, then use $d - d_0 = v_0 t + \frac{1}{2}at^2$.

10. Skydivers are affected by acceleration due to gravity and by air resistance. Eventually, they reach a speed where the force of gravity is almost equal to the force of air resistance. As they approach that point, their acceleration decreases in magnitude to near zero.
    Part A. Describe the shape of the graph of the magnitude of the acceleration versus time for a falling skydiver.
    Part B. Describe the shape of the graph of the magnitude of the velocity versus time for a falling skydiver.
    Part C. Describe the shape of the graph of the magnitude of the displacement versus time for a falling skydiver.
    a. Part A. Begins with a nonzero y-intercept with a downward slope that levels off at zero; Part B. Begins at zero with an upward slope that decreases in magnitude until the curve levels off; Part C. Begins at zero with an upward slope that increases in magnitude until it becomes a positive constant
    b. Part A. Begins with a nonzero y-intercept with an upward slope that levels off at zero; Part B. Begins at zero with an upward slope that decreases in magnitude until the curve levels off; Part C. Begins at zero with an upward slope that increases in magnitude until it becomes a positive constant
    c. Part A. Begins with a nonzero y-intercept with a downward slope that levels off at zero; Part B. Begins at zero with a downward slope that

decreases in magnitude until the curve levels off; Part C. Begins at zero with an upward slope that increases in magnitude until it becomes a positive constant

d. Part A. Begins with a nonzero y-intercept with an upward slope that levels off at zero; Part B. Begins at zero with a downward slope that decreases in magnitude until the curve levels off; Part C. Begins at zero with an upward slope that increases in magnitude until it becomes a positive constant

11. Which graph in the previous problem has a positive slope?
   a. Displacement versus time only
   b. Acceleration versus time and velocity versus time
   c. Velocity versus time and displacement versus time
   d. Acceleration versus time and displacement versus time

Problems

3.1 Acceleration

12. The driver of a sports car traveling at 10.0 m/s steps down hard on the accelerator for 5.0 s and the velocity increases to 30.0 m/s. What was the average acceleration of the car during the 5.0 s time interval?
   a. -1.0×102 m/s^2
   b. -4.0 m/s^2
   c. 4.0 m/s^2
   d. 1.0×102 m/s^2

13. A girl rolls a basketball across a basketball court. The ball slowly decelerates at a rate of -0.20 m/s^2. If the initial velocity was 2.0 m/s and the ball rolled to a stop at 5.0 sec after 12:00 p.m., at what time did she start the ball rolling?
   a. 0.1 seconds before noon
   b. 0.1 seconds after noon
   c. 5 seconds before noon
   d. 5 seconds after noon

3.2 Representing Acceleration with Equations and Graphs

14. A swan on a lake gets airborne by flapping its wings and running on top of the water. If the swan must reach a velocity of 6.00 m/s to take off and it accelerates from rest at an average rate of 0.350 m/s^2, how far will it travel before becoming airborne?
   a. -8.60 m
   b. 8.60 m
   c. -51.4 m
   d. 51.4 m

15. A swimmer bounces straight up from a diving board and falls feet first into a pool. She starts with a velocity of 4.00 m/s and her takeoff point is 8 m above the pool. How long are her feet in the air?
   a. 0.408 s
   b. 0.816 s
   c. 1.34 s
   d. 1.75 s
   e. 1.28 s

Performance Task

3.2 Representing Acceleration with Equations and Graphs

16. Design an experiment to measure displacement and elapsed time. Use the data to calculate final velocity, average velocity, acceleration, and acceleration.

Materials

- a small marble or ball bearing
- a garden hose
- a measuring tape
- a stopwatch or stopwatch software download
- a sloping driveway or lawn as long as the garden hose with a level area beyond

(a) How would you use the garden hose, stopwatch, marble, measuring tape, and slope to measure displacement and elapsed time? Hint—The marble is the accelerating object, and the length of the hose is total displacement.
(b) How would you use the displacement and time data to calculate velocity, average velocity, and acceleration? Which kinematic equations would you use?
(c) How would you use the materials, the measured and calculated data, and the flat area below the slope to determine the negative acceleration? What would you measure, and which kinematic equation would you use?

TEST PREP

Multiple Choice

3.1 Acceleration

17. Which variable represents displacement?
a. *a*
b. *d*
c. *t*
d. *v*

18. If a velocity increases from 0 to 20 m/s in 10 s, what is the average acceleration?
a. 0.5 m/s^2
b. 2 m/s^2
c. 10 m/s^2
d. 30 m/s^2

3.2 Representing Acceleration with Equations and Graphs

19. For the motion of a falling object, which graphs are straight lines?
a. Acceleration versus time only
b. Displacement versus time only
c. Displacement versus time and acceleration versus time
d. Velocity versus time and acceleration versus time

20. A bullet in a gun is accelerated from the firing chamber to the end of the barrel at an average rate of 6.30×10^5 m/s^2 for 8.10×10^{-4} s. What is the bullet's final velocity when it leaves the barrel, commonly known as the muzzle velocity?
a. 7.79 m/s
b. 51.0 m/s
c. 510 m/s
d. 1020 m/s

Short Answer

3.1 Acceleration

21. True or False—The vector for a negative acceleration points in the opposite direction when compared to the vector for a positive acceleration.
a. True
b. False

22. If a car decelerates from 20 m/s to 15 m/s in 5 s, what is Δv?
a. -5 m/s
b. -1 m/s
c. 1 m/s
d. 5 m/s

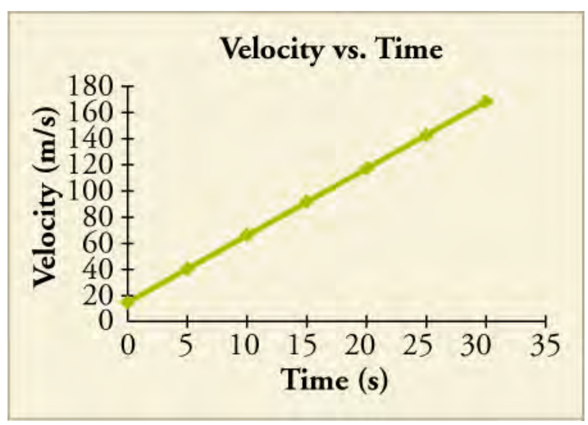
23. How is the vector arrow representing an acceleration of magnitude 3 m/s^2 different from the vector arrow representing a negative acceleration of magnitude 3 m/s^2?
a. They point in the same direction.
b. They are perpendicular, forming a 90° angle between each other.
c. They point in opposite directions.
d. They are perpendicular, forming a 270° angle between each other.

24. How long does it take to accelerate from 8.0 m/s to 20.0 m/s at a rate of acceleration of 3.0 m/s^2?
a. 0.25 s
b. 4.0 s
c. 9.33 s
d. 36 s

3.2 Representing Acceleration with Equations and Graphs

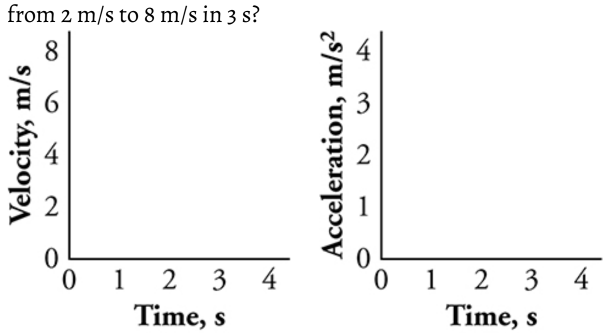
25. If a plot of displacement versus time is linear, what can be said about the acceleration?
a. Acceleration is 0.
b. Acceleration is a non-zero constant.
c. Acceleration is positive.
d. Acceleration is negative.

26.



True or False: —The image shows a velocity vs. time graph for a jet car. If you take the slope at any point on the graph, the jet car's acceleration will be 5.0 m/s^2.
a. True
b. False

27. When plotted on the blank plots, which answer choice would show the motion of an object that has uniformly accelerated from 2 m/s to 8 m/s in 3 s?



a. The plot on the left shows a line from (0,2) to (3,8) while the plot on the right shows a line from (0,2) to (3,2).
b. The plot on the left shows a line from (0,2) to (3,8) while the plot on the right shows a line from (0,3) to (3,3).
c. The plot on the left shows a line from (0,8) to (3,2) while the plot on the right shows a line from (0,2) to (3,2).
d. The plot on the left shows a line from (0,8) to (3,2) while the plot on the right shows a line from (0,3) to (3,3).

28. When is a plot of velocity versus time a straight line and when is it a curved line?
a. It is a straight line when acceleration is changing and is a curved line when acceleration is constant.
b. It is a straight line when acceleration is constant and is a curved line when acceleration is changing.
c. It is a straight line when velocity is constant and is a curved line when velocity is changing.
d. It is a straight line when velocity is changing and is a curved line when velocity is constant.

Extended Response

3.1 Acceleration

29. A test car carrying a crash test dummy accelerates from 0 to 30 m/s and then crashes into a brick wall. Describe the direction of the initial acceleration vector and compare the initial acceleration vector's magnitude with respect to the acceleration magnitude at the moment of the crash.
a. The direction of the initial acceleration vector will point towards the wall, and its magnitude will be less than the acceleration vector of the crash.
b. The direction of the initial acceleration vector will point away from the wall, and its magnitude will be less than the vector of the crash.
c. The direction of the initial acceleration vector will point towards the wall, and its magnitude will be more than the acceleration vector of the crash.
d. The direction of the initial acceleration vector will point away from the wall, and its magnitude will be more than the acceleration vector of the crash.

30. A car accelerates from rest at a stop sign at a rate of 3.0 m/s^2 to a speed of 21.0 m/s, and then immediately begins to decelerate to a stop at the next stop sign at a rate of 4.0 m/s^2. How long did it take the car to travel from the first stop sign to the second stop sign? Show your work.
a. 1.7 seconds
b. 5.3 seconds
c. 7.0 seconds
d. 12 seconds

3.2 Representing Acceleration with Equations and Graphs

31. True or False: Consider an object moving with constant acceleration. The plot of displacement versus time for such motion is a curved line while the plot of displacement versus time squared is a straight line.
a. True
b. False

32. You throw a ball straight up with an initial velocity of 15.0 m/s. It passes a tree branch on the way up at a height of 7.00 m. How much additional time will pass before the ball passes the tree branch on the way back down?
a. 0.574 s
b. 0.956 s
c. 1.53 s
d. 1.91 s

CHAPTER 4
Forces and Newton's Laws of Motion

Figure 4.1 Newton's laws of motion describe the motion of the dolphin's path. (Credit: Jin Jang)

Chapter Outline

INTRODUCTION Isaac Newton (1642–1727) was a natural philosopher; a great thinker who combined science and philosophy to try to explain the workings of nature on Earth and in the universe. His laws of motion were just one part of the monumental work that has made him legendary. The development of Newton's laws marks the transition from the Renaissance period of history to the modern era. This transition was characterized by a revolutionary change in the way people thought about the physical universe. Drawing upon earlier work by scientists Galileo Galilei and Johannes Kepler, Newton's laws of motion allowed motion on Earth and in space to be predicted mathematically. In this chapter you will learn about force as well as Newton's first, second, and third laws of motion.

4.1 Force

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Differentiate between force, net force, and dynamics
- Draw a free-body diagram

Section Key Terms

dynamics	external force	force
free-body diagram	net external force	net force

Defining Force and Dynamics

Force is the cause of motion, and motion draws our attention. Motion itself can be beautiful, such as a dolphin jumping out of the water, the flight of a bird, or the orbit of a satellite. The study of motion is called kinematics, but kinematics describes only the way objects move—their velocity and their acceleration. **Dynamics** considers the forces that affect the motion of moving objects and systems. Newton's laws of motion are the foundation of dynamics. These laws describe the way objects speed up, slow down, stay in motion, and interact with other objects. They are also universal laws: they apply everywhere on Earth as well as in space.

A force pushes or pulls an object. The object being moved by a force could be an inanimate object, a table, or an animate object, a person. The pushing or pulling may be done by a person, or even the gravitational pull of Earth. Forces have different magnitudes and directions; this means that some forces are stronger than others and can act in different directions. For example, a cannon exerts a strong force on the cannonball that is launched into the air. In contrast, a mosquito landing on your arm exerts only a small force on your arm.

When multiple forces act on an object, the forces combine. Adding together all of the forces acting on an object gives the total force, or **net force**. An **external force** is a force that acts on an object within the system *from outside* the system. This type of force is different than an internal force, which acts between two objects that are both within the system. **The net external force** combines these two definitions; it is the total combined external force. We discuss further details about net force, external force, and net external force in the coming sections.

In mathematical terms, two forces acting in opposite directions have opposite *signs* (positive or negative). By convention, the negative sign is assigned to any movement to the left or downward. If two forces pushing in opposite directions are added together, the larger force will be somewhat canceled out by the smaller force pushing in the opposite direction. It is important to be consistent with your chosen coordinate system within a problem; for example, if negative values are assigned to the downward direction for velocity, then distance, force, and acceleration should also be designated as being negative in the downward direction.

Free-Body Diagrams and Examples of Forces

For our first example of force, consider an object hanging from a rope. This example gives us the opportunity to introduce a useful tool known as a **free-body diagram**. A free-body diagram represents the object being acted upon—that is, the free body—as a single point. Only the forces acting *on* the body (that is, external forces) are shown and are represented by vectors (which are drawn as arrows). These forces are the only ones shown because only external forces acting on the body affect its motion. We can ignore any internal forces within the body because they cancel each other out, as explained in the section on Newton's third law of motion. Free-body diagrams are very useful for analyzing forces acting on an object.

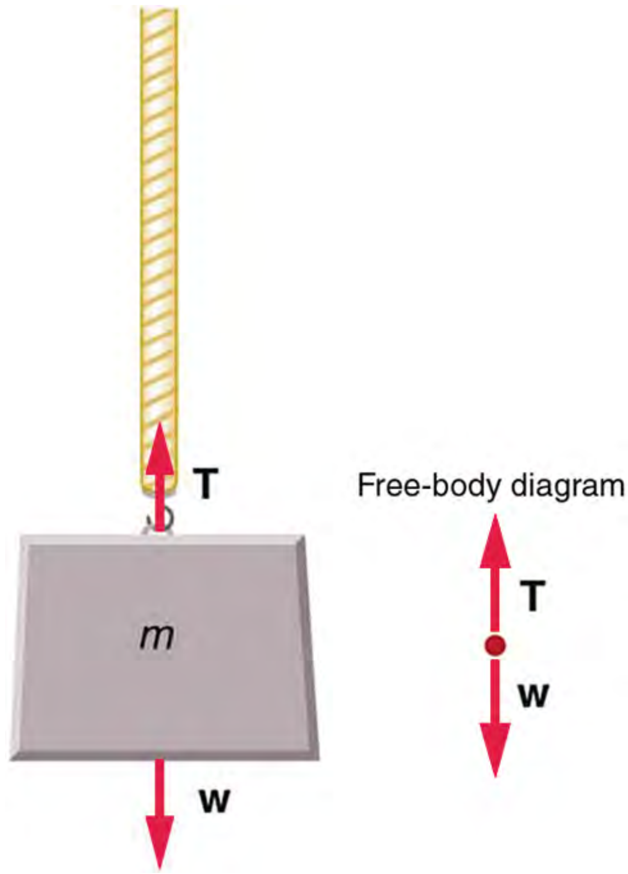


Figure 4.2 An object of mass, *m*, is held up by the force of tension.

Figure 4.2 shows the force of tension in the rope acting in the upward direction, opposite the force of gravity. The forces are indicated in the free-body diagram by an arrow pointing up, representing tension, and another arrow pointing down, representing gravity. In a free-body diagram, the lengths of the arrows show the relative magnitude (or strength) of the forces. Because forces are vectors, they add just like other vectors. Notice that the two arrows have equal lengths in Figure 4.2, which means that the forces of tension and weight are of equal magnitude. Because these forces of equal magnitude act in opposite directions, they are perfectly balanced, so they add together to give a net force of zero.

Not all forces are as noticeable as when you push or pull on an object. Some forces act without physical contact, such as the pull of a magnet (in the case of magnetic force) or the gravitational pull of Earth (in the case of gravitational force).

In the next three sections discussing Newton's laws of motion, we will learn about three specific types of forces: friction, the normal force, and the gravitational force. To analyze situations involving forces, we will create free-body diagrams to organize the framework of the mathematics for each individual situation.

TIPS FOR SUCCESS

Correctly drawing and labeling a free-body diagram is an important first step for solving a problem. It will help you visualize the problem and correctly apply the mathematics to solve the problem.

Check Your Understanding

1. What is kinematics?
   a. Kinematics is the study of motion.
   b. Kinematics is the study of the cause of motion.
   c. Kinematics is the study of dimensions.
   d. Kinematics is the study of atomic structures.

2. Do two bodies have to be in physical contact to exert a force upon one another?

a. No, the gravitational force is a field force and does not require physical contact to exert a force.
b. No, the gravitational force is a contact force and does not require physical contact to exert a force.
c. Yes, the gravitational force is a field force and requires physical contact to exert a force.
d. Yes, the gravitational force is a contact force and requires physical contact to exert a force.

3. What kind of physical quantity is force?
a. Force is a scalar quantity.
b. Force is a vector quantity.
c. Force is both a vector quantity and a scalar quantity.
d. Force is neither a vector nor a scalar quantity.

4. Which forces can be represented in a free-body diagram?
a. Internal forces
b. External forces
c. Both internal and external forces
d. A body that is not influenced by any force

4.2 Newton's First Law of Motion: Inertia

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Newton's first law and friction, and
- Discuss the relationship between mass and inertia.

Section Key Terms

friction	inertia	law of inertia
mass	Newton's first law of motion	system

Newton's First Law and Friction

Newton's first law of motion states the following:

1. A body at rest tends to remain at rest.
2. A body in motion tends to remain in motion at a constant velocity unless acted on by a net external force. (Recall that *constant velocity* means that the body moves in a straight line and at a constant speed.)

At first glance, this law may seem to contradict your everyday experience. You have probably noticed that a moving object will usually slow down and stop unless some effort is made to keep it moving. The key to understanding why, for example, a sliding box slows down (seemingly on its own) is to first understand that a net external force acts on the box to make the box slow down. Without this net external force, the box would continue to slide at a constant velocity (as stated in Newton's first law of motion). What force acts on the box to slow it down? This force is called **friction**. Friction is an external force that acts opposite to the direction of motion (see Figure 4.3). Think of friction as a resistance to motion that slows things down.

Consider an air hockey table. When the air is turned off, the puck slides only a short distance before friction slows it to a stop. However, when the air is turned on, it lifts the puck slightly, so the puck experiences very little friction as it moves over the surface. With friction almost eliminated, the puck glides along with very little change in speed. On a frictionless surface, the puck would experience no net external force (ignoring air resistance, which is also a form of friction). Additionally, if we know enough about friction, we can accurately predict how quickly objects will slow down.

Now let's think about another example. A man pushes a box across a floor at constant velocity by applying a force of +50 N. (The positive sign indicates that, by convention, the direction of motion is to the right.) What is the force of friction that opposes the motion? The force of friction must be –50 N. Why? According to Newton's first law of motion, any object moving at constant velocity has no net external force acting upon it, which means that the sum of the forces acting on the object must be zero. The mathematical way to say that no net external force acts on an object is $\mathbf{F}_{\text{net}} = 0$ or $\Sigma\mathbf{F} = 0$. So if the man applies +50 N of force, then the force of friction must be –50 N for the two forces to add up to zero (that is, for the two forces to *cancel* each other). Whenever you encounter the phrase *at constant velocity*, Newton's first law tells you that the net external force is zero.

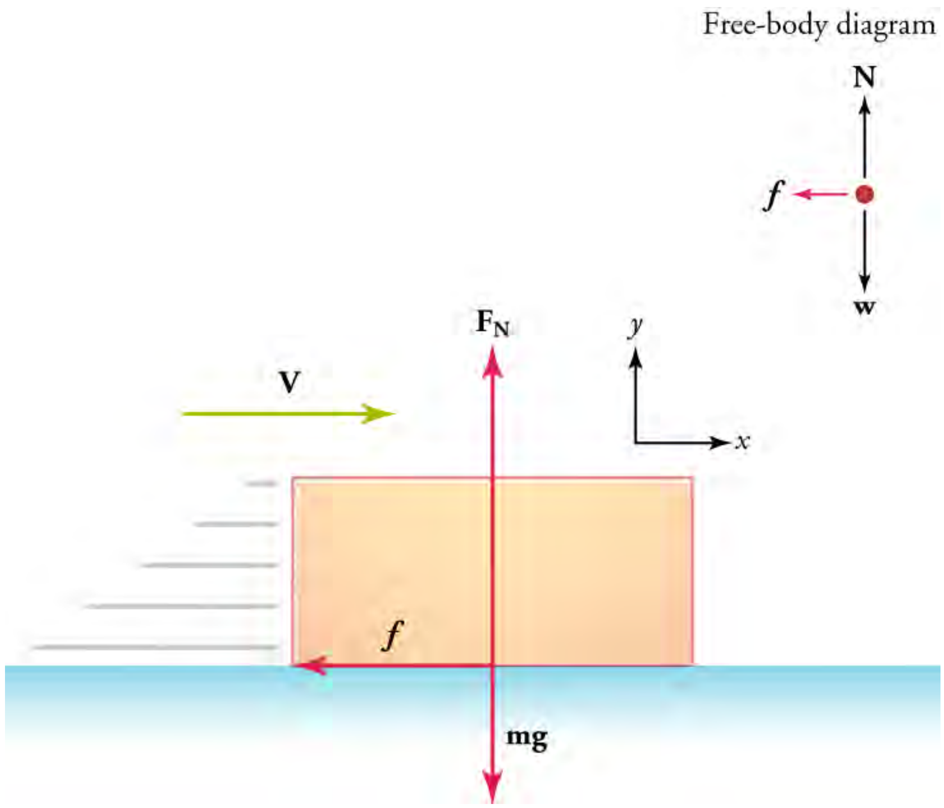


Figure 4.3 For a box sliding across a floor, friction acts in the direction opposite to the velocity.

The force of friction depends on two factors: the coefficient of friction and the normal force. For any two surfaces that are in contact with one another, the coefficient of friction is a constant that depends on the nature of the surfaces. The normal force is the force exerted by a surface that pushes on an object in response to gravity pulling the object down. In equation form, the force of friction is

$$\mathbf{f} = \mu \mathbf{N}, \tag{4.1}$$

where μ is the coefficient of friction and **N** is the normal force. (The coefficient of friction is discussed in more detail in another chapter, and the normal force is discussed in more detail in the section *Newton's Third Law of Motion*.)

Recall from the section on Force that a net external force acts from outside on the object of interest. A more precise definition is that it acts on the **system** of interest. A system is one or more objects that you choose to study. It is important to define the system at the beginning of a problem to figure out which forces are external and need to be considered, and which are internal and can be ignored.

For example, in Figure 4.4 (a), two children push a third child in a wagon at a constant velocity. The system of interest is the wagon plus the small child, as shown in part (b) of the figure. The two children behind the wagon exert external forces on this system ($\mathbf{F}_1$, $\mathbf{F}_2$). Friction *f* acting at the axles of the wheels and at the surface where the wheels touch the ground two other external forces acting on the system. Two more external forces act on the system: the weight **W** of the system pulling down and the normal force **N** of the ground pushing up. Notice that the wagon is not accelerating vertically, so Newton's first law tells us that the normal force balances the weight. Because the wagon is moving forward at a constant velocity, the force of friction must have the same strength as the sum of the forces applied by the two children.

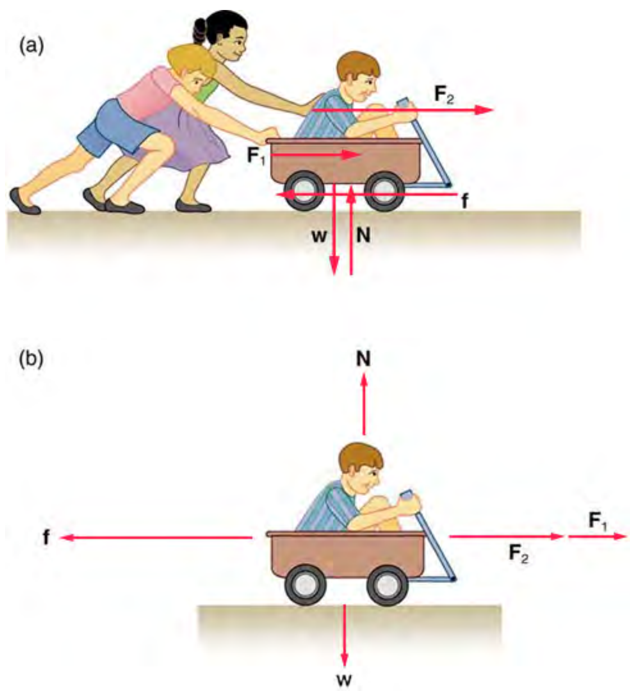


Figure 4.4 (a) The wagon and rider form a *system* that is acted on by external forces. (b) The two children pushing the wagon and child provide two external forces. Friction acting at the wheel axles and on the surface of the tires where they touch the ground provide an external force that act against the direction of motion. The weight **W** and the normal force **N** from the ground are two more external forces acting on the system. All external forces are represented in the figure by arrows. All of the external forces acting on the system add together, but because the wagon moves at a constant velocity, all of the forces must add up to zero.

Mass and Inertia

Inertia is the tendency for an object at rest to remain at rest, or for a moving object to remain in motion in a straight line with constant speed. This key property of objects was first described by Galileo. Later, Newton incorporated the concept of inertia into his first law, which is often referred to as the **law of inertia**.

As we know from experience, some objects have more inertia than others. For example, changing the motion of a large truck is more difficult than changing the motion of a toy truck. In fact, the inertia of an object is proportional to the mass of the object. **Mass** is a measure of the amount of matter (or *stuff*) in an object. The quantity or amount of matter in an object is determined by the number and types of atoms the object contains. Unlike weight (which changes if the gravitational force changes), mass does not depend on gravity. The mass of an object is the same on Earth, in orbit, or on the surface of the moon. In practice, it is very difficult to count and identify all of the atoms and molecules in an object, so mass is usually not determined this way. Instead, the mass of an object is determined by comparing it with the standard kilogram. Mass is therefore expressed in kilograms.

TIPS FOR SUCCESS

In everyday language, people often use the terms *weight* and *mass* interchangeably—but this is not correct. Weight is actually a force. (We cover this topic in more detail in the section *Newton's Second Law of Motion*.)

WATCH PHYSICS

Newton's First Law of Motion

This video contrasts the way we thought about motion and force in the time before Galileo's concept of inertia and Newton's first law of motion with the way we understand force and motion now.

[Click to view content (https://www.khanacademy.org/embed_video?v=5-ZFOhHQS68)](https://www.khanacademy.org/embed_video?v=5-ZFOhHQS68)

GRASP CHECK

Before we understood that objects have a tendency to maintain their velocity in a straight line unless acted upon by a net force, people thought that objects had a tendency to stop on their own. This happened because a specific force was not yet understood. What was that force?

a. Gravitational force
b. Electrostatic force
c. Nuclear force
d. Frictional force

Virtual Physics

Forces and Motion—Basics

In this simulation, you will first explore net force by placing blue people on the left side of a tug-of-war rope and red people on the right side of the rope (by clicking people and dragging them with your mouse). Experiment with changing the number and size of people on each side to see how it affects the outcome of the match and the net force. Hit the "Go!" button to start the match, and the "reset all" button to start over.

Next, click on the Friction tab. Try selecting different objects for the person to push. Slide the *applied force* button to the right to apply force to the right, and to the left to apply force to the left. The force will continue to be applied as long as you hold down the button. See the arrow representing friction change in magnitude and direction, depending on how much force you apply. Try increasing or decreasing the friction force to see how this change affects the motion.

Click to view content (https://phet.colorado.edu/sims/html/forces-and-motion-basics/latest/forces-and-motion-basics_en.html)

GRASP CHECK

Click on the tab for the *Acceleration Lab and check the Sum of Forces* option. Push the box to the right and then release. Notice which direction the sum of forces arrow points after the person stops pushing the box and lets it continue moving to the right on its own. At this point, in which direction is the net force, the sum of forces, pointing? Why?

a. The net force acts to the right because the applied external force acted to the right.
b. The net force acts to the left because the applied external force acted to the left.
c. The net force acts to the right because the frictional force acts to the right.
d. The net force acts to the left because the frictional force acts to the left.

Check Your Understanding

5. What does Newton's first law state?
   a. A body at rest tends to remain at rest and a body in motion tends to remain in motion at a constant acceleration unless acted on by a net external force.
   b. A body at rest tends to remain at rest and a body in motion tends to remain in motion at a constant velocity unless acted on by a net external force.
   c. The rate of change of momentum of a body is directly proportional to the external force applied to the body.
   d. The rate of change of momentum of a body is inversely proportional to the external force applied to the body.

6. According to Newton's first law, a body in motion tends to remain in motion at a constant velocity. However, when you slide an object across a surface, the object eventually slows down and stops. Why?
   a. The object experiences a frictional force exerted by the surface, which opposes its motion.
   b. The object experiences the gravitational force exerted by Earth, which opposes its motion
   c. The object experiences an internal force exerted by the body itself, which opposes its motion.
   d. The object experiences a pseudo-force from the body in motion, which opposes its motion.

7. What is inertia?
    a. Inertia is an object's tendency to maintain its mass.
    b. Inertia is an object's tendency to remain at rest.
    c. Inertia is an object's tendency to remain in motion
    d. Inertia is an object's tendency to remain at rest or, if moving, to remain in motion.

8. What is mass? What does it depend on?
    a. Mass is the weight of an object, and it depends on the gravitational force acting on the object.
    b. Mass is the weight of an object, and it depends on the number and types of atoms in the object.
    c. Mass is the quantity of matter contained in an object, and it depends on the gravitational force acting on the object.
    d. Mass is the quantity of matter contained in an object, and it depends on the number and types of atoms in the object.

4.3 Newton's Second Law of Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Newton's second law, both verbally and mathematically
- Use Newton's second law to solve problems

Section Key Terms

freefall	Newton's second law of motion	weight

Describing Newton's Second Law of Motion

Newton's first law considered bodies at rest or bodies in motion at a constant velocity. The other state of motion to consider is when an object is moving with a changing velocity, which means a change in the speed and/or the direction of motion. This type of motion is addressed by **Newton's second law of motion**, which states how force causes changes in motion. Newton's second law of motion is used to calculate what happens in situations involving forces and motion, and it shows the mathematical relationship between force, mass, and *acceleration*. Mathematically, the second law is most often written as

$$\mathbf{F}_{\text{net}} = m\mathbf{a} \text{ or } \Sigma\mathbf{F} = m\mathbf{a}, \tag{4.2}$$

where $\mathbf{F}_{\text{net}}$ (or $\sum\mathbf{F}$) is the net external force, m is the mass of the system, and $\mathbf{a}$ is the acceleration. Note that $\mathbf{F}_{\text{net}}$ and $\sum\mathbf{F}$ are the same because the net external force is the sum of all the external forces acting on the system.

First, what do we mean by *a change in motion*? A change in motion is simply a change in velocity: the speed of an object can become slower or faster, the direction in which the object is moving can change, or both of these variables may change. A change in velocity means, by definition, that an acceleration has occurred. Newton's first law says that only a nonzero net external force can cause a change in motion, so a net external force must cause an acceleration. Note that acceleration can refer to slowing down or to speeding up. Acceleration can also refer to a change in the direction of motion with no change in speed, because acceleration is the change in velocity divided by the time it takes for that change to occur, *and* velocity is defined by speed *and* direction.

From the equation $\mathrm{F}_{\text{net}} = m\mathrm{a}$, we see that force is directly proportional to both mass and acceleration, which makes sense. To accelerate two objects from rest to the same velocity, you would expect more force to be required to accelerate the more massive object. Likewise, for two objects of the same mass, applying a greater force to one would accelerate it to a greater velocity.

Now, let's rearrange Newton's second law to solve for acceleration. We get

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m} \text{ or } \mathbf{a} = \frac{\Sigma\mathbf{F}}{m}. \tag{4.3}$$

In this form, we can see that acceleration is directly proportional to force, which we write as

$$\mathbf{a} \propto \mathbf{F}_{\text{net}}, \tag{4.4}$$

where the symbol $\propto$ means *proportional to*.

This proportionality mathematically states what we just said in words: acceleration is directly proportional to the net external force. When two variables are directly proportional to each other, then if one variable doubles, the other variable must double. Likewise, if one variable is reduced by half, the other variable must also be reduced by half. In general, when one variable is multiplied by a number, the other variable is also multiplied by the same number. It seems reasonable that the acceleration of a system should be directly proportional to and in the same direction as the net external force acting on the system. An object experiences greater acceleration when acted on by a greater force.

It is also clear from the equation $\mathbf{a} = \mathbf{F}_{\text{net}}/m$ that acceleration is inversely proportional to mass, which we write as

$$\mathbf{a} \propto \frac{1}{m}. \tag{4.5}$$

Inversely proportional means that if one variable is multiplied by a number, the other variable must be *divided* by the same number. Now, it also seems reasonable that acceleration should be inversely proportional to the mass of the system. In other words, the larger the mass (the inertia), the smaller the acceleration produced by a given force. This relationship is illustrated in Figure 4.5, which shows that a given net external force applied to a basketball produces a much greater acceleration than when applied to a car.

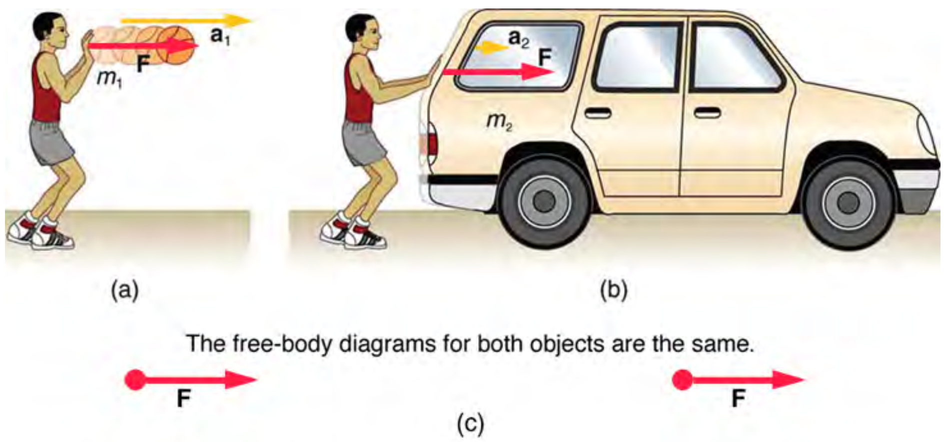


Figure 4.5 The same force exerted on systems of different masses produces different accelerations. (a) A boy pushes a basketball to make a pass. The effect of gravity on the ball is ignored. (b) The same boy pushing with identical force on a stalled car produces a far smaller acceleration (friction is negligible). Note that the free-body diagrams for the ball and for the car are identical, which allows us to compare the two situations.

Applying Newton's Second Law

Before putting Newton's second law into action, it is important to consider units. The equation $\mathbf{F}_{\text{net}} = m\mathbf{a}$ is used to define the units of force in terms of the three basic units of mass, length, and time (recall that acceleration has units of length divided by time squared). The SI unit of force is called the newton (abbreviated N) and is the force needed to accelerate a 1-kg system at the rate of 1 m/s^2. That is, because $\mathbf{F}_{\text{net}} = m\mathbf{a}$, we have

$$1\ \text{N} = 1\ \text{kg} \times 1\text{m/s}^2 = 1\frac{\text{kg} \cdot \text{m}}{\text{s}^2}. \tag{4.6}$$

One of the most important applications of Newton's second law is to calculate **weight** (also known as the gravitational force), which is usually represented mathematically as $\mathbf{W}$. When people talk about gravity, they don't always realize that it is an acceleration. When an object is dropped, it accelerates toward the center of Earth. Newton's second law states that the net external force acting on an object is responsible for the acceleration of the object. If air resistance is negligible, the net external force on a falling object is only the gravitational force (i.e., the weight of the object).

Weight can be represented by a vector because it has a direction. Down is defined as the direction in which gravity pulls, so weight is normally considered a downward force. By using Newton's second law, we can figure out the equation for weight.

Consider an object with mass m falling toward Earth. It experiences only the force of gravity (i.e., the gravitational force or weight), which is represented by W. Newton's second law states that $\mathbf{F}_{\text{net}} = m\mathbf{a}$. Because the only force acting on the object is the gravitational force, we have $\mathbf{F}_{\text{net}} = \mathbf{W}$. We know that the acceleration of an object due to gravity is $\mathbf{g}$, so we have $\mathbf{a} = \mathbf{g}$. Substituting these two expressions into Newton's second law gives

$$\mathbf{W} = m\mathbf{g}. \tag{4.7}$$

This is the equation for weight—the gravitational force on a mass m. On Earth, $\mathbf{g} = 9.80\ \text{m/s}^2$, so the weight (disregarding for now the direction of the weight) of a 1.0-kg object on Earth is

$$\mathbf{W} = m\mathbf{g} = (1.0\ \text{kg})(9.80\ \text{m/s}^2) = 9.8\ \ \text{N}. \tag{4.8}$$

Although most of the world uses newtons as the unit of force, in the United States the most familiar unit of force is the pound (lb), where 1 N = 0.225 lb.

Recall that although gravity acts downward, it can be assigned a positive or negative value, depending on what the positive direction is in your chosen coordinate system. Be sure to take this into consideration when solving problems with weight. When the downward direction is taken to be negative, as is often the case, acceleration due to gravity becomes $\mathbf{g} = -9.8\ \text{m/s}^2$.

When the net external force on an object is its weight, we say that it is in **freefall**. In this case, the only force acting on the object is the force of gravity. On the surface of Earth, when objects fall downward toward Earth, they are never truly in freefall because there is always some upward force due to air resistance that acts on the object (and there is also the buoyancy force of air, which is similar to the buoyancy force in water that keeps boats afloat).

Gravity varies slightly over the surface of Earth, so the weight of an object depends very slightly on its location on Earth. Weight varies dramatically away from Earth's surface. On the moon, for example, the acceleration due to gravity is only $1.67\ \text{m/s}^2$. Because weight depends on the force of gravity, a 1.0-kg mass weighs 9.8 N on Earth and only about 1.7 N on the moon.

It is important to remember that weight and mass are very different, although they are closely related. Mass is the quantity of matter (how much *stuff*) in an object and does not vary, but weight is the gravitational force on an object and is proportional to the force of gravity. It is easy to confuse the two, because our experience is confined to Earth, and the weight of an object is essentially the same no matter where you are on Earth. Adding to the confusion, the terms mass and weight are often used interchangeably in everyday language; for example, our medical records often show our weight in kilograms, but never in the correct unit of newtons.

Snap Lab

Mass and Weight

In this activity, you will use a scale to investigate mass and weight.

- 1 bathroom scale
- 1 table

1. What do bathroom scales measure?
2. When you stand on a bathroom scale, what happens to the scale? It depresses slightly. The scale contains springs that compress in proportion to your weight—similar to rubber bands expanding when pulled.
3. The springs provide a measure of your weight (provided you are not accelerating). This is a force in newtons (or pounds). In most countries, the measurement is now divided by 9.80 to give a reading in kilograms, which is a of mass. The scale detects weight but is calibrated to display mass.
4. If you went to the moon and stood on your scale, would it detect the same *mass* as it did on Earth?

GRASP CHECK

While standing on a bathroom scale, push down on a table next to you. What happens to the reading? Why?

a. The reading increases because part of your weight is applied to the table and the table exerts a matching force on you that acts in the direction of your weight.
b. The reading increases because part of your weight is applied to the table and the table exerts a matching force on you that acts in the direction opposite to your weight.
c. The reading decreases because part of your weight is applied to the table and the table exerts a matching force on you that acts in the direction of your weight.
d. The reading decreases because part of your weight is applied to the table and the table exerts a matching force on

you that acts in the direction opposite to your weight.

TIPS FOR SUCCESS

Only *net external force* impacts the acceleration of an object. If more than one force acts on an object and you calculate the acceleration by using only one of these forces, you will not get the correct acceleration for that object.

WATCH PHYSICS

Newton's Second Law of Motion

This video reviews Newton's second law of motion and how net external force and acceleration relate to one another and to mass. It also covers units of force, mass, and acceleration, and reviews a worked-out example.

[Click to view content (https://www.khanacademy.org/embed_video?v=ou9YMWlJgkE)](https://www.khanacademy.org/embed_video?v=ou9YMWlJgkE)

GRASP CHECK

True or False—If you want to reduce the acceleration of an object to half its original value, then you would need to reduce the net external force by half.

a. True
b. False

WORKED EXAMPLE

What Acceleration Can a Person Produce when Pushing a Lawn Mower?

Suppose that the net external force (push minus friction) exerted on a lawn mower is 51 N parallel to the ground. The mass of the mower is 240 kg. What is its acceleration?

Figure 4.6

Strategy

Because $\mathbf{F}_{\mathbf{net}}$ and m are given, the acceleration can be calculated directly from Newton's second law: $\mathbf{F}_{\mathbf{net}} = m\mathbf{a}$.

Solution

Solving Newton's second law for the acceleration, we find that the magnitude of the acceleration, $\mathbf{a}$, is $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$. Entering the given values for net external force and mass gives

$$\mathbf{a} = \frac{51 \text{ N}}{240 \text{ kg}} \tag{4.9}$$

Inserting the units $\text{kg} \cdot \text{m/s}^2$ for N yields

$$\mathbf{a} = \frac{51 \ \text{kg} \cdot \text{m/s}^2}{240 \ \text{kg}} = 0.21 \ \text{m/s}^2. \tag{4.10}$$

Discussion

The acceleration is in the same direction as the net external force, which is parallel to the ground and to the right. There is no information given in this example about the individual external forces acting on the system, but we can say something about their relative magnitudes. For example, the force exerted by the person pushing the mower must be greater than the friction opposing the motion, because we are given that the net external force is in the direction in which the person pushes. Also, the vertical forces must cancel if there is no acceleration in the vertical direction (the mower is moving only horizontally). The acceleration found is reasonable for a person pushing a mower; the mower's speed must increase by 0.21 m/s every second, which is possible. The time during which the mower accelerates would not be very long because the person's top speed would soon be reached. At this point, the person could push a little less hard, because he only has to overcome friction.

WORKED EXAMPLE

What Rocket Thrust Accelerates This Sled?

Prior to manned space flights, rocket sleds were used to test aircraft, missile equipment, and physiological effects on humans at high accelerations. Rocket sleds consisted of a platform mounted on one or two rails and propelled by several rockets. Calculate the magnitude of force exerted by each rocket, called its thrust, **T**, for the four-rocket propulsion system shown below. The sled's initial acceleration is $49 \ \text{m/s}^2$, the mass of the system is 2,100 kg, and the force of friction opposing the motion is 650 N.

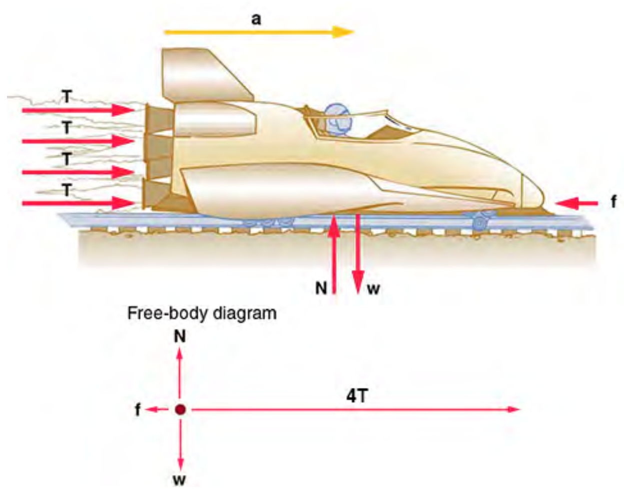


Figure 4.7

Strategy

The system of interest is the rocket sled. Although forces act vertically on the system, they must cancel because the system does not accelerate vertically. This leaves us with only horizontal forces to consider. We'll assign the direction to the right as the positive direction. See the free-body diagram in Figure 4.8.

Solution

We start with Newton's second law and look for ways to find the thrust **T** of the engines. Because all forces and acceleration are along a line, we need only consider the magnitudes of these quantities in the calculations. We begin with

$$\mathbf{F}_{\text{net}} = m\mathbf{a}, \tag{4.11}$$

where $\mathbf{F}_{\text{net}}$ is the net external force in the horizontal direction. We can see from Figure 4.8 that the engine thrusts are in the same direction (which we call the positive direction), whereas friction opposes the thrust. In equation form, the net external force is

$$\mathbf{F}_{\text{net}} = 4\mathbf{T} - \mathbf{f}. \tag{4.12}$$

Newton's second law tells us that $\mathbf{F}_{\text{net}} = m\mathbf{a}$, so we get

$$m\mathbf{a} = 4\mathbf{T} - \mathbf{f}. \tag{4.13}$$

After a little algebra, we solve for the total thrust 4**T**:

$$4\mathbf{T} = m\mathbf{a} + \mathbf{f}, \tag{4.14}$$

which means that the individual thrust is

$$\mathbf{T} = \frac{m\mathbf{a} + \mathbf{f}}{4}. \tag{4.15}$$

Inserting the known values yields

$$\mathbf{T} = \frac{(2100 \text{ kg})(49 \text{ m/s}^2) + 650 \text{ N}}{4} = 2.6 \times 10^4 \text{ N}. \tag{4.16}$$

Discussion

The numbers are quite large, so the result might surprise you. Experiments such as this were performed in the early 1960s to test the limits of human endurance and to test the apparatus designed to protect fighter pilots in emergency ejections. Speeds of 1000 km/h were obtained, with accelerations of 45 **g**. (Recall that g, the acceleration due to gravity, is 9.80 m/s^2. An acceleration of 45 **g** is $45 \times 9.80 \text{ m/s}^2$, which is approximately 440 m/s^2.) Living subjects are no longer used, and land speeds of 10,000 km/h have now been obtained with rocket sleds. In this example, as in the preceding example, the system of interest is clear. We will see in later examples that choosing the system of interest is crucial—and that the choice is not always obvious.

Practice Problems

9. If 1 N is equal to 0.225 lb, how many pounds is 5 N of force?
    a. 0.045 lb
    b. 1.125 lb
    c. 2.025 lb
    d. 5.000 lb

10. How much force needs to be applied to a 5-kg object for it to accelerate at 20 m/s^2?
    a. 1 N
    b. 10 N
    c. 100 N
    d. 1,000 N

Check Your Understanding

11. What is the mathematical statement for Newton's second law of motion?
    a. F = *m*a
    b. F = 2*m*a
    c. $F = \frac{m}{a}$
    d. F = *m*a^2

12. Newton's second law describes the relationship between which quantities?
    a. Force, mass, and time
    b. Force, mass, and displacement
    c. Force, mass, and velocity
    d. Force, mass, and acceleration

13. What is acceleration?
    a. Acceleration is the rate at which displacement changes.
    b. Acceleration is the rate at which force changes.
    c. Acceleration is the rate at which velocity changes.

d. Acceleration is the rate at which mass changes.

4.4 Newton's Third Law of Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Newton's third law, both verbally and mathematically
- Use Newton's third law to solve problems

Section Key Terms

Newton's third law of motion	normal force	tension	thrust

Describing Newton's Third Law of Motion

If you have ever stubbed your toe, you have noticed that although your toe initiates the impact, the surface that you stub it on exerts a force back on your toe. Although the first thought that crosses your mind is probably "ouch, that hurt" rather than "this is a great example of Newton's third law," both statements are true.

This is exactly what happens whenever one object exerts a force on another—each object experiences a force that is the same strength as the force acting on the other object but that acts in the opposite direction. Everyday experiences, such as stubbing a toe or throwing a ball, are all perfect examples of Newton's third law in action.

Newton's third law of motion states that whenever a first object exerts a force on a second object, the first object experiences a force equal in magnitude but opposite in direction to the force that it exerts.

Newton's third law of motion tells us that forces always occur in pairs, and one object cannot exert a force on another without experiencing the same strength force in return. We sometimes refer to these force pairs as *action-reaction* pairs, where the force exerted is the action, and the force experienced in return is the reaction (although which is which depends on your point of view).

Newton's third law is useful for figuring out which forces are external to a system. Recall that identifying external forces is important when setting up a problem, because the external forces must be added together to find the net force.

We can see Newton's third law at work by looking at how people move about. Consider a swimmer pushing off from the side of a pool, as illustrated in Figure 4.8. She pushes against the pool wall with her feet and accelerates in the direction opposite to her push. The wall has thus exerted on the swimmer a force of equal magnitude but in the direction opposite that of her push. You might think that two forces of equal magnitude but that act in opposite directions would cancel, *but they do not because they act on different systems.*

In this case, there are two different systems that we could choose to investigate: the swimmer or the wall. If we choose the swimmer to be the system of interest, as in the figure, then $\mathrm{F}_{\text{wall on feet}}$ is an external force on the swimmer and affects her motion. Because acceleration is in the same direction as the net external force, the swimmer moves in the direction of $\mathrm{F}_{\text{wall on feet}}$. Because the swimmer is our system (or object of interest) and not the wall, we do not need to consider the force $\mathrm{F}_{\text{feet on wall}}$ because it originates *from* the swimmer rather than *acting on* the swimmer. Therefore, $\mathrm{F}_{\text{feet on wall}}$ does not directly affect the motion of the system and does not cancel $\mathrm{F}_{\text{wall on feet}}$. Note that the swimmer pushes in the direction opposite to the direction in which she wants to move.

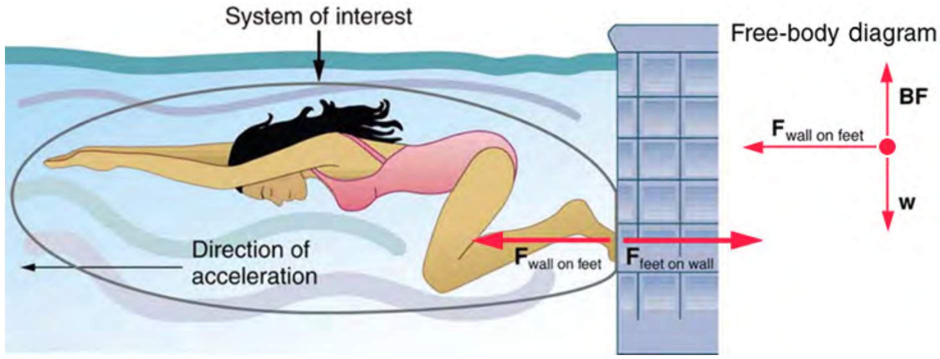


Figure 4.8 When the swimmer exerts a force $\mathbf{F}_{\text{feet on wall}}$ on the wall, she accelerates in the direction opposite to that of her push. This means that the net external force on her is in the direction opposite to $\mathbf{F}_{\text{feet on wall}}$. This opposition is the result of Newton's third law of motion, which dictates that the wall exerts a force $\mathbf{F}_{\text{wall on feet}}$ on the swimmer that is equal in magnitude but that acts in the direction opposite to the force that the swimmer exerts on the wall.

Other examples of Newton's third law are easy to find. As a teacher paces in front of a whiteboard, he exerts a force backward on the floor. The floor exerts a reaction force in the forward direction on the teacher that causes him to accelerate forward. Similarly, a car accelerates because the ground pushes forward on the car's wheels in reaction to the car's wheels pushing backward on the ground. You can see evidence of the wheels pushing backward when tires spin on a gravel road and throw rocks backward.

Another example is the force of a baseball as it makes contact with the bat. Helicopters create lift by pushing air down, creating an upward reaction force. Birds fly by exerting force on air in the direction opposite that in which they wish to fly. For example, the wings of a bird force air downward and backward in order to get lift and move forward. An octopus propels itself forward in the water by ejecting water backward through a funnel in its body, which is similar to how a jet ski is propelled. In these examples, the octopus or jet ski push the water backward, and the water, in turn, pushes the octopus or jet ski forward.

Applying Newton's Third Law

Forces are classified and given names based on their source, how they are transmitted, or their effects. In previous sections, we discussed the forces called *push*, *weight*, and *friction*. In this section, applying Newton's third law of motion will allow us to explore three more forces: the **normal force**, **tension**, and **thrust**. However, because we haven't yet covered vectors in depth, we'll only consider one-dimensional situations in this chapter. Another chapter will consider forces acting in two dimensions.

The gravitational force (or weight) acts on objects at all times and everywhere on Earth. We know from Newton's second law that a net force produces an acceleration; so, why is everything not in a constant state of freefall toward the center of Earth? The answer is the normal force. The normal force is the force that a surface applies to an object to support the weight of that object; it acts perpendicular to the surface upon which the object rests. If an object on a flat surface is not accelerating, the net external force is zero, and the normal force has the same magnitude as the weight of the system but acts in the opposite direction. In equation form, we write that

$$\mathbf{N} = m\mathbf{g}. \tag{4.17}$$

Note that this equation is only true for a horizontal surface.

The word *tension* comes from the Latin word meaning *to stretch*. Tension is the force along the length of a flexible connector, such as a string, rope, chain, or cable. Regardless of the type of connector attached to the object of interest, one must remember that the connector can only pull (or *exert tension*) in the direction *parallel* to its length. Tension is a pull that acts parallel to the connector, and that acts in opposite directions at the two ends of the connector. This is possible because a flexible connector is simply a long series of action-reaction forces, except at the two ends where outside objects provide one member of the action-reaction forces.

Consider a person holding a mass on a rope, as shown in Figure 4.9.

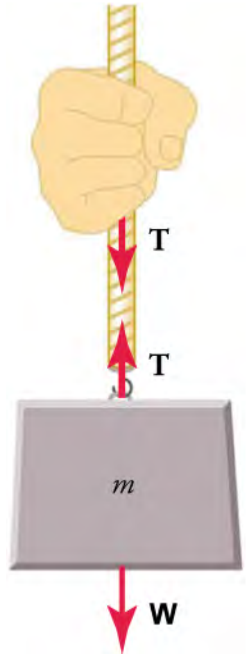


Figure 4.9 When a perfectly flexible connector (one requiring no force to bend it) such as a rope transmits a force **T**, this force must be parallel to the length of the rope, as shown. The pull that such a flexible connector exerts is a tension. Note that the rope pulls with equal magnitude force but in opposite directions to the hand and to the mass (neglecting the weight of the rope). This is an example of Newton's third law. The rope is the medium that transmits forces of equal magnitude between the two objects but that act in opposite directions.

Tension in the rope must equal the weight of the supported mass, as we can prove by using Newton's second law. If the 5.00 kg mass in the figure is stationary, then its acceleration is zero, so $\mathbf{F}_{\text{net}} = 0$. The only external forces acting on the mass are its weight **W** and the tension **T** supplied by the rope. Summing the external forces to find the net force, we obtain

$$\mathbf{F}_{\text{net}} = \mathbf{T} - \mathbf{W} = 0, \tag{4.18}$$

where **T** and **W** are the magnitudes of the tension and weight, respectively, and their signs indicate direction, with up being positive. By substituting $m\mathbf{g}$ for $\mathbf{F}_{\text{net}}$ and rearranging the equation, the tension equals the weight of the supported mass, just as you would expect

$$\mathbf{T} = \mathbf{W} = m\mathbf{g}. \tag{4.19}$$

For a 5.00-kg mass (neglecting the mass of the rope), we see that

$$\mathbf{T} = m\mathbf{g} = (5.00 \text{ kg})(9.80 \text{ m/s}^2) = 49.0 \text{ N}. \tag{4.20}$$

Another example of Newton's third law in action is thrust. Rockets move forward by expelling gas backward at a high velocity. This means that the rocket exerts a large force backward on the gas in the rocket combustion chamber, and the gas, in turn, exerts a large force forward on the rocket in response. This reaction force is called *thrust*.

TIPS FOR SUCCESS

A common misconception is that rockets propel themselves by pushing on the ground or on the air behind them. They actually work better in a vacuum, where they can expel exhaust gases more easily.

LINKS TO PHYSICS

Math: Problem-Solving Strategy for Newton's Laws of Motion

The basics of problem solving, presented earlier in this text, are followed here with specific strategies for applying Newton's laws of motion. These techniques also reinforce concepts that are useful in many other areas of physics.

First, identify the physical principles involved. If the problem involves forces, then Newton's laws of motion are involved, and it is important to draw a careful sketch of the situation. An example of a sketch is shown in Figure 4.10. Next, as in Figure 4.10, use vectors to represent all forces. Label the forces carefully, and make sure that their lengths are proportional to the magnitude of the forces and that the arrows point in the direction in which the forces act.

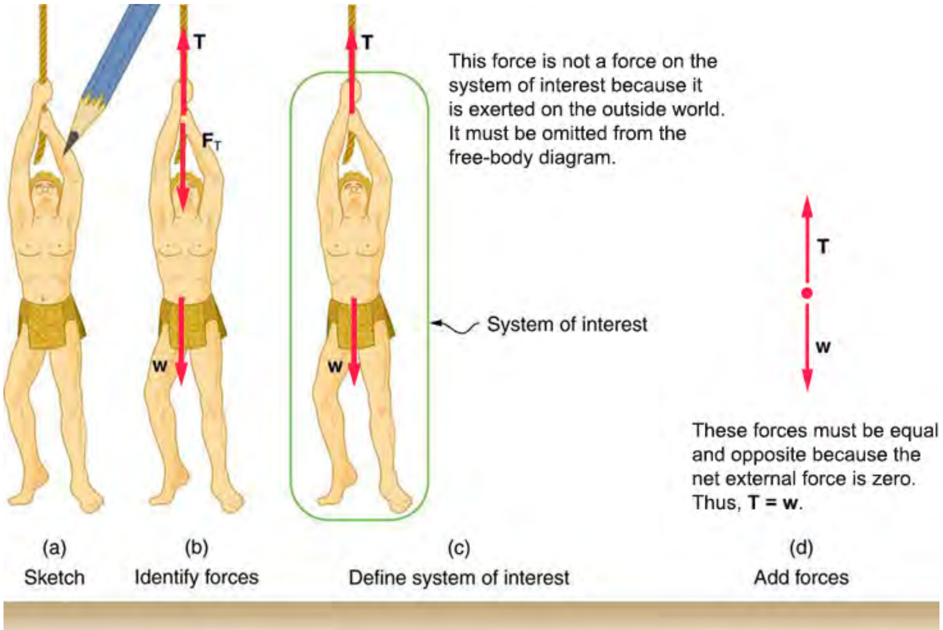


Figure 4.10 (a) A sketch of Tarzan hanging motionless from a vine. (b) Arrows are used to represent all forces. **T** is the tension exerted on Tarzan by the vine, $\mathbf{F}_\mathrm{T}$ is the force exerted on the vine by Tarzan, and **W** is Tarzan's weight (i.e., the force exerted on Tarzan by Earth's gravity). All other forces, such as a nudge of a breeze, are assumed to be negligible. (c) Suppose we are given Tarzan's mass and asked to find the tension in the vine. We define the system of interest as shown and draw a free-body diagram, as shown in (d). $\mathbf{F}_\mathrm{T}$ is no longer shown because it does not act on the system of interest; rather, $\mathbf{F}_\mathrm{T}$ acts on the outside world. (d) The free-body diagram shows only the external forces acting on Tarzan. For these to sum to zero, we must have $\mathbf{T} = \mathbf{W}$.

Next, make a list of knowns and unknowns and assign variable names to the quantities given in the problem. Figure out which variables need to be calculated; these are the unknowns. Now carefully define the system: which objects are of interest for the problem. This decision is important, because Newton's second law involves only external forces. Once the system is identified, it's possible to see which forces are external and which are internal (see Figure 4.10).

If the system acts on an object outside the system, then you know that the outside object exerts a force of equal magnitude but in the opposite direction on the system.

A diagram showing the system of interest and all the external forces acting on it is called a free-body diagram. Only external forces are shown on free-body diagrams, not acceleration or velocity. Figure 4.10 shows a free-body diagram for the system of interest.

After drawing a free-body diagram, apply Newton's second law to solve the problem. This is done in Figure 4.10 for the case of Tarzan hanging from a vine. When external forces are clearly identified in the free-body diagram, translate the forces into equation form and solve for the unknowns. Note that forces acting in opposite directions have opposite signs. By convention, forces acting downward or to the left are usually negative.

GRASP CHECK

If a problem has more than one system of interest, more than one free-body diagram is required to describe the external forces acting on the different systems.

a. True
b. False

WATCH PHYSICS

Newton's Third Law of Motion

This video explains Newton's third law of motion through examples involving push, normal force, and thrust (the force that propels a rocket or a jet).

Click to view content (https://www.openstax.org/l/astronaut)

GRASP CHECK

If the astronaut in the video wanted to move upward, in which direction should he throw the object? Why?

a. He should throw the object upward because according to Newton's third law, the object will then exert a force on him in the same direction (i.e., upward).
b. He should throw the object upward because according to Newton's third law, the object will then exert a force on him in the opposite direction (i.e., downward).
c. He should throw the object downward because according to Newton's third law, the object will then exert a force on him in the opposite direction (i.e., upward).
d. He should throw the object downward because according to Newton's third law, the object will then exert a force on him in the same direction (i.e., downward).

WORKED EXAMPLE

An Accelerating Subway Train

A physics teacher pushes a cart of demonstration equipment to a classroom, as in Figure 4.11. Her mass is 65.0 kg, the cart's mass is 12.0 kg, and the equipment's mass is 7.0 kg. To push the cart forward, the teacher's foot applies a force of 150 N in the opposite direction (backward) on the floor. Calculate the acceleration produced by the teacher. The force of friction, which opposes the motion, is 24.0 N.

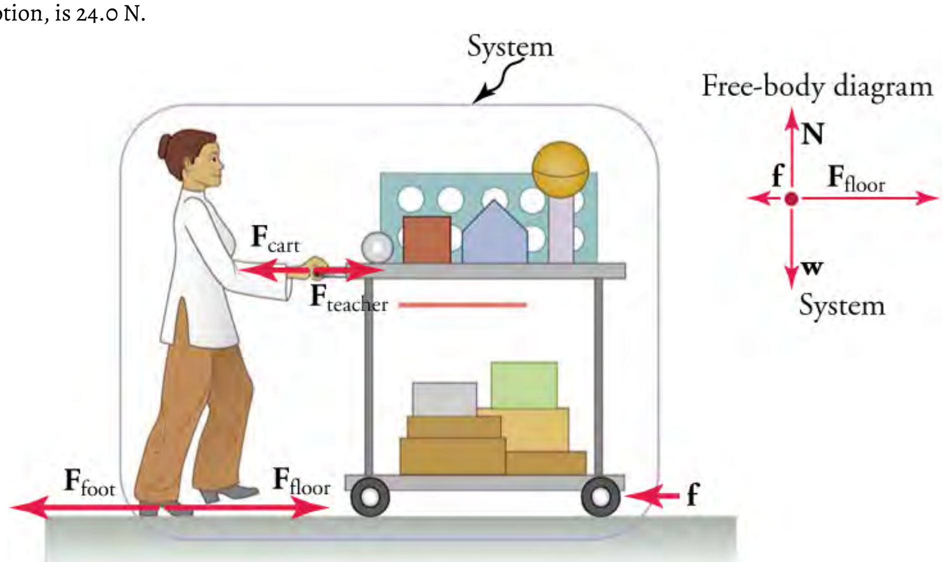


Figure 4.11

Strategy

Because they accelerate together, we define the system to be the teacher, the cart, and the equipment. The teacher pushes backward with a force $\mathbf{F}_{\text{foot}}$ of 150 N. According to Newton's third law, the floor exerts a forward force $\mathbf{F}_{\text{floor}}$ of 150 N on the system. Because all motion is horizontal, we can assume that no net force acts in the vertical direction, and the problem becomes one dimensional. As noted in the figure, the friction f opposes the motion and therefore acts opposite the direction of $\mathbf{F}_{\text{floor}}$.

We should not include the forces $\mathbf{F}_{\text{teacher}}$, $\mathbf{F}_{\text{cart}}$, or $\mathbf{F}_{\text{foot}}$ because these are exerted *by* the system, not *on* the system. We find the net external force by adding together the external forces acting on the system (see the free-body diagram in the figure) and then use Newton's second law to find the acceleration.

Solution

Newton's second law is

$$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}. \tag{4.21}$$

The net external force on the system is the sum of the external forces: the force of the floor acting on the teacher, cart, and equipment (in the horizontal direction) and the force of friction. Because friction acts in the opposite direction, we assign it a negative value. Thus, for the net force, we obtain

$$\mathbf{F}_{\text{net}} = \mathbf{F}_{\text{floor}} - \mathbf{f} = 150\ \text{N} - 24.0\ \text{N} = 126\ \text{N}. \tag{4.22}$$

The mass of the system is the sum of the mass of the teacher, cart, and equipment.

$$m = (65.0 + 12.0 + 7.0)\ \text{kg} = 84\ \text{kg} \tag{4.23}$$

Insert these values of net F and m into Newton's second law to obtain the acceleration of the system.

$$\begin{aligned} \mathbf{a} &= \frac{\mathbf{F}_{\text{net}}}{m} \\ a &= \frac{126\ \text{N}}{84\ \text{kg}} = 1.5\ \text{m/s}^2 \end{aligned} \tag{4.24}$$

$$F_1 < F_2 \tag{4.25}$$

Discussion

None of the forces between components of the system, such as between the teacher's hands and the cart, contribute to the net external force because they are internal to the system. Another way to look at this is to note that the forces between components of a system cancel because they are equal in magnitude and opposite in direction. For example, the force exerted by the teacher on the cart is of equal magnitude but in the opposite direction of the force exerted by the cart on the teacher. In this case, both forces act on the same system, so they cancel. Defining the system was crucial to solving this problem.

Practice Problems

14. What is the equation for the normal force for a body with mass m that is at rest on a horizontal surface?
  a. N = m
  b. N = mg
  c. N = mv
  d. N = g

15. An object with mass m is at rest on the floor. What is the magnitude and direction of the normal force acting on it?
  a. N = mv in upward direction
  b. N = mg in upward direction
  c. N = mv in downward direction
  d. N = mg in downward direction

Check Your Understanding

16. What is Newton's third law of motion?
  a. Whenever a first body exerts a force on a second body, the first body experiences a force that is twice the magnitude and acts in the direction of the applied force.
  b. Whenever a first body exerts a force on a second body, the first body experiences a force that is equal in magnitude and acts in the direction of the applied force.
  c. Whenever a first body exerts a force on a second body, the first body experiences a force that is twice the magnitude but acts in the direction opposite the direction of the applied force.
  d. Whenever a first body exerts a force on a second body, the first body experiences a force that is equal in magnitude but

acts in the direction opposite the direction of the applied force.

17. Considering Newton's third law, why don't two equal and opposite forces cancel out each other?

a. Because the two forces act in the same direction
b. Because the two forces have different magnitudes
c. Because the two forces act on different systems
d. Because the two forces act in perpendicular directions

KEY TERMS

dynamics the study of how forces affect the motion of objects and systems

external force a force acting on an object or system that originates outside of the object or system

force a push or pull on an object with a specific magnitude and direction; can be represented by vectors; can be expressed as a multiple of a standard force

free-body diagram a diagram showing all external forces acting on a body

freefall a situation in which the only force acting on an object is the force of gravity

friction an external force that acts in the direction opposite to the direction of motion

inertia the tendency of an object at rest to remain at rest, or for a moving object to remain in motion in a straight line and at a constant speed

law of inertia Newton's first law of motion: a body at rest remains at rest or, if in motion, remains in motion at a constant speed in a straight line, unless acted on by a net external force; also known as the law of inertia

mass the quantity of matter in a substance; measured in kilograms

net external force the sum of all external forces acting on an object or system

net force the sum of all forces acting on an object or system

Newton's first law of motion a body at rest remains at rest or, if in motion, remains in motion at a constant speed in a straight line, unless acted on by a net external force; also known as the law of inertia

Newton's second law of motion the net external force, $\mathbf{F}_{\text{net}}$, on an object is proportional to and in the same direction as the acceleration of the object, $\mathbf{a}$, and also proportional to the object's mass, m; defined mathematically as $\mathbf{F}_{\text{net}} = m\mathbf{a}$ or $\Sigma\mathbf{F} = m\mathbf{a}$.

Newton's third law of motion when one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts

normal force the force that a surface applies to an object; acts perpendicular and away from the surface with which the object is in contact

system one or more objects of interest for which only the forces acting on them from the outside are considered, but not the forces acting between them or inside them

tension a pulling force that acts along a connecting medium, especially a stretched flexible connector, such as a rope or cable; when a rope supports the weight of an object, the force exerted on the object by the rope is called tension

thrust a force that pushes an object forward in response to the backward ejection of mass by the object; rockets and airplanes are pushed forward by a thrust reaction force in response to ejecting gases backward

weight the force of gravity, $\mathbf{W}$, acting on an object of mass m; defined mathematically as $\mathbf{W} = m\mathbf{g}$, where $\mathbf{g}$ is the magnitude and direction of the acceleration due to gravity

SECTION SUMMARY

4.1 Force

- Dynamics is the study of how forces affect the motion of objects and systems.
- Force is a push or pull that can be defined in terms of various standards. It is a vector and so has both magnitude and direction.
- External forces are any forces outside of a body that act on the body. A free-body diagram is a drawing of all external forces acting on a body.

4.2 Newton's First Law of Motion: Inertia

- Newton's first law states that a body at rest remains at rest or, if moving, remains in motion in a straight line at a constant speed, unless acted on by a net external force. This law is also known as the law of inertia.
- Inertia is the tendency of an object at rest to remain at rest or, if moving, to remain in motion at constant velocity. Inertia is related to an object's mass.
- Friction is a force that opposes motion and causes an object or system to slow down.
- Mass is the quantity of matter in a substance.

4.3 Newton's Second Law of Motion

- Acceleration is a change in velocity, meaning a change in speed, direction, or both.
- An external force acts on a system from outside the system, as opposed to internal forces, which act between components within the system.
- Newton's second law of motion states that the acceleration of a system is directly proportional to and in the same direction as the net external force acting on the system, and inversely proportional to the system's mass.
- In equation form, Newton's second law of motion is $\mathbf{F}_{\text{net}} = m\mathbf{a}$ or $\Sigma\mathbf{F} = m\mathbf{a}$. This is sometimes written as $\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$ or $\mathbf{a} = \frac{\Sigma\mathbf{F}}{m}$.
- The weight of an object of mass m is the force of gravity that acts on it. From Newton's second law, weight is

given by $\mathbf{W} = m\mathbf{g}$.

- If the only force acting on an object is its weight, then the object is in freefall.

4.4 Newton's Third Law of Motion

- Newton's third law of motion states that when one body exerts a force on a second body, the first body experiences a force that is equal in magnitude and opposite in direction to the force that it exerts.
- When an object rests on a surface, the surface applies a force on the object that opposes the weight of the object. This force acts perpendicular to the surface and is called the normal force.
- The pulling force that acts along a stretched flexible connector, such as a rope or cable, is called tension. When a rope supports the weight of an object at rest, the tension in the rope is equal to the weight of the object.
- Thrust is a force that pushes an object forward in response to the backward ejection of mass by the object. Rockets and airplanes are pushed forward by thrust.

KEY EQUATIONS

4.2 Newton's First Law of Motion: Inertia

Newton's first law of motion	$\mathbf{F}_{\text{net}} = 0$ or $\Sigma\mathbf{F} = 0$

4.3 Newton's Second Law of Motion

Newton's second law of motion	$\mathbf{F}_{\text{net}} = m\mathbf{a}$ or $\Sigma\mathbf{F} = m\mathbf{a}$
Newton's second law of motion to solve acceleration	$\mathbf{a} = \frac{\mathbf{F}_{\text{net}}}{m}$ or $\mathbf{a} = \frac{\Sigma\mathbf{F}}{m}$
Newton's second law of motion to solve weight	$\mathbf{W} = m\mathbf{g}$

4.4 Newton's Third Law of Motion

normal force for a nonaccelerating horizontal surface	$\mathbf{N} = m\mathbf{g}$
tension for an object at rest	$\mathbf{T} = m\mathbf{g}$

CHAPTER REVIEW

Concept Items

4.1 Force

1. What is dynamics?

a. Dynamics is the study of internal forces.
b. Dynamics is the study of forces and their effect on motion.
c. Dynamics describes the motion of points, bodies, and systems without consideration of the cause of motion.
d. Dynamics describes the effect of forces on each other.

2. Two forces acting on an object are perpendicular to one another. How would you draw these in a free-body diagram?

a. The two force arrows will be drawn at a right angle to one another.
b. The two force arrows will be pointing in opposite directions.
c. The two force arrows will be at a 45° angle to one another.
d. The two force arrows will be at a 180° angle to one another.

3. A free-body diagram shows the forces acting on an object. How is that object represented in the diagram?

a. A single point
b. A square box
c. A unit circle
d. The object as it is

4.2 Newton's First Law of Motion: Inertia

4. A ball rolls along the ground, moving from north to south. What direction is the frictional force that acts on the ball?

a. North to south
b. South to north
c. West to east
d. East to west

5. The tires you choose to drive over icy roads will create more friction with the road than your summer tires. Give another example where more friction is desirable.

a. Children's slide
b. Air hockey table
c. Ice-skating rink
d. Jogging track

6. How do you express, mathematically, that no external force is acting on a body?
   a. $F_{net} = -1$
   b. $F_{net} = 0$
   c. $F_{net} = 1$
   d. $F_{net} = \infty$

4.3 Newton's Second Law of Motion

7. What does it mean for two quantities to be inversely proportional to each other?
   a. When one variable increases, the other variable decreases by a greater amount.
   b. When one variable increases, the other variable also increases.
   c. When one variable increases, the other variable decreases by the same factor.
   d. When one variable increases, the other variable also increases by the same factor.

8. True or False: Newton's second law can be interpreted based on Newton's first law.
   a. True
   b. False

4.4 Newton's Third Law of Motion

9. Which forces cause changes in the motion of a system?
   a. internal forces
   b. external forces
   c. both internal and external forces
   d. neither internal nor external forces

10. True or False—Newton's third law applies to the external forces acting on a system of interest.
    a. True
    b. False

11. A ball is dropped and hits the floor. What is the direction of the force exerted by the floor on the ball?
    a. Upward
    b. Downward
    c. Right
    d. Left

Critical Thinking Items

4.1 Force

12. Only two forces are acting on an object: force A to the left and force B to the right. If force B is greater than force A, in which direction will the object move?
    a. To the right
    b. To the left
    c. Upward
    d. The object does not move

13. In a free-body diagram, the arrows representing tension and weight have the same length but point away from one another. What does this indicate?
    a. They are equal in magnitude and act in the same direction.
    b. They are equal in magnitude and act in opposite directions.
    c. They are unequal in magnitude and act in the same direction.
    d. They are unequal in magnitude and act in opposite directions.

14. An object is at rest. Two forces, X and Y, are acting on it. Force X has a magnitude of x and acts in the downward direction. What is the magnitude and direction of Y?
    a. The magnitude is x and points in the upward direction.
    b. The magnitude is $2x$ and points in the upward direction.
    c. The magnitude is x and points in the downward direction.
    d. The magnitude is $2x$ and points in the downward direction.

15. Three forces, A, B, and C, are acting on the same object with magnitudes *a, b*, and *c*, respectively. Force A acts to the right, force B acts to the left, and force C acts downward. What is a necessary condition for the object to move straight down?
    a. The magnitude of force A must be greater than the magnitude of force B, so $a > b$.
    b. The magnitude of force A must be equal to the magnitude of force B, so $a = b$.
    c. The magnitude of force A must be greater than the magnitude of force C, so $A > C$.
    d. The magnitude of force C must be greater than the magnitude of forces A or B, so $A < C > B$.

4.2 Newton's First Law of Motion: Inertia

16. Two people push a cart on a horizontal surface by applying forces F_1 and F_2 in the same direction. Is the magnitude of the net force acting on the cart, F_{net}, equal to, greater than, or less than $F_1 + F_2$? Why?
    a. $F_{net} < F_1 + F_2$ because the net force will not include the frictional force.
    b. $F_{net} = F_1 + F_2$ because the net force will not include

the frictional force

c. $F_{net} < F_1 + F_2$ because the net force will include the component of frictional force

d. $F_{net} = F_1 + F_2$ because the net force will include the frictional force

17. True or False: A book placed on a balance scale is balanced by a standard 1-kg iron weight placed on the opposite side of the balance. If these objects are taken to the moon and a similar exercise is performed, the balance is still level because gravity is uniform on the moon's surface as it is on Earth's surface.

a. True

b. False

4.3 Newton's Second Law of Motion

18. From the equation for Newton's second law, we see that F_{net} is directly proportional to **a** and that the constant of proportionality is ***m***. What does this mean in a practical sense?

a. An increase in applied force will cause an increase in acceleration if the mass is constant.

b. An increase in applied force will cause a decrease in acceleration if the mass is constant.

c. An increase in applied force will cause an increase in acceleration, even if the mass varies.

d. An increase in applied force will cause an increase in acceleration and mass.

4.4 Newton's Third Law of Motion

19. True or False: A person accelerates while walking on the ground by exerting force. The ground in turn exerts force F_2 on the person. F_1 and F_2 are equal in magnitude but act in opposite directions. The person is able to walk because the two forces act on the different systems and the net force acting on the person is nonzero.

a. True

b. False

20. A helicopter pushes air down, which, in turn, pushes the helicopter up. Which force affects the helicopter's motion? Why?

a. Air pushing upward affects the helicopter's motion because it is an internal force that acts on the helicopter.

b. Air pushing upward affects the helicopter's motion because it is an external force that acts on the helicopter.

c. The downward force applied by the blades of the helicopter affects its motion because it is an internal force that acts on the helicopter.

d. The downward force applied by the blades of the helicopter affects its motion because it is an external force that acts on the helicopter.

Problems

4.3 Newton's Second Law of Motion

21. An object has a mass of 1 kg on Earth. What is its weight on the moon?

a. 1 N

b. 1.67 N

c. 9.8 N

d. 10 N

22. A bathroom scale shows your mass as 55 kg. What will it read on the moon?

a. 9.4 kg

b. 10.5 kg

c. 55.0 kg

d. 91.9 kg

4.4 Newton's Third Law of Motion

23. A person pushes an object of mass 5.0 kg along the floor by applying a force. If the object experiences a friction force of 10 N and accelerates at 18 m/s^2, what is the magnitude of the force exerted by the person?

a. −90 N

b. −80 N

c. 90 N

d. 100 N

Performance Task

4.4 Newton's Third Law of Motion

24. A car weighs 2,000 kg. It moves along a road by applying a force on the road with a parallel component of 560 N. There are two passengers in the car, each weighing 55 kg. If the magnitude of the force of friction experienced by the car is 45 N, what is the acceleration of the car?

a. 0.244 m/s^2

b. 0.265 m/s^2

c. 4.00 m/s^2

d. 4.10 m/s^2

TEST PREP

Multiple Choice

4.1 Force

25. Which of the following is a physical quantity that can be described by dynamics but not by kinematics?
a. Velocity
b. Acceleration
c. Force

26. Which of the following is used to represent an object in a free-body diagram?
a. A point
b. A line
c. A vector

4.2 Newton's First Law of Motion: Inertia

27. What kind of force is friction?
a. External force
b. Internal force
c. Net force

28. What is another name for Newton's first law?
a. Law of infinite motion
b. Law of inertia
c. Law of friction

29. True or False—A rocket is launched into space and escapes Earth's gravitational pull. It will continue to travel in a straight line until it is acted on by another force.
a. True
b. False

30. A 2,000-kg car is sitting at rest in a parking lot. A bike and rider with a total mass of 60 kg are traveling along a road at 10 km/h. Which system has more inertia? Why?
a. The car has more inertia, as its mass is greater than the mass of the bike.
b. The bike has more inertia, as its mass is greater than the mass of the car.
c. The car has more inertia, as its mass is less than the mass of the bike.
d. The bike has more inertia, as its mass is less than the mass of the car.

4.3 Newton's Second Law of Motion

31. In the equation for Newton's second law, what does F_{net} stand for?
a. Internal force
b. Net external force
c. Frictional force

32. What is the SI unit of force?
a. Kg
b. dyn
c. N

33. What is the net external force on an object in freefall on Earth if you were to neglect the effect of air?
a. The net force is zero.
b. The net force is upward with magnitude *mg*.
c. The net force is downward with magnitude *mg*.
d. The net force is downward with magnitude 9.8 N.

34. Two people push a 2,000-kg car to get it started. An acceleration of at least 5.0 m/s^2 is required to start the car. Assuming both people apply the same magnitude force, how much force will each need to apply if friction between the car and the road is 300 N?
a. 4850 N
b. 5150 N
c. 97000 N
d. 10300 N

4.4 Newton's Third Law of Motion

35. One object exerts a force of magnitude F_1 on another object and experiences a force of magnitude F_2 in return. What is true for F_1 and F_2?
a. $F_1 > F_2$
b. $F_1 < F_2$
c. $F_1 = F_2$

36. A weight is suspended with a rope and hangs freely. In what direction is the tension on the rope?
a. parallel to the rope
b. perpendicular to the rope

37. A person weighing 55 kg walks by applying 160 N of force on the ground, while pushing a 10-kg object. If the person accelerates at 2 m/s^2, what is the force of friction experienced by the system consisting of the person and the object?
a. 30 N
b. 50 N
c. 270 N
d. 290 N

38. A 65-kg swimmer pushes on the pool wall and accelerates at 6 m/s^2. The friction experienced by the swimmer is 100 N. What is the magnitude of the force that the swimmer applies on the wall?
a. −490 N
b. −290 N
c. 290 N
d. 490 N

Short Answer

4.1 Force

39. True or False—An external force is defined as a force generated outside the system of interest that acts on an object inside the system.
    a. True
    b. False

40. By convention, which sign is assigned to an object moving downward?
    a. A positive sign (+)
    b. A negative sign (−)
    c. Either a positive or negative sign (±)
    d. No sign is assigned

41. A body is pushed downward by a force of 5 units and upward by a force of 2 units. How would you draw a free-body diagram to represent this?
    a. Two force vectors acting at a point, both pointing up with lengths of 5 units and 2 units
    b. Two force vectors acting at a point, both pointing down with lengths of 5 units and 2 units
    c. Two force vectors acting at a point, one pointing up with a length of 5 units and the other pointing down with a length of 2 units
    d. Two force vectors acting at a point, one pointing down with a length of 5 units and the other pointing up with a length of 2 units

42. A body is pushed eastward by a force of four units and southward by a force of three units. How would you draw a free-body diagram to represent this?
    a. Two force vectors acting at a point, one pointing left with a length of 4 units and the other pointing down with a length of 3 units
    b. Two force vectors acting at a point, one pointing left with a length of 4 units and the other pointing up with a length of 3 units
    c. Two force vectors acting at a point, one pointing right with a length of 4 units and the other pointing down with a length of 3 units
    d. Two force vectors acting at a point, one pointing right with a length of 4 units and the other pointing up with a length of 3 units

4.2 Newton's First Law of Motion: Inertia

43. A body with mass m is pushed along a horizontal surface by a force F and is opposed by a frictional force *f*. How would you draw a free-body diagram to represent this situation?
    a. A dot with an arrow pointing right, labeled F, and an arrow pointing left, labeled *f*, that is of equal length or shorter than F
    b. A dot with an arrow pointing right, labeled F, and an arrow pointing right, labeled *f*, that is of equal length or shorter than F
    c. A dot with an arrow pointing right, labeled F, and a smaller arrow pointing up, labeled *f*, that is of equal length or longer than F
    d. A dot with an arrow pointing right, labeled F, and a smaller arrow pointing down, labeled *f*, that is of equal length or longer than F

44. Two objects rest on a uniform surface. A person pushes both with equal force. If the first object starts to move faster than the second, what can be said about their masses?
    a. The mass of the first object is less than that of the second object.
    b. The mass of the first object is equal to the mass of the second object.
    c. The mass of the first object is greater than that of the second object.
    d. No inference can be made because mass and force are not related to each other.

45. Two similar boxes rest on a table. One is empty and the other is filled with pebbles. Without opening or lifting either, how can you tell which box is full? Why?
    a. By applying an internal force; whichever box accelerates faster is lighter and so must be empty
    b. By applying an internal force; whichever box accelerates faster is heavier and so the other box must be empty
    c. By applying an external force; whichever box accelerates faster is lighter and so must be empty
    d. By applying an external force; whichever box accelerates faster is heavier and so the other box must be empty

46. True or False—An external force is required to set a stationary object in motion in outer space away from all gravitational influences and atmospheric friction.
    a. True
    b. False

4.3 Newton's Second Law of Motion

47. A steadily rolling ball is pushed in the direction from east to west, which causes the ball to move faster in the same direction. What is the direction of the acceleration?
    a. North to south
    b. South to north
    c. East to west
    d. West to east

48. A ball travels from north to south at 60 km/h. After being hit by a bat, it travels from west to east at 60 km/

h. Is there a change in velocity?
 a. Yes, because velocity is a scalar.
 b. Yes, because velocity is a vector.
 c. No, because velocity is a scalar.
 d. No, because velocity is a vector

49. What is the weight of a 5-kg object on Earth and on the moon?
 a. On Earth the weight is 1.67 N, and on the moon the weight is 1.67 N.
 b. On Earth the weight is 5 N, and on the moon the weight is 5 N.
 c. On Earth the weight is 49 N, and on the moon the weight is 8.35 N.
 d. On Earth the weight is 8.35 N, and on the moon the weight is 49 N.

50. An object weighs 294 N on Earth. What is its weight on the moon?
 a. 50.1 N
 b. 30.0 N
 c. 249 N
 d. 1461 N

4.4 Newton's Third Law of Motion

51. A large truck with mass 30 m crashes into a small sedan with mass *m*. If the truck exerts a force F on the sedan, what force will the sedan exert on the truck?
 a. $\frac{F}{30}$
 b. F
 c. 2F
 d. 30F

52. A fish pushes water backward with its fins. How does this propel the fish forward?
 a. The water exerts an internal force on the fish in the opposite direction, pushing the fish forward.
 b. The water exerts an external force on the fish in the opposite direction, pushing the fish forward.
 c. The water exerts an internal force on the fish in the same direction, pushing the fish forward.
 d. The water exerts an external force on the fish in the same direction, pushing the fish forward.

53. True or False—Tension is the result of opposite forces in a connector, such as a string, rope, chain or cable, that pulls each point of the connector apart in the direction parallel to the length of the connector. At the ends of the connector, the tension pulls toward the center of the connector.
 a. True
 b. False

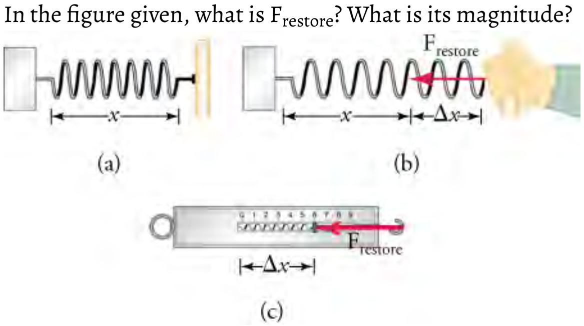
54. True or False—Normal reaction is the force that opposes the force of gravity and acts in the direction of the force of gravity.
 a. True
 b. False

Extended Response

4.1 Force

55. True or False—When two unequal forces act on a body, the body will not move in the direction of the weaker force.
 a. True
 b. False

56. In the figure given, what is $F_{restore}$? What is its magnitude?



 a. $F_{restore}$ is the force exerted by the hand on the spring, and it pulls to the right.
 b. $F_{restore}$ is the force exerted by the spring on the hand, and it pulls to the left.
 c. $F_{restore}$ is the force exerted by the hand on the spring, and it pulls to the left.
 d. $F_{restore}$ is the force exerted by the spring on the hand, and it pulls to the right.

4.2 Newton's First Law of Motion: Inertia

57. Two people apply the same force to throw two identical balls in the air. Will the balls necessarily travel the same distance? Why or why not?
 a. No, the balls will not necessarily travel the same distance because the gravitational force acting on them is different.
 b. No, the balls will not necessarily travel the same distance because the angle at which they are thrown may differ.
 c. Yes, the balls will travel the same distance because the gravitational force acting on them is the same.
 d. Yes, the balls will travel the same distance because the angle at which they are thrown may differ.

58. A person pushes a box from left to right and then lets the box slide freely across the floor. The box slows down as it slides across the floor. When the box is sliding freely, what is the direction of the net external force?

a. The net external force acts from left to right.
b. The net external force acts from right to left.
c. The net external force acts upward.
d. The net external force acts downward.

4.3 Newton's Second Law of Motion

59. A 55-kg lady stands on a bathroom scale inside an elevator. The scale reads 70 kg. What do you know about the motion of the elevator?

a. The elevator must be accelerating upward.
b. The elevator must be accelerating downward.
c. The elevator must be moving upward with a constant velocity.
d. The elevator must be moving downward with a constant velocity.

60. True or False—A skydiver initially accelerates in his jump. Later, he achieves a state of constant velocity called terminal velocity. Does this mean the skydiver becomes weightless?

a. Yes
b. No

4.4 Newton's Third Law of Motion

61. How do rockets propel themselves in space?

a. Rockets expel gas in the forward direction at high velocity, and the gas, which provides an internal force, pushes the rockets forward.
b. Rockets expel gas in the forward direction at high velocity, and the gas, which provides an external force, pushes the rockets forward.
c. Rockets expel gas in the backward direction at high velocity, and the gas, which is an internal force, pushes the rockets forward.
d. Rockets expel gas in the backward direction at high velocity, and the gas, which provides an external force, pushes the rockets forward.

62. Are rockets more efficient in Earth's atmosphere or in outer space? Why?

a. Rockets are more efficient in Earth's atmosphere than in outer space because the air in Earth's atmosphere helps to provide thrust for the rocket, and Earth has more air friction than outer space.
b. Rockets are more efficient in Earth's atmosphere than in outer space because the air in Earth's atmosphere helps to provide thrust to the rocket, and Earth has less air friction than the outer space.
c. Rockets are more efficient in outer space than in Earth's atmosphere because the air in Earth's atmosphere does not provide thrust but does create more air friction than in outer space.
d. Rockets are more efficient in outer space than in Earth's atmosphere because the air in Earth's atmosphere does not provide thrust but does create less air friction than in outer space.

CHAPTER 5
Motion in Two Dimensions

Figure 5.1 Billiard balls on a pool table are in motion after being hit with a cue stick. (Popperipopp, Wikimedia Commons)

Chapter Outline

INTRODUCTION In Chapter 2, we learned to distinguish between vectors and scalars; the difference being that a vector has magnitude and direction, whereas a scalar has only magnitude. We learned how to deal with vectors in physics by working straightforward one-dimensional vector problems, which may be treated mathematically in the same as scalars. In this chapter, we'll use vectors to expand our understanding of forces and motion into two dimensions. Most real-world physics problems (such as with the game of pool pictured here) are, after all, either two- or three-dimensional problems and physics is most useful when applied to real physical scenarios. We start by learning the practical skills of graphically adding and subtracting vectors (by using drawings) and analytically (with math). Once we're able to work with two-dimensional vectors, we apply these skills to problems of projectile motion, inclined planes, and harmonic motion.

5.1 Vector Addition and Subtraction: Graphical Methods

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the graphical method of vector addition and subtraction
- Use the graphical method of vector addition and subtraction to solve physics problems

Section Key Terms

graphical method	head (of a vector)	head-to-tail method	resultant
resultant vector	tail	vector addition	vector subtraction

The Graphical Method of Vector Addition and Subtraction

Recall that a vector is a quantity that has magnitude and direction. For example, displacement, velocity, acceleration, and force are all vectors. In one-dimensional or straight-line motion, the direction of a vector can be given simply by a plus or minus sign. Motion that is forward, to the right, or upward is usually considered to be *positive* (+); and motion that is backward, to the left, or downward is usually considered to be *negative* (–).

In two dimensions, a vector describes motion in two perpendicular directions, such as vertical and horizontal. For vertical and horizontal motion, each vector is made up of vertical and horizontal components. In a one-dimensional problem, one of the components simply has a value of zero. For two-dimensional vectors, we work with vectors by using a frame of reference such as a coordinate system. Just as with one-dimensional vectors, we graphically represent vectors with an arrow having a length proportional to the vector's magnitude and pointing in the direction that the vector points.

Figure 5.2 shows a graphical representation of a vector; the total displacement for a person walking in a city. The person first walks nine blocks east and then five blocks north. Her total displacement does not match her path to her final destination. The displacement simply connects her starting point with her ending point using a straight line, which is the shortest distance. We use the notation that a boldface symbol, such as **D**, stands for a vector. Its magnitude is represented by the symbol in italics, D, and its direction is given by an angle represented by the symbol θ. Note that her displacement would be the same if she had begun by first walking five blocks north and then walking nine blocks east.

TIPS FOR SUCCESS

In this text, we represent a vector with a boldface variable. For example, we represent a force with the vector **F**, which has both magnitude and direction. The magnitude of the vector is represented by the variable in italics, F, and the direction of the variable is given by the angle θ.

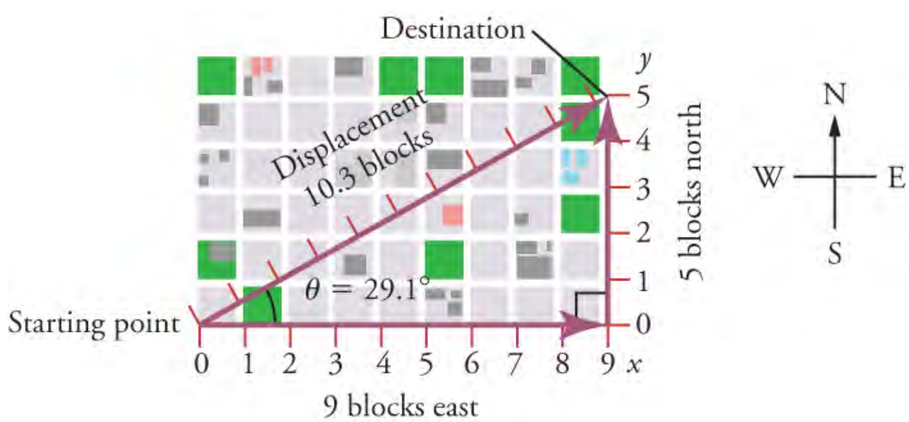


Figure 5.2 A person walks nine blocks east and five blocks north. The displacement is 10.3 blocks at an angle 29.1° north of east.

The **head-to-tail method** is a **graphical** way to add vectors. The **tail** of the vector is the starting point of the vector, and the **head** (or tip) of a vector is the pointed end of the arrow. The following steps describe how to use the head-to-tail method for graphical **vector addition**.

1. Let the x-axis represent the east-west direction. Using a ruler and protractor, draw an arrow to represent the first vector (nine blocks to the east), as shown in Figure 5.3(a).

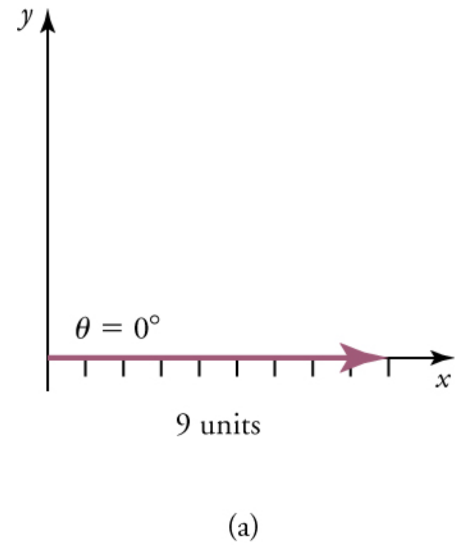


Figure 5.3 The diagram shows a vector with a magnitude of nine units and a direction of 0°.

2. Let the y-axis represent the north-south direction. Draw an arrow to represent the second vector (five blocks to the north). Place the tail of the second vector at the head of the first vector, as shown in Figure 5.4(b).

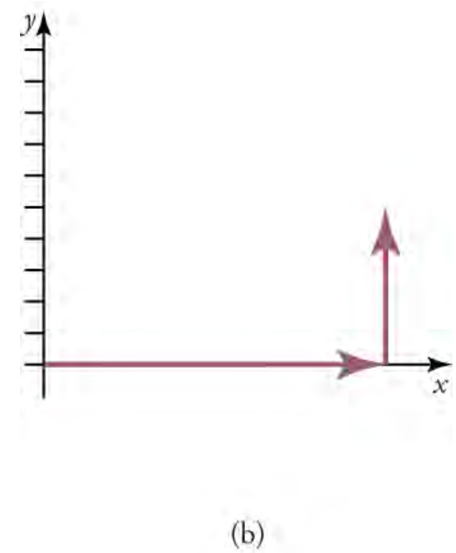


Figure 5.4 A vertical vector is added.

3. If there are more than two vectors, continue to add the vectors head-to-tail as described in step 2. In this example, we have only two vectors, so we have finished placing arrows tip to tail.
4. Draw an arrow from the tail of the first vector to the head of the last vector, as shown in Figure 5.5(c). This is the **resultant**, or the sum, of the vectors.

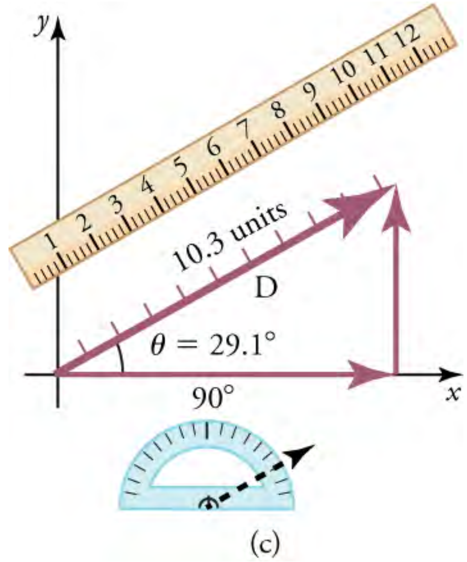


Figure 5.5 The diagram shows the resultant vector, a ruler, and protractor.

5. To find the magnitude of the resultant, measure its length with a ruler. When we deal with vectors analytically in the next section, the magnitude will be calculated by using the Pythagorean theorem.
6. To find the direction of the resultant, use a protractor to measure the angle it makes with the reference direction (in this case, the *x*-axis). When we deal with vectors analytically in the next section, the direction will be calculated by using trigonometry to find the angle.

WATCH PHYSICS

Visualizing Vector Addition Examples

This video shows four graphical representations of vector addition and matches them to the correct vector addition formula.

Click to view content (https://openstax.org/l/02addvector)

GRASP CHECK

There are two vectors $\vec{a}$ and $\vec{b}$. The head of vector $\vec{a}$ touches the tail of vector $\vec{b}$. The addition of vectors $\vec{a}$ and $\vec{b}$ gives a resultant vector $\vec{c}$. Can the addition of these two vectors can be represented by the following two equations? $\vec{a} + \vec{b} = \vec{c}$; $\vec{b} + \vec{a} = \vec{c}$

a. Yes, if we add the same two vectors in a different order it will still give the same resultant vector.
b. No, the resultant vector will change if we add the same vectors in a different order.

Vector subtraction is done in the same way as vector addition with one small change. We add the first vector to the negative of the vector that needs to be subtracted. A negative vector has the same magnitude as the original vector, but points in the opposite direction (as shown in Figure 5.6). Subtracting the vector **B** from the vector **A**, which is written as **A** – **B**, is the same as **A** + (–**B**). Since it does not matter in what order vectors are added, **A** – **B** is also equal to (–**B**) + **A**. This is true for scalars as well as vectors. For example, 5 – 2 = 5 + (–2) = (–2) + 5.

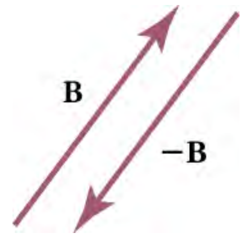


Figure 5.6 The diagram shows a vector, B, and the negative of this vector, –B.

Global angles are calculated in the counterclockwise direction. The clockwise direction is considered negative. For example, an angle of 30° south of west is the same as the global angle 210°, which can also be expressed as -150° from the positive x-axis.

Using the Graphical Method of Vector Addition and Subtraction to Solve Physics Problems

Now that we have the skills to work with vectors in two dimensions, we can apply vector addition to graphically determine the **resultant vector**, which represents the total force. Consider an example of force involving two ice skaters pushing a third as seen in Figure 5.7.

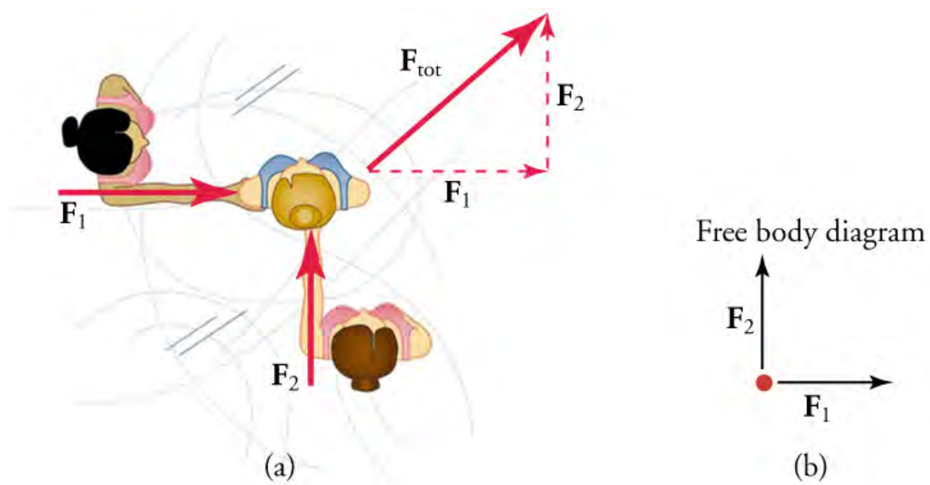


Figure 5.7 Part (a) shows an overhead view of two ice skaters pushing on a third. Forces are vectors and add like vectors, so the total force on the third skater is in the direction shown. In part (b), we see a free-body diagram representing the forces acting on the third skater.

In problems where variables such as force are already known, the forces can be represented by making the length of the vectors proportional to the magnitudes of the forces. For this, you need to create a scale. For example, each centimeter of vector length could represent 50 N worth of force. Once you have the initial vectors drawn to scale, you can then use the head-to-tail method to draw the resultant vector. The length of the resultant can then be measured and converted back to the original units using the scale you created.

You can tell by looking at the vectors in the free-body diagram in Figure 5.7 that the two skaters are pushing on the third skater with equal-magnitude forces, since the length of their force vectors are the same. Note, however, that the forces are not equal because they act in different directions. If, for example, each force had a magnitude of 400 N, then we would find the magnitude of the total external force acting on the third skater by finding the magnitude of the resultant vector. Since the forces act at a right angle to one another, we can use the Pythagorean theorem. For a triangle with sides a, b, and c, the Pythagorean theorem tells us that

$$a^2 + b^2 = c^2$$
$$c = \sqrt{a^2 + b^2}.$$

Applying this theorem to the triangle made by $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_{\text{tot}}$ in Figure 5.7, we get

$$\mathbf{F}_{\text{tot}}^2 = \sqrt{\mathbf{F}_1^2 + \mathbf{F}_1^2},$$

or

$$\mathbf{F}_{\text{tot}} = \sqrt{(400\ \text{N})^2 + (400\ \text{N})^2} = 566\ \text{N}.$$

Note that, if the vectors were not at a right angle to each other (90° to one another), we would not be able to use the Pythagorean theorem to find the magnitude of the resultant vector. Another scenario where adding two-dimensional vectors is necessary is for velocity, where the direction may not be purely east-west or north-south, but some combination of these two directions. In the next section, we cover how to solve this type of problem analytically. For now let's consider the problem graphically.

WORKED EXAMPLE

Adding Vectors Graphically by Using the Head-to-Tail Method: A Woman Takes a Walk

Use the graphical technique for adding vectors to find the total displacement of a person who walks the following three paths (displacements) on a flat field. First, he walks 25 m in a direction 49° north of east. Then, he walks 23 m heading 15° north of east. Finally, he turns and walks 32 m in a direction 68° south of east.

Strategy

Graphically represent each displacement vector with an arrow, labeling the first **A**, the second **B**, and the third **C**. Make the lengths proportional to the distance of the given displacement and orient the arrows as specified relative to an east-west line. Use the head-to-tail method outlined above to determine the magnitude and direction of the resultant displacement, which we'll call **R**.

Solution

(1) Draw the three displacement vectors, creating a convenient scale (such as 1 cm of vector length on paper equals 1 m in the problem), as shown in Figure 5.8.

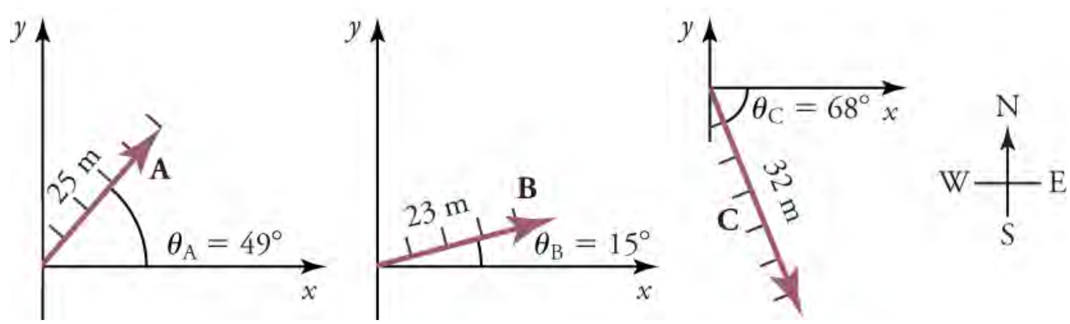


Figure 5.8 The three displacement vectors are drawn first.

(2) Place the vectors head to tail, making sure not to change their magnitude or direction, as shown in Figure 5.9.

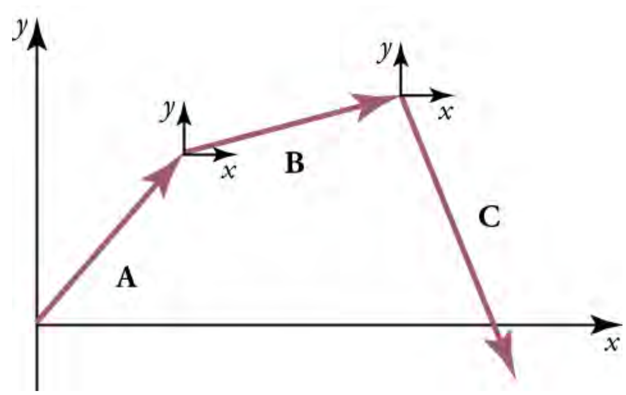


Figure 5.9 Next, the vectors are placed head to tail.

(3) Draw the **resultant vector R** from the tail of the first vector to the head of the last vector, as shown in Figure 5.10.

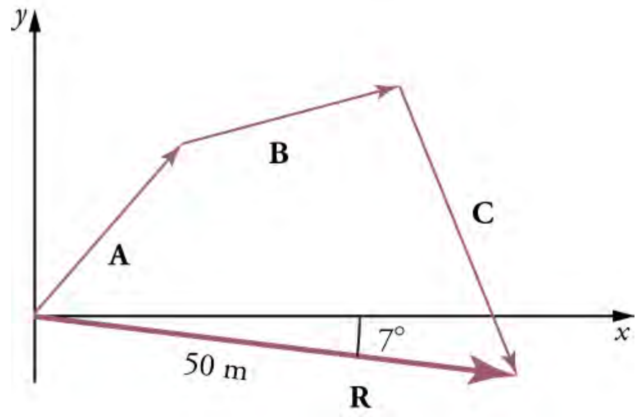


Figure 5.10 The resultant vector is drawn .

(4) Use a ruler to measure the magnitude of **R**, remembering to convert back to the units of meters using the scale. Use a protractor to measure the direction of **R**. While the direction of the vector can be specified in many ways, the easiest way is to measure the angle between the vector and the nearest horizontal or vertical axis. Since **R** is south of the eastward pointing axis (the x-axis), we flip the protractor upside down and measure the angle between the eastward axis and the vector, as illustrated in Figure 5.11.

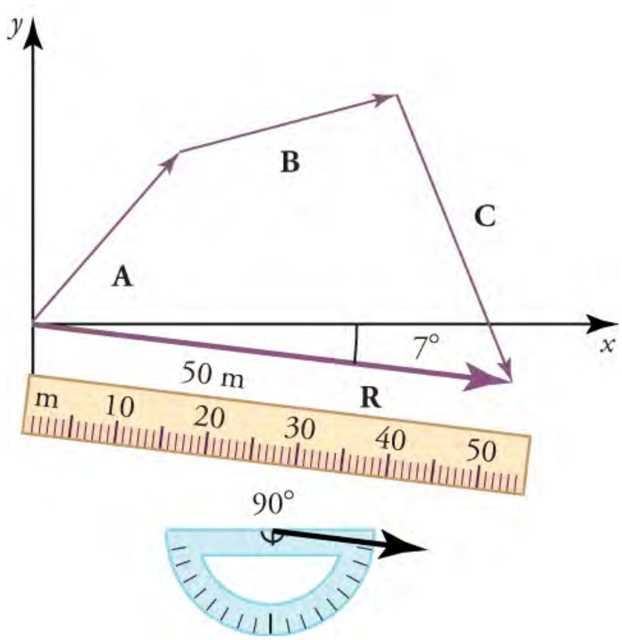


Figure 5.11 A ruler is used to measure the magnitude of **R**, and a protractor is used to measure the direction of **R**.

In this case, the total displacement **R** has a magnitude of 50 m and points 7° south of east. Using its magnitude and direction, this vector can be expressed as

$$R = 50 \text{ m} \tag{5.1}$$

and

$$\theta = 7^\circ \text{south of east.} \tag{5.2}$$

Discussion

The head-to-tail graphical method of vector addition works for any number of vectors. It is also important to note that it does not matter in what order the vectors are added. Changing the order does not change the resultant. For example, we could add the vectors as shown in Figure 5.12, and we would still get the same solution.

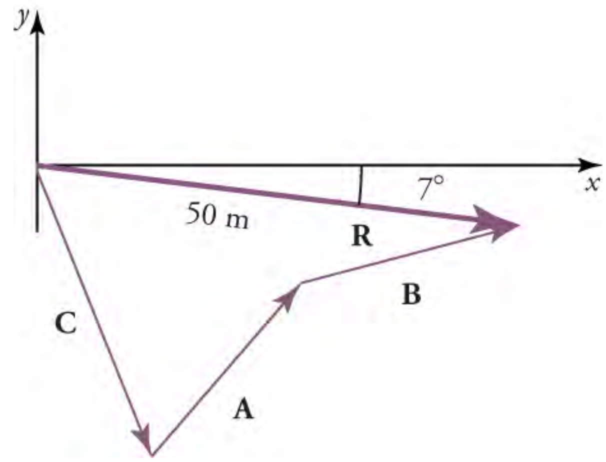


Figure 5.12 Vectors can be added in any order to get the same result.

WORKED EXAMPLE

Subtracting Vectors Graphically: A Woman Sailing a Boat

A woman sailing a boat at night is following directions to a dock. The instructions read to first sail 27.5 m in a direction 66.0° north of east from her current location, and then travel 30.0 m in a direction 112° north of east (or 22.0° west of north). If the woman makes a mistake and travels in the *opposite* direction for the second leg of the trip, where will she end up? The two legs of the woman's trip are illustrated in Figure 5.13.

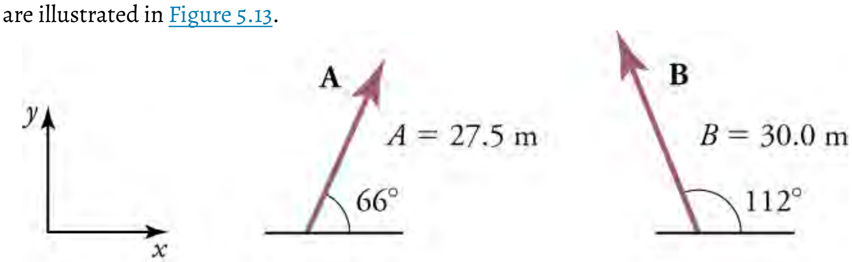


Figure 5.13 In the diagram, the first leg of the trip is represented by vector A and the second leg is represented by vector B.

Strategy

We can represent the first leg of the trip with a vector **A**, and the second leg of the trip that she was *supposed to* take with a vector **B**. Since the woman mistakenly travels in the opposite direction for the second leg of the journey, the vector for second leg of the trip she *actually* takes is –**B**. Therefore, she will end up at a location **A** + (–**B**), or **A** – **B**. Note that –**B** has the same magnitude as **B** (30.0 m), but is in the opposite direction, $68^\circ\,(180^\circ - 112^\circ)$ south of east, as illustrated in Figure 5.14.

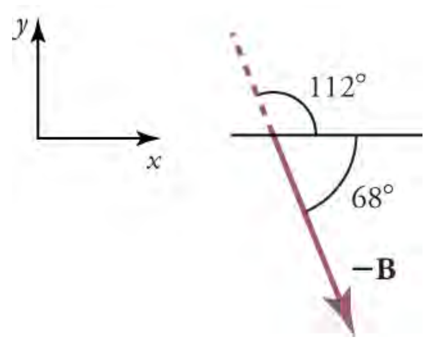


Figure 5.14 Vector –B represents traveling in the opposite direction of vector B.

We use graphical vector addition to find where the woman arrives **A** + (–**B**).

Solution

(1) To determine the location at which the woman arrives by accident, draw vectors **A** and –**B**.

(2) Place the vectors head to tail.

(3) Draw the resultant vector **R**.

(4) Use a ruler and protractor to measure the magnitude and direction of **R**.

These steps are demonstrated in Figure 5.15.

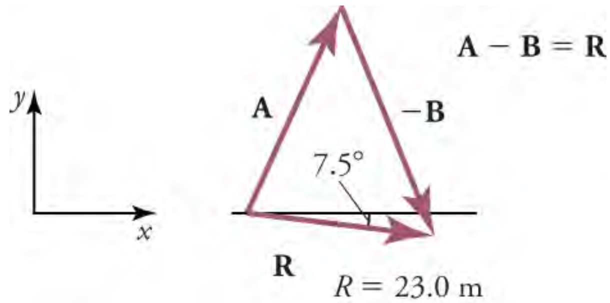


Figure 5.15 The vectors are placed head to tail.

In this case

$$R = 23.0\text{ m} \tag{5.3}$$

and

$$\theta = 7.5°\text{south of east.} \tag{5.4}$$

Discussion

Because subtraction of a vector is the same as addition of the same vector with the opposite direction, the graphical method for subtracting vectors works the same as for adding vectors.

WORKED EXAMPLE

Adding Velocities: A Boat on a River

A boat attempts to travel straight across a river at a speed of 3.8 m/s. The river current flows at a speed v_{river} of 6.1 m/s to the right. What is the total velocity and direction of the boat? You can represent each meter per second of velocity as one centimeter of vector length in your drawing.

Strategy

We start by choosing a coordinate system with its x-axis parallel to the velocity of the river. Because the boat is directed straight toward the other shore, its velocity is perpendicular to the velocity of the river. We draw the two vectors, $\mathbf{v}_{\text{boat}}$ and $\mathbf{v}_{\text{river}}$, as shown in Figure 5.16.

Using the head-to-tail method, we draw the resulting total velocity vector from the tail of $\mathbf{v}_{\text{boat}}$ to the head of $\mathbf{v}_{\text{river}}$.

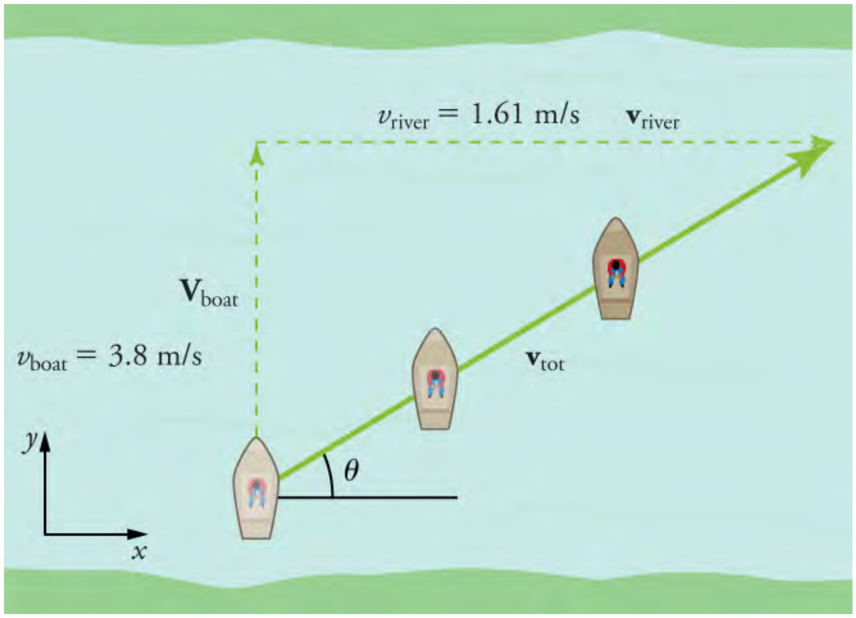


Figure 5.16 A boat attempts to travel across a river. What is the total velocity and direction of the boat?

Solution

By using a ruler, we find that the length of the resultant vector is 7.2 cm, which means that the magnitude of the total velocity is

$$v_{tot} = 7.2 \text{ m/s.} \tag{5.5}$$

By using a protractor to measure the angle, we find $\theta = 32.0^\circ$.

Discussion

If the velocity of the boat and river were equal, then the direction of the total velocity would have been 45°. However, since the velocity of the river is greater than that of the boat, the direction is less than 45° with respect to the shore, or *x* axis.

Practice Problems

1. Vector $\vec{A}$, having magnitude 2.5 m, pointing 37° south of east and vector $\vec{B}$ having magnitude 3.5 m, pointing 20° north of east are added. What is the magnitude of the resultant vector?
   a. 1.0 m
   b. 5.3 m
   c. 5.9 m
   d. 6.0 m

2. A person walks 32° north of west for 94 m and 35° east of south for 122 m. What is the magnitude of his displacement?
   a. 28 m
   b. 51 m
   c. 180 m
   d. 216 m

Virtual Physics

Vector Addition

In this simulation (https://archive.cnx.org/specials/d218bf9b-e50e-4d50-9a6c-b3db4dad0816/vector-addition/), you will experiment with adding vectors graphically. Click and drag the red vectors from the Grab One basket onto the graph in the middle of the screen. These red vectors can be rotated, stretched, or repositioned by clicking and dragging with your mouse. Check the Show Sum box to display the resultant vector (in green), which is the sum of all of the red vectors placed on the graph. To remove a red vector, drag it to the trash or click the Clear All button if you wish to start over. Notice that, if you click on any of the vectors, the $|\mathbf{R}|$ is its magnitude, θ is its direction with respect to the positive x-axis, $\mathbf{R_x}$ is its horizontal component, and R_y is its vertical component. You can check the resultant by lining up the vectors so that the head of the first vector touches the tail of the second. Continue until all of the vectors are aligned together head-to-tail. You will see that the resultant magnitude and angle is the same as the arrow drawn from the tail of the first vector to the head of the last vector. Rearrange the vectors in any order head-to-tail and compare. The resultant will always be the same.

Click to view content (https://archive.cnx.org/specials/d218bf9b-e50e-4d50-9a6c-b3db4dad0816/vector-addition/)

GRASP CHECK

True or False—The more long, red vectors you put on the graph, rotated in any direction, the greater the magnitude of the resultant green vector.

a. True
b. False

Check Your Understanding

3. While there is no single correct choice for the sign of axes, which of the following are conventionally considered positive?
   a. backward and to the left
   b. backward and to the right
   c. forward and to the right
   d. forward and to the left

4. True or False—A person walks 2 blocks east and 5 blocks north. Another person walks 5 blocks north and then two blocks east. The displacement of the first person will be more than the displacement of the second person.
   a. True
   b. False

5.2 Vector Addition and Subtraction: Analytical Methods

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Define components of vectors
- Describe the analytical method of vector addition and subtraction
- Use the analytical method of vector addition and subtraction to solve problems

Section Key Terms

analytical method	component (of a two-dimensional vector)

Components of Vectors

For the **analytical method** of vector addition and subtraction, we use some simple geometry and trigonometry, instead of using a ruler and protractor as we did for graphical methods. However, the graphical method will still come in handy to visualize the problem by drawing vectors using the head-to-tail method. The analytical method is more accurate than the graphical method, which is limited by the precision of the drawing. For a refresher on the definitions of the sine, cosine, and tangent of an angle, see Figure 5.17.

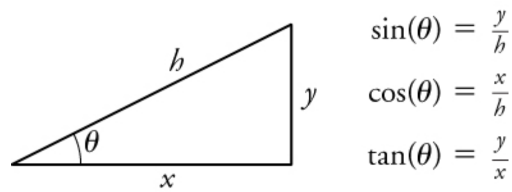


Figure 5.17 For a right triangle, the sine, cosine, and tangent of θ are defined in terms of the adjacent side, the opposite side, or the hypotenuse. In this figure, x is the adjacent side, y is the opposite side, and h is the hypotenuse.

Since, by definition, $\cos\theta = x/h$, we can find the length x if we know h and θ by using $x = h\cos\theta$. Similarly, we can find the length of y by using $y = h\sin\theta$. These trigonometric relationships are useful for adding vectors.

When a vector acts in more than one dimension, it is useful to break it down into its x and y components. For a two-dimensional vector, a **component** is a piece of a vector that points in either the x- or y-direction. Every 2-d vector can be expressed as a sum of its x and y components.

For example, given a vector like $\mathbf{A}$ in Figure 5.18, we may want to find what two perpendicular vectors, $\mathbf{A}_x$ and $\mathbf{A}_y$, add to produce it. In this example, $\mathbf{A}_x$ and $\mathbf{A}_y$ form a right triangle, meaning that the angle between them is 90 degrees. This is a common situation in physics and happens to be the least complicated situation trigonometrically.

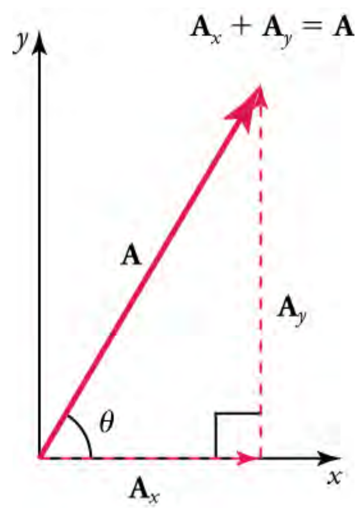


Figure 5.18 The vector $\mathbf{A}$, with its tail at the origin of an x- y-coordinate system, is shown together with its x- and y-components, $\mathbf{A}_x$ and $\mathbf{A}_y$. These vectors form a right triangle.

$\mathbf{A}_x$ and $\mathbf{A}_y$ are defined to be the components of $\mathbf{A}$ along the x- and y-axes. The three vectors, $\mathbf{A}$, $\mathbf{A}_x$, and $\mathbf{A}_y$, form a right triangle.

$$\mathbf{A_x} + \mathbf{A_y} = \mathbf{A}$$

If the vector $\mathbf{A}$ is known, then its magnitude A (its length) and its angle θ (its direction) are known. To find A_x and A_y, its x- and y-components, we use the following relationships for a right triangle:

$$A_x = A\cos\theta$$

and

$$A_y = A\sin\theta,$$

where A_x is the magnitude of **A** in the x-direction, A_y is the magnitude of **A** in the y-direction, and θ is the angle of the resultant with respect to the x-axis, as shown in Figure 5.19.

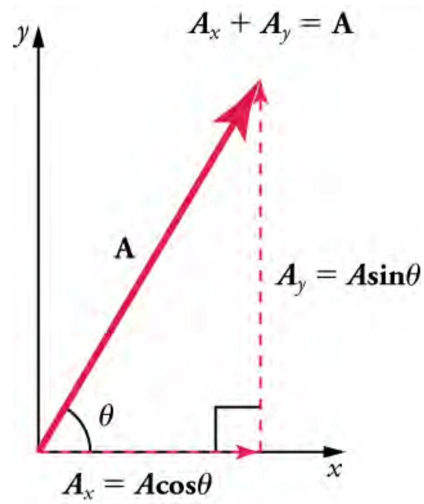


Figure 5.19 The magnitudes of the vector components $\mathbf{A}_x$ and $\mathbf{A}_y$ can be related to the resultant vector $\mathbf{A}$ and the angle θ with trigonometric identities. Here we see that $A_x = A\cos\theta$ and $A_y = A\sin\theta$.

Suppose, for example, that $\mathbf{A}$ is the vector representing the total displacement of the person walking in a city, as illustrated in Figure 5.20.

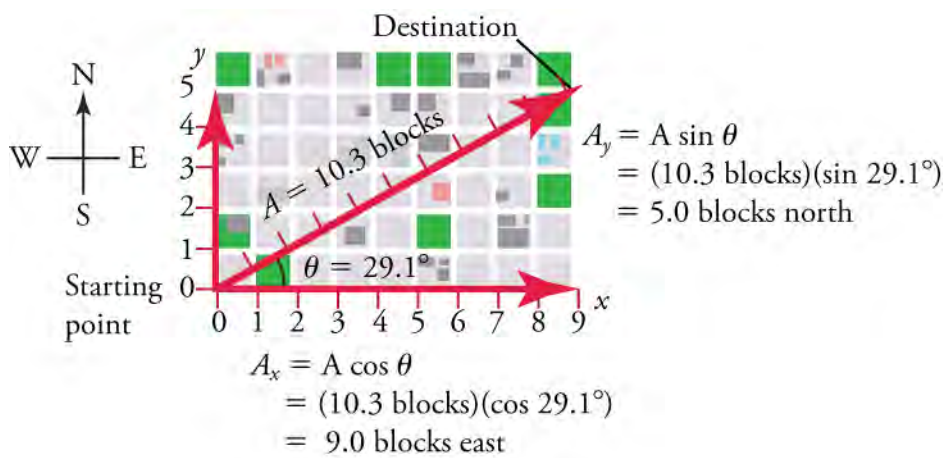


Figure 5.20 We can use the relationships $A_x = A\cos\theta$ and $A_y = A\sin\theta$ to determine the magnitude of the horizontal and vertical component vectors in this example.

Then A = 10.3 blocks and $\theta = 29.1^\circ$, so that

$$\begin{aligned} A_x &= A\cos\theta \\ &= (10.3 \;\; \text{blocks})(\cos 29.1^\circ) \\ &= (10.3 \;\; \text{blocks})(0.874) \\ &= 9.0 \text{ blocks.} \end{aligned} \tag{5.6}$$

This magnitude indicates that the walker has traveled 9 blocks to the east—in other words, a 9-block eastward displacement. Similarly,

$$\begin{aligned} A_y &= A\sin\theta \\ &= (10.3 \;\; \text{blocks})(\sin 29.1^\circ) \\ &= (10.3 \;\; \text{blocks})(0.846) \\ &= 5.0 \text{ blocks,} \end{aligned} \tag{5.7}$$

indicating that the walker has traveled 5 blocks to the north—a 5-block northward displacement.

Analytical Method of Vector Addition and Subtraction

Calculating a resultant vector (or vector addition) is the reverse of breaking the resultant down into its components. If the perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$ of a vector $\mathbf{A}$ are known, then we can find $\mathbf{A}$ analytically. How do we do this? Since, by definition,

$$\tan\theta = y/x \text{ (or in this case } \tan\theta = A_y/A_x\text{)},$$

we solve for θ to find the direction of the resultant.

$$\theta = \tan^{-1}(A_y/A_x)$$

Since this is a right triangle, the Pythagorean theorem ($x^2 + y^2 = h^2$) for finding the hypotenuse applies. In this case, it becomes

$$A^2 = A_x^2 + A_y^2.$$

Solving for A gives

$$A = \sqrt{{A_x}^2 + {A_y}^2}.$$

In summary, to find the magnitude A and direction θ of a vector from its perpendicular components $\mathbf{A}_x$ and $\mathbf{A}_y$, as illustrated in Figure 5.21, we use the following relationships:

$$A = \sqrt{{A_x}^2 + {A_y}^2}$$
$$\theta = \tan^{-1}(A_y/A_x)$$

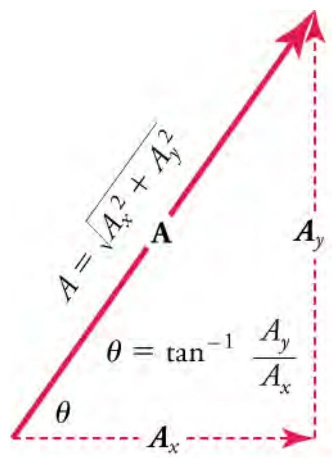


Figure 5.21 The magnitude and direction of the resultant vector $\mathbf{A}$ can be determined once the horizontal components $\mathbf{A}_x$ and $\mathbf{A}_y$ have been determined.

Sometimes, the vectors added are not perfectly perpendicular to one another. An example of this is the case below, where the vectors $\mathbf{A}$ and $\mathbf{B}$ are added to produce the resultant $\mathbf{R}$, as illustrated in Figure 5.22.

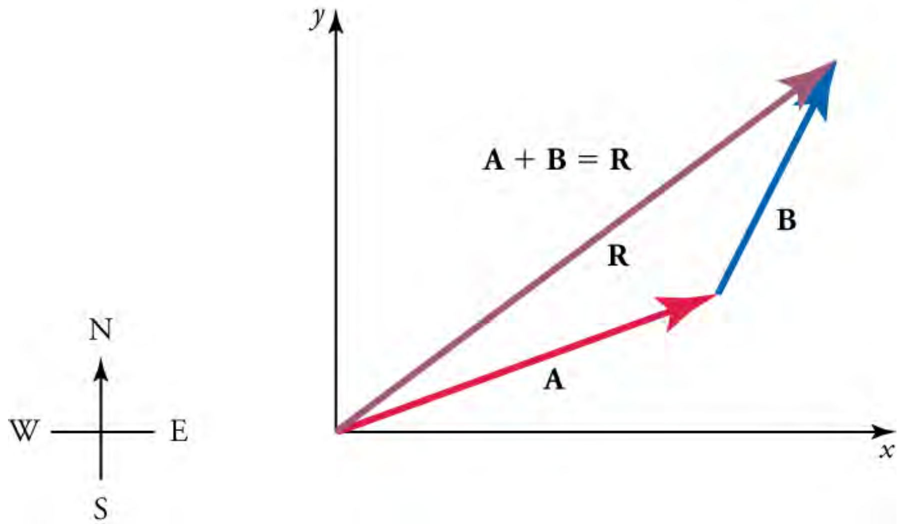


Figure 5.22 Vectors $\mathbf{A}$ and $\mathbf{B}$ are two legs of a walk, and $\mathbf{R}$ is the resultant or total displacement. You can use analytical methods to determine the magnitude and direction of $\mathbf{R}$.

If $\mathbf{A}$ and $\mathbf{B}$ represent two legs of a walk (two displacements), then $\mathbf{R}$ is the total displacement. The person taking the walk ends up at the tip of $\mathbf{R}$. There are many ways to arrive at the same point. The person could have walked straight ahead first in the *x*-direction and then in the *y*-direction. Those paths are the *x*- and *y*-components of the resultant, $\mathbf{R}_x$ and $\mathbf{R}_y$. If we know $\mathbf{R}_x$ and $\mathbf{R}_y$, we can find R and θ using the equations $R = \sqrt{R_x{}^2 + R_y{}^2}$ and $\theta = tan^{-1}(R_y/R_x)$.

1. Draw in the *x* and *y* components of each vector (including the resultant) with a dashed line. Use the equations $A_x = A\cos\theta$ and $A_y = A\sin\theta$ to find the components. In Figure 5.23, these components are A_x, A_y, B_x, and B_y. Vector $\mathbf{A}$ makes an angle of θ_A with the *x*-axis, and vector $\mathbf{B}$ makes and angle of θ_B with its own *x*-axis (which is slightly above the *x*-axis used by vector **A**).

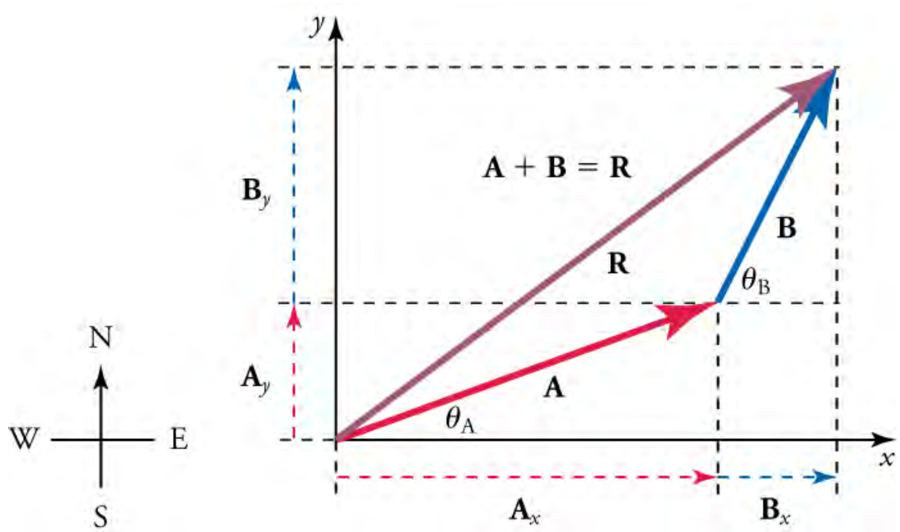


Figure 5.23 To add vectors $\mathbf{A}$ and $\mathbf{B}$, first determine the horizontal and vertical components of each vector. These are the dotted vectors $\mathbf{A}_x$, $\mathbf{A}_y$ $\mathbf{B}_y$ shown in the image.

2. Find the *x* component of the resultant by adding the *x* component of the vectors
$$R_x = A_x + B_x$$
and find the *y* component of the resultant (as illustrated in Figure 5.24) by adding the *y* component of the vectors.
$$R_y = A_y + B_y.$$

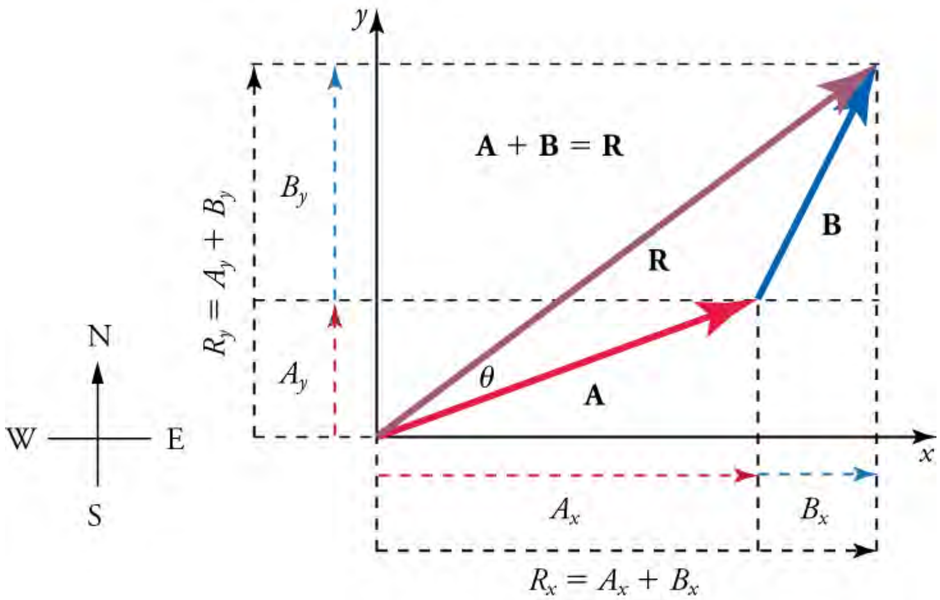


Figure 5.24 The vectors $\mathbf{A}_x$ and $\mathbf{B}_x$ add to give the magnitude of the resultant vector in the horizontal direction, R_x. Similarly, the vectors $\mathbf{A}_y$ and $\mathbf{B}_y$ add to give the magnitude of the resultant vector in the vertical direction, R_y.

Now that we know the components of $\mathbf{R}$, we can find its magnitude and direction.

3. To get the magnitude of the resultant R, use the Pythagorean theorem.

$$R = \sqrt{R_x^2 + R_y^2}$$

4. To get the direction of the resultant

$$\theta = \tan^{-1}(R_y/R_x).$$

WATCH PHYSICS

Classifying Vectors and Quantities Example

This video contrasts and compares three vectors in terms of their magnitudes, positions, and directions.

Click to view content (https://www.youtube.com/embed/YpoEhcVBxNU)

GRASP CHECK

Three vectors, $\vec{u}$, $\vec{v}$, and $\vec{w}$, have the same magnitude of 5 units. Vector $\vec{v}$ points to the northeast. Vector $\vec{w}$ points to the southwest exactly opposite to vector $\vec{u}$. Vector $\vec{u}$ points in the northwest. If the vectors $\vec{u}$, $\vec{v}$, and $\vec{w}$ were added together, what would be the magnitude of the resultant vector? Why?

a. 0 units. All of them will cancel each other out.
b. 5 units. Two of them will cancel each other out.
c. 10 units. Two of them will add together to give the resultant.
d. 15 units. All of them will add together to give the resultant.

TIPS FOR SUCCESS

In the video, the vectors were represented with an arrow above them rather than in bold. This is a common notation in math classes.

Using the Analytical Method of Vector Addition and Subtraction to Solve Problems

Figure 5.25 uses the analytical method to add vectors.

WORKED EXAMPLE

An Accelerating Subway Train

Add the vector $\mathbf{A}$ to the vector $\mathbf{B}$ shown in Figure 5.25, using the steps above. The *x*-axis is along the east–west direction, and the *y*-axis is along the north–south directions. A person first walks $53.0\ \text{m}$ in a direction $20.0°$ north of east, represented by vector $\mathbf{A}$. The person then walks $34.0\ \text{m}$ in a direction $63.0°$ north of east, represented by vector $\mathbf{B}$.

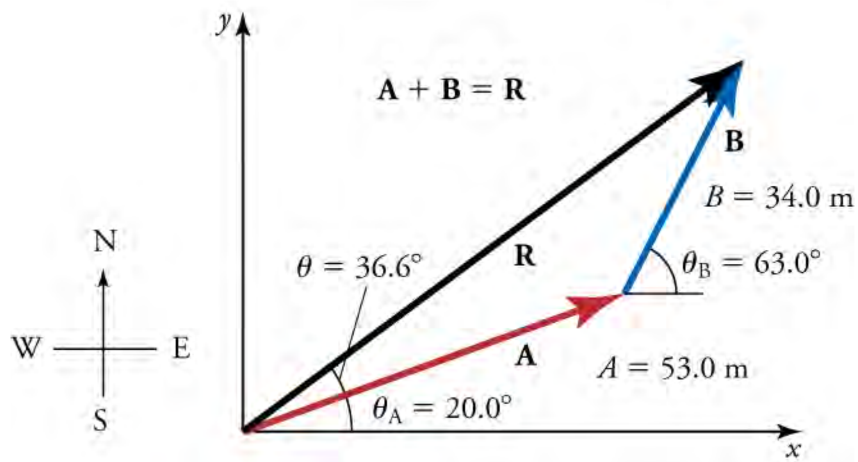


Figure 5.25 You can use analytical models to add vectors.

Strategy

The components of $\mathbf{A}$ and $\mathbf{B}$ along the *x*- and *y*-axes represent walking due east and due north to get to the same ending point. We will solve for these components and then add them in the x-direction and y-direction to find the resultant.

Solution

First, we find the components of $\mathbf{A}$ and $\mathbf{B}$ along the *x*- and *y*-axes. From the problem, we know that $A = 53.0\ \text{m}, \theta_\text{A} = 20.0°$, $B = 34.0\ \text{m}$, and $\theta_\text{B} = 63.0°$. We find the *x*-components by using $A_x = A\cos\theta$, which gives

$$\begin{aligned} A_x &= A\cos\theta_A = (53.0\ \text{m})(\cos 20.0°) \\ &= (53.0\ \text{m})(0.940) = 49.8\ \text{m} \end{aligned}$$

and

$$\begin{aligned} B_x &= B\cos\theta_B = (34.0\ \text{m})(\cos 63.0°) \\ &= (34.0\ \text{m})(0.454) = 15.4\ \text{m}. \end{aligned}$$

Similarly, the *y*-components are found using $A_y = A\sin\theta_A$

$$\begin{aligned} A_y &= A\sin\theta_A = (53.0\ \text{m})(\sin 20.0°) \\ &= (53.0\ \text{m})(0.342) = 18.1\ \text{m} \end{aligned}$$

and

$$\begin{aligned} B_y &= B\sin\theta_B = (34.0\ \text{m})(\sin 63.0°) \\ &= (34.0\ \text{m})(0.891) = 30.3\ \text{m}. \end{aligned}$$

The *x*- and *y*-components of the resultant are

$$R_x = A_x + B_x = 49.8\ \text{m} + 15.4\ \text{m} = 65.2\ \text{m}$$

and

$$R_y = A_y + B_y = 18.1 \text{ m} + 30.3 \text{ m} = 48.4 \text{ m}.$$

Now we can find the magnitude of the resultant by using the Pythagorean theorem

$$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(65.2)^2 + (48.4)^2} \text{ m} \tag{5.8}$$

so that

$$R = \sqrt{6601} \text{ m} = 81.2 \text{ m}.$$

Finally, we find the direction of the resultant

$$\theta = \tan^{-1}(R_y/R_x) = +\tan^{-1}(48.4/65.2).$$

This is

$$\theta = \tan^{-1}(0.742) = 36.6^\circ.$$

Discussion

This example shows vector addition using the analytical method. Vector subtraction using the analytical method is very similar. It is just the addition of a negative vector. That is, $\mathrm{A} - \mathrm{B} \equiv \mathrm{A} + (-\mathrm{B})$. The components of $-\mathrm{B}$ are the negatives of the components of B. Therefore, the *x*- and *y*-components of the resultant $\mathrm{A} - \mathrm{B} = \mathrm{R}$ are

$$R_x = A_x + -B_x$$

and

$$R_y = A_y + -B_y$$

and the rest of the method outlined above is identical to that for addition.

Practice Problems

5. What is the magnitude of a vector whose *x*-component is 4 cm and whose *y*-component is 3 cm?

a. 1 cm
b. 5 cm
c. 7 cm
d. 25 cm

6. What is the magnitude of a vector that makes an angle of 30° to the horizontal and whose *x*-component is 3 units?

a. 2.61 units
b. 3.00 units
c. 3.46 units
d. 6.00 units

LINKS TO PHYSICS

Atmospheric Science

Figure 5.26 This picture shows Bert Foord during a television Weather Forecast from the Meteorological Office in 1963. (BBC TV)

Atmospheric science is a physical science, meaning that it is a science based heavily on physics. Atmospheric science includes meteorology (the study of weather) and climatology (the study of climate). Climate is basically the average weather over a longer time scale. Weather changes quickly over time, whereas the climate changes more gradually.

The movement of air, water and heat is vitally important to climatology and meteorology. Since motion is such a major factor in weather and climate, this field uses vectors for much of its math.

Vectors are used to represent currents in the ocean, wind velocity and forces acting on a parcel of air. You have probably seen a weather map using vectors to show the strength (magnitude) and direction of the wind.

Vectors used in atmospheric science are often three-dimensional. We won't cover three-dimensional motion in this text, but to go from two-dimensions to three-dimensions, you simply add a third vector component. Three-dimensional motion is represented as a combination of x-, y- and z components, where z is the altitude.

Vector calculus combines vector math with calculus, and is often used to find the rates of change in temperature, pressure or wind speed over time or distance. This is useful information, since atmospheric motion is driven by changes in pressure or temperature. The greater the variation in pressure over a given distance, the stronger the wind to try to correct that imbalance. Cold air tends to be more dense and therefore has higher pressure than warm air. Higher pressure air rushes into a region of lower pressure and gets deflected by the spinning of the Earth, and friction slows the wind at Earth's surface.

Finding how wind changes over distance and multiplying vectors lets meteorologists, like the one shown in Figure 5.26, figure out how much rotation (spin) there is in the atmosphere at any given time and location. This is an important tool for tornado prediction. Conditions with greater rotation are more likely to produce tornadoes.

GRASP CHECK

Why are vectors used so frequently in atmospheric science?

a. Vectors have magnitude as well as direction and can be quickly solved through scalar algebraic operations.
b. Vectors have magnitude but no direction, so it becomes easy to express physical quantities involved in the atmospheric science.
c. Vectors can be solved very accurately through geometry, which helps to make better predictions in atmospheric science.
d. Vectors have magnitude as well as direction and are used in equations that describe the three dimensional motion of the atmosphere.

Check Your Understanding

7. Between the analytical and graphical methods of vector additions, which is more accurate? Why?
   a. The analytical method is less accurate than the graphical method, because the former involves geometry and

trigonometry.
 b. The analytical method is more accurate than the graphical method, because the latter involves some extensive calculations.
 c. The analytical method is less accurate than the graphical method, because the former includes drawing all figures to the right scale.
 d. The analytical method is more accurate than the graphical method, because the latter is limited by the precision of the drawing.

8. What is a component of a two dimensional vector?
 a. A component is a piece of a vector that points in either the x or y direction.
 b. A component is a piece of a vector that has half of the magnitude of the original vector.
 c. A component is a piece of a vector that points in the direction opposite to the original vector.
 d. A component is a piece of a vector that points in the same direction as original vector but with double of its magnitude.

9. How can we determine the global angle θ (measured counter-clockwise from positive x) if we know A_x and A_y?
 a. $\theta = \cos^{-1}\frac{A_y}{A_x}$
 b. $\theta = \cot^{-1}\frac{A_y}{A_x}$
 c. $\theta = \sin^{-1}\frac{A_y}{A_x}$
 d. $\theta = \tan^{-1}\frac{A_y}{A_x}$

10. How can we determine the magnitude of a vector if we know the magnitudes of its components?
 a. $\left|\vec{\mathrm{A}}\right| = A_x + A_y$
 b. $\left|\vec{\mathrm{A}}\right| = {A_x}^2 + {A_y}^2$
 c. $\left|\vec{\mathrm{A}}\right| = \sqrt{{A_x}^2 + {A_y}^2}$
 d. $\left|\vec{\mathrm{A}}\right| = \left({A_x}^2 + {A_y}^2\right)^2$

5.3 Projectile Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the properties of projectile motion
- Apply kinematic equations and vectors to solve problems involving projectile motion

Section Key Terms

air resistance	maximum height (of a projectile)	projectile
projectile motion	range	trajectory

Properties of Projectile Motion

Projectile motion is the motion of an object thrown (projected) into the air. After the initial force that launches the object, it only experiences the force of gravity. The object is called a **projectile**, and its path is called its **trajectory**. As an object travels through the air, it encounters a frictional force that slows its motion called **air resistance**. Air resistance does significantly alter trajectory motion, but due to the difficulty in calculation, it is ignored in introductory physics.

The most important concept in projectile motion is that *horizontal and vertical motions are independent*, meaning that they don't influence one another. Figure 5.27 compares a cannonball in free fall (in blue) to a cannonball launched horizontally in projectile motion (in red). You can see that the cannonball in free fall falls at the same rate as the cannonball in projectile motion. Keep in mind that if the cannon launched the ball with any vertical component to the velocity, the vertical displacements would not line up perfectly.

Since vertical and horizontal motions are independent, we can analyze them separately, along perpendicular axes. To do this, we separate projectile motion into the two components of its motion, one along the horizontal axis and the other along the vertical.

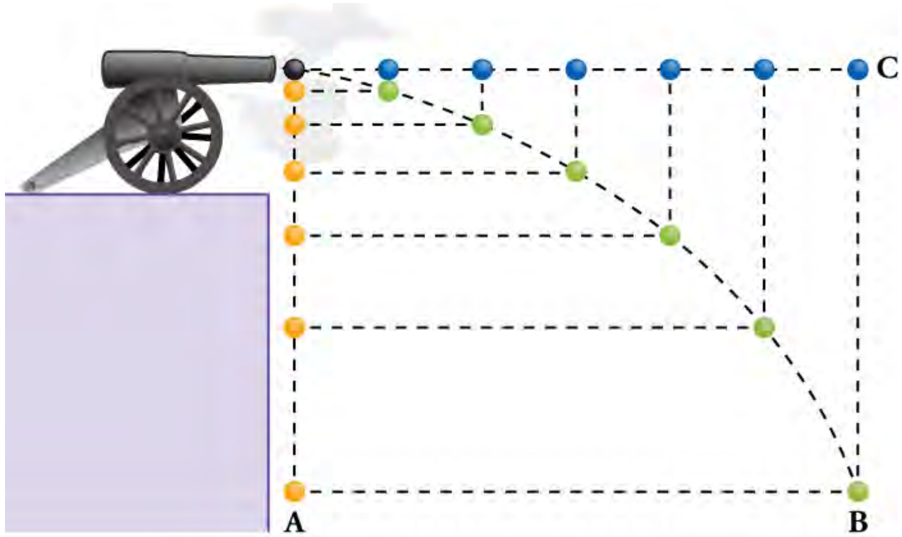


Figure 5.27 The diagram shows the projectile motion of a cannonball shot at a horizontal angle versus one dropped with no horizontal velocity. Note that both cannonballs have the same vertical position over time.

We'll call the horizontal axis the *x*-axis and the vertical axis the *y*-axis. For notation, **d** is the total displacement, and **x** and **y** are its components along the horizontal and vertical axes. The magnitudes of these vectors are *x* and *y*, as illustrated in Figure 5.28.

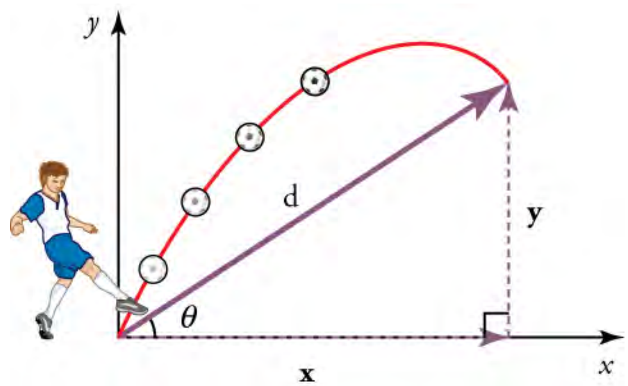


Figure 5.28 A boy kicks a ball at angle θ, and it is displaced a distance of **s** along its trajectory.

As usual, we use velocity, acceleration, and displacement to describe motion. We must also find the components of these variables along the *x*- and *y*-axes. The components of acceleration are then very simple $\mathbf{a_y} = -\mathbf{g} = -9.80\ \text{m/s}^2$. Note that this definition defines the upwards direction as positive. Because gravity is vertical, $\mathbf{a_x} = 0$. Both accelerations are constant, so we can use the kinematic equations. For review, the kinematic equations from a previous chapter are summarized in Table 5.1.

$\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_{\mathbf{avg}} t$ (when $\mathbf{a}=0$)
$\mathbf{v}_{\mathbf{avg}} = \frac{\mathbf{v}_0 + \mathbf{v}}{2}$ (when $\mathbf{a}=0$)
$\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$
$\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_0 t + \frac{1}{2}\mathbf{a}t^2$
$\mathbf{v}^2 = \mathbf{v}_0^2 + 2\mathbf{a}(\mathbf{x} - \mathbf{x}_0)$

Table 5.1 Summary of Kinematic Equations (constant a)

Where **x** is position, $\mathbf{x}_0$ is initial position, **v** is velocity, $\mathbf{v}_{avg}$ is average velocity, *t* is time and a is acceleration.

Solve Problems Involving Projectile Motion

The following steps are used to analyze projectile motion:

1. Separate the motion into horizontal and vertical components along the x- and y-axes. These axes are perpendicular, so $A_x = A\cos\theta$ and $A_y = A\sin\theta$ are used. The magnitudes of the displacement $\mathbf{s}$ along x- and y-axes are called x and y. The magnitudes of the components of the velocity $\mathbf{v}$ are $v_x = v \cos\theta$ and $v_y = v \sin\theta$, where v is the magnitude of the velocity and θ is its direction. Initial values are denoted with a subscript 0.
2. Treat the motion as two independent one-dimensional motions, one horizontal and the other vertical. The kinematic equations for horizontal and vertical motion take the following forms

$$\text{Horizontal Motion}(\mathbf{a}_x = 0)$$
$$x = x_0 + v_x t$$
$$v_x = v_{0x} = \mathbf{v}_\text{x} = \text{velocity is a constant.}$$

Vertical motion (assuming positive is up $\mathbf{a}_y = -\mathbf{g} = -9.80\text{ m/s}^2$)

$$\begin{aligned} y &= y_0 + \tfrac{1}{2}(v_{0y} + v_y)t \\ v_y &= v_{0y} - \mathbf{g}t \\ y &= y_0 + v_{0y}t - \tfrac{1}{2}\mathbf{g}t^2 \\ v_y^2 &= v_{0y}^2 - 2g(y - y_0) \end{aligned}$$

3. Solve for the unknowns in the two separate motions (one horizontal and one vertical). Note that the only common variable between the motions is time t . The problem solving procedures here are the same as for one-dimensional kinematics.
4. Recombine the two motions to find the total displacement $\mathbf{s}$ and velocity $\mathbf{v}$. We can use the analytical method of vector addition, which uses $A = \sqrt{A_x{}^2 + A_y{}^2}$ and $\theta = \tan^{-1}(A_y/A_x)$ to find the magnitude and direction of the total displacement and velocity.

$$\text{Displacement}$$
$$\mathbf{d} = \sqrt{x^2 + y^2}$$
$$\theta = \tan^{-1}(y/x)$$
$$\text{Velocity}$$
$$\mathbf{v} = \sqrt{\mathbf{v}_x^2 + \mathbf{v}_y^2}$$
$$\theta_v = \tan^{-1}(\mathbf{v}_y/\mathbf{v}_x)$$

θ is the direction of the displacement $\mathbf{d}$, and θ_v is the direction of the velocity $\mathbf{v}$. (See Figure 5.29

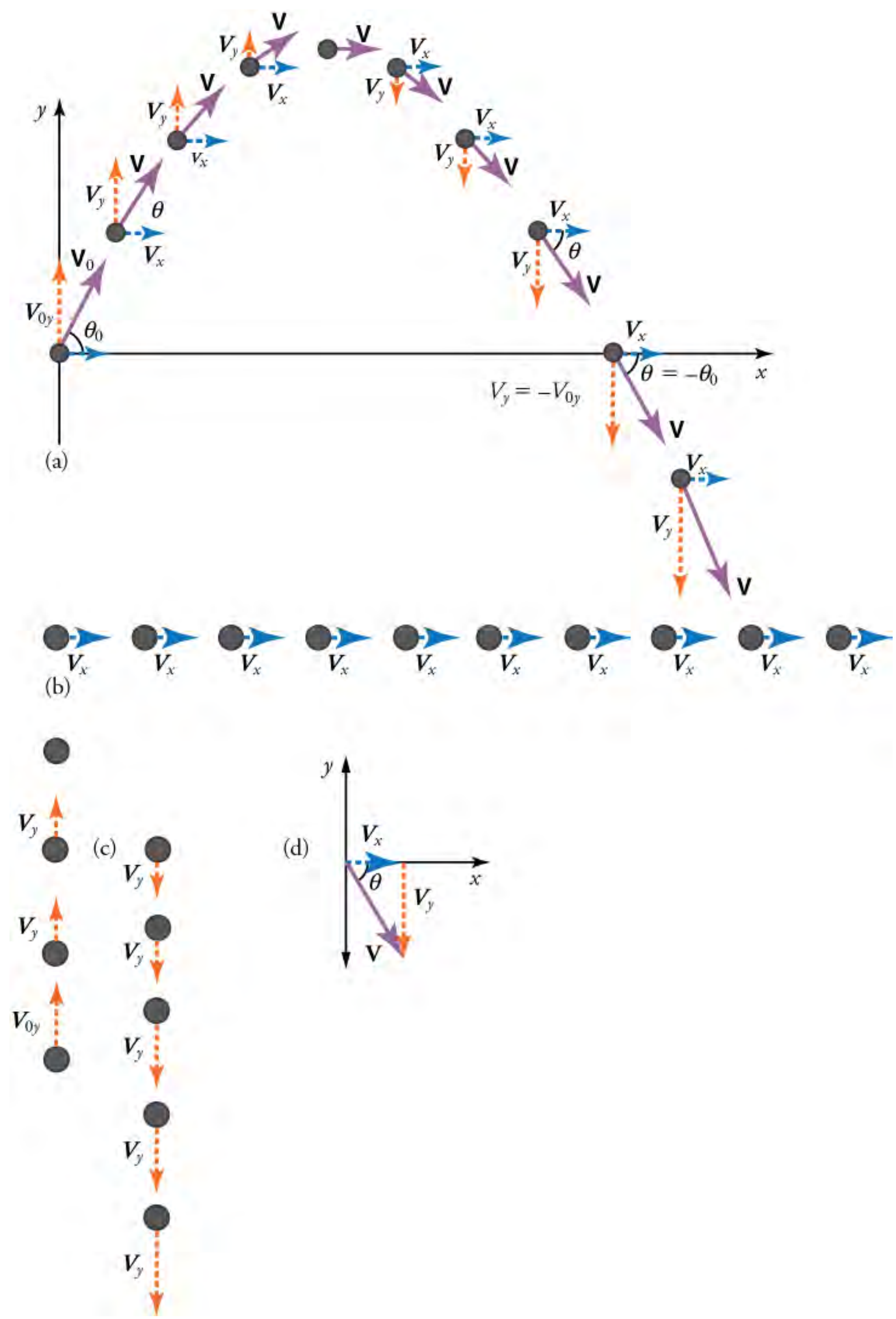


Figure 5.29 (a) We analyze two-dimensional projectile motion by breaking it into two independent one-dimensional motions along the vertical and horizontal axes. (b) The horizontal motion is simple, because $\mathbf{a}_x = 0$ and v_x is thus constant. (c) The velocity in the vertical direction begins to decrease as the object rises; at its highest point, the vertical velocity is zero. As the object falls towards the Earth again, the vertical velocity increases again in magnitude but points in the opposite direction to the initial vertical velocity. (d) The *x*- and *y*-motions are recombined to give the total velocity at any given point on the trajectory.

TIPS FOR SUCCESS

For problems of projectile motion, it is important to set up a coordinate system. The first step is to choose an initial position for $\mathbf{x}$ and $\mathbf{y}$. Usually, it is simplest to set the initial position of the object so that $\mathbf{x}_0 = 0$ and $\mathbf{y}_0 = 0$.

WATCH PHYSICS

Projectile at an Angle

This video presents an example of finding the displacement (or range) of a projectile launched at an angle. It also reviews basic trigonometry for finding the sine, cosine and tangent of an angle.

Click to view content (https://www.khanacademy.org/embed_video?v=ZZ39o1rAZWY)

GRASP CHECK

Assume the ground is uniformly level. If the horizontal component a projectile's velocity is doubled, but the vertical component is unchanged, what is the effect on the time of flight?

a. The time to reach the ground would remain the same since the vertical component is unchanged.
b. The time to reach the ground would remain the same since the vertical component of the velocity also gets doubled.
c. The time to reach the ground would be halved since the horizontal component of the velocity is doubled.
d. The time to reach the ground would be doubled since the horizontal component of the velocity is doubled.

WORKED EXAMPLE

A Fireworks Projectile Explodes High and Away

During a fireworks display like the one illustrated in Figure 5.30, a shell is shot into the air with an initial speed of 70.0 m/s at an angle of 75° above the horizontal. The fuse is timed to ignite the shell just as it reaches its highest point above the ground. (a) Calculate the height at which the shell explodes. (b) How much time passed between the launch of the shell and the explosion? (c) What is the horizontal displacement of the shell when it explodes?

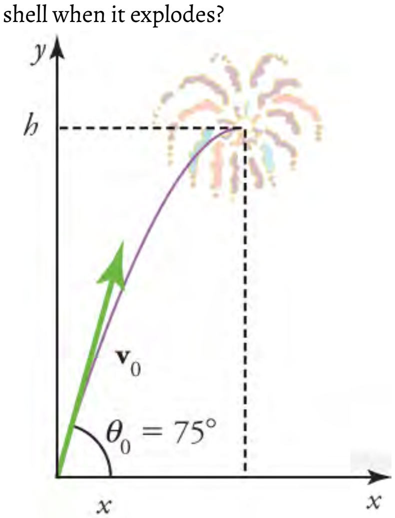


Figure 5.30 The diagram shows the trajectory of a fireworks shell.

Strategy

The motion can be broken into horizontal and vertical motions in which $\mathbf{a}_x = 0$ and $\mathbf{a}_y = \mathbf{g}$. We can then define $\mathbf{x}_0$ and $\mathbf{y}_0$ to be zero and solve for the **maximum height**.

Solution for (a)

By height we mean the altitude or vertical position $\mathbf{y}$ above the starting point. The highest point in any trajectory, the maximum height, is reached when $\mathbf{v}_y = 0$; this is the moment when the vertical velocity switches from positive (upwards) to negative (downwards). Since we know the initial velocity, initial position, and the value of $\mathbf{v}_y$ when the firework reaches its maximum height, we use the following equation to find $\mathbf{y}$

$$\mathbf{v}_y^2 = \mathbf{v}_{0y}^2 - 2\mathbf{g}(\mathbf{y} - y_0).$$

Because $\mathbf{y}_0$ and $\mathbf{v}_y$ are both zero, the equation simplifies to

$$0 = \mathbf{v}_{0y}^2 - 2\mathbf{gy}.$$

Solving for $\mathbf{y}$ gives

$$\mathbf{y} = \frac{\mathbf{v}_{0y}^2}{2\mathbf{g}}.$$

Now we must find $\mathbf{v}_{0y}$, the component of the initial velocity in the y-direction. It is given by $\mathbf{v}_{0y} = \mathbf{v}_0 \sin\theta$, where $\mathbf{v}_{0y}$ is the initial velocity of 70.0 m/s, and $\theta = 75^\circ$ is the initial angle. Thus,

$$\mathbf{v}_{0y} = \mathbf{v}_0 \sin\theta_0 = (70.0 \text{ m/s})(\sin 75^\circ) = 67.6 \text{ m/s}$$

and $\mathbf{y}$ is

$$\mathbf{y} = \frac{(67.6 \text{ m/s})^2}{2(9.80 \text{ m/s}^2),}$$

so that

$$\mathbf{y} = 233 \text{ m}.$$

Discussion for (a)

Since up is positive, the initial velocity and maximum height are positive, but the acceleration due to gravity is negative. The maximum height depends only on the vertical component of the initial velocity. The numbers in this example are reasonable for large fireworks displays, the shells of which do reach such heights before exploding.

Solution for (b)

There is more than one way to solve for the time to the highest point. In this case, the easiest method is to use $\mathbf{y} = \mathbf{y}_0 + \frac{1}{2}(\mathbf{v}_{0y} + \mathbf{v}_y)t$. Because y_0 is zero, this equation reduces to

$$\mathbf{y} = \frac{1}{2}(\mathbf{v}_{0y} + \mathbf{v}_y)t.$$

Note that the final vertical velocity, $\mathbf{v}_y$, at the highest point is zero. Therefore,

$$\begin{aligned} t &= \frac{2\mathbf{y}}{(\mathbf{v}_{0y}+\mathbf{v}_y)} = \frac{2(233 \text{ m})}{(67.6 \text{ m/s})} \\ &= 6.90 \text{ s}. \end{aligned}$$

Discussion for (b)

This time is also reasonable for large fireworks. When you are able to see the launch of fireworks, you will notice several seconds pass before the shell explodes. Another way of finding the time is by using $\mathbf{y} = \mathbf{y}_0 + \mathbf{v}_{0\mathbf{y}}t - \frac{1}{2}\mathbf{g}t^2$, and solving the quadratic equation for t .

Solution for (c)

Because air resistance is negligible, $\mathbf{a}_x = 0$ and the horizontal velocity is constant. The horizontal displacement is horizontal velocity multiplied by time as given by $\mathbf{x} = \mathbf{x}_0 + \mathbf{v}_x t$, where $\mathbf{x}_0$ is equal to zero

$$\mathbf{x} = \mathbf{v}_x t,$$

where $\mathbf{v}_x$ is the x-component of the velocity, which is given by $\mathbf{v}_x = \mathbf{v}_0 \cos\theta_0$. Now,

$$\mathbf{v}_x = \mathbf{v}_0 \cos\theta_0 = (70.0 \text{ m/s})(\cos 75^\circ) = 18.1 \text{ m/s}.$$

The time t for both motions is the same, and so $\mathbf{x}$ is

$$\mathbf{x} = (18.1 \text{ m/s})(6.90 \text{ s}) = 125 \text{ m}.$$

Discussion for (c)

The horizontal motion is a constant velocity in the absence of air resistance. The horizontal displacement found here could be useful in keeping the fireworks fragments from falling on spectators. Once the shell explodes, air resistance has a major effect, and many fragments will land directly below, while some of the fragments may now have a velocity in the –x direction due to the forces of the explosion.

The expression we found for $\mathbf{y}$ while solving part (a) of the previous problem works for any projectile motion problem where air resistance is negligible. Call the maximum height $\mathbf{y} = h$; then,

$$h = \frac{\mathbf{v}_{0y}^2}{2\mathbf{g}}.$$

This equation defines the **maximum height of a projectile**. The maximum height depends only on the vertical component of the initial velocity.

WORKED EXAMPLE

Calculating Projectile Motion: Hot Rock Projectile

Suppose a large rock is ejected from a volcano, as illustrated in Figure 5.31, with a speed of 25.0 m/s and at an angle $35°$ above the horizontal. The rock strikes the side of the volcano at an altitude 20.0 m lower than its starting point. (a) Calculate the time it takes the rock to follow this path.

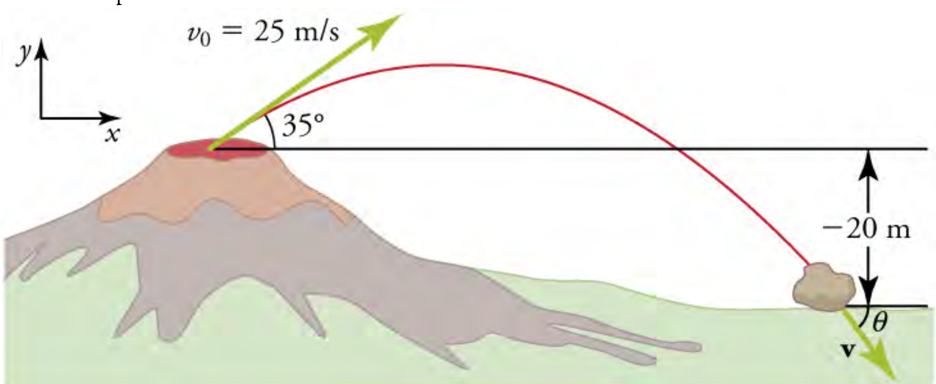


Figure 5.31 The diagram shows the projectile motion of a large rock from a volcano.

Strategy

Breaking this two-dimensional motion into two independent one-dimensional motions will allow us to solve for the time. The time a projectile is in the air depends only on its vertical motion.

Solution

While the rock is in the air, it rises and then falls to a final position 20.0 m lower than its starting altitude. We can find the time for this by using

$$\mathbf{y} = \mathbf{y}_0 + \mathbf{v}_{0\mathbf{y}}t - \frac{1}{2}\mathbf{g}t^2.$$

If we take the initial position $\mathbf{y}_0$ to be zero, then the final position is $\mathbf{y} = -20.0 \text{ m}$. Now the initial vertical velocity is the vertical component of the initial velocity, found from

$$\mathbf{v}_{0y} = \mathbf{v}_0 \sin\theta_0 = (25.0 \text{ m/s})(\sin 35°) = 14.3 \text{ m/s}. \quad (5.9)$$

Substituting known values yields

$$-20.0 \text{ m} = (14.3 \text{ m/s})t - \left(4.90 \text{ m/s}^2\right)t^2.$$

Rearranging terms gives a quadratic equation in t

$$\left(4.90 \text{ m/s}^2\right)t^2 - (14.3 \text{ m/s})t - (20.0 \text{ m}) = 0.$$

This expression is a quadratic equation of the form $at^2 + bt + c = 0$, where the constants are $a = 4.90$, $b = -14.3$, and $c = -20.0$. Its solutions are given by the quadratic formula

$$t = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}.$$

This equation yields two solutions $t = 3.96$ and $t = -1.03$. You may verify these solutions as an exercise. The time is t = 3.96 s or −1.03 s. The negative value of time implies an event before the start of motion, so we discard it. Therefore,

$$t = 3.96 \text{ s}.$$

Discussion

The time for projectile motion is completely determined by the vertical motion. So any projectile that has an initial vertical velocity of 14.3m/s and lands 20.0 m below its starting altitude will spend 3.96 s in the air.

Practice Problems

11. If an object is thrown horizontally, travels with an average x-component of its velocity equal to 5 m/s, and does not hit the ground, what will be the x-component of the displacement after 20 s?

a. -100 m
b. -4 m
c. 4 m
d. 100 m

12. If a ball is thrown straight up with an initial velocity of 20 m/s upward, what is the maximum height it will reach?

a. -20.4 m
b. -1.02 m
c. 1.02 m
d. 20.4 m

The fact that vertical and horizontal motions are independent of each other lets us predict the range of a projectile. The **range** is the horizontal distance **R** traveled by a projectile on level ground, as illustrated in Figure 5.32. Throughout history, people have been interested in finding the range of projectiles for practical purposes, such as aiming cannons.

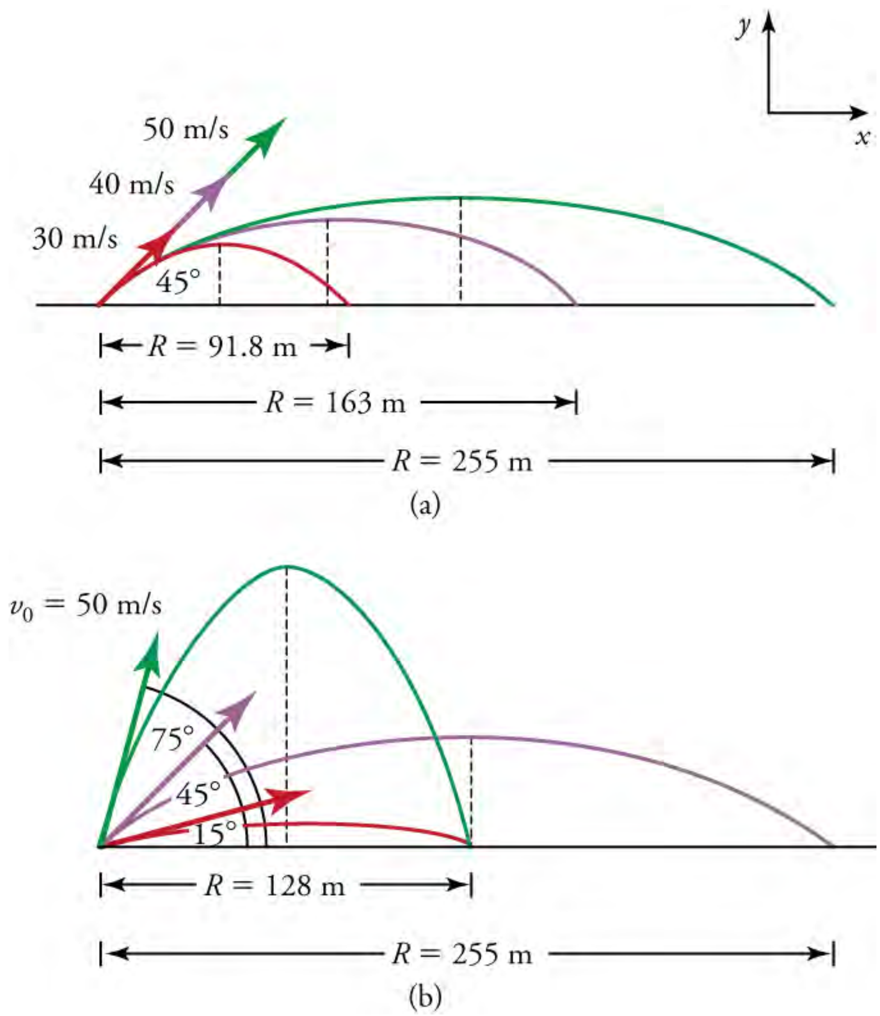


Figure 5.32 Trajectories of projectiles on level ground. (a) The greater the initial speed v_0, the greater the range for a given initial angle. (b) The effect of initial angle θ_0 on the range of a projectile with a given initial speed. Note that any combination of trajectories that add to 90 degrees will have the same range in the absence of air resistance, although the maximum heights of those paths are different.

How does the initial velocity of a projectile affect its range? Obviously, the greater the initial speed v_0, the greater the range, as shown in the figure above. The initial angle θ_0 also has a dramatic effect on the range. When air resistance is negligible, the range R of a projectile on *level ground* is

$$R = \frac{v_0^2 \sin 2\theta_0}{\mathbf{g}},$$

where v_0 is the initial speed and θ_0 is the initial angle relative to the horizontal. It is important to note that the range doesn't apply to problems where the initial and final y position are different, or to cases where the object is launched perfectly horizontally.

Virtual Physics

Projectile Motion

In this simulation you will learn about projectile motion by blasting objects out of a cannon. You can choose between objects such as a tank shell, a golf ball or even a Buick. Experiment with changing the angle, initial speed, and mass, and adding in air resistance. Make a game out of this simulation by trying to hit the target.

Click to view content (https://archive.cnx.org/specials/317dbd00-8e61-4065-b3eb-f2b80db9b7ed/projectile-motion/)

GRASP CHECK

If a projectile is launched on level ground, what launch angle maximizes the range of the projectile?

a. 0°
b. 30°
c. 45°
d. 60°

Check Your Understanding

13. What is projectile motion?

a. Projectile motion is the motion of an object projected into the air, which moves under the influence of gravity.
b. Projectile motion is the motion of an object projected into the air which moves independently of gravity.
c. Projectile motion is the motion of an object projected vertically upward into the air which moves under the influence of gravity.
d. Projectile motion is the motion of an object projected horizontally into the air which moves independently of gravity.

14. What is the force experienced by a projectile after the initial force that launched it into the air in the absence of air resistance?

a. The nuclear force
b. The gravitational force
c. The electromagnetic force
d. The contact force

5.4 Inclined Planes

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Distinguish between static friction and kinetic friction
- Solve problems involving inclined planes

Section Key Terms

kinetic friction	static friction

Static Friction and Kinetic Friction

Recall from the previous chapter that friction is a force that opposes motion, and is around us all the time. Friction allows us to move, which you have discovered if you have ever tried to walk on ice.

There are different types of friction—kinetic and static. **Kinetic friction** acts on an object in motion, while **static friction** acts on an object or system at rest. The maximum static friction is usually greater than the kinetic friction between the objects.

Imagine, for example, trying to slide a heavy crate across a concrete floor. You may push harder and harder on the crate and not move it at all. This means that the static friction responds to what you do—it increases to be equal to and in the opposite direction of your push. But if you finally push hard enough, the crate seems to slip suddenly and starts to move. Once in motion, it is easier to keep it in motion than it was to get it started because the kinetic friction force is less than the static friction force. If you were to add mass to the crate, (for example, by placing a box on top of it) you would need to push even harder to get it started and also to keep it moving. If, on the other hand, you oiled the concrete you would find it easier to get the crate started and keep it going.

Figure 5.33 shows how friction occurs at the interface between two objects. Magnifying these surfaces shows that they are rough on the microscopic level. So when you push to get an object moving (in this case, a crate), you must raise the object until it can skip along with just the tips of the surface hitting, break off the points, or do both. The harder the surfaces are pushed together (such as if another box is placed on the crate), the more force is needed to move them.

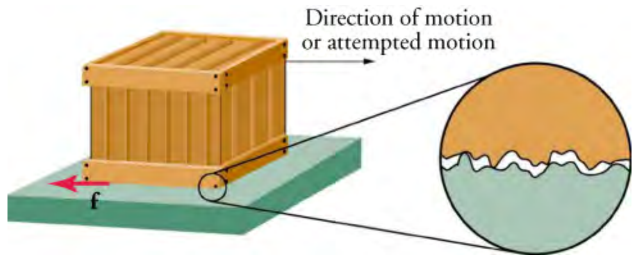


Figure 5.33 Frictional forces, such as **f**, always oppose motion or attempted motion between objects in contact. Friction arises in part because of the roughness of the surfaces in contact, as seen in the expanded view.

The magnitude of the frictional force has two forms: one for static friction, the other for kinetic friction. When there is no motion between the objects, the magnitude of static friction $\mathbf{f_s}$ is

$$\mathbf{f}_{\mathrm{s}} \leq \mu_{\mathrm{s}} \mathbf{N}_{\mathrm{s}},$$

where μ_{s} is the coefficient of static friction and **N** is the magnitude of the normal force. Recall that the normal force opposes the force of gravity and acts perpendicular to the surface in this example, but not always.

Since the symbol $\leq$ means less than or equal to, this equation says that static friction can have a maximum value of $\mu_{\mathrm{s}} \mathbf{N}$. That is,

$$\mathbf{f}_{\mathrm{s}}(\max) = \mu_{\mathrm{s}} \mathbf{N}.$$

Static friction is a responsive force that increases to be equal and opposite to whatever force is exerted, up to its maximum limit. Once the applied force exceeds **f**s(max), the object will move. Once an object is moving, the magnitude of kinetic friction $\mathbf{f}_{\mathrm{k}}$ is given by

$$\mathbf{f}_{\mathrm{k}} = \mu_{\mathrm{k}} \mathbf{N}.$$

where μ_{k} is the coefficient of kinetic friction.

Friction varies from surface to surface because different substances are rougher than others. Table 5.2 compares values of static and kinetic friction for different surfaces. The coefficient of the friction depends on the two surfaces that are in contact.

System	Static Friction μ_{s}	Kinetic Friction μ_{k}
Rubber on dry concrete	1.0	0.7
Rubber on wet concrete	0.7	0.5
Wood on wood	0.5	0.3
Waxed wood on wet snow	0.14	0.1
Metal on wood	0.5	0.3
Steel on steel (dry)	0.6	0.3
Steel on steel (oiled)	0.05	0.03
Teflon on steel	0.04	0.04
Bone lubricated by synovial fluid	0.016	0.015
Shoes on wood	0.9	0.7

Table 5.2 Coefficients of Static and Kinetic Friction

System	Static Friction μ_s	Kinetic Friction μ_k
Shoes on ice	0.1	0.05
Ice on ice	0.1	0.03
Steel on ice	0.4	0.02

Table 5.2 Coefficients of Static and Kinetic Friction

Since the direction of friction is always opposite to the direction of motion, friction runs parallel to the surface between objects and perpendicular to the normal force. For example, if the crate you try to push (with a force parallel to the floor) has a mass of 100 kg, then the normal force would be equal to its weight

$$\mathbf{W} = m\mathbf{g} = (100 \text{ kg})(9.80\text{m/s}^2) = 980 \text{ N},$$

perpendicular to the floor. If the coefficient of static friction is 0.45, you would have to exert a force parallel to the floor greater than

$$\mathbf{f}_\text{s}(\text{max}) = \mu_\text{s}\mathbf{N} = (0.45)(980 \text{ N}) = 440 \text{ N}$$

to move the crate. Once there is motion, friction is less and the coefficient of kinetic friction might be 0.30, so that a force of only 290 N

$$\mathbf{f}_\text{k} = \mu_\text{k}\mathbf{N} = (0.30)(980 \text{ N}) = 290 \text{ N}$$

would keep it moving at a constant speed. If the floor were lubricated, both coefficients would be much smaller than they would be without lubrication. The coefficient of friction is unitless and is a number usually between 0 and 1.0.

Working with Inclined Planes

We discussed previously that when an object rests on a horizontal surface, there is a normal force supporting it equal in magnitude to its weight. Up until now, we dealt only with normal force in one dimension, with gravity and normal force acting perpendicular to the surface in opposing directions (gravity downward, and normal force upward). Now that you have the skills to work with forces in two dimensions, we can explore what happens to weight and the normal force on a tilted surface such as an inclined plane. For inclined plane problems, it is easier breaking down the forces into their components if we rotate the coordinate system, as illustrated in Figure 5.34. The first step when setting up the problem is to break down the force of weight into components.

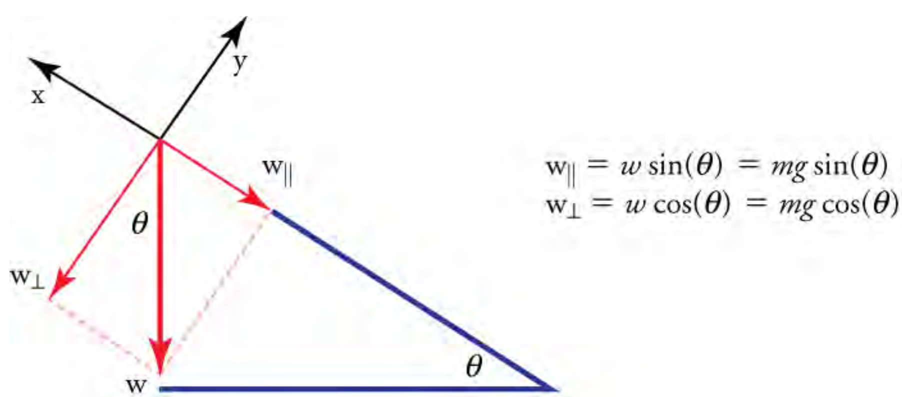


Figure 5.34 The diagram shows perpendicular and horizontal components of weight on an inclined plane.

When an object rests on an incline that makes an angle θ with the horizontal, the force of gravity acting on the object is divided into two components: A force acting perpendicular to the plane, $\mathbf{w}_\perp$, and a force acting parallel to the plane, $\mathbf{w}_{||}$. The perpendicular force of weight, $\mathbf{w}_\perp$, is typically equal in magnitude and opposite in direction to the normal force, $\mathbf{N}$. The force acting parallel to the plane, $\mathbf{w}_{||}$, causes the object to accelerate down the incline. The force of friction, $\mathbf{f}$, opposes the motion of the object, so it acts upward along the plane.

It is important to be careful when resolving the weight of the object into components. If the angle of the incline is at an angle θ to the horizontal, then the magnitudes of the weight components are

$$\mathbf{w}_{||} = \mathbf{w}sin(\theta) = m\mathbf{g}sin(\theta) \text{ and}$$
$$\mathbf{w}_{\perp} = \mathbf{w}cos(\theta) = m\mathbf{g}cos(\theta).$$

Instead of memorizing these equations, it is helpful to be able to determine them from reason. To do this, draw the right triangle formed by the three weight vectors. Notice that the angle of the incline is the same as the angle formed between $\mathbf{w}$ and $\mathbf{w}_{\perp}$. Knowing this property, you can use trigonometry to determine the magnitude of the weight components

$$cos(\theta) = \frac{\mathbf{w}_{\perp}}{\mathbf{w}}$$
$$\mathbf{w}_{\perp} = \mathbf{w}cos(\theta) = m\mathbf{g}cos(\theta)$$
$$sin(\theta) = \frac{\mathbf{w}_{||}}{\mathbf{w}}$$
$$\mathbf{w}_{||} = \mathbf{w}sin(\theta) = m\mathbf{g}sin(\theta).$$

WATCH PHYSICS

Inclined Plane Force Components

This video (https://www.khanacademy.org/embed_video?v=TC23wD34C7k) shows how the weight of an object on an inclined plane is broken down into components perpendicular and parallel to the surface of the plane. It explains the geometry for finding the angle in more detail.

GRASP CHECK

Click to view content (https://www.youtube.com/embed/TC23wD34C7k)

This video shows how the weight of an object on an inclined plane is broken down into components perpendicular and parallel to the surface of the plane. It explains the geometry for finding the angle in more detail.

When the surface is flat, you could say that one of the components of the gravitational force is zero; Which one? As the angle of the incline gets larger, what happens to the magnitudes of the perpendicular and parallel components of gravitational force?

a. When the angle is zero, the parallel component is zero and the perpendicular component is at a maximum. As the angle increases, the parallel component decreases and the perpendicular component increases. This is because the cosine of the angle shrinks while the sine of the angle increases.
b. When the angle is zero, the parallel component is zero and the perpendicular component is at a maximum. As the angle increases, the parallel component decreases and the perpendicular component increases. This is because the cosine of the angle increases while the sine of the angle shrinks.
c. When the angle is zero, the parallel component is zero and the perpendicular component is at a maximum. As the angle increases, the parallel component increases and the perpendicular component decreases. This is because the cosine of the angle shrinks while the sine of the angle increases.
d. When the angle is zero, the parallel component is zero and the perpendicular component is at a maximum. As the angle increases, the parallel component increases and the perpendicular component decreases. This is because the cosine of the angle increases while the sine of the angle shrinks.

TIPS FOR SUCCESS

Normal force is represented by the variable $\mathbf{N}$. This should not be confused with the symbol for the newton, which is also represented by the letter N. It is important to tell apart these symbols, especially since the units for normal force ($\mathbf{N}$) happen to be newtons (N). For example, the normal force, $\mathbf{N}$, that the floor exerts on a chair might be $\mathbf{N} = 100 \text{ N}$. One important difference is that normal force is a vector, while the newton is simply a unit. Be careful not to confuse these letters in your calculations!

To review, the process for solving inclined plane problems is as follows:

1. Draw a sketch of the problem.
2. Identify known and unknown quantities, and identify the system of interest.
3. Draw a free-body diagram (which is a sketch showing all of the forces acting on an object) with the coordinate system rotated at the same angle as the inclined plane. Resolve the vectors into horizontal and vertical components and draw them on the free-body diagram.
4. Write Newton's second law in the horizontal and vertical directions and add the forces acting on the object. If the object does not accelerate in a particular direction (for example, the x -direction) then $\mathbf{F}$net $x = 0$. If the object does accelerate in that direction, $\mathbf{F}$net $x = m\mathbf{a}$.
5. Check your answer. Is the answer reasonable? Are the units correct?

WORKED EXAMPLE

Finding the Coefficient of Kinetic Friction on an Inclined Plane

A skier, illustrated in Figure 5.35(a), with a mass of 62 kg is sliding down a snowy slope at an angle of 25 degrees. Find the coefficient of kinetic friction for the skier if friction is known to be 45.0 N.

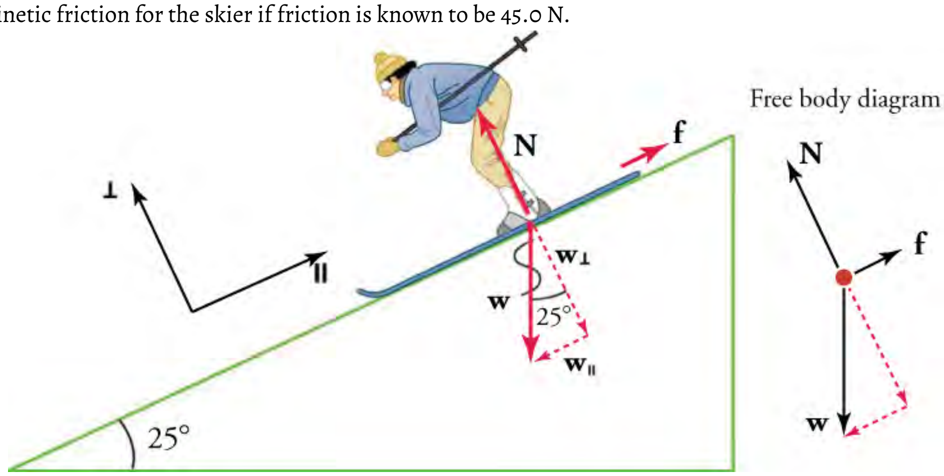


Figure 5.35 Use the diagram to help find the coefficient of kinetic friction for the skier.

Strategy

The magnitude of kinetic friction was given as 45.0 N. Kinetic friction is related to the normal force **N** as $\mathbf{f}_\mathrm{k} = \mu_\mathrm{k}\mathbf{N}$. Therefore, we can find the coefficient of kinetic friction by first finding the normal force of the skier on a slope. The normal force is always perpendicular to the surface, and since there is no motion perpendicular to the surface, the normal force should equal the component of the skier's weight perpendicular to the slope.

That is,

$$\mathbf{N} = \mathbf{w}_\perp = \mathbf{w}\cos(25^\circ) = m\mathbf{g}\cos(25^\circ).$$

Substituting this into our expression for kinetic friction, we get

$$\mathbf{f}_\mathrm{k} = \mu_\mathrm{k} m\mathbf{g}\cos 25^\circ,$$

which can now be solved for the coefficient of kinetic friction μ_k.

Solution

Solving for μ_k gives

$$\mu_\mathrm{k} = \frac{\mathbf{f}_\mathrm{k}}{\mathbf{w}\cos 25^\circ} = \frac{\mathbf{f}_\mathrm{k}}{m\mathbf{g}\cos 25^\circ}.$$

Substituting known values on the right-hand side of the equation,

$$\mu_\mathrm{k} = \frac{45.0\ \mathrm{N}}{(62\ \mathrm{kg})(9.80\ \mathrm{m/s^2})(0.906)} = 0.082.$$

Discussion

This result is a little smaller than the coefficient listed in Table 5.2 for waxed wood on snow, but it is still reasonable since values of the coefficients of friction can vary greatly. In situations like this, where an object of mass m slides down a slope that makes an angle θ with the horizontal, friction is given by $\mathbf{f}_\mathrm{k} = \mu_\mathrm{k} m\mathbf{g} \cos\theta$.

WORKED EXAMPLE

Weight on an Incline, a Two-Dimensional Problem

The skier's mass, including equipment, is 60.0 kg. (See Figure 5.36(b).) (a) What is her acceleration if friction is negligible? (b) What is her acceleration if the frictional force is 45.0 N?

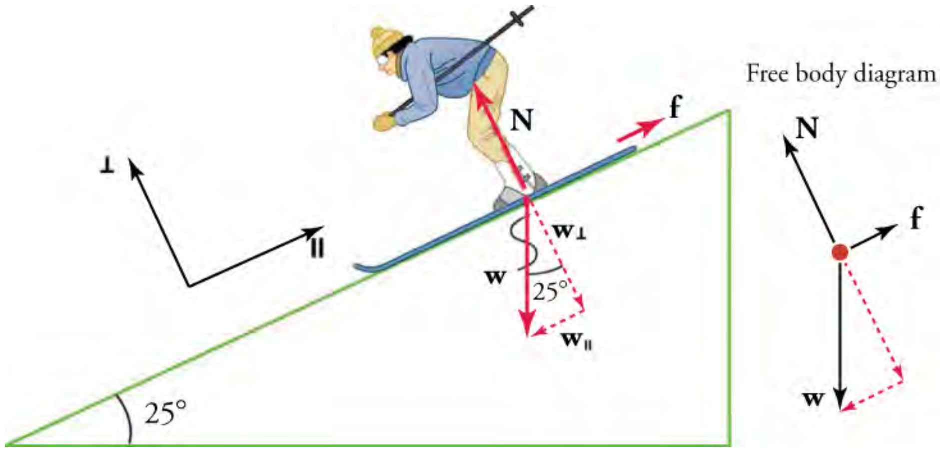


Figure 5.36 Now use the diagram to help find the skier's acceleration if friction is negligible and if the frictional force is 45.0 N.

Strategy

The most convenient coordinate system for motion on an incline is one that has one coordinate parallel to the slope and one perpendicular to the slope. Remember that motions along perpendicular axes are independent. We use the symbol $\perp$ to mean perpendicular, and $||$ to mean parallel.

The only external forces acting on the system are the skier's weight, friction, and the normal force exerted by the ski slope, labeled $\mathbf{w}$, $\mathbf{f}$, and $\mathbf{N}$ in the free-body diagram. $\mathbf{N}$ is always perpendicular to the slope and $\mathbf{f}$ is parallel to it. But $\mathbf{w}$ is not in the direction of either axis, so we must break it down into components along the chosen axes. We define $\mathbf{w}_{||}$ to be the component of weight parallel to the slope and $\mathbf{w}_\perp$ the component of weight perpendicular to the slope. Once this is done, we can consider the two separate problems of forces parallel to the slope and forces perpendicular to the slope.

Solution

The magnitude of the component of the weight parallel to the slope is $\mathbf{w}_{||} = \mathbf{w}\sin(25°) = m\mathbf{g}\sin(25°)$, and the magnitude of the component of the weight perpendicular to the slope is $\mathbf{w}_\perp = \mathbf{w}\cos(25°) = m\mathbf{g}\cos(25°\ )$.

(a) Neglecting friction: Since the acceleration is parallel to the slope, we only need to consider forces parallel to the slope. Forces perpendicular to the slope add to zero, since there is no acceleration in that direction. The forces parallel to the slope are the amount of the skier's weight parallel to the slope $\mathbf{w}_{||}$ and friction $\mathbf{f}$. Assuming no friction, by Newton's second law the acceleration parallel to the slope is

$$\mathbf{a}_{||} = \frac{\mathbf{F}_{\mathrm{net}\ ||}}{m},$$

Where the net force parallel to the slope $\mathbf{F}_{\mathrm{net}\ ||} = \mathbf{w}_{||} = m\mathbf{g}\sin(25°)$, so that

$$\begin{aligned} \mathbf{a}_{||} &= \frac{\mathbf{F}_{\mathrm{net}\ ||}}{m} = \frac{m\mathbf{g}\sin(25°)}{m} = \mathbf{g}\sin(25°) \\ &= (9.80\ \mathrm{m/s^2})(0.423) = 4.14\ \mathrm{m/s^2} \end{aligned}$$

is the acceleration.

(b) Including friction: Here we now have a given value for friction, and we know its direction is parallel to the slope and it opposes motion between surfaces in contact. So the net external force is now

$$\mathbf{F}_{\text{net } ||} = \mathbf{w}_{||} - \mathbf{f},$$

and substituting this into Newton's second law, $a_{||} = \frac{\mathbf{F}_{\text{net } ||}}{m}$ gives

$$\mathbf{a}_{||} = \frac{\mathbf{F}_{\text{net } ||}}{m} = \frac{\mathbf{w}_{||} - \mathbf{f}}{m} = \frac{m\mathbf{g}\sin(25°) - \mathbf{f}}{m}.$$

We substitute known values to get

$$\mathbf{a}_{||} = \frac{(60.0 \text{ kg})(9.80 \text{ m/s}^2)(0.423) - 45.0 \text{ N}}{60.0 \text{ kg}},$$

or

$$\mathbf{a}_{||} = 3.39 \text{ m/s}^2,$$

which is the acceleration parallel to the incline when there is 45 N opposing friction.

Discussion
Since friction always opposes motion between surfaces, the acceleration is smaller when there is friction than when there is not.

Practice Problems

15. When an object sits on an inclined plane that makes an angle θ with the horizontal, what is the expression for the component of the objects weight force that is parallel to the incline?
   a. $w_{||} = w\cos\theta$
   b. $w_{||} = w\sin\theta$
   c. $w_{||} = w\sin\theta - \cos\theta$
   d. $w_{||} = w\cos\theta - \sin\theta$

16. An object with a mass of 5 kg rests on a plane inclined $30°$ from horizontal. What is the component of the weight force that is parallel to the incline?
   a. 4.33 N
   b. 5.0 N
   c. 24.5 N
   d. 42.43 N

Snap Lab

Friction at an Angle: Sliding a Coin

An object will slide down an inclined plane at a constant velocity if the net force on the object is zero. We can use this fact to measure the coefficient of kinetic friction between two objects. As shown in the first Worked Example, the kinetic friction on a slope $\mathbf{f}_\text{k} = \mu_\text{k} m\mathbf{g} \cos\theta$, and the component of the weight down the slope is equal to $m\mathbf{g} \sin\theta$. These forces act in opposite directions, so when they have equal magnitude, the acceleration is zero. Writing these out

$$\begin{aligned} \mathbf{f}_\text{k} &= \mathbf{Fg}_x \\ \mu_\text{k} m\mathbf{g} \cos\theta &= m\mathbf{g} \sin\theta. \end{aligned}$$

Solving for μ_k, since $\tan\theta = \sin\theta/\cos\theta$ we find that

$$\mu_\text{k} = \frac{m\mathbf{g} \sin\theta}{m\mathbf{g} \cos\theta} = \tan\theta. \qquad (5.10)$$

- 1 coin
- 1 book
- 1 protractor

1. Put a coin flat on a book and tilt it until the coin slides at a constant velocity down the book. You might need to tap the book lightly to get the coin to move.

2. Measure the angle of tilt relative to the horizontal and find μ_k .

GRASP CHECK

True or False—If only the angles of two vectors are known, we can find the angle of their resultant addition vector.

a. True
b. False

Check Your Understanding

17. What is friction?

a. Friction is an internal force that opposes the relative motion of an object.
b. Friction is an internal force that accelerates an object's relative motion.
c. Friction is an external force that opposes the relative motion of an object.
d. Friction is an external force that increases the velocity of the relative motion of an object.

18. What are the two varieties of friction? What does each one act upon?

a. Kinetic and static friction both act on an object in motion.
b. Kinetic friction acts on an object in motion, while static friction acts on an object at rest.
c. Kinetic friction acts on an object at rest, while static friction acts on an object in motion.
d. Kinetic and static friction both act on an object at rest.

19. Between static and kinetic friction between two surfaces, which has a greater value? Why?

a. The kinetic friction has a greater value because the friction between the two surfaces is more when the two surfaces are in relative motion.
b. The static friction has a greater value because the friction between the two surfaces is more when the two surfaces are in relative motion.
c. The kinetic friction has a greater value because the friction between the two surfaces is less when the two surfaces are in relative motion.
d. The static friction has a greater value because the friction between the two surfaces is less when the two surfaces are in relative motion.

5.5 Simple Harmonic Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Hooke's law and Simple Harmonic Motion
- Describe periodic motion, oscillations, amplitude, frequency, and period
- Solve problems in simple harmonic motion involving springs and pendulums

Section Key Terms

amplitude	deformation	equilibrium position	frequency
Hooke's law	oscillate	period	periodic motion
restoring force	simple harmonic motion	simple pendulum	

Hooke's Law and Simple Harmonic Motion

Imagine a car parked against a wall. If a bulldozer pushes the car into the wall, the car will not move but it will noticeably change shape. A change in shape due to the application of a force is a **deformation**. Even very small forces are known to cause some deformation. For small deformations, two important things can happen. First, unlike the car and bulldozer example, the object returns to its original shape when the force is removed. Second, the size of the deformation is proportional to the force. This second property is known as **Hooke's law**. In equation form, Hooke's law is

$$\mathbf{F} = -\mathbf{kx},$$

where **x** is the amount of deformation (the change in length, for example) produced by the restoring force **F**, and **k** is a constant that depends on the shape and composition of the object. The restoring force is the force that brings the object back to its equilibrium position; the minus sign is there because the restoring force acts in the direction opposite to the displacement. Note that the restoring force is proportional to the deformation **x**. The deformation can also be thought of as a displacement from equilibrium. It is a change in position due to a force. In the absence of force, the object would rest at its equilibrium position. The force constant **k** is related to the stiffness of a system. The larger the force constant, the stiffer the system. A stiffer system is more difficult to deform and requires a greater restoring force. The units of **k** are newtons per meter (N/m). One of the most common uses of Hooke's law is solving problems involving springs and pendulums, which we will cover at the end of this section.

Oscillations and Periodic Motion

What do an ocean buoy, a child in a swing, a guitar, and the beating of hearts all have in common? They all **oscillate**. That is, they move back and forth between two points, like the ruler illustrated in Figure 5.37. All oscillations involve force. For example, you push a child in a swing to get the motion started.

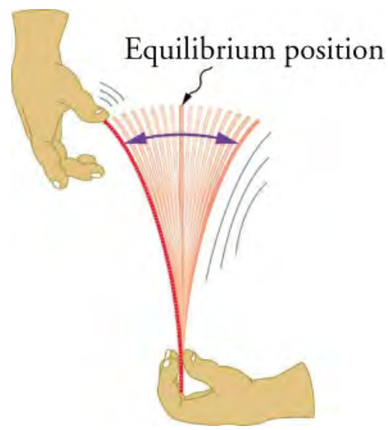


Figure 5.37 A ruler is displaced from its equilibrium position.

Newton's first law implies that an object oscillating back and forth is experiencing forces. Without force, the object would move in a straight line at a constant speed rather than oscillate. Consider, for example, plucking a plastic ruler to the left as shown in Figure 5.38. The deformation of the ruler creates a force in the opposite direction, known as a **restoring force**. Once released, the restoring force causes the ruler to move back toward its stable equilibrium position, where the net force on it is zero. However, by the time the ruler gets there, it gains momentum and continues to move to the right, producing the opposite deformation. It is then forced to the left, back through equilibrium, and the process is repeated until it gradually loses all of its energy. The simplest oscillations occur when the restoring force is directly proportional to displacement. Recall that Hooke's law describes this situation with the equation **F** = –**kx**. Therefore, Hooke's law describes and applies to the simplest case of oscillation, known as **simple harmonic motion**.

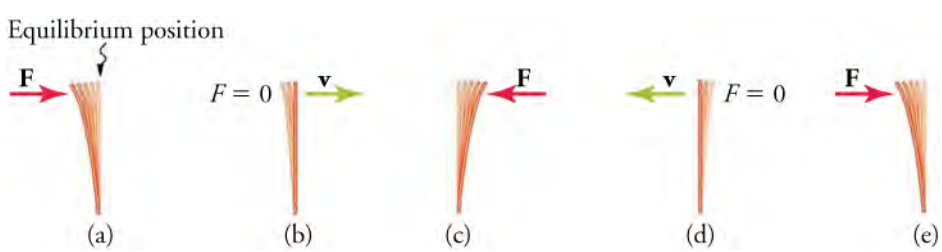


Figure 5.38 (a) The plastic ruler has been released, and the restoring force is returning the ruler to its equilibrium position. (b) The net force is zero at the equilibrium position, but the ruler has momentum and continues to move to the right. (c) The restoring force is in the opposite direction. It stops the ruler and moves it back toward equilibrium again. (d) Now the ruler has momentum to the left. (e) In the absence of damping (caused by frictional forces), the ruler reaches its original position. From there, the motion will repeat itself.

When you pluck a guitar string, the resulting sound has a steady tone and lasts a long time. Each vibration of the string takes the same time as the previous one. **Periodic motion** is a motion that repeats itself at regular time intervals, such as with an object bobbing up and down on a spring or a pendulum swinging back and forth. The time to complete one oscillation (a complete cycle of motion) remains constant and is called the **period** T. Its units are usually seconds.

Frequency f is the number of oscillations per unit time. The SI unit for frequency is the hertz (Hz), defined as the number of oscillations per second. The relationship between frequency and period is

$$f = 1/T.$$

As you can see from the equation, frequency and period are different ways of expressing the same concept. For example, if you get a paycheck twice a month, you could say that the frequency of payment is two per month, or that the period between checks is half a month.

If there is no friction to slow it down, then an object in simple motion will oscillate forever with equal displacement on either side of the equilibrium position. The **equilibrium position** is where the object would naturally rest in the absence of force. The maximum displacement from equilibrium is called the **amplitude X**. The units for amplitude and displacement are the same, but depend on the type of oscillation. For the object on the spring, shown in Figure 5.39, the units of amplitude and displacement are meters.

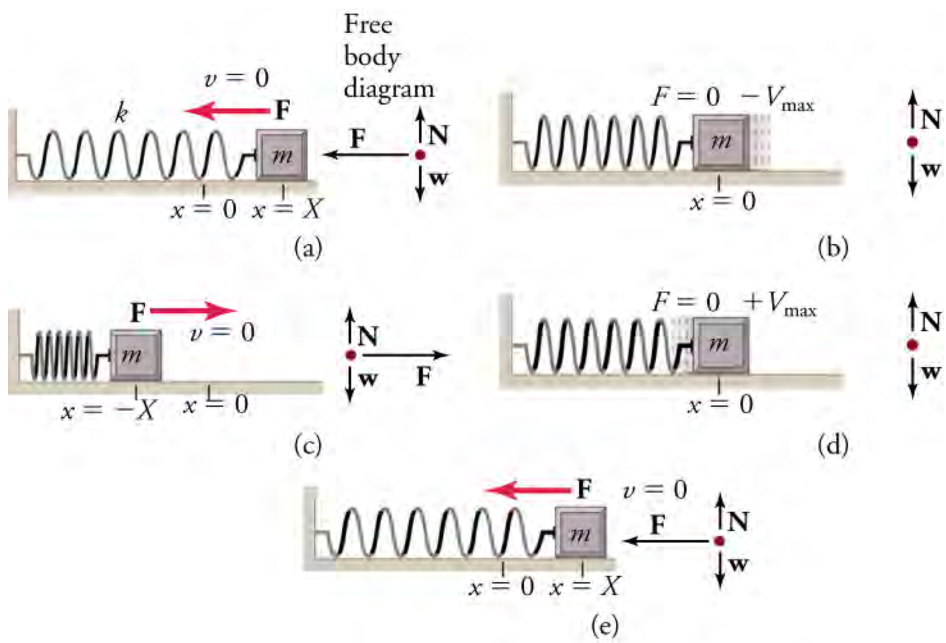


Figure 5.39 An object attached to a spring sliding on a frictionless surface is a simple harmonic oscillator. When displaced from equilibrium, the object performs simple harmonic motion that has an amplitude **X** and a period T. The object's maximum speed occurs as it passes through equilibrium. The stiffer the spring is, the smaller the period T. The greater the mass of the object is, the greater the period T.

The mass m and the force constant **k** are the *only* factors that affect the period and frequency of simple harmonic motion. The period of a simple harmonic oscillator is given by

$$T = 2\pi\sqrt{\frac{m}{\mathbf{k}}}$$

and, because $f = 1/T$, the frequency of a simple harmonic oscillator is

$$f = \frac{1}{2\pi}\sqrt{\frac{\mathbf{k}}{m}}.$$

WATCH PHYSICS

Introduction to Harmonic Motion

This video shows how to graph the displacement of a spring in the x-direction over time, based on the period. Watch the first 10 minutes of the video (you can stop when the narrator begins to cover calculus).

Click to view content (https://www.khanacademy.org/embed_video?v=Nk2q-_jkJVs)

GRASP CHECK

If the amplitude of the displacement of a spring were larger, how would this affect the graph of displacement over time? What would happen to the graph if the period was longer?

a. Larger amplitude would result in taller peaks and troughs and a longer period would result in greater separation in time between peaks.
b. Larger amplitude would result in smaller peaks and troughs and a longer period would result in greater distance between peaks.
c. Larger amplitude would result in taller peaks and troughs and a longer period would result in shorter distance between peaks.
d. Larger amplitude would result in smaller peaks and troughs and a longer period would result in shorter distance between peaks.

Solving Spring and Pendulum Problems with Simple Harmonic Motion

Before solving problems with springs and pendulums, it is important to first get an understanding of how a pendulum works. Figure 5.40 provides a useful illustration of a simple pendulum.

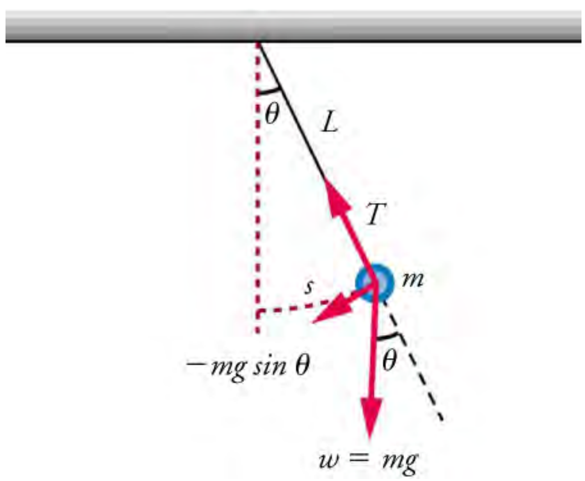


Figure 5.40 A simple pendulum has a small-diameter bob and a string that has a very small mass but is strong enough not to stretch. The linear displacement from equilibrium is s, the length of the arc. Also shown are the forces on the bob, which result in a net force of $-mg \sin\theta$ toward the equilibrium position—that is, a restoring force.

Everyday examples of pendulums include old-fashioned clocks, a child's swing, or the sinker on a fishing line. For small displacements of less than 15 degrees, a pendulum experiences simple harmonic oscillation, meaning that its restoring force is directly proportional to its displacement. A pendulum in simple harmonic motion is called a **simple pendulum**. A pendulum has an object with a small mass, also known as the pendulum bob, which hangs from a light wire or string. The equilibrium position for a pendulum is where the angle θ is zero (that is, when the pendulum is hanging straight down). It makes sense that without any force applied, this is where the pendulum bob would rest.

The displacement of the pendulum bob is the arc length s. The weight $m\mathbf{g}$ has components $m\mathbf{g} \cos\theta$ along the string and $m\mathbf{g} \sin\theta$ tangent to the arc. Tension in the string exactly cancels the component $m\mathbf{g} \cos\theta$ parallel to the string. This leaves a *net* restoring force back toward the equilibrium position that runs tangent to the arc and equals $-m\mathbf{g} \sin\theta$.

For a simple pendulum, The period is $T = 2\pi\sqrt{\frac{L}{\mathbf{g}}}$.

The only things that affect the period of a simple pendulum are its length and the acceleration due to gravity. The period is completely independent of other factors, such as mass or amplitude. However, note that *T* does depend on **g**. This means that if we know the length of a pendulum, we can actually use it to measure gravity! This will come in useful in Figure 5.40.

TIPS FOR SUCCESS

Tension is represented by the variable **T**, and period is represented by the variable *T*. It is important not to confuse the two, since tension is a force and period is a length of time.

WORKED EXAMPLE

Measuring Acceleration due to Gravity: The Period of a Pendulum

What is the acceleration due to gravity in a region where a simple pendulum having a length 75.000 cm has a period of 1.7357 s?

Strategy

We are asked to find **g** given the period *T* and the length *L* of a pendulum. We can solve $T = 2\pi\sqrt{\frac{L}{\mathbf{g}}}$ for **g**, assuming that the angle of deflection is less than 15 degrees. Recall that when the angle of deflection is less than 15 degrees, the pendulum is considered to be in simple harmonic motion, allowing us to use this equation.

Solution

1. Square $T = 2\pi\sqrt{\frac{L}{\mathbf{g}}}$ and solve for *g*.

$$\mathbf{g} = 4\pi^2 \frac{L}{T^2}$$

2. Substitute known values into the new equation.

$$\mathbf{g} = 4\pi^2 \frac{0.75000 \text{ m}}{(1.7357 \text{ s})^2}$$

3. Calculate to find **g**.

$$\mathbf{g} = 9.8281 \text{ m/s}^2$$

Discussion

This method for determining **g** can be very accurate. This is why length and period are given to five digits in this example.

WORKED EXAMPLE

Hooke's Law: How Stiff Are Car Springs?

What is the force constant for the suspension system of a car, like that shown in Figure 5.41, that settles 1.20 cm when an 80.0-kg person gets in?

Figure 5.41 A car in a parking lot. (exfordy, Flickr)

Strategy

Consider the car to be in its equilibrium position $\mathbf{x} = 0$ before the person gets in. The car then settles down 1.20 cm, which means it is displaced to a position $\mathbf{x} = -1.20 \times 10^{-2}$ m.

At that point, the springs supply a restoring force **F** equal to the person's weight

$\mathbf{w} = m\mathbf{g} = (80.0 \text{ kg})(9.80 \text{ m/s}^2) = 784$ N. We take this force to be **F** in Hooke's law.

Knowing **F** and **x**, we can then solve for the force constant **k**.

Solution

Solve Hooke's law, $\mathbf{F} = -\mathbf{kx}$, for **k**.

$$\mathbf{k} = \frac{\mathbf{F}}{\mathbf{x}}$$

Substitute known values and solve for **k**.

$$\begin{aligned} \mathbf{k} &= \frac{-784 \text{ N}}{-1.20 \times 10^{-2} \text{ m}} \\ &= 6.53 \times 10^4 \text{ N/m} \end{aligned}$$

Discussion

Note that **F** and **x** have opposite signs because they are in opposite directions—the restoring force is up, and the displacement is down. Also, note that the car would oscillate up and down when the person got in, if it were not for the shock absorbers. Bouncing cars are a sure sign of bad shock absorbers.

Practice Problems

20. A force of 70 N applied to a spring causes it to be displaced by 0.3 m. What is the force constant of the spring?

a. -233 N/m
b. -21 N/m
c. 21 N/m
d. 233 N/m

21. What is the force constant for the suspension system of a car that settles 3.3 cm when a 65 kg person gets in?

a. 1.93×10^4 N/m
b. 1.97×10^3 N/m

c. 1.93×10^2 N/m
d. 1.97×10^1 N/m

Snap Lab

Finding Gravity Using a Simple Pendulum

Use a simple pendulum to find the acceleration due to gravity **g** in your home or classroom.

- 1 string
- 1 stopwatch
- 1 small dense object

1. Cut a piece of a string or dental floss so that it is about 1 m long.
2. Attach a small object of high density to the end of the string (for example, a metal nut or a car key).
3. Starting at an angle of less than 10 degrees, allow the pendulum to swing and measure the pendulum's period for 10 oscillations using a stopwatch.
4. Calculate **g**.

GRASP CHECK

How accurate is this measurement for *g*? How might it be improved?

a. Accuracy for value of *g* will increase with an increase in the mass of a dense object.
b. Accuracy for the value of *g* will increase with increase in the length of the pendulum.
c. The value of *g* will be more accurate if the angle of deflection is more than 15°.
d. The value of *g* will be more accurate if it maintains simple harmonic motion.

Check Your Understanding

22. What is deformation?

a. Deformation is the magnitude of the restoring force.
b. Deformation is the change in shape due to the application of force.
c. Deformation is the maximum force that can be applied on a spring.
d. Deformation is regaining the original shape upon the removal of an external force.

23. According to Hooke's law, what is deformation proportional to?

a. Force
b. Velocity
c. Displacement
d. Force constant

24. What are oscillations?

a. Motion resulting in small displacements
b. Motion which repeats itself periodically
c. Periodic, repetitive motion between two points
d. motion that is the opposite to the direction of the restoring force

25. True or False—Oscillations can occur without force.

a. True
b. False

KEY TERMS

air resistance a frictional force that slows the motion of objects as they travel through the air; when solving basic physics problems, air resistance is assumed to be zero

amplitude the maximum displacement from the equilibrium position of an object oscillating around the equilibrium position

analytical method the method of determining the magnitude and direction of a resultant vector using the Pythagorean theorem and trigonometric identities

component (of a 2-dimensional vector) a piece of a vector that points in either the vertical or the horizontal direction; every 2-d vector can be expressed as a sum of two vertical and horizontal vector components

deformation displacement from equilibrium, or change in shape due to the application of force

equilibrium position where an object would naturally rest in the absence of force

frequency number of events per unit of time

graphical method drawing vectors on a graph to add them using the head-to-tail method

head (of a vector) the end point of a vector; the location of the vector's arrow; also referred to as the tip

head-to-tail method a method of adding vectors in which the tail of each vector is placed at the head of the previous vector

Hooke's law proportional relationship between the force $\mathbf{F}$ on a material and the deformation ΔL it causes, $\mathbf{F} = \mathbf{k}\Delta L$

kinetic friction a force that opposes the motion of two systems that are in contact and moving relative to one another

maximum height (of a projectile) the highest altitude, or maximum displacement in the vertical position reached in the path of a projectile

oscillate moving back and forth regularly between two points

period time it takes to complete one oscillation

periodic motion motion that repeats itself at regular time intervals

projectile an object that travels through the air and experiences only acceleration due to gravity

projectile motion the motion of an object that is subject only to the acceleration of gravity

range the maximum horizontal distance that a projectile travels

restoring force force acting in opposition to the force caused by a deformation

resultant the sum of the a collection of vectors

resultant vector the vector sum of two or more vectors

simple harmonic motion the oscillatory motion in a system where the net force can be described by Hooke's law

simple pendulum an object with a small mass suspended from a light wire or string

static friction a force that opposes the motion of two systems that are in contact and are not moving relative to one another

tail the starting point of a vector; the point opposite to the head or tip of the arrow

trajectory the path of a projectile through the air

vector addition adding together two or more vectors

SECTION SUMMARY

5.1 Vector Addition and Subtraction: Graphical Methods

- The graphical method of adding vectors $\mathbf{A}$ and $\mathbf{B}$ involves drawing vectors on a graph and adding them by using the head-to-tail method. The resultant vector $\mathbf{R}$ is defined such that $\mathbf{A} + \mathbf{B} = \mathbf{R}$. The magnitude and direction of $\mathbf{R}$ are then determined with a ruler and protractor.
- The graphical method of subtracting vectors $\mathbf{A}$ and $\mathbf{B}$ involves adding the opposite of vector $\mathbf{B}$, which is defined as $-\mathbf{B}$. In this case, $\mathbf{A} - \mathbf{B} = \mathbf{A} + (-\mathbf{B}) = \mathbf{R}$. Next, use the head-to-tail method as for vector addition to obtain the resultant vector $\mathbf{R}$.
- Addition of vectors is independent of the order in which they are added; $\mathbf{A} + \mathbf{B} = \mathbf{B} + \mathbf{A}$.
- The head-to-tail method of adding vectors involves drawing the first vector on a graph and then placing the tail of each subsequent vector at the head of the previous vector. The resultant vector is then drawn from the tail of the first vector to the head of the final vector.
- Variables in physics problems, such as force or velocity, can be represented with vectors by making the length of the vector proportional to the magnitude of the force or velocity.
- Problems involving displacement, force, or velocity may be solved graphically by measuring the resultant vector's magnitude with a ruler and measuring the direction with a protractor.

5.2 Vector Addition and Subtraction: Analytical Methods

- The analytical method of vector addition and subtraction uses the Pythagorean theorem and trigonometric identities to determine the magnitude and direction of a resultant vector.

- The steps to add vectors A and B using the analytical method are as follows:

1. Determine the coordinate system for the vectors. Then, determine the horizontal and vertical components of each vector using the equations
$$A_x = A\cos\theta$$
$$B_x = B\cos\theta$$
and
$$A_y = A\sin\theta$$
$$B_y = B\sin\theta.$$
2. Add the horizontal and vertical components of each vector to determine the components R_x and R_y of the resultant vector, R.
$$R_x = A_x + B_x$$
and
$$R_y = A_y + B_y.$$
3. Use the Pythagorean theorem to determine the magnitude, R, of the resultant vector R.
$$R = \sqrt{R_x^2 + R_y^2}$$
4. Use a trigonometric identity to determine the direction, θ, of R.
$$\theta = \tan^{-1}(R_y/R_x)$$

5.3 Projectile Motion

- Projectile motion is the motion of an object through the air that is subject only to the acceleration of gravity.
- Projectile motion in the horizontal and vertical directions are independent of one another.
- The maximum height of an projectile is the highest altitude, or maximum displacement in the vertical position reached in the path of a projectile.
- The range is the maximum horizontal distance traveled by a projectile.
- To solve projectile problems: choose a coordinate system; analyze the motion in the vertical and horizontal direction separately; then, recombine the horizontal and vertical components using vector addition equations.

5.4 Inclined Planes

- Friction is a contact force between systems that opposes the motion or attempted motion between them. Simple friction is proportional to the normal force **N** pushing the systems together. A normal force is always perpendicular to the contact surface between systems. Friction depends on both of the materials involved.
- μ_s is the coefficient of static friction, which depends on both of the materials.
- μ_k is the coefficient of kinetic friction, which also depends on both materials.
- When objects rest on an inclined plane that makes an angle θ with the horizontal surface, the weight of the object can be broken into components that act perpendicular ($\mathbf{w}_\perp$) and parallel ($\mathbf{w}_{||}$) to the surface of the plane.

5.5 Simple Harmonic Motion

- An oscillation is a back and forth motion of an object between two points of deformation.
- An oscillation may create a wave, which is a disturbance that propagates from where it was created.
- The simplest type of oscillations are related to systems that can be described by Hooke's law.
- Periodic motion is a repetitious oscillation.
- The time for one oscillation is the period T.
- The number of oscillations per unit time is the frequency
- A mass m suspended by a wire of length L is a simple pendulum and undergoes simple harmonic motion for amplitudes less than about 15 degrees.

KEY EQUATIONS

5.2 Vector Addition and Subtraction: Analytical Methods

resultant magnitude	$R = \sqrt{R_x^2 + R_y^2}$
resultant direction	$\theta = \tan^{-1}(R_y/R_x)$
x-component of a vector A (when an angle is given relative to the horizontal)	$A_x = A\cos\theta$
y-component of a vector A (when an angle is given relative to the horizontal)	$A_y = A\sin\theta$
addition of vectors	$\mathbf{A_x} + \mathbf{A_y} = \mathbf{A}$

5.3 Projectile Motion

angle of displacement	$\theta = \tan^{-1}(\mathbf{y}/\mathbf{x})$

velocity	$\mathbf{v} = \sqrt{\mathbf{v}_x^2 + \mathbf{v}_y^2}$
angle of velocity	$\theta_v = \tan^{-1}(\mathbf{v}_y/\mathbf{v}_x)$
maximum height	$h = \frac{\mathbf{v}_{0y}^2}{2\mathbf{g}}$
range	$R = \frac{\mathbf{v}_0^2 \sin 2\theta_0}{\mathbf{g}}$

5.4 Inclined Planes

force of static friction	$\mathbf{f}_\mathrm{s} \leq \mu_\mathrm{s}\mathbf{N}$
force of kinetic friction	$\mathbf{f}_\mathrm{k} = \mu_\mathrm{k}\mathbf{N}$
perpendicular component of weight on an inclined plane	$\mathbf{w}_\perp = \mathbf{w}\cos(\theta) = m\mathbf{g}\cos(\theta)$
parallel component of weight on an inclined plane	$\mathbf{w}_{\|\|} = \mathbf{w}\sin(\theta) = m\mathbf{g}\sin(\theta)$

5.5 Simple Harmonic Motion

Hooke's law	$\mathbf{F} = -\mathbf{kx}$
period in simple harmonic motion	$T = 2\pi\sqrt{\frac{m}{\mathbf{k}}}$
frequency in simple harmonic motion	$f = \frac{1}{2\pi}\sqrt{\frac{\mathbf{k}}{m}}$
period of a simple pendulum	$T = 2\pi\sqrt{\frac{L}{\mathbf{g}}}$

CHAPTER REVIEW

Concept Items

5.1 Vector Addition and Subtraction: Graphical Methods

1. There is a vector $\overrightarrow{\mathrm{A}}$, with magnitude 5 units pointing towards west and vector $\overrightarrow{\mathrm{B}}$, with magnitude 3 units, pointing towards south. Using vector addition, calculate the magnitude of the resultant vector.
   a. 4.0
   b. 5.8
   c. 6.3
   d. 8.0

2. If you draw two vectors using the head-to-tail method, how can you then draw the resultant vector?
   a. By joining the head of the first vector to the head of the last
   b. By joining the head of the first vector with the tail of the last
   c. By joining the tail of the first vector to the head of the last
   d. By joining the tail of the first vector with the tail of the last

3. What is the global angle of 20° south of west?
   a. 110°
   b. 160°
   c. 200°
   d. 290°

5.2 Vector Addition and Subtraction: Analytical Methods

4. What is the angle between the x and y components of a vector?
   a. 0°
   b. 45°
   c. 90°
   d. 180°

5. Two vectors are equal in magnitude and opposite in direction. What is the magnitude of their resultant vector?
   a. The magnitude of the resultant vector will be zero.
   b. The magnitude of resultant vector will be twice the magnitude of the original vector.
   c. The magnitude of resultant vector will be same as magnitude of the original vector.
   d. The magnitude of resultant vector will be half the magnitude of the original vector.

6. How can we express the x and y-components of a vector in terms of its magnitude, A, and direction, global angle θ?
   a. $A_x = A\cos\theta\, A_y = A\sin\theta$
   b. $A_x = A\cos\theta\, A_y = A\cos\theta$
   c. $A_x = A\sin\theta\, A_y = A\cos\theta$

d. $A_x = A \sin \theta A_y = A \sin \theta$

7. True or False—Every 2-D vector can be expressed as the product of its x and y-components.
   a. True
   b. False

5.3 Projectile Motion

8. Horizontal and vertical motions of a projectile are independent of each other. What is meant by this?
   a. Any object in projectile motion falls at the same rate as an object in freefall, regardless of its horizontal velocity.
   b. All objects in projectile motion fall at different rates, regardless of their initial horizontal velocities.
   c. Any object in projectile motion falls at the same rate as its initial vertical velocity, regardless of its initial horizontal velocity.
   d. All objects in projectile motion fall at different rates and the rate of fall of the object is independent of the initial velocity.

9. Using the conventional choice for positive and negative axes described in the text, what is the y-component of the acceleration of an object experiencing projectile motion?
   a. -9.8 m/s
   b. -9.8 m/s^2
   c. 9.8 m/s
   d. 9.8 m/s^2

5.4 Inclined Planes

10. True or False—Kinetic friction is less than the limiting static friction because once an object is moving, there are fewer points of contact, and the friction is reduced. For this reason, more force is needed to start moving an object than to keep it in motion.
    a. True
    b. False

11. When there is no motion between objects, what is the relationship between the magnitude of the static friction f_s and the normal force N?
    a. $f_s \leq N$
    b. $f_s \leq \mu_s N$
    c. $f_s \geq N$
    d. $f_s \geq \mu_s N$

12. What equation gives the magnitude of kinetic friction?
    a. $f_k = \mu_s N$
    b. $f_k = \mu_k N$
    c. $f_k \leq \mu_s N$
    d. $f_k \leq \mu_k N$

5.5 Simple Harmonic Motion

13. Why is there a negative sign in the equation for Hooke's law?
    a. The negative sign indicates that displacement decreases with increasing force.
    b. The negative sign indicates that the direction of the applied force is opposite to that of displacement.
    c. The negative sign indicates that the direction of the restoring force is opposite to that of displacement.
    d. The negative sign indicates that the force constant must be negative.

14. With reference to simple harmonic motion, what is the equilibrium position?
    a. The position where velocity is the minimum
    b. The position where the displacement is maximum
    c. The position where the restoring force is the maximum
    d. The position where the object rests in the absence of force

15. What is Hooke's law?
    a. Restoring force is directly proportional to the displacement from the mean position and acts in the the opposite direction of the displacement.
    b. Restoring force is directly proportional to the displacement from the mean position and acts in the same direction as the displacement.
    c. Restoring force is directly proportional to the square of the displacement from the mean position and acts in the opposite direction of the displacement.
    d. Restoring force is directly proportional to the square of the displacement from the mean position and acts in the same direction as the displacement.

Critical Thinking Items

5.1 Vector Addition and Subtraction: Graphical Methods

16. True or False—A person is following a set of directions. He has to walk 2 km east and then 1 km north. He takes a wrong turn and walks in the opposite direction for the second leg of the trip. The magnitude of his total displacement will be the same as it would have been had he followed directions correctly.
    a. True
    b. False

5.2 Vector Addition and Subtraction: Analytical Methods

17. What is the magnitude of a vector whose x-component is 2 units and whose angle is $60°$?
  a. 1.0 units
  b. 2.0 units
  c. 2.3 units
  d. 4.0 units

18. Vectors $\vec{A}$ and $\vec{B}$ are equal in magnitude and opposite in direction. Does $\vec{A} - \vec{B}$ have the same direction as vector $\vec{A}$ or $\vec{B}$?
  a. $\vec{A}$
  b. $\vec{B}$

5.3 Projectile Motion

19. Two identical items, object 1 and object 2, are dropped from the top of a 50.0 m building. Object 1 is dropped with an initial velocity of 0 m/s, while object 2 is thrown straight downward with an initial velocity of 13.0 m/s. What is the difference in time, in seconds rounded to the nearest tenth, between when the two objects hit the ground?
  a. Object 1 will hit the ground 3.2 s after object 2.
  b. Object 1 will hit the ground 2.1 s after object 2.
  c. Object 1 will hit the ground at the same time as object 2.
  d. Object 1 will hit the ground 1.1 s after object 2.

20. An object is launched into the air. If the y-component of its acceleration is 9.8 m/s^2, which direction is defined as positive?
  a. Vertically upward in the coordinate system
  b. Vertically downward in the coordinate system
  c. Horizontally to the right side of the coordinate system
  d. Horizontally to the left side of the coordinate system

5.4 Inclined Planes

21. A box weighing 500 N is at rest on the floor. A person pushes against it and it starts moving when 100 N force is applied to it. What can be said about the coefficient of kinetic friction between the box and the floor?
  a. $\mu_k = 0$
  b. $\mu_k = 0.2$
  c. $\mu_k < 0.2$
  d. $\mu_k > 0.2$

22. The component of the weight parallel to an inclined plane of an object resting on an incline that makes an angle of $70.0°$ with the horizontal is 100.0 N. What is the object's mass?
  a. 10.9 kg
  b. 29.8 kg
  c. 106 kg
  d. 292 kg

5.5 Simple Harmonic Motion

23. Two springs are attached to two hooks. Spring A has a greater force constant than spring B. Equal weights are suspended from both. Which of the following statements is true?
  a. Spring A will have more extension than spring B.
  b. Spring B will have more extension than spring A.
  c. Both springs will have equal extension.
  d. Both springs are equally stiff.

24. Two simple harmonic oscillators are constructed by attaching similar objects to two different springs. The force constant of the spring on the left is 5 N/m and that of the spring on the right is 4 N/m. If the same force is applied to both, which of the following statements is true?
  a. The spring on the left will oscillate faster than spring on the right.
  b. The spring on the right will oscillate faster than the spring on the left.
  c. Both the springs will oscillate at the same rate.
  d. The rate of oscillation is independent of the force constant.

Problems

5.1 Vector Addition and Subtraction: Graphical Methods

25. A person attempts to cross a river in a straight line by navigating a boat at 15 m/s. If the river flows at 5.0 m/s from his left to right, what would be the magnitude of the boat's resultant velocity? In what direction would the boat go, relative to the straight line across it?
  a. The resultant velocity of the boat will be 10.0 m/s. The boat will go toward his right at an angle of $26.6°$ to a line drawn across the river.
  b. The resultant velocity of the boat will be 10.0 m/s. The boat will go toward his left at an angle of $26.6°$ to a line drawn across the river.
  c. The resultant velocity of the boat will be 15.8 m/s. The boat will go toward his right at an angle of

18.4° to a line drawn across the river.

d. The resultant velocity of the boat will be 15.8 m/s. The boat will go toward his left at an angle of 18.4° to a line drawn across the river.

26. A river flows in a direction from south west to north east at a velocity of 7.1 m/s. A boat captain wants to cross this river to reach a point on the opposite shore due east of the boat's current position. The boat moves at 13 m/s . Which direction should it head towards if the resultant velocity is 19.74 m/s?

a. It should head in a direction 22.6° east of south.
b. It should head in a direction 22.6° south of east.
c. It should head in a direction 45.0° east of south.
d. It should head in a direction 45.0° south of east.

5.2 Vector Addition and Subtraction: Analytical Methods

27. A person walks 10.0 m north and then 2.00 m east. Solving analytically, what is the resultant displacement of the person?

a. $\vec{R} = 10.2$ m, $\theta = 78.7°$ east of north
b. $\vec{R} = 10.2$ m, $\theta = 78.7°$ north of east
c. $\vec{R} = 12.0$ m, $\theta = 78.7°$ east of north
d. $\vec{R} = 12.0$ m, $\theta = 78.7°$ north of east

28. A person walks 12.0° north of west for 55.0 m and 63.0° south of west for 25.0 m. What is the magnitude of his displacement? Solve analytically.

a. 10.84 m
b. 65.1 m
c. 66.04 m
d. 80.00 m

5.3 Projectile Motion

29. A water balloon cannon is fired at 30 m/s at an angle of 50° above the horizontal. How far away will it fall?

a. 2.35 m
b. 3.01 m
c. 70.35 m
d. 90.44 m

30. A person wants to fire a water balloon cannon such that it hits a target 100 m away. If the cannon can only be launched at 45° above the horizontal, what should be the initial speed at which it is launched?

a. 31.3 m/s
b. 37.2 m/s
c. 980.0 m/s
d. 1,385.9 m/s

5.4 Inclined Planes

31. A coin is sliding down an inclined plane at constant velocity. If the angle of the plane is 10° to the horizontal, what is the coefficient of kinetic friction?

a. $\mu_{\mathrm{k}} = 0$
b. $\mu_{\mathrm{k}} = 0.18$
c. $\mu_{\mathrm{k}} = 5.88$
d. $\mu_{\mathrm{k}} = \infty$

32. A skier with a mass of 55 kg is skiing down a snowy slope that has an incline of 30°. Find the coefficient of kinetic friction for the skier if friction is known to be 25 N .

a. $\mu k = 0$
b. $\mu k = 0.05$
c. $\mu k = 0.09$
d. $\mu k = \infty$

5.5 Simple Harmonic Motion

33. What is the time period of a 6 cm long pendulum on earth?

a. 0.08 s
b. 0.49 s
c. 4.9 s
d. 80 s

34. A simple harmonic oscillator has time period 4 s. If the mass of the system is 2 kg, what is the force constant of the spring used?

a. 0.125 N/m
b. 0.202 N/m
c. 0.81 N/m
d. 4.93 N/m

Performance Task

5.5 Simple Harmonic Motion

35. Construct a seconds pendulum (pendulum with time period 2 seconds).

TEST PREP

Multiple Choice

5.1 Vector Addition and Subtraction: Graphical Methods

36. True or False—We can use Pythagorean theorem to calculate the length of the resultant vector obtained from the addition of two vectors which are at right angles to each other.
a. True
b. False

37. True or False—The direction of the resultant vector depends on both the magnitude and direction of added vectors.
a. True
b. False

38. A plane flies north at 200 m/s with a headwind blowing from the north at 70 m/s. What is the resultant velocity of the plane?
a. 130 m/s north
b. 130 m/s south
c. 270 m/s north
d. 270 m/s south

39. Two hikers take different routes to reach the same spot. The first one goes 255 m southeast, then turns and goes 82 m at $14°$ south of east. The second hiker goes 200 m south. How far and in which direction must the second hiker travel now, in order to reach the first hiker's location destination?
a. 200 m east
b. 200 m south
c. 260 m east
d. 260 m south

5.2 Vector Addition and Subtraction: Analytical Methods

40. When will the *x*-component of a vector with angle θ be greater than its y-component?
a. $0° < \theta < 45°$
b. $\theta = 45°$
c. $45° < \theta < 60°$
d. $60° < \theta < 90°$

41. The resultant vector of the addition of vectors $\overrightarrow{\text{a}}$ and $\overrightarrow{\text{b}}$ is $\overrightarrow{\text{r}}$. The magnitudes of $\overrightarrow{\text{a}}$, $\overrightarrow{\text{b}}$, and $\overrightarrow{\text{r}}$ are A, B, and R, respectively. Which of the following is true?
a. $R_x + R_y = 0$
b. $A_x + A_y = \overrightarrow{\text{A}}$
c. $A_x + B_y = B_x + A_y$
d. $A_x + B_x = R_x$

42. What is the dimensionality of vectors used in the study of atmospheric sciences?
a. One-dimensional
b. Two-dimensional
c. Three-dimensional

5.3 Projectile Motion

43. After a projectile is launched in the air, in which direction does it experience constant, non-zero acceleration, ignoring air resistance?
a. The x direction
b. The y direction
c. Both the x and y directions
d. Neither direction

44. Which is true when the height of a projectile is at its maximum?
a. $v_y = 0$
b. $v_y = \text{maximum}$
c. $v_x = \text{maximum}$

45. A ball is thrown in the air at an angle of 40°. If the maximum height it reaches is 10 m, what must be its initial speed?
a. 17.46 m/s
b. 21.78 m/s
c. 304.92 m/s
d. 474.37 m/s

46. A large rock is ejected from a volcano with a speed of 30 m/s and at an angle $60°$ above the horizontal. The rock strikes the side of the volcano at an altitude of 10.0 m lower than its starting point. Calculate the horizontal displacement of the rock.
a. 84.90 m
b. 96.59 m
c. 169.80 m
d. 193.20 m

5.4 Inclined Planes

47. For objects of identical masses but made of different materials, which of the following experiences the most static friction?
a. Shoes on ice
b. Metal on wood
c. Teflon on steel

48. If an object sits on an inclined plane and no other object makes contact with the object, what is typically equal in magnitude to the component of the weight perpendicular to the plane?

a. The normal force
b. The total weight
c. The parallel force of weight

49. A 5 kg box is at rest on the floor. The coefficient of static friction between the box and the floor is 0.4. A horizontal force of 50 N is applied to the box. Will it move?
a. No, because the applied force is less than the maximum limiting static friction.
b. No, because the applied force is more than the maximum limiting static friction.
c. Yes, because the applied force is less than the maximum limiting static friction.
d. Yes, because the applied force is more than the maximum limiting static friction.

50. A skier with a mass of 67 kg is skiing down a snowy slope with an incline of 37°. Find the friction if the coefficient of kinetic friction is 0.07.
a. 27.66 N
b. 34.70 N
c. 36.71 N
d. 45.96 N

5.5 Simple Harmonic Motion

51. A change in which of the following is an example of deformation?
a. Velocity
b. Length
c. Mass
d. Weight

52. The units of amplitude are the same as those for which of the following measurements?
a. Speed
b. Displacement
c. Acceleration
d. Force

53. Up to approximately what angle is simple harmonic motion a good model for a pendulum?
a. $15°$
b. $45°$
c. $75°$
d. $90°$

54. How would simple harmonic motion be different in the absence of friction?
a. Oscillation will not happen in the absence of friction.
b. Oscillation will continue forever in the absence of friction.
c. Oscillation will have changing amplitude in the absence of friction.
d. Oscillation will cease after a certain amount of time in the absence of friction.

55. What mass needs to be attached to a spring with a force constant of $7\ \mathrm{N/m}$ in order to make a simple harmonic oscillator oscillate with a time period of 3 s?
a. $0.03\ \mathrm{kg}$
b. $1.60\ \mathrm{kg}$
c. $30.7\ \mathrm{kg}$
d. $63.0\ \mathrm{kg}$

Short Answer

5.1 Vector Addition and Subtraction: Graphical Methods

56. Find $\vec{A} - \vec{B}$ for the following vectors:
$\vec{A} = (122\ \mathrm{cm}, \angle 145°)\ \vec{B} = (110\ \mathrm{cm}, \angle 270°)$
a. 108 cm, $\theta = 119.0°$
b. 108 cm, $\theta = 125.0°$
c. 206 cm, $\theta = 119.0°$
d. 206 cm, $\theta = 125.0°$

57. Find $\vec{A} + \vec{B}$ for the following vectors:
$\vec{A} = (122\ \mathrm{cm}, \angle 145°)\ \vec{B} = (110\ \mathrm{cm}, \angle 270°)$
a. 108 cm, $\theta = 119.1°$
b. 108 cm, $\theta = 201.8°$
c. 232 cm, $\theta = 119.1°$
d. 232 cm, $\theta = 201.8°$

58. Consider six vectors of 2 cm each, joined from head to tail making a hexagon. What would be the magnitude of the addition of these vectors?
a. Zero
b. Six
c. Eight
d. Twelve

59. Two people pull on ropes tied to a trolley, each applying 44 N of force. The angle the ropes form with each other is 39.5°. What is the magnitude of the net force exerted on the trolley?
a. 0.0 N
b. 79.6 N
c. 82.8 N
d. 88.0 N

5.2 Vector Addition and Subtraction: Analytical Methods

60. True or False—A vector can form the shape of a right angle triangle with its x and y components.
a. True

b. False

61. True or False—All vectors have positive x and y components.
a. True
b. False

62. Consider $\vec{A} - \vec{B} = \vec{R}$. What is R_x in terms of A_x and B_x?
a. $R_x = \frac{A_x}{B_x}$
b. $R_x = \frac{B_x}{A_x}$
c. $R_x = A_x + B_x$
d. $R_x = A_x - B_x$

63. Consider $\vec{A} - \vec{B} = \vec{R}$. What is R_y in terms of A_y and B_y?
a. $R_y = \frac{A_y}{B_y}$
b. $R_y = \frac{B_y}{A_y}$
c. $R_y = A_y + B_y$
d. $R_y = A_y - B_y$

64. When a three dimensional vector is used in the study of atmospheric sciences, what is z?
a. Altitude
b. Heat
c. Temperature
d. Wind speed

65. Which method is not an application of vector calculus?
a. To find the rate of change in atmospheric temperature
b. To study changes in wind speed and direction
c. To predict changes in atmospheric pressure
d. To measure changes in average rainfall

5.3 Projectile Motion

66. How can you express the velocity, $\vec{v}$, of a projectile in terms of its initial velocity, $\vec{v_0}$, acceleration, $\vec{a}$, and time, *t*?
a. $\vec{v} = \vec{a}t$
b. $\vec{v} = \vec{v_0} + \vec{a}t$
c. $\vec{v} + \vec{v_0} = \vec{a}t$
d. $\vec{v_0} + \vec{v} + \vec{a}t$

67. In the equation for the maximum height of a projectile, what does v_{0y} stand for? $h = \frac{{v_{0y}}^2}{2g}$
a. Initial velocity in the x direction
b. Initial velocity in the y direction
c. Final velocity in the x direction
d. Final velocity in the y direction

68. True or False—Range is defined as the maximum vertical distance travelled by a projectile.
a. True
b. False

69. For what angle of a projectile is its range equal to zero?
a. $0°$ or $30°$
b. $0°$ or $45°$
c. $90°$ or $0°$
d. $90°$ or $45°$

5.4 Inclined Planes

70. What are the units of the coefficient of friction?
a. N
b. m/s
c. m/s^2
d. unitless

71. Two surfaces in contact are moving slowly past each other. As the relative speed between the two surfaces in contact increases, what happens to the magnitude of their coefficient of kinetic friction?
a. It increases with the increase in the relative motion.
b. It decreases with the increase in the relative motion.
c. It remains constant and is independent of the relative motion.

72. When will an object slide down an inclined plane at constant velocity?
a. When the magnitude of the component of the weight along the slope is equal to the magnitude of the frictional force.
b. When the magnitude of the component of the weight along the slope is greater than the magnitude of the frictional force.
c. When the magnitude of the component of the weight perpendicular to the slope is less than the magnitude of the frictional force.
d. When the magnitude of the component of the weight perpendicular to the slope is equal to the magnitude of the frictional force.

73. A box is sitting on an inclined plane. At what angle of incline is the perpendicular component of the box's weight at its maximum?
a. $0°$
b. $30°$
c. $60°$
d. $90°$

5.5 Simple Harmonic Motion

74. What is the term used for changes in shape due to the application of force?
a. Amplitude

b. Deformation
c. Displacement
d. Restoring force

75. What is the restoring force?
a. The normal force on the surface of an object
b. The weight of a mass attached to an object
c. Force which is applied to deform an object from its original shape
d. Force which brings an object back to its equilibrium position

76. For a given oscillator, what are the factors that affect its period and frequency?
a. Mass only
b. Force constant only
c. Applied force and mass
d. Mass and force constant

77. For an object in simple harmonic motion, when does the maximum speed occur?
a. At the extreme positions
b. At the equilibrium position
c. At the moment when the applied force is removed
d. Midway between the extreme and equilibrium positions

78. What is the equilibrium position of a pendulum?
a. When the tension in the string is zero
b. When the pendulum is hanging straight down
c. When the tension in the string is maximum
d. When the weight of the mass attached is minimum

79. If a pendulum is displaced by an angle θ, what is the net restoring force it experiences?
a. $mg\sin\theta$
b. $mg\cos\theta$
c. $-mg\sin\theta$
d. $-mg\cos\theta$

Extended Response

5.1 Vector Addition and Subtraction: Graphical Methods

80. True or False—For vectors the order of addition is important.
a. True
b. False

81. Consider five vectors *a, b, c, d,* and *e.* Is it true or false that their addition always results in a vector with a greater magnitude than if only two of the vectors were added?
a. True
b. False

5.2 Vector Addition and Subtraction: Analytical Methods

82. For what angle of a vector is it possible that its magnitude will be equal to its y-component?
a. $\theta = 0°$
b. $\theta = 45°$
c. $\theta = 60°$
d. $\theta = 90°$

83. True or False—If only the angles of two vectors are known, we can find the angle of their resultant addition vector.
a. True
b. False

84. True or false—We can find the magnitude and direction of the resultant vector if we know the angles of two vectors and the magnitude of one.

a. True
b. False

5.3 Projectile Motion

85. Ignoring drag, what is the x-component of the acceleration of a projectile? Why?
a. The x-component of the acceleration of a projectile is 0 because acceleration of a projectile is due to gravity, which acts in the y direction.
b. The x component of the acceleration of a projectile is g because acceleration of a projectile is due to gravity, which acts in the x direction.
c. The x-component of the acceleration of a projectile is 0 because acceleration of a projectile is due to gravity, which acts in the x direction.
d. The x-component of the acceleration of a projectile is g because acceleration of a projectile is due to gravity, which acts in the y direction.

86. What is the optimum angle at which a projectile should be launched in order to cover the maximum distance?
a. 0°
b. 45°
c. 60°
d. 90°

5.4 Inclined Planes

87. True or False—Friction varies from surface to surface because different substances have different degrees of roughness or smoothness.
a. True
b. False

88. As the angle of the incline gets larger, what happens to the magnitudes of the perpendicular and parallel components of gravitational force?

a. Both the perpendicular and the parallel component will decrease.
b. The perpendicular component will decrease and the parallel component will increase.
c. The perpendicular component will increase and the parallel component will decrease.
d. Both the perpendicular and the parallel component will increase.

5.5 Simple Harmonic Motion

89. What physical characteristic of a system is its force constant related to?

a. The force constant k is related to the stiffness of a system: The larger the force constant, the stiffer the system.
b. The force constant k is related to the stiffness of a system: The larger the force constant, the looser the system.
c. The force constant k is related to the friction in the system: The larger the force constant, the greater the friction in the system.
d. The force constant k is related to the friction in the system: The larger the force constant, the lower the friction in the system.

90. How or why does a pendulum oscillate?

a. A pendulum oscillates due to applied force.
b. A pendulum oscillates due to the elastic nature of the string.
c. A pendulum oscillates due to restoring force arising from gravity.
d. A pendulum oscillates due to restoring force arising from tension in the string.

91. If a pendulum from earth is taken to the moon, will its frequency increase or decrease? Why?

a. It will increase because g on the Moon is less than g on Earth.
b. It will decrease because g on the Moon is less than g on Earth.
c. It will increase because g on the Moon is greater than g on Earth.
d. It will decrease because g on the Moon is greater than g on Earth.

CHAPTER 6
Circular and Rotational Motion

Figure 6.1 This Australian Grand Prix Formula 1 race car moves in a circular path as it makes the turn. Its wheels also spin rapidly. The same physical principles are involved in both of these motions. (Richard Munckton).

Chapter Outline

INTRODUCTION You may recall learning about various aspects of motion along a straight line: kinematics (where we learned about displacement, velocity, and acceleration), projectile motion (a special case of two-dimensional kinematics), force, and Newton's laws of motion. In some ways, this chapter is a continuation of Newton's laws of motion. Recall that Newton's first law tells us that objects move along a straight line at constant speed unless a net external force acts on them. Therefore, if an object moves along a circular path, such as the car in the photo, it must be experiencing an external force. In this chapter, we explore both circular motion and rotational motion.

6.1 Angle of Rotation and Angular Velocity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the angle of rotation and relate it to its linear counterpart
- Describe angular velocity and relate it to its linear counterpart
- Solve problems involving angle of rotation and angular velocity

Section Key Terms

angle of rotation	angular velocity	arc length	circular motion
radius of curvature	rotational motion	spin	tangential velocity

Angle of Rotation

What exactly do we mean by *circular motion* or *rotation*? **Rotational motion** is the circular motion of an object about an axis of rotation. We will discuss specifically circular motion and spin. **Circular motion** is when an object moves in a circular path. Examples of circular motion include a race car speeding around a circular curve, a toy attached to a string swinging in a circle around your head, or the circular *loop-the-loop* on a roller coaster. **Spin** is rotation about an axis that goes through the center of mass of the object, such as Earth rotating on its axis, a wheel turning on its axle, the spin of a tornado on its path of destruction, or a figure skater spinning during a performance at the Olympics. Sometimes, objects will be spinning while in circular motion, like the Earth spinning on its axis while revolving around the Sun, but we will focus on these two motions separately.

When solving problems involving rotational motion, we use variables that are similar to linear variables (distance, velocity, acceleration, and force) but take into account the curvature or rotation of the motion. Here, we define the **angle of rotation**, which is the angular equivalence of distance; and **angular velocity**, which is the angular equivalence of linear velocity.

When objects rotate about some axis—for example, when the CD in Figure 6.2 rotates about its center—each point in the object follows a circular path.

Figure 6.2 All points on a CD travel in circular paths. The pits (dots) along a line from the center to the edge all move through the same angle $\Delta\theta$ in time Δt .

The **arc length,** , is the distance traveled along a circular path. The **radius of curvature**, **r**, is the radius of the circular path. Both are shown in Figure 6.3.

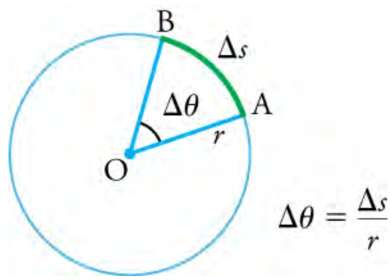


Figure 6.3 The radius (r) of a circle is rotated through an angle $\Delta\theta$. The arc length, Δs, is the distance covered along the circumference.

Consider a line from the center of the CD to its edge. In a given time, each *pit* (used to record information) on this line moves through the same angle. The angle of rotation is the amount of rotation and is the angular analog of distance. The angle of rotation $\Delta\theta$ is the arc length divided by the radius of curvature.

$$\Delta\theta = \frac{\Delta s}{r}$$

The angle of rotation is often measured by using a unit called the radian. (Radians are actually dimensionless, because a radian is defined as the ratio of two distances, radius and arc length.) A revolution is one complete rotation, where every point on the circle returns to its original position. One revolution covers 2π radians (or 360 degrees), and therefore has an angle of rotation of 2π radians, and an arc length that is the same as the circumference of the circle. We can convert between radians, revolutions, and degrees using the relationship

1 revolution = 2π rad = 360°. See Table 6.1 for the conversion of degrees to radians for some common angles.

$$\begin{aligned} 2\pi \text{ rad} &= 360° \\ 1 \text{ rad} &= \frac{360°}{2\pi} \approx 57.3° \end{aligned} \quad (6.1)$$

Degree Measures	Radian Measures
30°	$\frac{\pi}{6}$
60°	$\frac{\pi}{3}$
90°	$\frac{\pi}{2}$
120°	$\frac{2\pi}{3}$
135°	$\frac{3\pi}{4}$
180°	π

Table 6.1 Commonly Used Angles in Terms of Degrees and Radians

Angular Velocity

How fast is an object rotating? We can answer this question by using the concept of angular velocity. Consider first the angular speed (ω) is the rate at which the angle of rotation changes. In equation form, the angular speed is

$$\omega = \frac{\Delta\theta}{\Delta t}, \quad (6.2)$$

which means that an angular rotation ($\Delta\theta$) occurs in a time, Δt . If an object rotates through a greater angle of rotation in a given time, it has a greater angular speed. The units for angular speed are radians per second (rad/s).

Now let's consider the direction of the angular speed, which means we now must call it the angular velocity. The direction of the angular velocity is along the axis of rotation. For an object rotating clockwise, the angular velocity points away from you along the axis of rotation. For an object rotating counterclockwise, the angular velocity points toward you along the axis of rotation.

Angular velocity (ω) is the angular version of linear velocity **v**. **Tangential velocity** is the instantaneous linear velocity of an object in rotational motion**.** To get the precise relationship between angular velocity and tangential velocity, consider again a pit on the rotating CD. This pit moves through an arc length (Δs) in a *short* time (Δt) so its tangential *speed* is

$$v = \frac{\Delta s}{\Delta t}. \tag{6.3}$$

From the definition of the angle of rotation, $\Delta\theta = \frac{\Delta s}{r}$, we see that $\Delta s = r\Delta\theta$. Substituting this into the expression for *v* gives

$$v = \frac{\mathrm{r}\Delta\theta}{\Delta t} = r\omega.$$

The equation $v = r\omega$ says that the tangential speed *v* is proportional to the distance *r* from the center of rotation. Consequently, tangential speed is greater for a point on the outer edge of the CD (with larger *r*) than for a point closer to the center of the CD (with smaller *r*). This makes sense because a point farther out from the center has to cover a longer arc length in the same amount of time as a point closer to the center. Note that both points will still have the same angular speed, regardless of their distance from the center of rotation. See Figure 6.4.

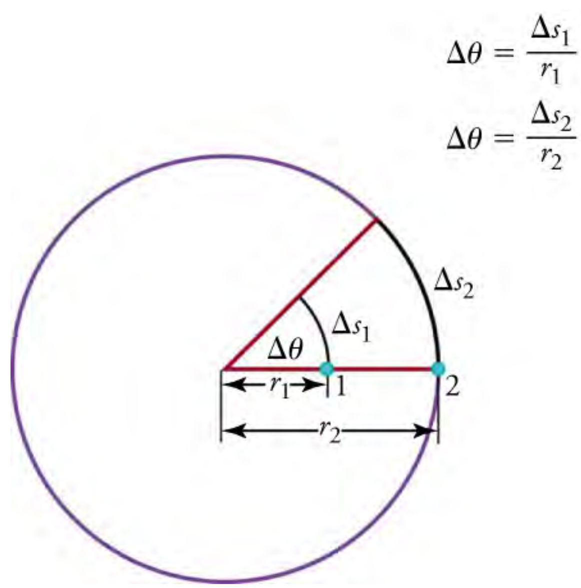


Figure 6.4 Points 1 and 2 rotate through the same angle ($\Delta\theta$), but point 2 moves through a greater arc length (Δs_2) because it is farther from the center of rotation.

Now, consider another example: the tire of a moving car (see Figure 6.5). The faster the tire spins, the faster the car moves—large ω means large *v* because $v = r\omega$. Similarly, a larger-radius tire rotating at the same angular velocity, **ω**, will produce a greater linear (tangential) velocity, **v,** for the car. This is because a larger radius means a longer arc length must contact the road, so the car must move farther in the same amount of time.

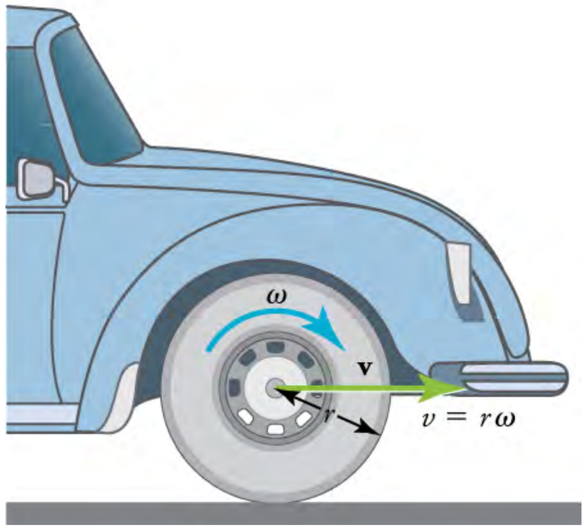


Figure 6.5 A car moving at a velocity, **v,** to the right has a tire rotating with angular velocity **ω** . The speed of the tread of the tire relative to the axle is *v*, the same as if the car were jacked up and the wheels spinning without touching the road. Directly below the axle, where the tire touches the road, the tire tread moves backward with respect to the axle with tangential velocity $v = r\omega$, where *r* is the tire radius. Because the road is stationary with respect to this point of the tire, the car must move forward at the linear velocity **v**. A larger angular velocity for the tire means a greater linear velocity for the car.

However, there are cases where linear velocity and tangential velocity are not equivalent, such as a car spinning its tires on ice. In this case, the linear velocity will be less than the tangential velocity. Due to the lack of friction under the tires of a car on ice, the arc length through which the tire treads move is greater than the linear distance through which the car moves. It's similar to running on a treadmill or pedaling a stationary bike; you are literally going nowhere fast.

TIPS FOR SUCCESS

Angular velocity **ω** and tangential velocity **v** are vectors, so we must include magnitude and direction. The direction of the angular velocity is along the axis of rotation, and points away from you for an object rotating clockwise, and toward you for an object rotating counterclockwise. In mathematics this is described by the right-hand rule. Tangential velocity is usually described as up, down, left, right, north, south, east, or west, as shown in Figure 6.6.

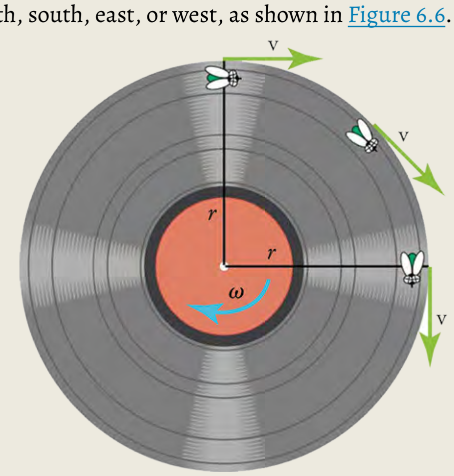


Figure 6.6 As the fly on the edge of an old-fashioned vinyl record moves in a circle, its instantaneous velocity is always at a tangent to the circle. The direction of the angular velocity is into the page this case.

WATCH PHYSICS

Relationship between Angular Velocity and Speed

This video reviews the definition and units of angular velocity and relates it to linear speed. It also shows how to convert between revolutions and radians.

Click to view content (https://www.youtube.com/embed/zAx61CO5mDw)

GRASP CHECK

For an object traveling in a circular path at a constant angular speed, would the linear speed of the object change if the radius of the path increases?

a. Yes, because tangential speed is independent of the radius.
b. Yes, because tangential speed depends on the radius.
c. No, because tangential speed is independent of the radius.
d. No, because tangential speed depends on the radius.

Solving Problems Involving Angle of Rotation and Angular Velocity

Snap Lab

Measuring Angular Speed

In this activity, you will create and measure uniform circular motion and then contrast it with circular motions with different radii.

- One string (1 m long)
- One object (two-hole rubber stopper) to tie to the end
- One timer

Procedure

1. Tie an object to the end of a string.
2. Swing the object around in a horizontal circle above your head (swing from your wrist). It is important that the circle be horizontal!
3. Maintain the object at uniform speed as it swings.
4. Measure the angular speed of the object in this manner. Measure the time it takes in seconds for the object to travel 10 revolutions. Divide that time by 10 to get the angular speed in revolutions per second, which you can convert to radians per second.
5. What is the approximate linear speed of the object?
6. Move your hand up the string so that the length of the string is 90 cm. Repeat steps 2–5.
7. Move your hand up the string so that its length is 80 cm. Repeat steps 2–5.
8. Move your hand up the string so that its length is 70 cm. Repeat steps 2–5.
9. Move your hand up the string so that its length is 60 cm. Repeat steps 2–5
10. Move your hand up the string so that its length is 50 cm. Repeat steps 2–5
11. Make graphs of angular speed vs. radius (i.e. string length) and linear speed vs. radius. Describe what each graph looks like.

GRASP CHECK

If you swing an object slowly, it may rotate at less than one revolution per second. What would be the revolutions per second for an object that makes one revolution in five seconds? What would be its angular speed in radians per second?

a. The object would spin at $\frac{1}{5}$ rev/s. The angular speed of the object would be $\frac{2\pi}{5}$ rad/s.
b. The object would spin at $\frac{1}{5}$ rev/s. The angular speed of the object would be $\frac{\pi}{5}$ rad/s.

c. The object would spin at 5 rev/s. The angular speed of the object would be 10π rad/s.
d. The object would spin at 5 rev/s. The angular speed of the object would be 5π rad/s.

Now that we have an understanding of the concepts of angle of rotation and angular velocity, we'll apply them to the real-world situations of a clock tower and a spinning tire.

WORKED EXAMPLE

Angle of rotation at a Clock Tower

The clock on a clock tower has a radius of 1.0 m. (a) What angle of rotation does the hour hand of the clock travel through when it moves from 12 p.m. to 3 p.m.? (b) What's the arc length along the outermost edge of the clock between the hour hand at these two times?

Strategy

We can figure out the angle of rotation by multiplying a full revolution (2π radians) by the fraction of the 12 hours covered by the hour hand in going from 12 to 3. Once we have the angle of rotation, we can solve for the arc length by rearranging the equation $\Delta\theta = \frac{\Delta s}{r}$ since the radius is given.

Solution to (a)

In going from 12 to 3, the hour hand covers 1/4 of the 12 hours needed to make a complete revolution. Therefore, the angle between the hour hand at 12 and at 3 is $\frac{1}{4} \times 2\ \pi\text{rad} = \frac{\pi}{2}$ (i.e., 90 degrees).

Solution to (b)

Rearranging the equation

$$\Delta\theta = \frac{\Delta s}{r}, \tag{6.4}$$

we get

$$\Delta s = r\Delta\theta. \tag{6.5}$$

Inserting the known values gives an arc length of

$$\begin{aligned} \Delta s &= (1.0 \text{ m})\left(\tfrac{\pi}{2}\text{rad}\right) \\ &= 1.6 \text{ m} \end{aligned} \tag{6.6}$$

Discussion

We were able to drop the radians from the final solution to part (b) because radians are actually dimensionless. This is because the radian is defined as the ratio of two distances (radius and arc length). Thus, the formula gives an answer in units of meters, as expected for an arc length.

WORKED EXAMPLE

How Fast Does a Car Tire Spin?

Calculate the angular speed of a 0.300 m radius car tire when the car travels at 15.0 m/s (about 54 km/h). See Figure 6.5.

Strategy

In this case, the speed of the tire tread with respect to the tire axle is the same as the speed of the car with respect to the road, so we have v = 15.0 m/s. The radius of the tire is r = 0.300 m. Since we know v and r, we can rearrange the equation $v = r\omega$, to get $\omega = \frac{v}{r}$ and find the angular speed.

Solution

To find the angular speed, we use the relationship: $\omega = \frac{v}{r}$.

Inserting the known quantities gives

$$\begin{aligned}\omega &= \frac{15.0\ \text{m/s}}{0.300\ \text{m}} \\ &= 50.0\ \text{rad/s}.\end{aligned} \tag{6.7}$$

Discussion

When we cancel units in the above calculation, we get 50.0/s (i.e., 50.0 per second, which is usually written as 50.0 s^{-1}). But the angular speed must have units of rad/s. Because radians are dimensionless, we can insert them into the answer for the angular speed because we know that the motion is circular. Also note that, if an earth mover with much larger tires, say 1.20 m in radius, were moving at the same speed of 15.0 m/s, its tires would rotate more slowly. They would have an angular speed of

$$\begin{aligned}\omega &= \frac{15.0\ \text{m/s}}{1.20\ \text{m}} \\ &= 12.5\ \text{rad/s}\end{aligned} \tag{6.8}$$

Practice Problems

1. What is the angle in degrees between the hour hand and the minute hand of a clock showing 9:00 a.m.?
   a. 0°
   b. 90°
   c. 180°
   d. 360°

2. What is the approximate value of the arc length between the hour hand and the minute hand of a clock showing 10:00 a.m if the radius of the clock is 0.2 m?
   a. 0.1 m
   b. 0.2 m
   c. 0.3 m
   d. 0.6 m

Check Your Understanding

3. What is circular motion?
   a. Circular motion is the motion of an object when it follows a linear path.
   b. Circular motion is the motion of an object when it follows a zigzag path.
   c. Circular motion is the motion of an object when it follows a circular path.
   d. Circular motion is the movement of an object along the circumference of a circle or rotation along a circular path.

4. What is meant by radius of curvature when describing rotational motion?
   a. The radius of curvature is the radius of a circular path.
   b. The radius of curvature is the diameter of a circular path.
   c. The radius of curvature is the circumference of a circular path.
   d. The radius of curvature is the area of a circular path.

5. What is angular velocity?
   a. Angular velocity is the rate of change of the diameter of the circular path.
   b. Angular velocity is the rate of change of the angle subtended by the circular path.
   c. Angular velocity is the rate of change of the area of the circular path.
   d. Angular velocity is the rate of change of the radius of the circular path.

6. What equation defines angular velocity, ω? Take that r is the radius of curvature, θ is the angle, and t is time.
   a. $\omega = \frac{\Delta\theta}{\Delta t}$
   b. $\omega = \frac{\Delta t}{\Delta\theta}$
   c. $\omega = \frac{\Delta r}{\Delta t}$
   d. $\omega = \frac{\Delta t}{\Delta r}$

7. Identify three examples of an object in circular motion.

a. an artificial satellite orbiting the Earth, a race car moving in the circular race track, and a top spinning on its axis
b. an artificial satellite orbiting the Earth, a race car moving in the circular race track, and a ball tied to a string being swung in a circle around a person's head
c. Earth spinning on its own axis, a race car moving in the circular race track, and a ball tied to a string being swung in a circle around a person's head
d. Earth spinning on its own axis, blades of a working ceiling fan, and a top spinning on its own axis

8. What is the relative orientation of the radius and tangential velocity vectors of an object in uniform circular motion?
a. Tangential velocity vector is always parallel to the radius of the circular path along which the object moves.
b. Tangential velocity vector is always perpendicular to the radius of the circular path along which the object moves.
c. Tangential velocity vector is always at an acute angle to the radius of the circular path along which the object moves.
d. Tangential velocity vector is always at an obtuse angle to the radius of the circular path along which the object moves.

6.2 Uniform Circular Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe centripetal acceleration and relate it to linear acceleration
- Describe centripetal force and relate it to linear force
- Solve problems involving centripetal acceleration and centripetal force

Section Key Terms

centrifugal force	centripetal acceleration	centripetal force	uniform circular motion

Centripetal Acceleration

In the previous section, we defined circular motion. The simplest case of circular motion is **uniform circular motion**, where an object travels a circular path *at a constant speed*. Note that, unlike speed, the linear velocity of an object in circular motion is constantly changing because it is always changing direction. We know from kinematics that acceleration is a change in velocity, either in magnitude or in direction or both. Therefore, an object undergoing uniform circular motion is always accelerating, even though the magnitude of its velocity is constant.

You experience this acceleration yourself every time you ride in a car while it turns a corner. If you hold the steering wheel steady during the turn and move at a constant speed, you are executing uniform circular motion. What you notice is a feeling of sliding (or being flung, depending on the speed) away from the center of the turn. This isn't an actual force that is acting on you—it only happens because your body wants to continue moving in a straight line (as per Newton's first law) whereas the car is turning off this straight-line path. Inside the car it appears as if you are forced away from the center of the turn. This fictitious force is known as the **centrifugal force**. The sharper the curve and the greater your speed, the more noticeable this effect becomes.

Figure 6.7 shows an object moving in a circular path at constant speed. The direction of the instantaneous tangential velocity is shown at two points along the path. Acceleration is in the direction of the change in velocity; in this case it points roughly toward the center of rotation. (The center of rotation is at the center of the circular path). If we imagine Δs becoming smaller and smaller, then the acceleration would point *exactly* toward the center of rotation, but this case is hard to draw. We call the acceleration of an object moving in uniform circular motion the **centripetal acceleration $\mathbf{a}_c$** because centripetal means *center seeking*.

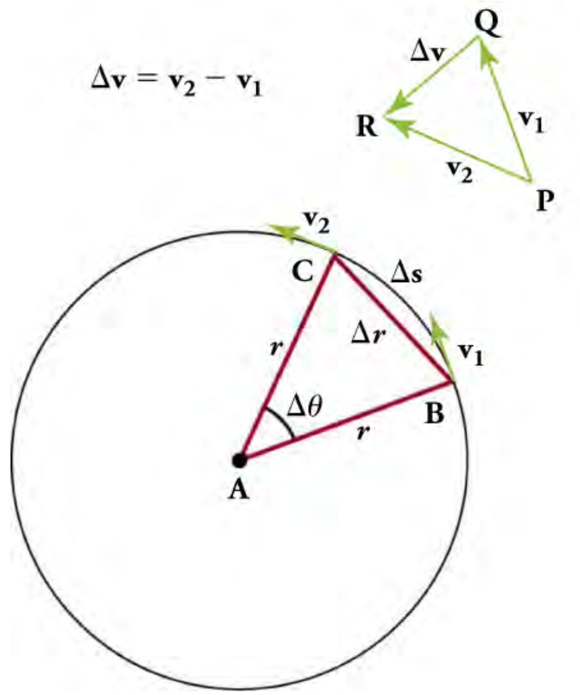


Figure 6.7 The directions of the velocity of an object at two different points are shown, and the change in velocity $\Delta \mathbf{v}$ is seen to point approximately toward the center of curvature (see small inset). For an extremely small value of Δs, $\Delta \mathbf{v}$ points exactly toward the center of the circle (but this is hard to draw). Because $\mathbf{a}_c = \Delta \mathbf{v}/\Delta t$, the acceleration is also toward the center, so $\mathbf{a}_c$ is called centripetal acceleration.

Now that we know that the direction of centripetal acceleration is toward the center of rotation, let's discuss the magnitude of centripetal acceleration. For an object traveling at speed v in a circular path with radius r, the magnitude of centripetal acceleration is

$$\mathbf{a}_c = \frac{v^2}{r}.$$

Centripetal acceleration is greater at high speeds and in sharp curves (smaller radius), as you may have noticed when driving a car, because the car actually pushes you toward the center of the turn. But it is a bit surprising that $\mathbf{a}_c$ is proportional to the speed squared. This means, for example, that the acceleration is four times greater when you take a curve at 100 km/h than at 50 km/h.

We can also express $\mathbf{a}_c$ in terms of the magnitude of angular velocity. Substituting $v = r\omega$ into the equation above, we get $a_c = \frac{(r\omega)^2}{r} = r\omega^2$. Therefore, the magnitude of centripetal acceleration in terms of the magnitude of angular velocity is

$$\mathbf{a}_c = r\omega^2. \tag{6.9}$$

TIPS FOR SUCCESS

The equation expressed in the form $a_c = r\omega^2$ is useful for solving problems where you know the angular velocity rather than the tangential velocity.

Virtual Physics

Ladybug Motion in 2D

In this simulation, you experiment with the position, velocity, and acceleration of a ladybug in circular and elliptical motion. Switch the type of motion from linear to circular and observe the velocity and acceleration vectors. Next, try elliptical motion and notice how the velocity and acceleration vectors differ from those in circular motion.

Click to view content (https://archive.cnx.org/specials/317a2b1e-2fbd-11e5-99b5-e38ffb545fe6/ladybug-motion/)

GRASP CHECK

In uniform circular motion, what is the angle between the acceleration and the velocity? What type of acceleration does a body experience in the uniform circular motion?

a. The angle between acceleration and velocity is 0°, and the body experiences linear acceleration.
b. The angle between acceleration and velocity is 0°, and the body experiences centripetal acceleration.
c. The angle between acceleration and velocity is 90°, and the body experiences linear acceleration.
d. The angle between acceleration and velocity is 90°, and the body experiences centripetal acceleration.

Centripetal Force

Because an object in uniform circular motion undergoes constant acceleration (by changing direction), we know from Newton's second law of motion that there must be a constant net external force acting on the object.

Any force or combination of forces can cause a centripetal acceleration. Just a few examples are the tension in the rope on a tether ball, the force of Earth's gravity on the Moon, the friction between a road and the tires of a car as it goes around a curve, or the normal force of a roller coaster track on the cart during a loop-the-loop.

Any net force causing uniform circular motion is called a **centripetal force**. The direction of a centripetal force is toward the center of rotation, the same as for centripetal acceleration. According to Newton's second law of motion, a net force causes the acceleration of mass according to $\mathbf{F}_{\text{net}} = m\mathbf{a}$. For uniform circular motion, the acceleration is centripetal acceleration: $\mathbf{a} = \mathbf{a}_\text{c}$. Therefore, the magnitude of centripetal force, $\mathbf{F}_\text{c}$, is $\mathbf{F}_\text{c} = m\mathbf{a}_\text{c}$.

By using the two different forms of the equation for the magnitude of centripetal acceleration, $\mathbf{a}_\text{c} = v^2/r$ and $\mathbf{a}_c = r\omega^2$, we get two expressions involving the magnitude of the centripetal force $\mathbf{F}_\text{c}$. The first expression is in terms of tangential speed, the second is in terms of angular speed: $\mathbf{F}_\text{c} = m\frac{v^2}{r}$ and $\mathbf{F}_\text{c} = mr\omega^2$.

Both forms of the equation depend on mass, velocity, and the radius of the circular path. You may use whichever expression for centripetal force is more convenient. Newton's second law also states that the object will accelerate in the same direction as the net force. By definition, the centripetal force is directed towards the center of rotation, so the object will also accelerate towards the center. A straight line drawn from the circular path to the center of the circle will always be perpendicular to the tangential velocity. Note that, if you solve the first expression for r, you get

$$r = \frac{mv^2}{\mathbf{F}_\text{c}}.$$

From this expression, we see that, for a given mass and velocity, a large centripetal force causes a small radius of curvature—that is, a tight curve.

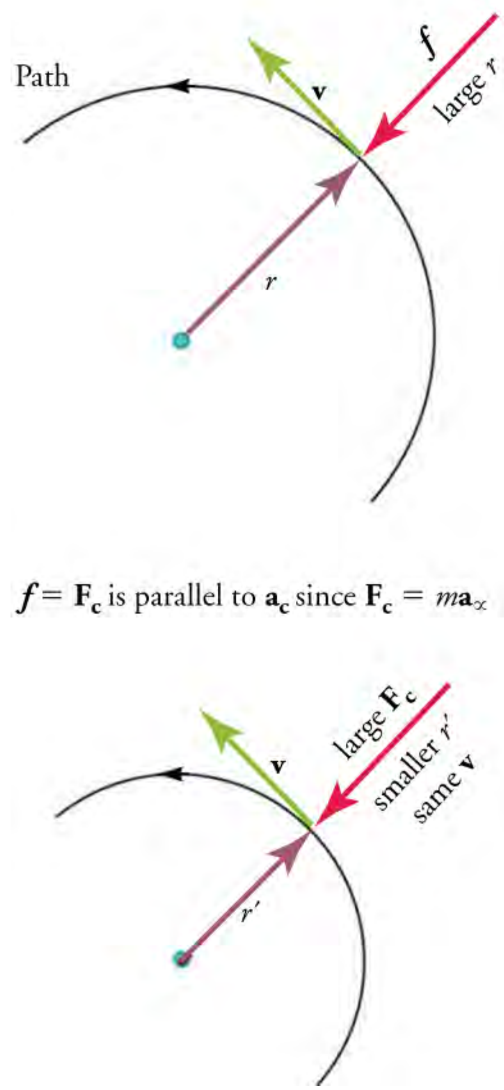


Figure 6.8 In this figure, the frictional force f serves as the centripetal force $\mathbf{F}_c$. Centripetal force is perpendicular to tangential velocity and causes uniform circular motion. The larger the centripetal force $\mathbf{F}_c$, the smaller is the radius of curvature r and the sharper is the curve. The lower curve has the same velocity **v**, but a larger centripetal force $\mathbf{F}_c$ produces a smaller radius r' .

WATCH PHYSICS

Centripetal Force and Acceleration Intuition

This video explains why a centripetal force creates centripetal acceleration and uniform circular motion. It also covers the difference between speed and velocity and shows examples of uniform circular motion.

[Click to view content (https://www.youtube.com/embed/vZOk8NnjILg)](https://www.youtube.com/embed/vZOk8NnjILg)

GRASP CHECK

Imagine that you are swinging a yoyo in a vertical clockwise circle in front of you, perpendicular to the direction you are facing. Now, imagine that the string breaks just as the yoyo reaches its bottommost position, nearest the floor. Which of the following describes the path of the yoyo after the string breaks?

a. The yoyo will fly upward in the direction of the centripetal force.
b. The yoyo will fly downward in the direction of the centripetal force.

c. The yoyo will fly to the left in the direction of the tangential velocity.
d. The yoyo will fly to the right in the direction of the tangential velocity.

Solving Centripetal Acceleration and Centripetal Force Problems

To get a feel for the typical magnitudes of centripetal acceleration, we'll do a lab estimating the centripetal acceleration of a tennis racket and then, in our first Worked Example, compare the centripetal acceleration of a car rounding a curve to gravitational acceleration. For the second Worked Example, we'll calculate the force required to make a car round a curve.

Snap Lab

Estimating Centripetal Acceleration

In this activity, you will measure the swing of a golf club or tennis racket to estimate the centripetal acceleration of the end of the club or racket. You may choose to do this in slow motion. Recall that the equation for centripetal acceleration is $\mathbf{a}_c = \frac{v^2}{r}$ or $\mathbf{a}_c = r\omega^2$.

- One tennis racket or golf club
- One timer
- One ruler or tape measure

Procedure

1. Work with a partner. Stand a safe distance away from your partner as he or she swings the golf club or tennis racket.
2. Describe the motion of the swing—is this uniform circular motion? Why or why not?
3. Try to get the swing as close to uniform circular motion as possible. What adjustments did your partner need to make?
4. Measure the radius of curvature. What did you physically measure?
5. By using the timer, find either the linear or angular velocity, depending on which equation you decide to use.
6. What is the approximate centripetal acceleration based on these measurements? How accurate do you think they are? Why? How might you and your partner make these measurements more accurate?

GRASP CHECK

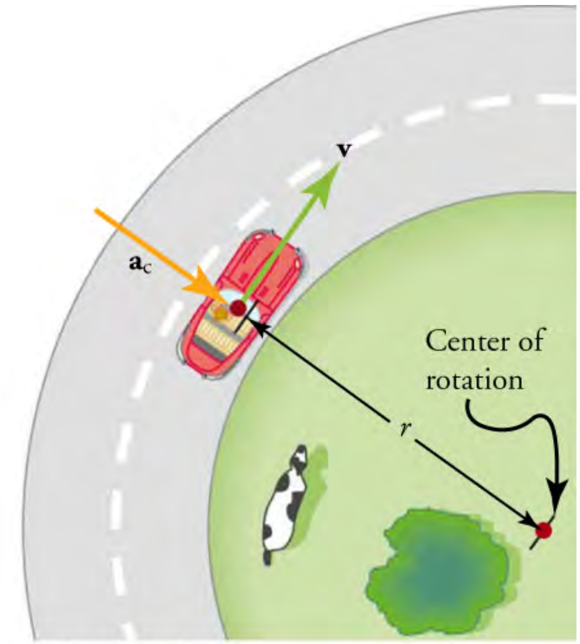
Was it more useful to use the equation $a_c = \frac{v^2}{r}$ or $a_c = r\omega^2$ in this activity? Why?

a. It should be simpler to use $a_c = r\omega^2$ because measuring angular velocity through observation would be easier.
b. It should be simpler to use $a_c = \frac{v^2}{r}$ because measuring tangential velocity through observation would be easier.
c. It should be simpler to use $a_c = r\omega^2$ because measuring angular velocity through observation would be difficult.
d. It should be simpler to use $a_c = \frac{v^2}{r}$ because measuring tangential velocity through observation would be difficult.

WORKED EXAMPLE

Comparing Centripetal Acceleration of a Car Rounding a Curve with Acceleration Due to Gravity

A car follows a curve of radius 500 m at a speed of 25.0 m/s (about 90 km/h). What is the magnitude of the car's centripetal acceleration? Compare the centripetal acceleration for this fairly gentle curve taken at highway speed with acceleration due to gravity (g).



Car around corner

Strategy

Because linear rather than angular speed is given, it is most convenient to use the expression $\mathbf{a}_c = \frac{v^2}{r}$ to find the magnitude of the centripetal acceleration.

Solution

Entering the given values of $v = 25.0$ m/s and $r = 500$ m into the expression for $\mathbf{a}_c$ gives

$$\begin{aligned}\mathbf{a}_c &= \frac{v^2}{r}\\ &= \frac{(25.0\ \text{m/s})^2}{500\ \text{m}}\\ &= 1.25\text{m/s}^2.\end{aligned}$$

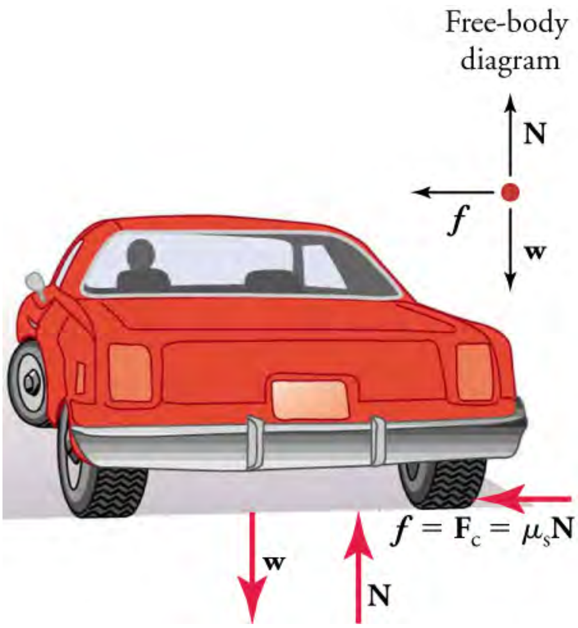
Discussion

To compare this with the acceleration due to gravity ($g = 9.80\ \text{m/s}^2$), we take the ratio $\mathbf{a}_c/g = (1.25\ \text{m/s}^2)/(9.80\text{m/s}^2) = 0.128$. Therefore, $\mathbf{a}_c = 0.128g$, which means that the centripetal acceleration is about one tenth the acceleration due to gravity.

WORKED EXAMPLE

Frictional Force on Car Tires Rounding a Curve

a. Calculate the centripetal force exerted on a 900 kg car that rounds a 600-m-radius curve on horizontal ground at 25.0 m/s.
b. Static friction prevents the car from slipping. Find the magnitude of the frictional force between the tires and the road that allows the car to round the curve without sliding off in a straight line.



Strategy and Solution for (a)

We know that $\mathbf{F}_c = m\frac{v^2}{r}$. Therefore,

$$\begin{aligned}\mathbf{F}_c &= m\frac{v^2}{r} \\ &= \frac{(900\ \text{kg})(25.0\ \text{m/s})^2}{600\ \text{m}} \\ &= 938\ \text{N}.\end{aligned}$$

Strategy and Solution for (b)

The image above shows the forces acting on the car while rounding the curve. In this diagram, the car is traveling into the page as shown and is turning to the left. Friction acts toward the left, accelerating the car toward the center of the curve. Because friction is the only horizontal force acting on the car, it provides all of the centripetal force in this case. Therefore, the force of friction is the centripetal force in this situation and points toward the center of the curve.

$$f = \mathbf{F}_c = 938\ \text{N}$$

Discussion

Since we found the force of friction in part (b), we could also solve for the coefficient of friction, since $f = \mu_s \text{N} = \mu_s mg$.

Practice Problems

9. What is the centripetal acceleration of an object with speed $12\ \text{m/s}$ going along a path of radius $2.0\ \text{m}$?
   a. $6\ \text{m/s}$
   b. $72\ \text{m/s}$
   c. $6\ \text{m/s}^2$
   d. $72\ \text{m/s}^2$

10. Calculate the centripetal acceleration of an object following a path with a radius of a curvature of 0.2 m and at an angular velocity of 5 rad/s.
    a. 1 m/s
    b. 5 m/s
    c. $1\ \text{m/s}^2$
    d. $5\ \text{m/s}^2$

Check Your Understanding

11. What is uniform circular motion?

a. Uniform circular motion is when an object accelerates on a circular path at a constantly increasing velocity.
b. Uniform circular motion is when an object travels on a circular path at a variable acceleration.
c. Uniform circular motion is when an object travels on a circular path at a constant speed.
d. Uniform circular motion is when an object travels on a circular path at a variable speed.

12. What is centripetal acceleration?
a. The acceleration of an object moving in a circular path and directed radially toward the center of the circular orbit
b. The acceleration of an object moving in a circular path and directed tangentially along the circular path
c. The acceleration of an object moving in a linear path and directed in the direction of motion of the object
d. The acceleration of an object moving in a linear path and directed in the direction opposite to the motion of the object

13. Is there a net force acting on an object in uniform circular motion?
a. Yes, the object is accelerating, so a net force must be acting on it.
b. Yes, because there is no acceleration.
c. No, because there is acceleration.
d. No, because there is no acceleration.

14. Identify two examples of forces that can cause centripetal acceleration.
a. The force of Earth's gravity on the moon and the normal force
b. The force of Earth's gravity on the moon and the tension in the rope on an orbiting tetherball
c. The normal force and the force of friction acting on a moving car
d. The normal force and the tension in the rope on a tetherball

6.3 Rotational Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe rotational kinematic variables and equations and relate them to their linear counterparts
- Describe torque and lever arm
- Solve problems involving torque and rotational kinematics

Section Key Terms

angular acceleration	kinematics of rotational motion	lever arm
tangential acceleration	torque	

Rotational Kinematics

In the section on uniform circular motion, we discussed motion in a circle at constant speed and, therefore, constant angular velocity. However, there are times when angular velocity is not constant—rotational motion can speed up, slow down, or reverse directions. Angular velocity is not constant when a spinning skater pulls in her arms, when a child pushes a merry-go-round to make it rotate, or when a CD slows to a halt when switched off. In all these cases, **angular acceleration** occurs because the angular velocity $\boldsymbol{\omega}$ changes. The faster the change occurs, the greater is the angular acceleration. Angular acceleration $\boldsymbol{\alpha}$ is the rate of change of angular velocity. In equation form, angular acceleration is

$$\boldsymbol{\alpha} = \frac{\Delta \boldsymbol{\omega}}{\Delta t},$$

where $\Delta\boldsymbol{\omega}$ is the change in angular velocity and Δt is the change in time. The units of angular acceleration are (rad/s)/s, or rad/s^2. If $\boldsymbol{\omega}$ increases, then $\boldsymbol{\alpha}$ is positive. If $\boldsymbol{\omega}$ decreases, then $\boldsymbol{\alpha}$ is negative. Keep in mind that, by convention, counterclockwise is the positive direction and clockwise is the negative direction. For example, the skater in Figure 6.9 is rotating counterclockwise as seen from above, so her angular velocity is positive. Acceleration would be negative, for example, when an object that is rotating counterclockwise slows down. It would be positive when an object that is rotating counterclockwise speeds up.

Figure 6.9 A figure skater spins in the counterclockwise direction, so her angular velocity is normally considered to be positive. (Luu, Wikimedia Commons)

The relationship between the magnitudes of **tangential acceleration**, **a**, and angular acceleration,

$$\boldsymbol{\alpha}, \mathrm{is}\mathbf{a} = r\boldsymbol{\alpha}\mathrm{or}\boldsymbol{\alpha} = \frac{\mathbf{a}}{r}. \tag{6.10}$$

These equations mean that the magnitudes of tangential acceleration and angular acceleration are directly proportional to each other. The greater the angular acceleration, the larger the change in tangential acceleration, and vice versa. For example, consider riders in their pods on a Ferris wheel at rest. A Ferris wheel with greater angular acceleration will give the riders greater tangential acceleration because, as the Ferris wheel increases its rate of spinning, it also increases its tangential velocity. Note that the radius of the spinning object also matters. For example, for a given angular acceleration $\boldsymbol{\alpha}$, a smaller Ferris wheel leads to a smaller tangential acceleration for the riders.

TIPS FOR SUCCESS

Tangential acceleration is sometimes denoted $\mathbf{a}_t$. It is a linear acceleration in a direction tangent to the circle at the point of interest in circular or rotational motion. Remember that tangential acceleration is parallel to the tangential velocity (either in the same direction or in the opposite direction.) Centripetal acceleration is always perpendicular to the tangential velocity.

So far, we have defined three rotational variables: θ, $\boldsymbol{\omega}$, and $\boldsymbol{\alpha}$. These are the angular versions of the linear variables x, **v**, and **a**. Table 6.2 shows how they are related.

Rotational	Linear	Relationship
θ	x	$\theta = \frac{x}{r}$

Table 6.2 Rotational and Linear Variables

Rotational	Linear	Relationship
$\boldsymbol{\omega}$	$\mathbf{v}$	$\boldsymbol{\omega} = \frac{\mathbf{v}}{r}$
$\boldsymbol{\alpha}$	$\mathbf{a}$	$\boldsymbol{\alpha} = \frac{\mathbf{a}}{r}$

Table 6.2 Rotational and Linear Variables

We can now begin to see how rotational quantities like θ, $\boldsymbol{\omega}$, and $\boldsymbol{\alpha}$ are related to each other. For example, if a motorcycle wheel that starts at rest has a large angular acceleration for a fairly long time, it ends up spinning rapidly and rotates through many revolutions. Putting this in terms of the variables, if the wheel's angular acceleration $\boldsymbol{\alpha}$ is large for a long period of time t, then the final angular velocity $\boldsymbol{\omega}$ and angle of rotation θ are large. In the case of linear motion, if an object starts at rest and undergoes a large linear acceleration, then it has a large final velocity and will have traveled a large distance.

The **kinematics of rotational motion** describes the relationships between the angle of rotation, angular velocity, angular acceleration, and time. It only *describes* motion—it does not include any forces or masses that may affect rotation (these are part of dynamics). Recall the kinematics equation for linear motion: $\mathbf{v} = \mathbf{v}_0 + \mathbf{a}t$ (constant $\mathbf{a}$).

As in linear kinematics, we assume $\mathbf{a}$ is constant, which means that angular acceleration $\boldsymbol{\alpha}$ is also a constant, because $\mathbf{a} = r\boldsymbol{\alpha}$. The equation for the kinematics relationship between $\boldsymbol{\omega}$, $\boldsymbol{\alpha}$, and t is

$$\boldsymbol{\omega} = \boldsymbol{\omega}_0 + \boldsymbol{\alpha}t (\text{constant}\alpha),$$

where $\boldsymbol{\omega}_0$ is the initial angular velocity. Notice that the equation is identical to the linear version, except with angular analogs of the linear variables. In fact, all of the linear kinematics equations have rotational analogs, which are given in Table 6.3. These equations can be used to solve rotational or linear kinematics problem in which $\mathbf{a}$ and $\boldsymbol{\alpha}$ are constant.

Rotational	Linear	
$\theta = \overline{\boldsymbol{\omega}}t$	$x = \overline{\mathbf{v}}t$	
$\boldsymbol{\omega} = \boldsymbol{\omega}_0 + \boldsymbol{\alpha}t$	$\mathbf{v} = \mathbf{v}_0 + \boldsymbol{\alpha}t$	constant $\boldsymbol{\alpha}$, $\mathbf{a}$
$\theta = \boldsymbol{\omega}_0 t + \frac{1}{2}\boldsymbol{\alpha}t^2$	$x = \mathbf{v}_0 t + \frac{1}{2}\boldsymbol{\alpha}t^2$	constant $\boldsymbol{\alpha}$, $\mathbf{a}$
$\boldsymbol{\omega}^2 = \boldsymbol{\omega}_0{}^2 + 2\boldsymbol{\alpha}\theta$	$\mathbf{v}^2 = \mathbf{v}_0{}^2 + 2\boldsymbol{\alpha}x$	constant $\boldsymbol{\alpha}$, $\mathbf{a}$

Table 6.3 Equations for Rotational Kinematics

In these equations, $\boldsymbol{\omega}_0$ and $\mathbf{v}_0$ are initial values, t_0 is zero, and the average angular velocity $\overline{\boldsymbol{\omega}}$ and average velocity $\overline{\mathbf{v}}$ are

$$\overline{\boldsymbol{\omega}} = \frac{\boldsymbol{\omega}_0 + \boldsymbol{\omega}}{2} \text{and} \overline{\mathbf{v}} = \frac{\mathbf{v}_0 + \mathbf{v}}{2}. \tag{6.11}$$

FUN IN PHYSICS

Storm Chasing

Figure 6.10 Tornadoes descend from clouds in funnel-like shapes that spin violently. (Daphne Zaras, U.S. National Oceanic and Atmospheric Administration)

Storm chasers tend to fall into one of three groups: Amateurs chasing tornadoes as a hobby, atmospheric scientists gathering data for research, weather watchers for news media, or scientists having fun under the guise of work. Storm chasing is a dangerous pastime because tornadoes can change course rapidly with little warning. Since storm chasers follow in the wake of the destruction left by tornadoes, changing flat tires due to debris left on the highway is common. The most active part of the world for tornadoes, called *tornado alley*, is in the central United States, between the Rocky Mountains and Appalachian Mountains.

Tornadoes are perfect examples of rotational motion in action in nature. They come out of severe thunderstorms called supercells, which have a column of air rotating around a horizontal axis, usually about four miles across. The difference in wind speeds between the strong cold winds higher up in the atmosphere in the jet stream and weaker winds traveling north from the Gulf of Mexico causes the column of rotating air to shift so that it spins around a vertical axis, creating a tornado.

Tornadoes produce wind speeds as high as 500 km/h (approximately 300 miles/h), particularly at the bottom where the funnel is narrowest because the rate of rotation increases as the radius decreases. They blow houses away as if they were made of paper and have been known to pierce tree trunks with pieces of straw.

GRASP CHECK

What is the physics term for the eye of the storm? Why would winds be weaker at the eye of the tornado than at its outermost edge?

a. The eye of the storm is the center of rotation. Winds are weaker at the eye of a tornado because tangential velocity is directly proportional to radius of curvature.
b. The eye of the storm is the center of rotation. Winds are weaker at the eye of a tornado because tangential velocity is inversely proportional to radius of curvature.
c. The eye of the storm is the center of rotation. Winds are weaker at the eye of a tornado because tangential velocity is directly proportional to the square of the radius of curvature.
d. The eye of the storm is the center of rotation. Winds are weaker at the eye of a tornado because tangential velocity is inversely proportional to the square of the radius of curvature.

Torque

If you have ever spun a bike wheel or pushed a merry-go-round, you know that force is needed to change angular velocity. The farther the force is applied from the pivot point (or fulcrum), the greater the angular acceleration. For example, a door opens slowly if you push too close to its hinge, but opens easily if you push far from the hinges. Furthermore, we know that the more massive the door is, the more slowly it opens; this is because angular acceleration is inversely proportional to mass. These relationships are very similar to the relationships between force, mass, and acceleration from Newton's second law of motion. Since we have already covered the angular versions of distance, velocity and time, you may wonder what the angular version of force is, and how it relates to linear force.

The angular version of force is **torque $\boldsymbol{\tau}$**, which is the turning effectiveness of a force. See Figure 6.11. The equation for the magnitude of torque is

$$\boldsymbol{\tau} = r\mathbf{F}\sin\theta,$$

where r is the magnitude of the **lever arm**, $\mathbf{F}$ is the magnitude of the linear force, and θ is the angle between the lever arm and the force. The lever arm is the vector from the point of rotation (pivot point or fulcrum) to the location where force is applied. Since the magnitude of the lever arm is a distance, its units are in meters, and torque has units of N·m. Torque is a vector quantity and has the same direction as the angular acceleration that it produces.

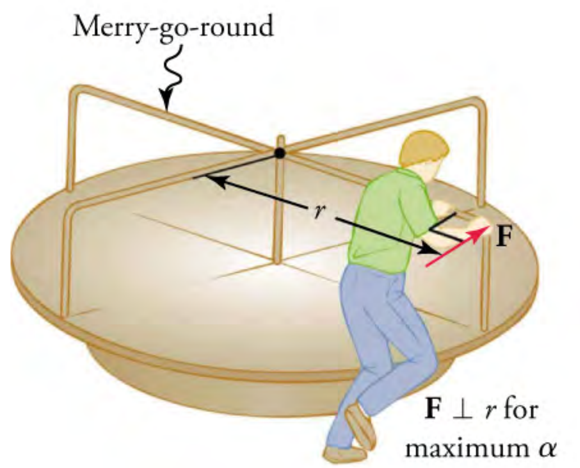


Figure 6.11 A man pushes a merry-go-round at its edge and perpendicular to the lever arm to achieve maximum torque.

Applying a stronger torque will produce a greater angular acceleration. For example, the harder the man pushes the merry-go-round in Figure 6.11, the faster it accelerates. Furthermore, the more massive the merry-go-round is, the slower it accelerates for the same torque. If the man wants to maximize the effect of his force on the merry-go-round, he should push as far from the center as possible to get the largest lever arm and, therefore, the greatest torque and angular acceleration. Torque is also maximized when the force is applied perpendicular to the lever arm.

Solving Rotational Kinematics and Torque Problems

Just as linear forces can balance to produce zero net force and no linear acceleration, the same is true of rotational motion. When two torques of equal magnitude act in opposing directions, there is no net torque and no angular acceleration, as you can see in the following video. If zero net torque acts on a system spinning at a constant angular velocity, the system will continue to spin at the same angular velocity.

WATCH PHYSICS

Introduction to Torque

This video (https://www.khanacademy.org/science/physics/torque-angular-momentum/torque-tutorial/v/introduction-to-torque) defines torque in terms of moment arm (which is the same as lever arm). It also covers a problem with forces acting in opposing directions about a pivot point. (At this stage, you can ignore Sal's references to work and mechanical advantage.)

GRASP CHECK

Click to view content (https://www.openstax.org/l/28torque)

If the net torque acting on the ruler from the example was positive instead of zero, what would this say about the angular acceleration? What would happen to the ruler over time?

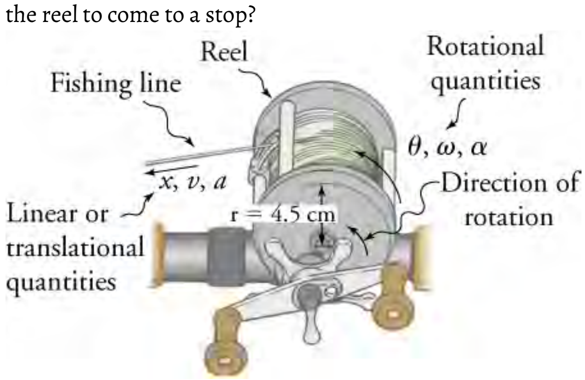
a. The ruler is in a state of rotational equilibrium so it will not rotate about its center of mass. Thus, the angular acceleration will be zero.
b. The ruler is not in a state of rotational equilibrium so it will not rotate about its center of mass. Thus, the angular acceleration will be zero.
c. The ruler is not in a state of rotational equilibrium so it will rotate about its center of mass. Thus, the angular acceleration will be non-zero.
d. The ruler is in a state of rotational equilibrium so it will rotate about its center of mass. Thus, the angular acceleration will be non-zero.

Now let's look at examples applying rotational kinematics to a fishing reel and the concept of torque to a merry-go-round.

WORKED EXAMPLE

Calculating the Time for a Fishing Reel to Stop Spinning

A deep-sea fisherman uses a fishing rod with a reel of radius 4.50 cm. A big fish takes the bait and swims away from the boat, pulling the fishing line from his fishing reel. As the fishing line unwinds from the reel, the reel spins at an angular velocity of 220 rad/s. The fisherman applies a brake to the spinning reel, creating an angular acceleration of –300 rad/s². How long does it take the reel to come to a stop?



Strategy

We are asked to find the time t for the reel to come to a stop. The magnitude of the initial angular velocity is $\boldsymbol{\omega}_0 = 220$ rad/s, and the magnitude of the final angular velocity $\boldsymbol{\omega} = 0$. The signed magnitude of the angular acceleration is $\boldsymbol{\alpha} = -300$ rad/s², where the minus sign indicates that it acts in the direction opposite to the angular velocity. Looking at the rotational kinematic equations, we see all quantities but t are known in the equation $\boldsymbol{\omega} = \boldsymbol{\omega}_0 + \boldsymbol{\alpha}t$, making it the easiest equation to use for this problem.

Solution

The equation to use is $\boldsymbol{\omega} = \boldsymbol{\omega}_0 + \boldsymbol{\alpha}t$.

We solve the equation algebraically for t, and then insert the known values.

$$\begin{aligned} t &= \frac{\omega - \omega_0}{\alpha} \\ &= \frac{0 - 220\ \text{rad/s}}{-300 \text{rad/s}^2} \\ &= 0.733\ \text{s} \end{aligned} \tag{6.12}$$

Discussion

The time to stop the reel is fairly small because the acceleration is fairly large. Fishing lines sometimes snap because of the forces involved, and fishermen often let the fish swim for a while before applying brakes on the reel. A tired fish will be slower, requiring a smaller acceleration and therefore a smaller force.

WORKED EXAMPLE

Calculating the Torque on a Merry-Go-Round

Consider the man pushing the playground merry-go-round in Figure 6.11. He exerts a force of 250 N at the edge of the merry-go-round and perpendicular to the radius, which is 1.50 m. How much torque does he produce? Assume that friction acting on the merry-go-round is negligible.

Strategy

To find the torque, note that the applied force is perpendicular to the radius and that friction is negligible.

Solution

$$\begin{aligned} \tau &= r\mathbf{F}\sin\theta \\ &= (1.50\text{ m})(250\text{ N})\sin\left(\tfrac{\pi}{2}\right). \\ &= 375\text{ N}\cdot\text{m} \end{aligned} \tag{6.13}$$

Discussion

The man maximizes the torque by applying force perpendicular to the lever arm, so that $\theta = \frac{\pi}{2}$ and $\sin\theta = 1$. The man also maximizes his torque by pushing at the outer edge of the merry-go-round, so that he gets the largest-possible lever arm.

Practice Problems

15. How much torque does a person produce if he applies a 12 N force 1.0 m away from the pivot point, perpendicularly to the lever arm?
    a. $\frac{1}{144}$ N-m
    b. $\frac{1}{12}$ N-m
    c. 12 N-m
    d. 144 N-m

16. An object's angular velocity changes from 3 rad/s clockwise to 8 rad/s clockwise in 5 s. What is its angular acceleration?
    a. 0.6 rad/s^2
    b. 1.6 rad/s^2
    c. 1 rad/s^2
    d. 5 rad/s^2

Check Your Understanding

17. What is angular acceleration?
    a. Angular acceleration is the rate of change of the angular displacement.
    b. Angular acceleration is the rate of change of the angular velocity.
    c. Angular acceleration is the rate of change of the linear displacement.
    d. Angular acceleration is the rate of change of the linear velocity.

18. What is the equation for angular acceleration, α? Assume θ is the angle, ω is the angular velocity, and t is time.
    a. $\alpha = \frac{\Delta\omega}{\Delta t}$
    b. $\alpha = \Delta\omega\Delta t$
    c. $\alpha = \frac{\Delta\theta}{\Delta t}$
    d. $\alpha = \Delta\theta\Delta t$

19. Which of the following best describes torque?
    a. It is the rotational equivalent of a force.
    b. It is the force that affects linear motion.
    c. It is the rotational equivalent of acceleration.
    d. It is the acceleration that affects linear motion.

20. What is the equation for torque?

a. $\tau = F\, cos\theta\, r$
b. $\tau = \frac{F \sin\theta}{r}$
c. $\tau = rF\cos\theta$
d. $\tau = rF\sin\theta$

KEY TERMS

angle of rotation the ratio of the arc length to the radius of curvature of a circular path

angular acceleration the rate of change of angular velocity with time

angular velocity ($\boldsymbol{\omega}$) the rate of change in the angular position of an object following a circular path

arc length (Δs) the distance traveled by an object along a circular path

centrifugal force a fictitious force that acts in the direction opposite the centripetal acceleration

centripetal acceleration the acceleration of an object moving in a circle, directed toward the center of the circle

centripetal force any force causing uniform circular motion

circular motion the motion of an object along a circular path

kinematics of rotational motion the relationships between rotation angle, angular velocity, angular acceleration, and time

lever arm the distance between the point of rotation (pivot point) and the location where force is applied

radius of curvature the distance between the center of a circular path and the path

rotational motion the circular motion of an object about an axis of rotation

spin rotation about an axis that goes through the center of mass of the object

tangential acceleration the acceleration in a direction tangent to the circular path of motion and in the same direction or opposite direction as the tangential velocity

tangential velocity the instantaneous linear velocity of an object in circular or rotational motion

torque the effectiveness of a force to change the rotational speed of an object

uniform circular motion the motion of an object in a circular path at constant speed

SECTION SUMMARY

6.1 Angle of Rotation and Angular Velocity

- Circular motion is motion in a circular path.
- The angle of rotation $\Delta\theta$ is defined as the ratio of the arc length to the radius of curvature.
- The arc length Δs is the distance traveled along a circular path and r is the radius of curvature of the circular path.
- The angle of rotation $\Delta\theta$ is measured in units of radians (rad), where $2\ \pi\text{rad} = 360° = 1$ revolution.
- Angular velocity $\boldsymbol{\omega}$ is the rate of change of an angle, where a rotation $\Delta\theta$ occurs in a time Δt .
- The units of angular velocity are radians per second (rad/s).
- Tangential speed v and angular speed ω are related by $v = r\omega$, and tangential velocity has units of m/s.
- The direction of angular velocity is along the axis of rotation, toward (away) from you for clockwise (counterclockwise) motion.

6.2 Uniform Circular Motion

- Centripetal acceleration $\mathbf{a}_c$ is the acceleration experienced while in uniform circular motion.
- Centripetal acceleration force is a *center-seeking* force that always points toward the center of rotation, perpendicular to the linear velocity, in the same direction as the net force, and in the direction opposite that of the radius vector.
- The standard unit for centripetal acceleration is m/s^2.
- Centripetal force $\mathbf{F}_c$ is any net force causing uniform circular motion.

6.3 Rotational Motion

- Kinematics is the description of motion.
- The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, angular acceleration, and time.
- Torque is the effectiveness of a force to change the rotational speed of an object. Torque is the rotational analog of force.
- The lever arm is the distance between the point of rotation (pivot point) and the location where force is applied.
- Torque is maximized by applying force perpendicular to the lever arm and at a point as far as possible from the pivot point or fulcrum. If torque is zero, angular acceleration is zero.

KEY EQUATIONS

6.1 Angle of Rotation and Angular Velocity

Angle of rotation	$\Delta\theta = \frac{\Delta s}{r}$
Angular speed:	$\omega = \frac{\Delta\theta}{\Delta t}$
Tangential speed:	$v = r\omega$

6.2 Uniform Circular Motion

Centripetal acceleration	$\mathbf{a}_c = \frac{v^2}{r}$ or $\mathbf{a}_c = r\omega^2$
Centripetal force	$\mathbf{F}_c = m\mathbf{a}_c, \mathbf{F}_c = m\frac{v^2}{r}, \mathbf{F}_c = mr\omega^2$

6.3 Rotational Motion

Angular acceleration	$\boldsymbol{\alpha} = \frac{\Delta\boldsymbol{\omega}}{\Delta t}$
Rotational kinematic equations	$\theta = \boldsymbol{\omega} t, \boldsymbol{\omega} = \boldsymbol{\omega}_0 + \boldsymbol{\alpha} t,$ $\theta = \boldsymbol{\omega}_0 t + \frac{1}{2}\boldsymbol{\alpha} t^2,$ $\boldsymbol{\omega}^2 = {\boldsymbol{\omega}_0}^2 + 2\boldsymbol{\alpha}\theta$
Tangential (linear) acceleration	$\mathbf{a} = r\boldsymbol{\alpha}$
Torque	$\boldsymbol{\tau} = r\mathbf{F}\sin\theta$

CHAPTER REVIEW

Concept Items

6.1 Angle of Rotation and Angular Velocity

1. One revolution is equal to how many radians? Degrees?
   a. $1 \text{ rev} = \pi \text{ rad} = 180°$
   b. $1 \text{ rev} = \pi \text{ rad} = 360°$
   c. $1 \text{ rev} = 2\pi \text{ rad} = 180°$
   d. $1 \text{ rev} = 2\pi \text{ rad} = 360°$

2. What is tangential velocity?
   a. Tangential velocity is the average linear velocity of an object in a circular motion.
   b. Tangential velocity is the instantaneous linear velocity of an object undergoing rotational motion.
   c. Tangential velocity is the average angular velocity of an object in a circular motion.
   d. Tangential velocity is the instantaneous angular velocity of an object in a circular motion.

3. What kind of motion is called *spin*?
   a. Spin is rotational motion of an object about an axis parallel to the axis of the object.
   b. Spin is translational motion of an object about an axis parallel to the axis of the object.
   c. Spin is the rotational motion of an object about its center of mass.
   d. Spin is translational motion of an object about its own axis.

6.2 Uniform Circular Motion

4. What is the equation for centripetal acceleration in terms of angular velocity and the radius?
   a. $a_c = \frac{\omega^2}{r}$
   b. $a_c = \frac{\omega}{r}$
   c. $a_c = r\omega^2$
   d. $a_c = r\omega$

5. How can you express centripetal force in terms of centripetal acceleration?
   a. $F_c = \frac{a_c^2}{m}$
   b. $F_c = \frac{a_c}{m}$
   c. $F_c = ma_c^2$
   d. $F_c = ma_c$

6. What is meant by the word centripetal?
   a. center-seeking
   b. center-avoiding
   c. central force
   d. central acceleration

6.3 Rotational Motion

7. Conventionally, for which direction of rotation of an object is angular acceleration considered positive?
   a. the positive x direction of the coordinate system
   b. the negative x direction of the coordinate system
   c. the counterclockwise direction
   d. the clockwise direction

8. When you push a door closer to the hinges, why does it open more slowly?
   a. It opens slowly, because the lever arm is shorter so the torque is large.
   b. It opens slowly because the lever arm is longer so the torque is large.
   c. It opens slowly, because the lever arm is shorter so the torque is less.
   d. It opens slowly, because the lever arm is longer so the torque is less.

9. When is angular acceleration negative?
   a. Angular acceleration is the rate of change of the displacement and is negative when ω increases.
   b. Angular acceleration is the rate of change of the displacement and is negative when ω decreases.
   c. Angular acceleration is the rate of change of angular velocity and is negative when ω increases.
   d. Angular acceleration is the rate of change of angular velocity and is negative when ω decreases.

Critical Thinking Items

6.1 Angle of Rotation and Angular Velocity

10. When the radius of the circular path of rotational motion increases, what happens to the arc length for a given angle of rotation?
    a. The arc length is directly proportional to the radius of the circular path, and it increases with the radius.
    b. The arc length is inversely proportional to the radius of the circular path, and it decreases with the radius.
    c. The arc length is directly proportional to the radius of the circular path, and it decreases with the radius.
    d. The arc length is inversely proportional to the radius of the circular path, and it increases with the radius.

11. Consider a CD spinning clockwise. What is the sum of the instantaneous velocities of two points on both ends of its diameter?
    a. $2v$
    b. $\frac{v}{2}$
    c. $-v$
    d. 0

6.2 Uniform Circular Motion

12. What are the directions of the velocity and acceleration of an object in uniform circular motion?
    a. Velocity is tangential, and acceleration is radially outward.
    b. Velocity is tangential, and acceleration is radially inward.
    c. Velocity is radially outward, and acceleration is tangential.
    d. Velocity is radially inward, and acceleration is tangential.

13. Suppose you have an object tied to a rope and are rotating it over your head in uniform circular motion. If you increase the length of the rope, would you have to apply more or less force to maintain the same speed?
    a. More force is required, because the force is inversely proportional to the radius of the circular orbit.
    b. More force is required because the force is directly proportional to the radius of the circular orbit.
    c. Less force is required because the force is inversely proportional to the radius of the circular orbit.
    d. Less force is required because the force is directly proportional to the radius of the circular orbit.

6.3 Rotational Motion

14. Consider two spinning tops with different radii. Both have the same linear instantaneous velocities at their edges. Which top has a higher angular velocity?
    a. the top with the smaller radius because the radius of curvature is inversely proportional to the angular velocity
    b. the top with the smaller radius because the radius of curvature is directly proportional to the angular velocity
    c. the top with the larger radius because the radius of curvature is inversely proportional to the angular velocity
    d. The top with the larger radius because the radius of curvature is directly proportional to the angular velocity

15. A person tries to lift a stone by using a lever. If the lever arm is constant and the mass of the stone increases, what is true of the torque necessary to lift it?
    a. It increases, because the torque is directly proportional to the mass of the body.
    b. It increases because the torque is inversely proportional to the mass of the body.
    c. It decreases because the torque is directly proportional to the mass of the body.
    d. It decreases, because the torque is inversely proportional to the mass of the body.

Problems

6.1 Angle of Rotation and Angular Velocity

16. What is the angle of rotation (in degrees) between two hands of a clock, if the radius of the clock is 0.70 m and the arc length separating the two hands is 1.0 m?
   a. $40°$
   b. $80°$
   c. $81°$
   d. $163°$

17. A clock has radius of 0.5 m. The outermost point on its minute hand travels along the edge. What is its tangential speed?
   a. $9 \times 10^{-4}\text{ m/s}$
   b. $3.4 \times 10^{-3}\text{ m/s}$
   c. $8.5 \times 10^{-4}\text{ m/s}$
   d. $1.3 \times 10^{1}\text{ m/s}$

6.2 Uniform Circular Motion

18. What is the centripetal force exerted on a 1,600 kg car that rounds a 100 m radius curve at 12 m/s?
   a. 192 N
   b. 1, 111 N
   c. 2, 300 N
   d. 13, 333 N

19. Find the frictional force between the tires and the road that allows a 1,000 kg car traveling at 30 m/s to round a 20 m radius curve.
   a. 22 N
   b. 667 N
   c. 1, 500 N
   d. 45, 000 N

6.3 Rotational Motion

20. An object's angular acceleration is 36 rad/s^2. If it were initially spinning with a velocity of 6.0 m/s, what would its angular velocity be after 5.0 s?
   a. 186 rad/s
   b. 190 rad/s^2
   c. −174 rad/s
   d. −174 rad/s^2

21. When a fan is switched on, it undergoes an angular acceleration of 150 rad/s^2. How long will it take to achieve its maximum angular velocity of 50 rad/s?
   a. −0.3 s
   b. 0.3 s
   c. 3.0 s

Performance Task

6.3 Rotational Motion

22. Design a lever arm capable of lifting a 0.5 kg object such as a stone. The force for lifting should be provided by placing coins on the other end of the lever. How many coins would you need? What happens if you shorten or lengthen the lever arm? What does this say about torque?

TEST PREP

Multiple Choice

6.1 Angle of Rotation and Angular Velocity

23. What is 1 radian approximately in degrees?
   a. 57.3°
   b. 360°
   c. $\pi°$
   d. $2\pi°$

24. If the following objects are spinning at the same angular velocities, the edge of which one would have the highest speed?
   a. Mini CD
   b. Regular CD
   c. Vinyl record

25. What are possible units for tangential velocity?
   a. m/s
   b. rad/s
   c. °/s

26. What is $30°$ in radians?
   a. $\frac{\pi}{12}$
   b. $\frac{\pi}{9}$
   c. $\frac{\pi}{6}$
   d. $\frac{\pi}{3}$

27. For a given object, what happens to the arc length as the angle of rotation increases?
   a. The arc length is directly proportional to the angle of rotation, so it increases with the angle of rotation.
   b. The arc length is inversely proportional to the angle of rotation, so it decreases with the angle of rotation.
   c. The arc length is directly proportional to the angle of rotation, so it decreases with the angle of rotation.

d. The arc length is inversely proportional to the angle of rotation, so it increases with the angle of rotation.

6.2 Uniform Circular Motion

28. Which of these quantities is constant in uniform circular motion?
a. Speed
b. Velocity
c. Acceleration
d. Displacement

29. Which of these quantities impact centripetal force?
a. Mass and speed only
b. Mass and radius only
c. Speed and radius only
d. Mass, speed, and radius all impact centripetal force

30. An increase in the magnitude of which of these quantities causes a reduction in centripetal force?
a. Mass
b. Radius of curvature
c. Speed

31. What happens to centripetal acceleration as the radius of curvature decreases and the speed is constant, and why?
a. It increases, because the centripetal acceleration is inversely proportional to the radius of the curvature.
b. It increases, because the centripetal acceleration is directly proportional to the radius of curvature.
c. It decreases, because the centripetal acceleration is inversely proportional to the radius of the curvature.
d. It decreases, because the centripetal acceleration is directly proportional to the radius of the curvature.

32. Why do we experience more sideways acceleration while driving around sharper curves?
a. Centripetal acceleration is inversely proportional to the radius of curvature, so it increases as the radius of curvature decreases.
b. Centripetal acceleration is directly proportional to the radius of curvature, so it decreases as the radius of curvature decreases.
c. Centripetal acceleration is directly proportional to the radius of curvature, so it decreases as the radius of curvature increases.
d. Centripetal acceleration is directly proportional to the radius of curvature, so it increases as the radius of curvature increases.

6.3 Rotational Motion

33. Which of these quantities is not described by the kinematics of rotational motion?
a. Rotation angle
b. Angular acceleration
c. Centripetal force
d. Angular velocity

34. In the equation $\tau = rF\sin\theta$, what is F?
a. Linear force
b. Centripetal force
c. Angular force

35. What happens when two torques act equally in opposite directions?
a. Angular velocity is zero.
b. Angular acceleration is zero.

36. What is the mathematical relationship between angular and linear accelerations?
a. $a = r\alpha$
b. $a = \frac{\alpha}{r}$
c. $a = r^2\alpha$
d. $a = \frac{\alpha}{r^2}$

Short Answer

6.1 Angle of Rotation and Angular Velocity

37. What is the rotational analog of linear velocity?
a. Angular displacement
b. Angular velocity
c. Angular acceleration
d. Angular momentum

38. What is the rotational analog of distance?
a. Rotational angle
b. Torque
c. Angular velocity
d. Angular momentum

39. What is the equation that relates the linear speed of a point on a rotating object with the object's angular quantities?
a. $v = \frac{\omega}{r}$
b. $v = r\omega$
c. $v = \frac{\alpha}{r}$
d. $v = r\alpha$

40. As the angular velocity of an object increases, what happens to the linear velocity of a point on that object?
a. It increases, because linear velocity is directly proportional to angular velocity.
b. It increases, because linear velocity is inversely proportional to angular velocity.

c. It decreases because linear velocity is directly proportional to angular velocity.
d. It decreases because linear velocity is inversely proportional to angular velocity.

41. What is angular speed in terms of tangential speed and the radius?
a. $\omega = \frac{v^2}{r}$
b. $\omega = \frac{v}{r}$
c. $\omega = rv$
d. $\omega = rv^2$

42. Why are radians dimensionless?
a. Radians are dimensionless, because they are defined as a ratio of distances. They are defined as the ratio of the arc length to the radius of the circle.
b. Radians are dimensionless because they are defined as a ratio of distances. They are defined as the ratio of the area to the radius of the circle.
c. Radians are dimensionless because they are defined as multiplication of distance. They are defined as the multiplication of the arc length to the radius of the circle.
d. Radians are dimensionless because they are defined as multiplication of distance. They are defined as the multiplication of the area to the radius of the circle.

6.2 Uniform Circular Motion

43. What type of quantity is centripetal acceleration?
a. Scalar quantity; centripetal acceleration has magnitude only but no direction
b. Scalar quantity; centripetal acceleration has magnitude as well as direction
c. Vector quantity; centripetal acceleration has magnitude only but no direction
d. Vector quantity; centripetal acceleration has magnitude as well as direction

44. What are the standard units for centripetal acceleration?
a. m/s
b. m/s^2
c. m^2/s
d. m^2/s^2

45. What is the angle formed between the vectors of tangential velocity and centripetal force?
a. 0°
b. 30°
c. 90°
d. 180°

46. What is the angle formed between the vectors of centripetal acceleration and centripetal force?
a. 0°
b. 30°
c. 90°
d. 180°

47. What are the standard units for centripetal force?
a. m
b. m/s
c. m/s^2
d. newtons

48. As the mass of an object in uniform circular motion increases, what happens to the centripetal force required to keep it moving at the same speed?
a. It increases, because the centripetal force is directly proportional to the mass of the rotating body.
b. It increases, because the centripetal force is inversely proportional to the mass of the rotating body.
c. It decreases, because the centripetal force is directly proportional to the mass of the rotating body.
d. It decreases, because the centripetal force is inversely proportional to the mass of the rotating body.

6.3 Rotational Motion

49. The relationships between which variables are described by the kinematics of rotational motion?
a. The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, and angular acceleration.
b. The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, angular acceleration, and angular momentum.
c. The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, angular acceleration, and time.
d. The kinematics of rotational motion describes the relationships between rotation angle, angular velocity, angular acceleration, torque, and time.

50. What is the kinematics relationship between ω, α, and t ?
a. $\omega = \alpha t$
b. $\omega = \omega_0 - \alpha t$
c. $\omega = \omega_0 + \alpha t$
d. $\omega = \omega_0 + \frac{1}{2}\alpha t$

51. What kind of quantity is torque?
a. Scalar
b. Vector

c. Dimensionless
d. Fundamental quantity

52. If a linear force is applied to a lever arm farther away from the pivot point, what happens to the resultant torque?
a. It decreases.
b. It increases.
c. It remains the same.
d. It changes the direction.

53. How can the same force applied to a lever produce different torques?
a. By applying the force at different points of the lever arm along the length of the lever or by changing the angle between the lever arm and the applied force.
b. By applying the force at the same point of the lever arm along the length of the lever or by changing the angle between the lever arm and the applied force.
c. By applying the force at different points of the lever arm along the length of the lever or by maintaining the same angle between the lever arm and the applied force.
d. By applying the force at the same point of the lever arm along the length of the lever or by maintaining the same angle between the lever arm and the applied force.

Extended Response

6.1 Angle of Rotation and Angular Velocity

54. Consider two pits on a CD, one close to the center and one close to the outer edge. When the CD makes one full rotation, which pit would have gone through a greater angle of rotation? Which one would have covered a greater arc length?
a. The one close to the center would go through the greater angle of rotation. The one near the outer edge would trace a greater arc length.
b. The one close to the center would go through the greater angle of rotation. The one near the center would trace a greater arc length.
c. Both would go through the same angle of rotation. The one near the outer edge would trace a greater arc length.
d. Both would go through the same angle of rotation. The one near the center would trace a greater arc length.

55. Consider two pits on a CD, one close to the center and one close to the outer edge. For a given angular velocity of the CD, which pit has a higher angular velocity? Which has a higher tangential velocity?
a. The point near the center would have the greater angular velocity and the point near the outer edge would have the higher linear velocity.
b. The point near the edge would have the greater angular velocity and the point near the center would have the higher linear velocity.
c. Both have the same angular velocity and the point near the outer edge would have the higher linear velocity.
d. Both have the same angular velocity and the point near the center would have the higher linear velocity.

56. What happens to tangential velocity as the radius of an object increases provided the angular velocity remains the same?
a. It increases because tangential velocity is directly proportional to the radius.
b. It increases because tangential velocity is inversely proportional to the radius.
c. It decreases because tangential velocity is directly proportional to the radius.
d. It decreases because tangential velocity is inversely proportional to the radius.

6.2 Uniform Circular Motion

57. Is an object in uniform circular motion accelerating? Why or why not?
a. Yes, because the velocity is not constant.
b. No, because the velocity is not constant.
c. Yes, because the velocity is constant.
d. No, because the velocity is constant.

58. An object is in uniform circular motion. Suppose the centripetal force was removed. In which direction would the object now travel?
a. In the direction of the centripetal force
b. In the direction opposite to the direction of the centripetal force
c. In the direction of the tangential velocity
d. In the direction opposite to the direction of the tangential velocity

59. An object undergoes uniform circular motion. If the radius of curvature and mass of the object are constant, what is the centripetal force proportional to?
a. $F_c \propto \frac{1}{v}$
b. $F_c \propto \frac{1}{v^2}$
c. $F_c \propto v$
d. $F_c \propto v^2$

6.3 Rotational Motion

60. Why do tornadoes produce more wind speed at the bottom of the funnel?

a. Wind speed is greater at the bottom because rate of rotation increases as the radius increases.
b. Wind speed is greater at the bottom because rate of rotation increases as the radius decreases.
c. Wind speed is greater at the bottom because rate of rotation decreases as the radius increases.
d. Wind speed is greater at the bottom because rate of rotation decreases as the radius increases.

61. How can you maximize the torque applied to a given lever arm without applying more force?

a. The force should be applied perpendicularly to the lever arm as close as possible from the pivot point.
b. The force should be applied perpendicularly to the lever arm as far as possible from the pivot point.
c. The force should be applied parallel to the lever arm as far as possible from the pivot point.
d. The force should be applied parallel to the lever arm as close as possible from the pivot point.

62. When will an object continue spinning at the same angular velocity?

a. When net torque acting on it is zero
b. When net torque acting on it is non zero
c. When angular acceleration is positive
d. When angular acceleration is negative

CHAPTER 7
Newton's Law of Gravitation

Figure 7.1 Johannes Kepler (left) showed how the planets move, and Isaac Newton (right) discovered that gravitational force caused them to move that way. ((left) unknown, Public Domain; (right) Sir Godfrey Kneller, Public Domain)

Chapter Outline

INTRODUCTION What do a falling apple and the orbit of the moon have in common? You will learn in this chapter that each is caused by gravitational force. The motion of all celestial objects, in fact, is determined by the gravitational force, which depends on their mass and separation.

Johannes Kepler discovered three laws of planetary motion that all orbiting planets and moons follow. Years later, Isaac Newton found these laws useful in developing his law of universal gravitation. This law relates gravitational force to the masses of objects and the distance between them. Many years later still, Albert Einstein showed there was a little more to the gravitation story when he published his theory of general relativity.

7.1 Kepler's Laws of Planetary Motion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain Kepler's three laws of planetary motion
- Apply Kepler's laws to calculate characteristics of orbits

Section Key Terms

aphelion	Copernican model	eccentricity
Kepler's laws of planetary motion	perihelion	Ptolemaic model

Concepts Related to Kepler's Laws of Planetary Motion

Examples of orbits abound. Hundreds of artificial satellites orbit Earth together with thousands of pieces of debris. The moon's orbit around Earth has intrigued humans from time immemorial. The orbits of planets, asteroids, meteors, and comets around the sun are no less interesting. If we look farther, we see almost unimaginable numbers of stars, galaxies, and other celestial objects orbiting one another and interacting through gravity.

All these motions are governed by gravitational force. The orbital motions of objects in our own solar system are simple enough to describe with a few fairly simple laws. The orbits of planets and moons satisfy the following two conditions:

- The mass of the orbiting object, m, is small compared to the mass of the object it orbits, M.
- The system is isolated from other massive objects.

Based on the motion of the planets about the sun, Kepler devised a set of three classical laws, called **Kepler's laws of planetary motion**, that describe the orbits of all bodies satisfying these two conditions:

1. The orbit of each planet around the sun is an ellipse with the sun at one focus.
2. Each planet moves so that an imaginary line drawn from the sun to the planet sweeps out equal areas in equal times.
3. The ratio of the squares of the periods of any two planets about the sun is equal to the ratio of the cubes of their average distances from the sun.

These descriptive laws are named for the German astronomer Johannes Kepler (1571–1630). He devised them after careful study (over some 20 years) of a large amount of meticulously recorded observations of planetary motion done by Tycho Brahe (1546–1601). Such careful collection and detailed recording of methods and data are hallmarks of good science. Data constitute the evidence from which new interpretations and meanings can be constructed. Let's look closer at each of these laws.

Kepler's First Law

The orbit of each planet about the sun is an ellipse with the sun at one focus, as shown in Figure 7.2. The planet's closest approach to the sun is called **aphelion** and its farthest distance from the sun is called **perihelion**.

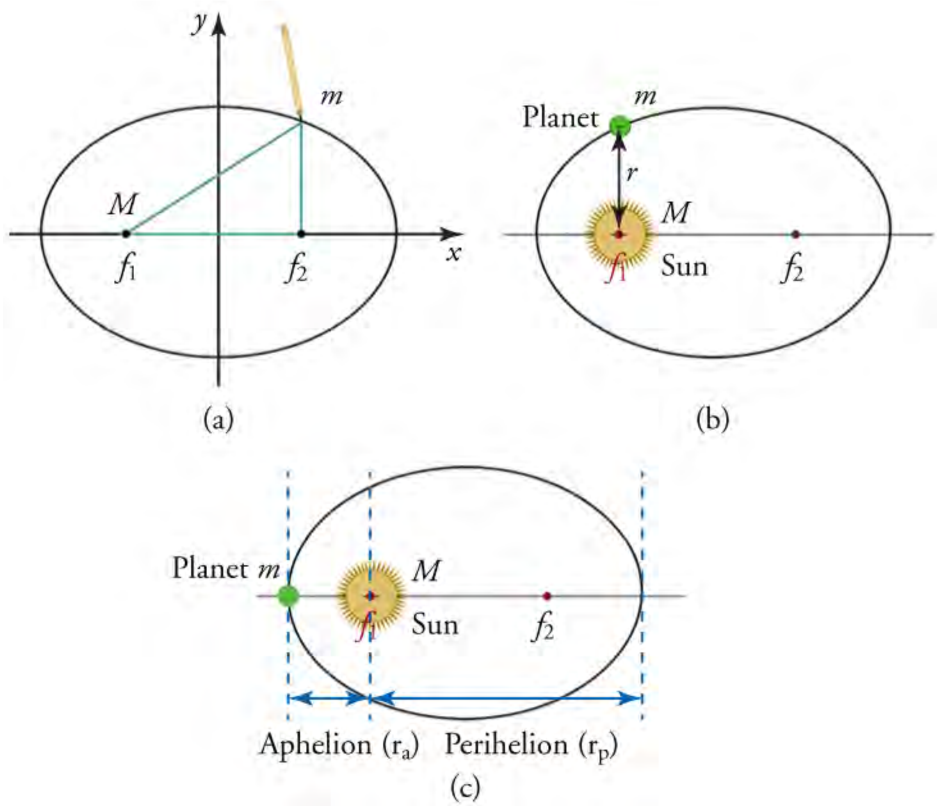


Figure 7.2 (a) An ellipse is a closed curve such that the sum of the distances from a point on the curve to the two foci (f_1 and f_2) is constant. (b) For any closed orbit, *m* follows an elliptical path with *M* at one focus. (c) The aphelion (*r*a) is the closest distance between the planet and the sun, while the perihelion (*r*p) is the farthest distance from the sun.

If you know the aphelion (r_a) and perihelion (r_p) distances, then you can calculate the semi-major axis (a) and semi-minor axis (b).

$$a = \frac{(r_a + r_p)}{2}$$

$$b = \sqrt{r_a r_p}$$

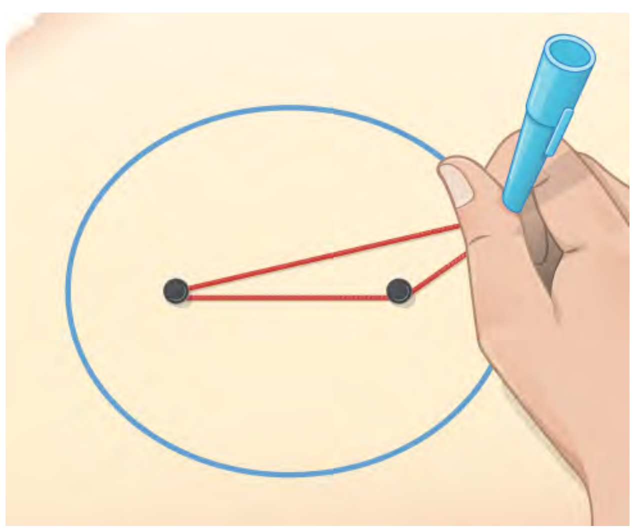


Figure 7.3 You can draw an ellipse as shown by putting a pin at each focus, and then placing a loop of string around a pen and the pins and tracing a line on the paper.

Kepler's Second Law

Each planet moves so that an imaginary line drawn from the sun to the planet sweeps out equal areas in equal times, as shown in Figure 7.4.

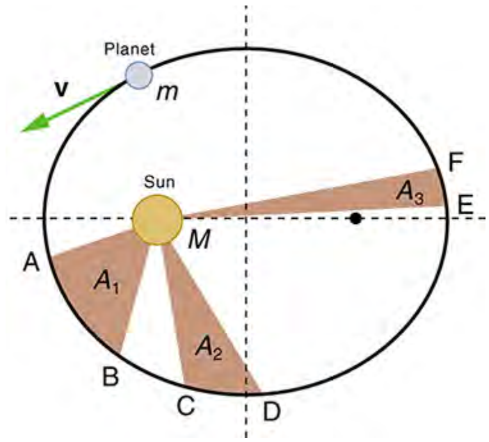


Figure 7.4 The shaded regions have equal areas. The time for *m* to go from A to B is the same as the time to go from C to D and from E to F. The mass *m* moves fastest when it is closest to *M*. Kepler's second law was originally devised for planets orbiting the sun, but it has broader validity.

TIPS FOR SUCCESS

Note that while, for historical reasons, Kepler's laws are stated for planets orbiting the sun, they are actually valid for all bodies satisfying the two previously stated conditions.

Kepler's Third Law

The ratio of the periods squared of any two planets around the sun is equal to the ratio of their average distances from the sun cubed. In equation form, this is

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3},$$

where T is the period (time for one orbit) and r is the average distance (also called orbital radius). This equation is valid only for comparing two small masses orbiting a single large mass. Most importantly, this is only a descriptive equation; it gives no information about the cause of the equality.

LINKS TO PHYSICS

History: Ptolemy vs. Copernicus

Before the discoveries of Kepler, Copernicus, Galileo, Newton, and others, the solar system was thought to revolve around Earth as shown in Figure 7.5 **(a)**. This is called the **Ptolemaic model**, named for the Greek philosopher Ptolemy who lived in the second century AD. The Ptolemaic model is characterized by a list of facts for the motions of planets, with no explanation of cause and effect. There tended to be a different rule for each heavenly body and a general lack of simplicity.

Figure 7.5 **(b)** represents the modern or **Copernican model**. In this model, a small set of rules and a single underlying force explain not only all planetary motion in the solar system, but also all other situations involving gravity. The breadth and simplicity of the laws of physics are compelling.

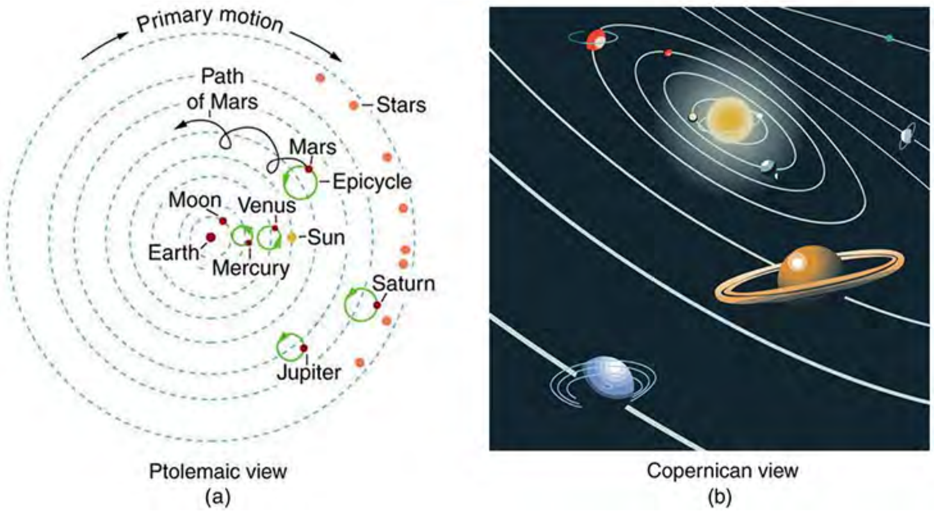


Figure 7.5 (a) The Ptolemaic model of the universe has Earth at the center with the moon, the planets, the sun, and the stars revolving about it in complex circular paths. This geocentric (Earth-centered) model, which can be made progressively more accurate by adding more circles, is purely descriptive, containing no hints about the causes of these motions. (b) The Copernican heliocentric (sun-centered) model is a simpler and more accurate model.

Nicolaus Copernicus (1473–1543) first had the idea that the planets circle the sun, in about 1514. It took him almost 20 years to work out the mathematical details for his model. He waited another 10 years or so to publish his work. It is thought he hesitated because he was afraid people would make fun of his theory. Actually, the reaction of many people was more one of fear and anger. Many people felt the Copernican model threatened their basic belief system. About 100 years later, the astronomer Galileo was put under house arrest for providing evidence that planets, including Earth, orbited the sun. In all, it took almost 300 years for everyone to admit that Copernicus had been right all along.

GRASP CHECK

Explain why Earth does actually appear to be the center of the solar system.

a. Earth appears to be the center of the solar system because Earth is at the center of the universe, and everything revolves around it in a circular orbit.
b. Earth appears to be the center of the solar system because, in the reference frame of Earth, the sun, moon, and planets all appear to move across the sky as if they were circling Earth.
c. Earth appears to be at the center of the solar system because Earth is at the center of the solar system and all the heavenly bodies revolve around it.
d. Earth appears to be at the center of the solar system because Earth is located at one of the foci of the elliptical orbit of the sun, moon, and other planets.

Virtual Physics

Acceleration

This simulation allows you to create your own solar system so that you can see how changing distances and masses determines the orbits of planets. Click *Help* for instructions.

[Click to view content (https://archive.cnx.org/specials/ee816dff-0b5f-4e6f-8250-f9fb9e39d716/my-solar-system/)](https://archive.cnx.org/specials/ee816dff-0b5f-4e6f-8250-f9fb9e39d716/my-solar-system/)

GRASP CHECK

When the central object is off center, how does the speed of the orbiting object vary?

a. The orbiting object moves fastest when it is closest to the central object and slowest when it is farthest away.
b. The orbiting object moves slowest when it is closest to the central object and fastest when it is farthest away.

c. The orbiting object moves with the same speed at every point on the circumference of the elliptical orbit.
d. There is no relationship between the speed of the object and the location of the planet on the circumference of the orbit.

Calculations Related to Kepler's Laws of Planetary Motion

Kepler's First Law

Refer back to Figure 7.2 **(a)**. Notice which distances are constant. The foci are fixed, so distance $\overline{f_1 f_2}$ is a constant. The definition of an ellipse states that the sum of the distances $\overline{f_1 m} + \overline{mf_2}$ is also constant. These two facts taken together mean that the perimeter of triangle $\Delta f_1 mf_2$ must also be constant. Knowledge of these constants will help you determine positions and distances of objects in a system that includes one object orbiting another.

Kepler's Second Law

Refer back to Figure 7.4. The second law says that the segments have equal area and that it takes equal time to sweep through each segment. That is, the time it takes to travel from A to B equals the time it takes to travel from C to D, and so forth. Velocity **v** equals distance *d* divided by time *t*: $\mathbf{v} = d/t$. Then, $t = d/\mathbf{v}$, so distance divided by velocity is also a constant. For example, if we know the average velocity of Earth on June 21 and December 21, we can compare the distance Earth travels on those days.

The degree of elongation of an elliptical orbit is called its **eccentricity** (*e*). Eccentricity is calculated by dividing the distance *f* from the center of an ellipse to one of the foci by half the long axis *a*.

$$(e) = f/a \tag{7.1}$$

When $e = 0$, the ellipse is a circle.

The area of an ellipse is given by $A = \pi ab$, where *b* is half the short axis. If you know the axes of Earth's orbit and the area Earth sweeps out in a given period of time, you can calculate the fraction of the year that has elapsed.

WORKED EXAMPLE

Kepler's First Law

At its closest approach, a moon comes within 200,000 km of the planet it orbits. At that point, the moon is 300,000 km from the other focus of its orbit, f_2. The planet is focus f_1 of the moon's elliptical orbit. How far is the moon from the planet when it is 260,000 km from f_2?

Strategy

Show and label the ellipse that is the orbit in your solution. Picture the triangle f_1mf_2 collapsed along the major axis and add up the lengths of the three sides. Find the length of the unknown side of the triangle when the moon is 260,000 km from f_2.

Solution

Perimeter of $f_1 mf_2 = 200,000 \text{ km} + 100{,}000 \text{ km} + 300{,}000 \text{ km} = 600{,}000 \text{ km}.$

$$mf_1 = 600{,}000 \text{ km} - (100{,}000 \text{ km} + 200{,}000 \text{ km}) = 240{,}000 \text{ km}.$$

Discussion

The perimeter of triangle f_1mf_2 must be constant because the distance between the foci does not change and Kepler's first law says the orbit is an ellipse. For any ellipse, the sum of the two sides of the triangle, which are f_1m and mf_2, is constant.

WORKED EXAMPLE

Kepler's Second Law

Figure 7.6 shows the major and minor axes of an ellipse. The semi-major and semi-minor axes are half of these, respectively.

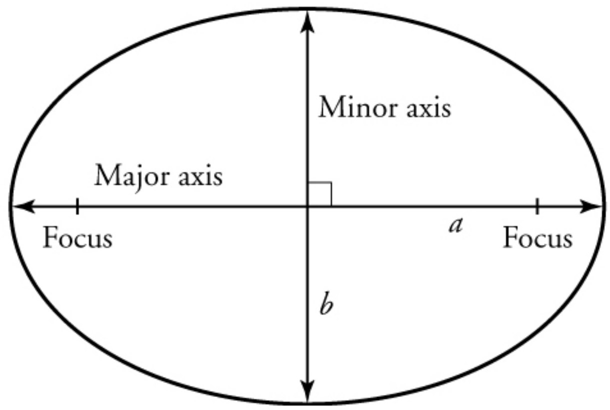


Figure 7.6 The major axis is the length of the ellipse, and the minor axis is the width of the ellipse. The semi-major axis is half the major axis, and the semi-minor axis is half the minor axis.

Earth's orbit is slightly elliptical, with a semi-major axis of 152 million km and a semi-minor axis of 147 million km. If Earth's period is 365.26 days, what area does an Earth-to-sun line sweep past in one day?

Strategy

Each day, Earth sweeps past an equal-sized area, so we divide the total area by the number of days in a year to find the area swept past in one day. For total area use $A = \pi ab$. Calculate A, the area inside Earth's orbit and divide by the number of days in a year (i.e., its period).

Solution

$$\begin{aligned} \text{area per day} &= \frac{\text{total area}}{\text{total number of days}} \\ &= \frac{\pi ab}{365\ \text{d}} \\ &= \frac{\pi\left(1.47\times10^{8}\ \text{km}\right)\left(1.52\times10^{3}\ \text{km}\right)}{365\ \text{d}} \\ &= 1.92\times10^{14}\ \text{km}^2/\text{d} \end{aligned} \tag{7.2}$$

The area swept out in one day is thus $1.92 \times 10^{14}\text{km}^2$.

Discussion

The answer is based on Kepler's law, which states that a line from a planet to the sun sweeps out equal areas in equal times.

Kepler's Third Law

Kepler's third law states that the ratio of the squares of the periods of any two planets (T_1, T_2) is equal to the ratio of the cubes of their average orbital distance from the sun (r_1, r_2). Mathematically, this is represented by

$$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}.$$

From this equation, it follows that the ratio r^3/T^2 is the same for all planets in the solar system. Later we will see how the work of Newton leads to a value for this constant.

WORKED EXAMPLE

Kepler's Third Law

Given that the moon orbits Earth each 27.3 days and that it is an average distance of $3.84 \times 10^8\,\text{m}$ from the center of Earth, calculate the period of an artificial satellite orbiting at an average altitude of 1,500 km above Earth's surface.

Strategy

The period, or time for one orbit, is related to the radius of the orbit by Kepler's third law, given in mathematical form by

$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$. Let us use the subscript 1 for the moon and the subscript 2 for the satellite. We are asked to find T_2. The given information tells us that the orbital radius of the moon is $r_1 = 3.84 \times 10^8\,\text{m}$, and that the period of the moon is $T_1 = 27.3\ \text{days}$. The height of the artificial satellite above Earth's surface is given, so to get the distance r_2 from the center of Earth we must add the height to the radius of Earth (6380 km). This gives $r_2 = 1500\ \text{km} + 6380\ \text{km} = 7880\ \text{km}$. Now all quantities are known, so T_2 can be found.

Solution

To solve for T_2, we cross-multiply and take the square root, yielding

$$T_2^2 = T_1^2\left(\frac{r_2}{r_1}\right)^3;\ T_2 = T_1\left(\frac{r_2}{r_1}\right)^{\frac{3}{2}}$$
$$T_2 = (27.3\ \text{d})\left(\frac{24.0\ \text{h}}{\text{d}}\right)\left(\frac{7880\ \text{km}}{3.84\times10^5\ \text{km}}\right)^{\frac{3}{2}} = 1.93\ \text{h}. \tag{7.3}$$

Discussion

This is a reasonable period for a satellite in a fairly low orbit. It is interesting that any satellite at this altitude will complete one orbit in the same amount of time.

Practice Problems

1. A planet with no axial tilt is located in another solar system. It circles its sun in a very elliptical orbit so that the temperature varies greatly throughout the year. If the year there has 612 days and the inhabitants celebrate the coldest day on day 1 of their calendar, when is the warmest day?
   a. Day 1
   b. Day 153
   c. Day 306
   d. Day 459

2. A geosynchronous Earth satellite is one that has an orbital period of precisely 1 day. Such orbits are useful for communication and weather observation because the satellite remains above the same point on Earth (provided it orbits in the equatorial plane in the same direction as Earth's rotation). The ratio $\frac{r^3}{T^2}$ for the moon is $1.01 \times 10^{18}\ \frac{\text{km}^3}{y^2}$. Calculate the radius of the orbit of such a satellite.
   a. 2.75×10^3 km
   b. 1.96×10^4 km
   c. 1.40×10^5 km
   d. 1.00×10^6 km

Check Your Understanding

3. Are Kepler's laws purely descriptive, or do they contain causal information?
   a. Kepler's laws are purely descriptive.
   b. Kepler's laws are purely causal.
   c. Kepler's laws are descriptive as well as causal.
   d. Kepler's laws are neither descriptive nor causal.

4. True or false—According to Kepler's laws of planetary motion, a satellite increases its speed as it approaches its parent body and decreases its speed as it moves away from the parent body.
   a. True
   b. False

5. Identify the locations of the foci of an elliptical orbit.
   a. One focus is the parent body, and the other is located at the opposite end of the ellipse, at the same distance from the center as the parent body.
   b. One focus is the parent body, and the other is located at the opposite end of the ellipse, at half the distance from the center as the parent body.

c. One focus is the parent body and the other is located outside of the elliptical orbit, on the line on which is the semi-major axis of the ellipse.
d. One focus is on the line containing the semi-major axis of the ellipse, and the other is located anywhere on the elliptical orbit of the satellite.

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain Newton's law of universal gravitation and compare it to Einstein's theory of general relativity
- Perform calculations using Newton's law of universal gravitation

Section Key Terms

Einstein's theory of general relativity	gravitational constant	Newton's universal law of gravitation

Concepts Related to Newton's Law of Universal Gravitation

Sir Isaac Newton was the first scientist to precisely define the gravitational force, and to show that it could explain both falling bodies and astronomical motions. See Figure 7.7. But Newton was not the first to suspect that the same force caused both our weight and the motion of planets. His forerunner, Galileo Galilei, had contended that falling bodies and planetary motions had the same cause. Some of Newton's contemporaries, such as Robert Hooke, Christopher Wren, and Edmund Halley, had also made some progress toward understanding gravitation. But Newton was the first to propose an exact mathematical form and to use that form to show that the motion of heavenly bodies should be conic sections—circles, ellipses, parabolas, and hyperbolas. This theoretical prediction was a major triumph. It had been known for some time that moons, planets, and comets follow such paths, but no one had been able to propose an explanation of the mechanism that caused them to follow these paths and not others.

Figure 7.7 The popular legend that Newton suddenly discovered the law of universal gravitation when an apple fell from a tree and hit him on the head has an element of truth in it. A more probable account is that he was walking through an orchard and wondered why all the apples fell in the same direction with the same acceleration. Great importance is attached to it because Newton's universal law of gravitation and his laws of motion answered very old questions about nature and gave tremendous support to the notion of underlying simplicity and unity in nature. Scientists still expect underlying simplicity to emerge from their ongoing inquiries into nature.

The gravitational force is relatively simple. It is always attractive, and it depends only on the masses involved and the distance between them. Expressed in modern language, **Newton's universal law of gravitation** states that every object in the universe attracts every other object with a force that is directed along a line joining them. The force is directly proportional to the product of their masses and inversely proportional to the square of the distance between them. This attraction is illustrated by Figure 7.8.

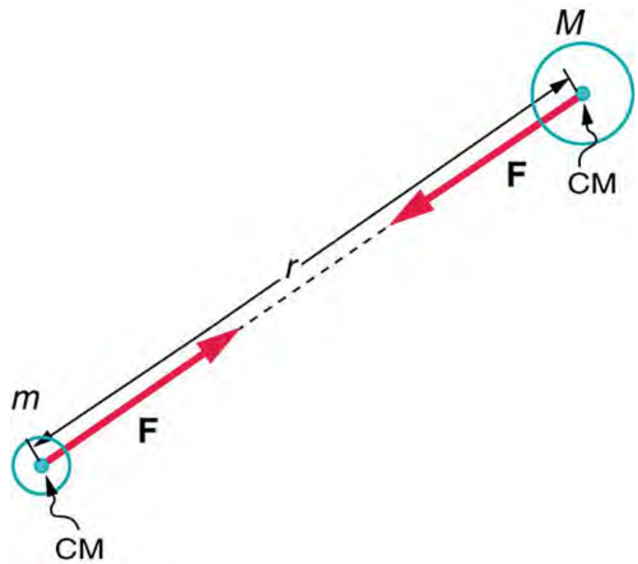


Figure 7.8 Gravitational attraction is along a line joining the centers of mass (CM) of the two bodies. The magnitude of the force on each body is the same, consistent with Newton's third law (action-reaction).

For two bodies having masses m and M with a distance r between their centers of mass, the equation for Newton's universal law of gravitation is

$$\mathbf{F} = G\frac{mM}{r^2}$$

where **F** is the magnitude of the gravitational force and G is a proportionality factor called the **gravitational constant**. G is a universal constant, meaning that it is thought to be the same everywhere in the universe. It has been measured experimentally to be $G = 6.673 \times 10^{-11}\,\mathrm{N \cdot m^2/kg^2}$.

If a person has a mass of 60.0 kg, what would be the force of gravitational attraction on him at Earth's surface? G is given above, Earth's mass M is 5.97×10^{24} kg, and the radius r of Earth is 6.38×10^{6} m. Putting these values into Newton's universal law of gravitation gives

$$\mathbf{F} = G\frac{mM}{r^2} = \left(6.673 \times 10^{-11}\,\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)\left(\frac{(60.0\ \mathrm{kg})\left(5.97 \times 10^{24}\ \mathrm{kg}\right)}{\left(6.38 \times 10^{6}\ \mathrm{m}\right)^2}\right) = 584\ \mathrm{N}$$

We can check this result with the relationship: $\mathbf{F} = m\mathbf{g} = (60\ \mathrm{kg})(9.8\mathrm{m/s^2}) = 588\ \mathrm{N}$

You may remember that **g**, the acceleration due to gravity, is another important constant related to gravity. By substituting **g** for **a** in the equation for Newton's second law of motion we get $\mathbf{F} = m\mathbf{g}$. Combining this with the equation for universal gravitation gives

$$m\mathbf{g} = G\frac{mM}{r^2}$$

Cancelling the mass m on both sides of the equation and filling in the values for the gravitational constant and mass and radius of the Earth, gives the value of g, which may look familiar.

$$\mathbf{g} = G\frac{M}{r^2} = \left(6.67 \times 10^{-11}\,\frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}\right)\left(\frac{5.98 \times 10^{24}\mathrm{kg}}{\left(6.38 \times 10^{6}\mathrm{m}\right)^2}\right) = 9.80\mathrm{m/s^2}$$

This is a good point to recall the difference between mass and weight. Mass is the amount of matter in an object; weight is the force of attraction between the mass within two objects. Weight can change because *g* is different on every moon and planet. An object's mass *m* does not change but its weight $m\mathbf{g}$ can.

Virtual Physics

Gravity and Orbits

Move the sun, Earth, moon and space station in this simulation to see how it affects their gravitational forces and orbital paths. Visualize the sizes and distances between different heavenly bodies. Turn off gravity to see what would happen without it!

[Click to view content (https://archive.cnx.org/specials/a14085c8-96b8-4d04-bb5a-56d9ccbe6e69/gravity-and-orbits/)](https://archive.cnx.org/specials/a14085c8-96b8-4d04-bb5a-56d9ccbe6e69/gravity-and-orbits/)

GRASP CHECK

Why doesn't the Moon travel in a smooth circle around the Sun?

a. The Moon is not affected by the gravitational field of the Sun.
b. The Moon is not affected by the gravitational field of the Earth.
c. The Moon is affected by the gravitational fields of both the Earth and the Sun, which are always additive.
d. The moon is affected by the gravitational fields of both the Earth and the Sun, which are sometimes additive and sometimes opposite.

Snap Lab

Take-Home Experiment: Falling Objects

In this activity you will study the effects of mass and air resistance on the acceleration of falling objects. Make predictions (hypotheses) about the outcome of this experiment. Write them down to compare later with results.

- Four sheets of $8\text{ -}1/2 \times 11$ -inch paper

Procedure

- Take four identical pieces of paper.
  - Crumple one up into a small ball.
  - Leave one uncrumpled.
  - Take the other two and crumple them up together, so that they make a ball of exactly twice the mass of the other crumpled ball.
  - Now compare which ball of paper lands first when dropped simultaneously from the same height.
    1. Compare crumpled one-paper ball with crumpled two-paper ball.
    2. Compare crumpled one-paper ball with uncrumpled paper.

GRASP CHECK

Why do some objects fall faster than others near the surface of the earth if all mass is attracted equally by the force of gravity?

a. Some objects fall faster because of air resistance, which acts in the direction of the motion of the object and exerts more force on objects with less surface area.
b. Some objects fall faster because of air resistance, which acts in the direction opposite the motion of the object and exerts more force on objects with less surface area.
c. Some objects fall faster because of air resistance, which acts in the direction of motion of the object and exerts more force on objects with more surface area.
d. Some objects fall faster because of air resistance, which acts in the direction opposite the motion of the object and exerts more force on objects with more surface area.

It is possible to derive Kepler's third law from Newton's law of universal gravitation. Applying Newton's second law of motion to angular motion gives an expression for centripetal force, which can be equated to the expression for force in the universal gravitation equation. This expression can be manipulated to produce the equation for Kepler's third law. We saw earlier that the expression r^3/T^2 is a constant for satellites orbiting the same massive object. The derivation of Kepler's third law from Newton's law of universal gravitation and Newton's second law of motion yields that constant:

$$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$$

where M is the mass of the central body about which the satellites orbit (for example, the sun in our solar system). The usefulness of this equation will be seen later.

The universal gravitational constant G is determined experimentally. This definition was first done accurately in 1798 by English scientist Henry Cavendish (1731–1810), more than 100 years after Newton published his universal law of gravitation. The measurement of G is very basic and important because it determines the strength of one of the four forces in nature. Cavendish's experiment was very difficult because he measured the tiny gravitational attraction between two ordinary-sized masses (tens of kilograms at most) by using an apparatus like that in Figure 7.9. Remarkably, his value for G differs by less than 1% from the modern value.

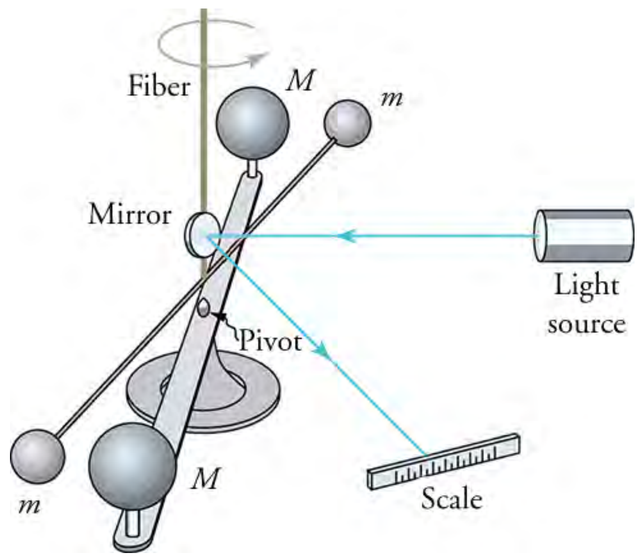


Figure 7.9 Cavendish used an apparatus like this to measure the gravitational attraction between two suspended spheres (*m*) and two spheres on a stand (*M*) by observing the amount of torsion (twisting) created in the fiber. The distance between the masses can be varied to check the dependence of the force on distance. Modern experiments of this type continue to explore gravity.

Einstein's Theory of General Relativity

Einstein's theory of general relativity explained some interesting properties of gravity not covered by Newton's theory. Einstein based his theory on the postulate that acceleration and gravity have the same effect and cannot be distinguished from each other. He concluded that light must fall in both a gravitational field and in an accelerating reference frame. Figure 7.10 shows this effect (greatly exaggerated) in an accelerating elevator. In Figure 7.10**(a)**, the elevator accelerates upward in zero gravity. In Figure 7.10**(b)**, the room is not accelerating but is subject to gravity. The effect on light is the same: it "falls" downward in both situations. The person in the elevator cannot tell whether the elevator is accelerating in zero gravity or is stationary and subject to gravity. Thus, gravity affects the path of light, even though we think of gravity as acting between masses, while photons are massless.

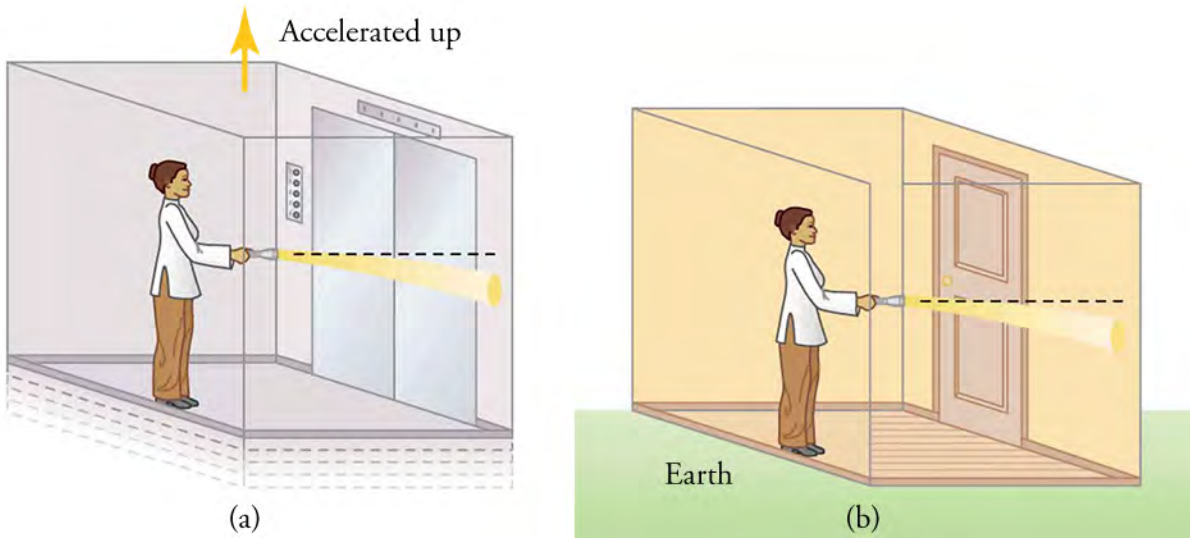


Figure 7.10 (a) A beam of light emerges from a flashlight in an upward-accelerating elevator. Since the elevator moves up during the time the light takes to reach the wall, the beam strikes lower than it would if the elevator were not accelerated. (b) Gravity must have the same effect on light, since it is not possible to tell whether the elevator is accelerating upward or is stationary and acted upon by gravity.

Einstein's theory of general relativity got its first verification in 1919 when starlight passing near the sun was observed during a solar eclipse. (See Figure 7.11.) During an eclipse, the sky is darkened and we can briefly see stars. Those on a line of sight nearest the sun should have a shift in their apparent positions. Not only was this shift observed, but it agreed with Einstein's predictions well within experimental uncertainties. This discovery created a scientific and public sensation. Einstein was now a folk hero as well as a very great scientist. The bending of light by matter is equivalent to a bending of space itself, with light following the curve. This is another radical change in our concept of space and time. It is also another connection that any particle with mass *or* energy (e.g., massless photons) is affected by gravity.

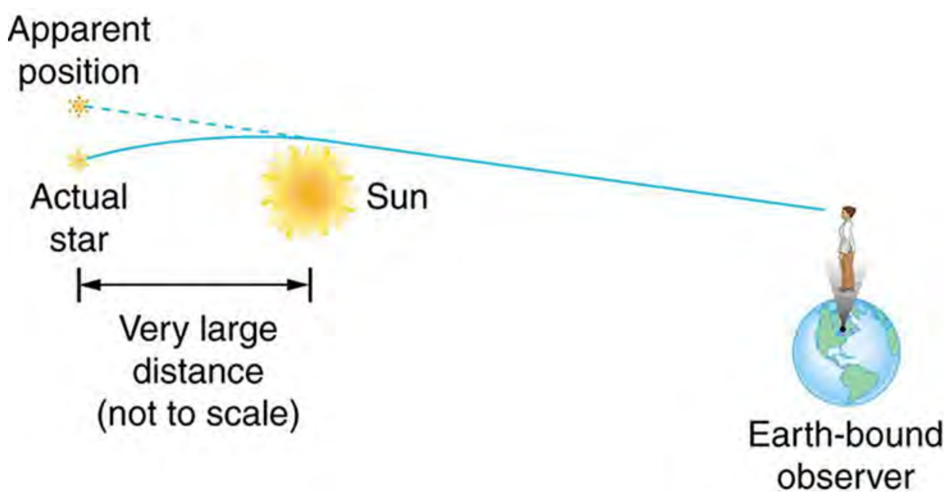


Figure 7.11 This schematic shows how light passing near a massive body like the sun is curved toward it. The light that reaches the Earth then seems to be coming from different locations than the known positions of the originating stars. Not only was this effect observed, but the amount of bending was precisely what Einstein predicted in his general theory of relativity.

To summarize the two views of gravity, Newton envisioned gravity as a tug of war along the line connecting any two objects in the universe. In contrast, Einstein envisioned gravity as a bending of space-time by mass.

BOUNDLESS PHYSICS

NASA gravity probe B

NASA's Gravity Probe B (GP-B) mission has confirmed two key predictions derived from Albert Einstein's general theory of relativity. The probe, shown in Figure 7.12 was launched in 2004. It carried four ultra-precise gyroscopes designed to measure two effects hypothesized by Einstein's theory:

- The geodetic effect, which is the warping of space and time by the gravitational field of a massive body (in this case, Earth)
- The frame-dragging effect, which is the amount by which a spinning object pulls space and time with it as it rotates

Figure 7.12 Artist concept of Gravity Probe B spacecraft in orbit around the Earth. (credit: NASA/MSFC)

Both effects were measured with unprecedented precision. This was done by pointing the gyroscopes at a single star while orbiting Earth in a polar orbit. As predicted by relativity theory, the gyroscopes experienced very small, but measureable, changes in the direction of their spin caused by the pull of Earth's gravity.

The principle investigator suggested imagining Earth spinning in honey. As Earth rotates it drags space and time with it as it would a surrounding sea of honey.

GRASP CHECK

According to the general theory of relativity, a gravitational field bends light. What does this have to do with time and space?

a. Gravity has no effect on the space-time continuum, and gravity only affects the motion of light.
b. The space-time continuum is distorted by gravity, and gravity has no effect on the motion of light.
c. Gravity has no effect on either the space-time continuum or on the motion of light.
d. The space-time continuum is distorted by gravity, and gravity affects the motion of light.

Calculations Based on Newton's Law of Universal Gravitation

TIPS FOR SUCCESS

When performing calculations using the equations in this chapter, use units of kilograms for mass, meters for distances, newtons for force, and seconds for time.

The mass of an object is constant, but its weight varies with the strength of the gravitational field. This means the value of **g** varies from place to place in the universe. The relationship between force, mass, and acceleration from the second law of motion can be written in terms of **g**.

$$\mathbf{F} = m\mathbf{a} = m\mathbf{g}$$

In this case, the force is the weight of the object, which is caused by the gravitational attraction of the planet or moon on which the object is located. We can use this expression to compare weights of an object on different moons and planets.

WATCH PHYSICS

Mass and Weight Clarification

This video shows the mathematical basis of the relationship between mass and weight. The distinction between mass and weight are clearly explained. The mathematical relationship between mass and weight are shown mathematically in terms of the equation for Newton's law of universal gravitation and in terms of his second law of motion.

[Click to view content (https://www.khanacademy.org/embed_video?v=IuBoeDihLUc)](https://www.khanacademy.org/embed_video?v=IuBoeDihLUc)

GRASP CHECK

Would you have the same mass on the moon as you do on Earth? Would you have the same weight?

a. You would weigh more on the moon than on Earth because gravity on the moon is stronger than gravity on Earth.
b. You would weigh less on the moon than on Earth because gravity on the moon is weaker than gravity on Earth.
c. You would weigh less on the moon than on Earth because gravity on the moon is stronger than gravity on Earth.
d. You would weigh more on the moon than on Earth because gravity on the moon is weaker than gravity on Earth.

Two equations involving the gravitational constant, G, are often useful. The first is Newton's equation, $\mathbf{F} = G\frac{mM}{r^2}$. Several of the values in this equation are either constants or easily obtainable. **F** is often the weight of an object on the surface of a large object with mass M, which is usually known. The mass of the smaller object, m, is often known, and G is a universal constant with the same value anywhere in the universe. This equation can be used to solve problems involving an object on or orbiting Earth or other massive celestial object. Sometimes it is helpful to equate the right-hand side of the equation to $m\mathbf{g}$ and cancel the m on both sides.

The equation $\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$ is also useful for problems involving objects in orbit. Note that there is no need to know the mass of the object. Often, we know the radius r or the period T and want to find the other. If these are both known, we can use the equation to calculate the mass of a planet or star.

WATCH PHYSICS

Mass and Weight Clarification

This video demonstrates calculations involving Newton's universal law of gravitation.

[Click to view content (https://www.khanacademy.org/embed_video?v=391txUI76gM)](https://www.khanacademy.org/embed_video?v=391txUI76gM)

GRASP CHECK

Identify the constants g and G.

a. g and G are both the acceleration due to gravity
b. g is acceleration due to gravity on Earth and G is the universal gravitational constant.
c. g is the gravitational constant and G is the acceleration due to gravity on Earth.
d. g and G are both the universal gravitational constant.

WORKED EXAMPLE

Change in g

The value of **g** on the planet Mars is 3.71 m/s^2. If you have a mass of 60.0 kg on Earth, what would be your mass on Mars? What would be your weight on Mars?

Strategy

Weight equals acceleration due to gravity times mass: $\mathbf{W} = m\mathbf{g}$. An object's mass is constant. Call acceleration due to gravity on Mars $\mathbf{g}_M$ and weight on Mars $\mathbf{W}_M$.

Solution

Mass on Mars would be the same, 60 kg.

$$\mathbf{W}_M = m\mathbf{g}_M = (60.0\text{ kg})\left(3.71\text{m/s}^2\right) = 223\text{ N} \tag{7.4}$$

Discussion

The value of **g** on any planet depends on the mass of the planet and the distance from its center. If the material below the surface varies from point to point, the value of **g** will also vary slightly.

WORKED EXAMPLE

Earth's g at the Moon

Find the acceleration due to Earth's gravity at the distance of the moon.

$$\begin{aligned} G &= 6.67 \times 10^{-11}\text{ N}\cdot\text{m}^2/\text{kg}^2 \\ \text{Earth-moon distance} &= 3.84 \times 10^8\text{ m} \\ \text{Earth's mass} &= 5.98 \times 10^{24}\text{ kg} \end{aligned} \tag{7.5}$$

Express the force of gravity in terms of g.

$$\text{F} = \text{W} = ma = mg \tag{7.6}$$

Combine with the equation for universal gravitation.

$$m\mathbf{g} = mG\frac{M}{r^2} \tag{7.7}$$

Solution

Cancel m and substitute.

$$\mathbf{g} = G\frac{M}{r^2} = \left(6.67 \times 10^{-11}\frac{\text{N}\cdot\text{m}^2}{\text{kg}^2}\right)\left(\frac{5.98 \times 10^{24}\text{ kg}}{\left(3.84 \times 10^8\text{ m}\right)^2}\right) = 2.70 \times 10^{-3}\text{m/s}^2 \tag{7.8}$$

Discussion

The value of **g** for the moon is 1.62 m/s^2. Comparing this value to the answer, we see that Earth's gravitational influence on an object on the moon's surface would be insignificant.

Practice Problems

6. What is the mass of a person who weighs 600 N?

a. 6.00 kg
b. 61.2 kg
c. 600 kg
d. 610 kg

7. Calculate Earth's mass given that the acceleration due to gravity at the North Pole is 9.830m/s^2 and the radius of the Earth is 6371 km from pole to center.

a. 5.94×10^{17} kg
b. 5.94×10^{24} kg
c. 9.36×10^{17} kg
d. 9.36×10^{24} kg

Check Your Understanding

8. Some of Newton's predecessors and contemporaries also studied gravity and proposed theories. What important advance did Newton make in the study of gravity that the other scientists had failed to do?

a. He gave an exact mathematical form for the theory.

b. He added a correction term to a previously existing formula.
c. Newton found the value of the universal gravitational constant.
d. Newton showed that gravitational force is always attractive.

9. State the law of universal gravitation in words only.
a. Gravitational force between two objects is directly proportional to the sum of the squares of their masses and inversely proportional to the square of the distance between them.
b. Gravitational force between two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.
c. Gravitational force between two objects is directly proportional to the sum of the squares of their masses and inversely proportional to the distance between them.
d. Gravitational force between two objects is directly proportional to the product of their masses and inversely proportional to the distance between them.

10. Newton's law of universal gravitation explains the paths of what?
a. A charged particle
b. A ball rolling on a plane surface
c. A planet moving around the sun
d. A stone tied to a string and whirled at constant speed in a horizontal circle

KEY TERMS

aphelion closest distance between a planet and the sun (called apoapsis for other celestial bodies)

Copernican model the model of the solar system where the sun is at the center of the solar system and all the planets orbit around it; this is also called the heliocentric model

eccentricity a measure of the separation of the foci of an ellipse

Einstein's theory of general relativity the theory that gravitational force results from the bending of spacetime by an object's mass

gravitational constant the proportionality constant in Newton's law of universal gravitation

Kepler's laws of planetary motion three laws derived by Johannes Kepler that describe the properties of all orbiting satellites

Newton's universal law of gravitation states that gravitational force between two objects is directly proportional to the product of their masses and inversely proportional to the square of the distance between them.

perihelion farthest distance between a planet and the sun (called periapsis for other celestial bodies)

Ptolemaic model the model of the solar system where Earth is at the center of the solar system and the sun and all the planets orbit around it; this is also called the geocentric model

SECTION SUMMARY

7.1 Kepler's Laws of Planetary Motion

- All satellites follow elliptical orbits.
- The line from the satellite to the parent body sweeps out equal areas in equal time.
- The radius cubed divided by the period squared is a constant for all satellites orbiting the same parent body.

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

- Newton's law of universal gravitation provides a mathematical basis for gravitational force and Kepler's laws of planetary motion.
- Einstein's theory of general relativity shows that gravitational fields change the path of light and warp space and time.
- An object's mass is constant, but its weight changes when acceleration due to gravity, **g**, changes.

KEY EQUATIONS

7.1 Kepler's Laws of Planetary Motion

Kepler's third law	$\frac{T_1^2}{T_2^2} = \frac{r_1^3}{r_2^3}$
eccentricity	$e = \frac{f}{a}$
area of an ellipse	$A = \pi ab$
semi-major axis of an ellipse	$a = \left(r_a + r_p\right)/2$
semi-minor axis of an ellipse	$b = \sqrt{r_a r_p}$

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

Newton's second law of motion	$\mathbf{F} = m\mathbf{a} = m\mathbf{g}$
Newton's universal law of gravitation	$\mathbf{F} = G\frac{mM}{r^2}$
acceleration due to gravity	$\mathbf{g} = G\frac{M}{r^2}$
constant for satellites orbiting the same massive object	$\frac{r^3}{T^2} = \frac{GM}{4\pi^2}$

CHAPTER REVIEW

Concept Items

7.1 Kepler's Laws of Planetary Motion

1. A circle is a special case of an ellipse. Explain how a circle is different from other ellipses.
   a. The foci of a circle are at the same point and are located at the center of the circle.
   b. The foci of a circle are at the same point and are

located at the circumference of the circle.
   c. The foci of a circle are at the same point and are located outside of the circle.
   d. The foci of a circle are at the same point and are located anywhere on the diameter, except on its midpoint.

2. Comets have very elongated elliptical orbits with the sun at one focus. Using Kepler's Law, explain why a comet travels much faster near the sun than it does at the other end of the orbit.
   a. Because the satellite sweeps out equal areas in equal times
   b. Because the satellite sweeps out unequal areas in equal times
   c. Because the satellite is at the other focus of the ellipse
   d. Because the square of the period of the satellite is proportional to the cube of its average distance from the sun

3. True or False—A planet-satellite system must be isolated from other massive objects to follow Kepler's laws of planetary motion.
   a. True
   b. False

4. Explain why the string, pins, and pencil method works for drawing an ellipse.
   a. The string, pins, and pencil method works because the length of the two sides of the triangle remains constant as you are drawing the ellipse.
   b. The string, pins, and pencil method works because the area of the triangle remains constant as you are drawing the ellipse.
   c. The string, pins, and pencil method works because the perimeter of the triangle remains constant as you are drawing the ellipse.
   d. The string, pins, and pencil method works because the volume of the triangle remains constant as you are drawing the ellipse.

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

5. Describe the postulate on which Einstein based the theory of general relativity and describe an everyday experience that illustrates this postulate.
   a. Gravity and velocity have the same effect and cannot be distinguished from each other. An acceptable illustration of this is any description of the feeling of constant velocity in a situation where no outside frame of reference is considered.
   b. Gravity and velocity have different effects and can be distinguished from each other. An acceptable illustration of this is any description of the feeling of constant velocity in a situation where no outside frame of reference is considered.
   c. Gravity and acceleration have the same effect and cannot be distinguished from each other. An acceptable illustration of this is any description of the feeling of acceleration in a situation where no outside frame of reference is considered.
   d. Gravity and acceleration have different effects and can be distinguished from each other. An acceptable illustration of this is any description of the feeling of acceleration in a situation where no outside frame of reference is considered.

6. Titan, with a radius of 2.58×10^6 m, is the largest moon of the planet Saturn. If the mass of Titan is 1.35×10^{23} kg, what is the acceleration due to gravity on the surface of this moon?
   a. $1.35\ \text{m/s}^2$
   b. $3.49\ \text{m/s}^2$
   c. $3.49 \times 10^6\ \text{m/s}^2$
   d. $1.35 \times 10^6\ \text{m/s}^2$

7. Saturn's moon Titan has an orbital period of 15.9 days. If Saturn has a mass of 5.68×10^{23} kg, what is the average distance from Titan to the center of Saturn?
   a. 1.22×10^6 m
   b. 4.26×10^7 m
   c. 5.25×10^4 km
   d. 4.26×10^{10} km

8. Explain why doubling the mass of an object doubles its weight, but doubling its distance from the center of Earth reduces its weight fourfold.
   a. The weight is two times the gravitational force between the object and Earth.
   b. The weight is half the gravitational force between the object and Earth.
   c. The weight is equal to the gravitational force between the object and Earth, and the gravitational force is inversely proportional to the distance squared between the object and Earth.
   d. The weight is directly proportional to the square of the gravitational force between the object and Earth.

9. Explain why a star on the other side of the Sun might appear to be in a location that is not its true location.
   a. It can be explained by using the concept of atmospheric refraction.
   b. It can be explained by using the concept of the special theory of relativity.
   c. It can be explained by using the concept of the general theory of relativity.
   d. It can be explained by using the concept of light

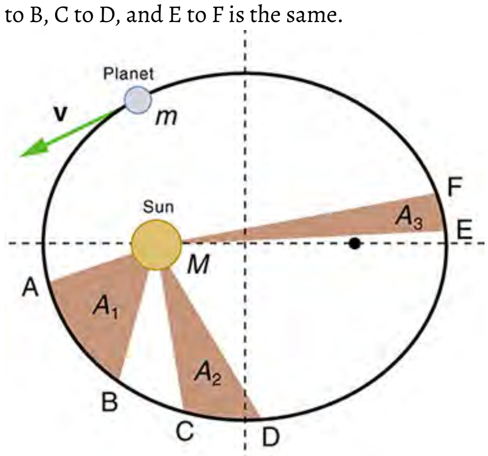
scattering in the atmosphere.

10. The Cavendish experiment marked a milestone in the study of gravity.
Part A. What important value did the experiment determine?
Part B. Why was this so difficult in terms of the masses used in the apparatus and the strength of the gravitational force?
   a. Part A. The experiment measured the acceleration due to gravity, g. Part B. Gravity is a very weak force but despite this limitation, Cavendish was able to measure the attraction between very massive objects.
   b. Part A. The experiment measured the gravitational constant, G. Part B. Gravity is a very weak force but, despite this limitation, Cavendish was able to measure the attraction between very massive objects.
   c. Part A. The experiment measured the acceleration due to gravity, g. Part B. Gravity is a very weak force but despite this limitation, Cavendish was able to measure the attraction between less massive objects.
   d. Part A. The experiment measured the gravitational constant, G. Part B. Gravity is a very weak force but despite this limitation, Cavendish was able to measure the attraction between less massive objects.

Critical Thinking Items

7.1 Kepler's Laws of Planetary Motion

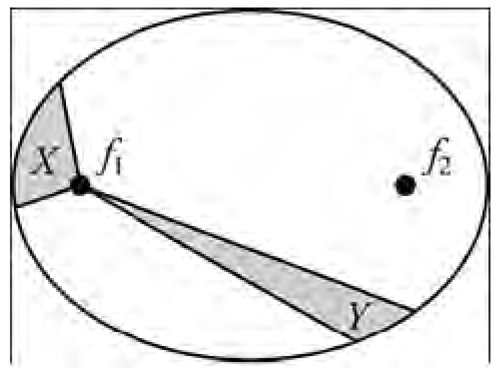
11. In the figure, the time it takes for the planet to go from A to B, C to D, and E to F is the same.



Compare the areas A_1, A_2, and A_3 in terms of size.
   a. $A_1 \neq A_2 \neq A_3$
   b. $A_1 = A_2 = A_3$
   c. $A_1 = A_2 > A_3$
   d. $A_1 > A_2 = A_3$

12. A moon orbits a planet in an elliptical orbit. The foci of the ellipse are 50, 000 km apart. The closest approach of the moon to the planet is 400, 000 km. What is the length of the major axis of the orbit?
   a. 400, 000 km
   b. 450, 000, km
   c. 800, 000 km
   d. 850, 000 km

13. In this figure, if f_1 represents the parent body, which set of statements holds true?



   a. Area X < Area Y; the speed is greater for area X.
   b. Area X > Area Y; the speed is greater for area Y.
   c. Area X = Area Y; the speed is greater for area X.
   d. Area X = Area Y; the speed is greater for area Y.

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

14. Rhea, with a radius of 7.63×10^5 m, is the second-largest moon of the planet Saturn. If the mass of Rhea is 2.31×10^{21} kg, what is the acceleration due to gravity on the surface of this moon?
   a. 2.65×10^{-1} m/s
   b. 2.02×10^5 m/s
   c. 2.65×10^{-1} m/s^2
   d. 2.02×10^5 m/s^2

15. Earth has a mass of 5.971×10^{24} kg and a radius of 6.371×10^6 m. Use the data to check the value of the gravitational constant.
   a. $6.66 \times 10^{-11}\ \frac{\text{N} \cdot \text{m}}{\text{kg}^2}$, it matches the value of the gravitational constant G.
   b. $1.05 \times 10^{-17}\ \frac{\text{N} \cdot \text{m}}{\text{kg}^2}$, it matches the value of the gravitational constant G.
   c. $6.66 \times 10^{-11}\ \frac{\text{N} \cdot \text{m}^2}{\text{kg}^2}$, it matches the value of the gravitational constant G.

d. $1.05 \times 10^{-17}\ \frac{\mathrm{N \cdot m^2}}{\mathrm{kg^2}}$, it matches the value of the gravitational constant G.

16. The orbit of the planet Mercury has a period of 88.0 days and an average radius of 5.791×10^{10} m. What is the mass of the sun?

a. 3.43×10^{19} kg
b. 1.99×10^{30} kg
c. 2.56×10^{29} kg
d. 1.48×10^{40} kg

Problems

7.1 Kepler's Laws of Planetary Motion

17. The closest Earth comes to the sun is 1.47×10^{8} km, and Earth's farthest distance from the sun is 1.52×10^{8} km. What is the area inside Earth's orbit?

a. 2.23×10^{16} km^2
b. 6.79×10^{16} km^2
c. 7.02×10^{16} km^2
d. 7.26×10^{16} km^2

18. Earth is 1.496×10^{8} km from the sun, and Neptune is 4.490×10^{9} km from the sun. What best represents the number of Earth years it takes for Neptune to complete one orbit around the sun?

a. 10 years
b. 30 years
c. 160 years
d. 900 years

Performance Task

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

19. Design an experiment to test whether magnetic force is inversely proportional to the square of distance. Gravitational, magnetic, and electrical fields all act at a distance, but do they all follow the inverse square law? One difference in the forces related to these fields is that gravity is only attractive, but the other two can repel as well. In general, the inverse square law says that force *F* equals a constant *C* divided by the distance between objects, *d*, squared: $F = C/d^2$.
Incorporate these materials into your design:

- Two strong, permanent bar magnets
- A spring scale that can measure small forces
- A short ruler calibrated in millimeters

Use the magnets to study the relationship between attractive force and distance.

a. What will be the independent variable?
b. What will be the dependent variable?
c. How will you measure each of these variables?
d. If you plot the independent variable versus the dependent variable and the inverse square law is upheld, will the plot be a straight line? Explain.
e. Which plot would be a straight line if the inverse square law were upheld?

TEST PREP

Multiple Choice

7.1 Kepler's Laws of Planetary Motion

20. A planet of mass m circles a sun of mass M. Which distance changes throughout the planet's orbit?

a. $\overline{f_1 f_2}$
b. $\overline{mM}$
c. $\overline{Mf_2}$
d. $\overline{Mf_1}$

21. The focal point of the elliptical orbit of a moon is $50,000$ km from the center of the orbit. If the eccentricity of the orbit is 0.25, what is the length of the semi-major axis?

a. $12,500$ km
b. $100,000$ km
c. $200,000$ km
d. $400,000$ km

22. An artificial satellite orbits the Earth at a distance of 1.45×10^{4} km from Earth's center. The moon orbits the Earth at a distance of 3.84×10^{5} km once every 27.3 days. How long does it take the satellite to orbit the Earth?

a. 0.200 days
b. 3.07 days
c. 243 days
d. 3721 days

23. Earth is 1.496×10^{8} km from the sun, and Venus is 1.08×10^{8} km from the sun. One day on Venus is 243 Earth days long. What best represents the number of Venusian days in a Venusian year?

a. 0.78 days
b. 0.92 days
c. 1.08 days
d. 1.21 days

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

24. What did the Cavendish experiment measure?
  a. The mass of Earth
  b. The gravitational constant
  c. Acceleration due to gravity
  d. The eccentricity of Earth's orbit

25. You have a mass of 55 kg and you have just landed on one of the moons of Jupiter where you have a weight of 67.9 N. What is the acceleration due to gravity, g, on the moon you are visiting?
  a. $.810 \text{m/s}^2$
  b. 1.23m/s^2
  c. 539m/s^2
  d. 3735 m/s^2

26. A person is in an elevator that suddenly begins to descend. The person knows, intuitively, that the feeling of suddenly becoming lighter is because the elevator is accelerating downward. What other change would produce the same feeling? How does this demonstrate Einstein's postulate on which he based the theory of general relativity?
  a. It would feel the same if the force of gravity suddenly became weaker. This illustrates Einstein's postulates that gravity and acceleration are indistinguishable.
  b. It would feel the same if the force of gravity suddenly became stronger. This illustrates Einstein's postulates that gravity and acceleration are indistinguishable.
  c. It would feel the same if the force of gravity suddenly became weaker. This illustrates Einstein's postulates that gravity and acceleration are distinguishable.
  d. It would feel the same if the force of gravity suddenly became stronger. This illustrates Einstein's postulates that gravity and acceleration are distinguishable.

Short Answer

7.1 Kepler's Laws of Planetary Motion

27. Explain how the masses of a satellite and its parent body must compare in order to apply Kepler's laws of planetary motion.
  a. The mass of the parent body must be much less than that of the satellite.
  b. The mass of the parent body must be much greater than that of the satellite.
  c. The mass of the parent body must be equal to the mass of the satellite.
  d. There is no specific relationship between the masses for applying Kepler's laws of planetary motion.

28. Hyperion is a moon of the planet Saturn. Its orbit has an eccentricity of 0.123 and a semi-major axis of $1.48 \times 10^6 \text{ km}$. How far is the center of the orbit from the center of Saturn?
  a. $1.82 \times 10^5 \text{ km}$
  b. $3.64 \times 10^5 \text{ km}$
  c. $1.20 \times 10^7 \text{ km}$
  d. $2.41 \times 10^7 \text{ km}$

29. The orbits of satellites are elliptical. Define an ellipse.
  a. An ellipse is an open curve wherein the sum of the distance from the foci to any point on the curve is constant.
  b. An ellipse is a closed curve wherein the sum of the distance from the foci to any point on the curve is constant.
  c. An ellipse is an open curve wherein the distances from the two foci to any point on the curve are equal.
  d. An ellipse is a closed curve wherein the distances from the two foci to any point on the curve are equal.

30. Mars has two moons, Deimos and Phobos. The orbit of Deimos has a period of 1.26 days and an average radius of $2.35 \times 10^3 \text{ km}$. The average radius of the orbit of Phobos is $9.374 \times 10^3 \text{ km}$. According to Kepler's third law of planetary motion, what is the period of Phobos?
  a. 0.16 d
  b. 0.50 d
  c. 3.17 d
  d. 10.0 d

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

31. Newton's third law of motion says that, for every action force, there is a reaction force equal in magnitude but that acts in the opposite direction. Apply this law to gravitational forces acting between the Washington Monument and Earth.
  a. The monument is attracted to Earth with a force equal to its weight, and Earth is attracted to the monument with a force equal to Earth's weight. The situation can be represented with two force vectors of unequal magnitude and pointing in the same direction.

b. The monument is attracted to Earth with a force equal to its weight, and Earth is attracted to the monument with a force equal to Earth's weight. The situation can be represented with two force vectors of unequal magnitude but pointing in opposite directions.
c. The monument is attracted to Earth with a force equal to its weight, and Earth is attracted to the monument with an equal force. The situation can be represented with two force vectors of equal magnitude and pointing in the same direction.
d. The monument is attracted to Earth with a force equal to its weight, and Earth is attracted to the monument with an equal force. The situation can be represented with two force vectors of equal magnitude but pointing in opposite directions.

32. True or false—Gravitational force is the attraction of the mass of one object to the mass of another. Light, either as a particle or a wave, has no rest mass. Despite this fact gravity bends a beam of light.
a. True
b. False

33. The average radius of Earth is 6.37×10^6 m. What is Earth's mass?
a. 9.35×10^{17} kg
b. 5.96×10^{24} kg
c. 3.79×10^{31} kg
d. 2.42×10^{38} kg

34. What is the gravitational force between two 60.0 kg people sitting 100 m apart?
a. 2.4×10^{-11} N
b. 2.4×10^{-9} N
c. 3.6×10^{-1} N
d. 3.6×10^{1} N

Extended Response

7.1 Kepler's Laws of Planetary Motion

35. The orbit of Halley"s Comet has an eccentricity of 0.967 and stretches to the edge of the solar system.
Part A. Describe the shape of the comet's orbit.
Part B. Compare the distance traveled per day when it is near the sun to the distance traveled per day when it is at the edge of the solar system.
Part C. Describe variations in the comet's speed as it completes an orbit. Explain the variations in terms of Kepler's second law of planetary motion.
a. Part A. The orbit is circular, with the sun at the center. Part B. The comet travels much farther when it is near the sun than when it is at the edge of the solar system. Part C. The comet decelerates as it approaches the sun and accelerates as it leaves the sun.
b. Part A. The orbit is circular, with the sun at the center. Part B. The comet travels much farther when it is near the sun than when it is at the edge of the solar system. Part C. The comet accelerates as it approaches the sun and decelerates as it leaves the sun.
c. Part A. The orbit is very elongated, with the sun near one end. Part B. The comet travels much farther when it is near the sun than when it is at the edge of the solar system. Part C. The comet decelerates as it approaches the sun and accelerates as it moves away from the sun.

36. For convenience, astronomers often use astronomical units (AU) to measure distances within the solar system. One AU equals the average distance from Earth to the sun. Halley's Comet returns once every 75.3 years. What is the average radius of the orbit of Halley's Comet in AU?
a. 0.002 AU
b. 0.056 AU
c. 17.8 AU
d. 653 AU

7.2 Newton's Law of Universal Gravitation and Einstein's Theory of General Relativity

37. It took scientists a long time to arrive at the understanding of gravity as explained by Galileo and Newton. They were hindered by two ideas that seemed like common sense but were serious misconceptions. First was the fact that heavier things fall faster than light things. Second, it was believed impossible that forces could act at a distance. Explain why these ideas persisted and why they prevented advances.
a. Heavier things fall faster than light things if they have less surface area and greater mass density. In the Renaissance and before, forces that acted at a distance were considered impossible, so people were skeptical about scientific theories that invoked such forces.
b. Heavier things fall faster than light things because they have greater surface area and less mass density. In the Renaissance and before, forces that act at a distance were considered impossible, so people were skeptical about scientific theories that invoked such forces.
c. Heavier things fall faster than light things because they have less surface area and greater mass density. In the Renaissance and before, forces that

act at a distance were considered impossible, so people were quick to accept scientific theories that invoked such forces.

d. Heavier things fall faster than light things because they have larger surface area and less mass density. In the Renaissance and before, forces that act at a distance were considered impossible because of people's faith in scientific theories.

38. The masses of Earth and the moon are 5.97×10^{24} kg and 7.35×10^{22} kg, respectively. The distance from Earth to the moon is 3.80×10^{5} km. At what point between the Earth and the moon are the opposing gravitational forces equal? (Use subscripts e and m to represent Earth and moon.)

a. 3.42×10^{5} km from the center of Earth
b. 3.80×10^{5} km from the center of Earth
c. 3.42×10^{6} km from the center of Earth
d. 3.10×10^{7} km from the center of Earth

CHAPTER 8
Momentum

Figure 8.1 NFC defensive backs Ronde Barber and Roy Williams along with linebacker Jeremiah Trotter gang tackle AFC running back LaDainian Tomlinson during the 2006 Pro Bowl in Hawaii. (United States Marine Corps)

Chapter Outline

INTRODUCTION We know from everyday use of the word *momentum* that it is a tendency to continue on course in the same direction. Newscasters speak of sports teams or politicians gaining, losing, or maintaining the momentum to win. As we learned when studying about inertia, which is Newton's first law of motion, every object or system has inertia—that is, a tendency for an object in motion to remain in motion or an object at rest to remain at rest. Mass is a useful variable that lets us quantify inertia. Momentum is mass in motion.

Momentum is important because it is conserved in isolated systems; this fact is convenient for solving problems where objects collide. The magnitude of momentum grows with greater mass and/or speed. For example, look at the football players in the photograph (Figure 8.1). They collide and fall to the ground. During their collisions, momentum will play a large part. In this chapter, we will learn about momentum, the different types of collisions, and how to use momentum equations to solve collision problems.

8.1 Linear Momentum, Force, and Impulse

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe momentum, what can change momentum, impulse, and the impulse-momentum theorem
- Describe Newton's second law in terms of momentum
- Solve problems using the impulse-momentum theorem

Section Key Terms

change in momentum	impulse	impulse–momentum theorem	linear momentum

Momentum, Impulse, and the Impulse-Momentum Theorem

Linear momentum is the product of a system's mass and its velocity. In equation form, linear momentum $\mathbf{p}$ is

$$\mathbf{p} = m\mathbf{v}.$$

You can see from the equation that momentum is directly proportional to the object's mass (m) and velocity ($\mathbf{v}$). Therefore, the greater an object's mass or the greater its velocity, the greater its momentum. A large, fast-moving object has greater momentum than a smaller, slower object.

Momentum is a vector and has the same direction as velocity $\mathbf{v}$. Since mass is a scalar, when velocity is in a negative direction (i.e., opposite the direction of motion), the momentum will also be in a negative direction; and when velocity is in a positive direction, momentum will likewise be in a positive direction. The SI unit for momentum is kg m/s.

Momentum is so important for understanding motion that it was called the *quantity of motion* by physicists such as Newton. Force influences momentum, and we can rearrange Newton's second law of motion to show the relationship between force and momentum.

Recall our study of Newton's second law of motion ($\mathbf{F}_{\text{net}} = m\mathbf{a}$). Newton actually stated his second law of motion in terms of momentum: The net external force equals the **change in momentum** of a system divided by the time over which it changes. The change in momentum is the difference between the final and initial values of momentum.

In equation form, this law is

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t},$$

where $\mathbf{F}_{\text{net}}$ is the net external force, $\Delta \mathbf{p}$ is the change in momentum, and Δt is the change in time.

We can solve for $\Delta \mathbf{p}$ by rearranging the equation

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$$

to be

$$\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t.$$

$\mathbf{F}_{\text{net}} \Delta t$ is known as **impulse** and this equation is known as the **impulse-momentum theorem**. From the equation, we see that the impulse equals the average net external force multiplied by the time this force acts. It is equal to the change in momentum. *The effect of a force on an object depends on how long it acts, as well as the strength of the force.* Impulse is a useful concept because it quantifies the effect of a force. A very large force acting for a short time can have a great effect on the momentum of an object, such as the force of a racket hitting a tennis ball. A small force could cause the same change in momentum, but it would have to act for a much longer time.

Newton's Second Law in Terms of Momentum

When Newton's second law is expressed in terms of momentum, it can be used for solving problems where mass varies, since $\Delta \mathbf{p} = \Delta(m\mathbf{v})$. In the more traditional form of the law that you are used to working with, mass is assumed to be constant. In fact, this traditional form is a special case of the law, where mass is constant. $\mathbf{F}_{\text{net}} = m\mathbf{a}$ is actually derived from the equation:

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$$

For the sake of understanding the relationship between Newton's second law in its two forms, let's recreate the derivation of $\mathbf{F}_{\text{net}} = m\mathbf{a}$ from

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$$

by substituting the definitions of acceleration and momentum.

The change in momentum $\Delta \mathbf{p}$ is given by

$$\Delta \mathbf{p} = \Delta(m\mathbf{v}).$$

If the mass of the system is constant, then

$$\Delta(m\mathbf{v}) = m\Delta \mathbf{v}.$$

By substituting $m\Delta \mathbf{v}$ for $\Delta \mathbf{p}$, Newton's second law of motion becomes

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t} = \frac{m\Delta \mathbf{v}}{\Delta t}$$

for a constant mass.

Because

$$\frac{\Delta \mathbf{v}}{\Delta t} = \mathbf{a},$$

we can substitute to get the familiar equation

$$\mathbf{F}_{\text{net}} = m\mathbf{a}$$

when the mass of the system is constant.

TIPS FOR SUCCESS

We just showed how $\mathbf{F}_{\text{net}} = m\mathbf{a}$ applies only when the mass of the system is constant. An example of when this formula would not apply would be a moving rocket that burns enough fuel to significantly change the mass of the rocket. In this case, you would need to use Newton's second law expressed in terms of momentum to account for the changing mass.

Snap Lab

Hand Movement and Impulse

In this activity you will experiment with different types of hand motions to gain an intuitive understanding of the relationship between force, time, and impulse.

- one ball
- one tub filled with water

Procedure:

1. Try catching a ball while *giving* with the ball, pulling your hands toward your body.
2. Next, try catching a ball while keeping your hands still.
3. Hit water in a tub with your full palm. Your full palm represents a swimmer doing a belly flop.
4. After the water has settled, hit the water again by diving your hand with your fingers first into the water. Your diving hand represents a swimmer doing a dive.
5. Explain what happens in each case and why.

GRASP CHECK

What are some other examples of motions that impulse affects?

a. a football player colliding with another, or a car moving at a constant velocity
b. a car moving at a constant velocity, or an object moving in the projectile motion
c. a car moving at a constant velocity, or a racket hitting a ball
d. a football player colliding with another, or a racket hitting a ball

LINKS TO PHYSICS

Engineering: Saving Lives Using the Concept of Impulse

Cars during the past several decades have gotten much safer. Seat belts play a major role in automobile safety by preventing people from flying into the windshield in the event of a crash. Other safety features, such as airbags, are less visible or obvious, but are also effective at making auto crashes less deadly (see Figure 8.2). Many of these safety features make use of the concept of impulse from physics. Recall that impulse is the net force multiplied by the duration of time of the impact. This was expressed mathematically as $\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t$.

Figure 8.2 Vehicles have safety features like airbags and seat belts installed.

Airbags allow the net force on the occupants in the car to act over a much longer time when there is a sudden stop. The momentum change is the same for an occupant whether an airbag is deployed or not. But the force that brings the occupant to a stop will be much less if it acts over a larger time. By rearranging the equation for impulse to solve for force $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$, you can see how increasing Δt while $\Delta \mathbf{p}$ stays the same will decrease $\mathbf{F}_{\text{net}}$. This is another example of an inverse relationship. Similarly, a padded dashboard increases the time over which the force of impact acts, thereby reducing the force of impact.

Cars today have many plastic components. One advantage of plastics is their lighter weight, which results in better gas mileage. Another advantage is that a car will crumple in a collision, especially in the event of a head-on collision. A longer collision time means the force on the occupants of the car will be less. Deaths during car races decreased dramatically when the rigid frames of racing cars were replaced with parts that could crumple or collapse in the event of an accident.

GRASP CHECK

You may have heard the advice to bend your knees when jumping. In this example, a friend dares you to jump off of a park bench onto the ground without bending your knees. You, of course, refuse. Explain to your friend why this would be a foolish thing. Show it using the impulse-momentum theorem.

a. Bending your knees increases the time of the impact, thus decreasing the force.
b. Bending your knees decreases the time of the impact, thus decreasing the force.
c. Bending your knees increases the time of the impact, thus increasing the force.
d. Bending your knees decreases the time of the impact, thus increasing the force.

Solving Problems Using the Impulse-Momentum Theorem

WORKED EXAMPLE

Calculating Momentum: A Football Player and a Football

(a) Calculate the momentum of a 110 kg football player running at 8 m/s. (b) Compare the player's momentum with the momentum of a 0.410 kg football thrown hard at a speed of 25 m/s.

Strategy

No information is given about the direction of the football player or the football, so we can calculate only the magnitude of the momentum, *p*. (A symbol in italics represents magnitude.) In both parts of this example, the magnitude of momentum can be calculated directly from the definition of momentum:

$$\mathbf{p} = m\mathbf{v}$$

Solution for (a)

To find the player's momentum, substitute the known values for the player's mass and speed into the equation.

$$\mathbf{p}_{\text{player}} = (110\ \text{ kg})(8\ \text{ m/s}) = 880\ \text{kg} \cdot \text{m/s}$$

Solution for (b)

To find the ball's momentum, substitute the known values for the ball's mass and speed into the equation.

$$\mathbf{p}_{\text{ball}} = (0.410\ \text{ kg})(25\ \text{ m/s}) = 10.25\ \text{kg} \cdot \text{m/s}$$

The ratio of the player's momentum to the ball's momentum is

$$\frac{\mathbf{p}_{\text{player}}}{\mathbf{p}_{\text{ball}}} = \frac{880}{10.3} = 85.9\ .$$

Discussion

Although the ball has greater velocity, the player has a much greater mass. Therefore, the momentum of the player is about 86 times greater than the momentum of the football.

WORKED EXAMPLE

Calculating Force: Venus Williams' Racquet

During the 2007 French Open, Venus Williams (Figure 8.3) hit the fastest recorded serve in a premier women's match, reaching a speed of 58 m/s (209 km/h). What was the average force exerted on the 0.057 kg tennis ball by Williams' racquet? Assume that the ball's speed just after impact was 58 m/s, the horizontal velocity before impact is negligible, and that the ball remained in contact with the racquet for 5 ms (milliseconds).

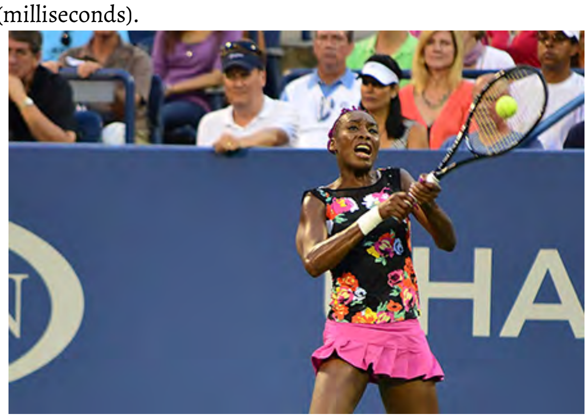


Figure 8.3 Venus Williams playing in the 2013 US Open (Edwin Martinez, Flickr)

Strategy

Recall that Newton's second law stated in terms of momentum is

$$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}.$$

As noted above, when mass is constant, the change in momentum is given by

$$\Delta \mathbf{p} = m\Delta \mathbf{v} = m(\mathbf{v}_{\text{f}} - \mathbf{v}_{\text{i}}),$$

where $\mathbf{v}_{\text{f}}$ is the final velocity and $\mathbf{v}_{\text{i}}$ is the initial velocity. In this example, the velocity just after impact and the change in time are given, so after we solve for $\Delta \mathbf{p}$, we can use $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$ to find the force.

Solution

To determine the change in momentum, substitute the values for mass and the initial and final velocities into the equation above.

$$\begin{aligned} \Delta \mathbf{p} &= m(\mathbf{v}_f - \mathbf{v}_i) \\ &= (0.057 \text{ kg})(58 \text{ m/s} - 0 \text{ m/s}) \\ &= 3.306 \text{ kg·m/s} \approx 3.3 \text{ kg·m/s} \end{aligned} \tag{8.1}$$

Now we can find the magnitude of the net external force using $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$

$$\begin{aligned} \mathbf{F}_{\text{net}} &= \frac{\Delta \mathbf{p}}{\Delta t} = \frac{3.306}{5 \times 10^{-3}} \\ &= 661 \text{ N} \\ &\approx 660 \text{ N.} \end{aligned} \tag{8.2}$$

Discussion

This quantity was the average force exerted by Venus Williams' racquet on the tennis ball during its brief impact. This problem could also be solved by first finding the acceleration and then using $\mathbf{F}_{\text{net}} = m\mathbf{a}$, but we would have had to do one more step. In this case, using momentum was a shortcut.

Practice Problems

1. What is the momentum of a bowling ball with mass 5 kg and velocity 10 m/s?
   a. $0.5 \text{ kg} \cdot \text{m/s}$
   b. $2 \text{ kg} \cdot \text{m/s}$
   c. $15 \text{ kg} \cdot \text{m/s}$
   d. $50 \text{ kg} \cdot \text{m/s}$

2. What will be the change in momentum caused by a net force of 120 N acting on an object for 2 seconds?
   a. $60 \text{ kg} \cdot \text{m/s}$
   b. $118 \text{ kg} \cdot \text{m/s}$
   c. $122 \text{ kg} \cdot \text{m/s}$
   d. $240 \text{ kg} \cdot \text{m/s}$

Check Your Understanding

3. What is linear momentum?
   a. the sum of a system's mass and its velocity
   b. the ratio of a system's mass to its velocity
   c. the product of a system's mass and its velocity
   d. the product of a system's moment of inertia and its velocity

4. If an object's mass is constant, what is its momentum proportional to?
   a. Its velocity
   b. Its weight
   c. Its displacement
   d. Its moment of inertia

5. What is the equation for Newton's second law of motion, in terms of mass, velocity, and time, when the mass of the system is

constant?

a. $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{v}}{\Delta m \Delta t}$
b. $\mathbf{F}_{\text{net}} = \frac{m \Delta t}{\Delta \mathbf{v}}$
c. $\mathbf{F}_{\text{net}} = \frac{m \Delta \mathbf{v}}{\Delta t}$
d. $\mathbf{F}_{\text{net}} = \frac{\Delta m \Delta \mathbf{v}}{\Delta t}$

6. Give an example of a system whose mass is not constant.
a. A spinning top
b. A baseball flying through the air
c. A rocket launched from Earth
d. A block sliding on a frictionless inclined plane

8.2 Conservation of Momentum

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the law of conservation of momentum verbally and mathematically

Section Key Terms

angular momentum	isolated system	law of conservation of momentum

Conservation of Momentum

It is important we realize that momentum is conserved during collisions, explosions, and other events involving objects in motion. To say that a quantity is conserved means that it is constant throughout the event. In the case of conservation of momentum, the total momentum in the system remains the same before and after the collision.

You may have noticed that momentum was *not* conserved in some of the examples previously presented in this chapter. where forces acting on the objects produced large changes in momentum. Why is this? The systems of interest considered in those problems were not inclusive enough. If the systems were expanded to include more objects, then momentum would in fact be conserved in those sample problems. It is always possible to find a larger system where momentum is conserved, even though momentum changes for individual objects within the system.

For example, if a football player runs into the goalpost in the end zone, a force will cause him to bounce backward. His momentum is obviously greatly changed, and considering only the football player, we would find that momentum is not conserved. However, the system can be expanded to contain the entire Earth. Surprisingly, Earth also recoils—conserving momentum—because of the force applied to it through the goalpost. The effect on Earth is not noticeable because it is so much more massive than the player, but the effect is real.

Next, consider what happens if the masses of two colliding objects are more similar than the masses of a football player and Earth—in the example shown in Figure 8.4 of one car bumping into another. Both cars are coasting in the same direction when the lead car, labeled m_2, is bumped by the trailing car, labeled m_1. The only unbalanced force on each car is the force of the collision, assuming that the effects due to friction are negligible. Car *m*1 slows down as a result of the collision, losing some momentum, while car *m*2 speeds up and gains some momentum. If we choose the system to include both cars and assume that friction is negligible, then the momentum of the two-car system should remain constant. Now we will prove that the total momentum of the two-car system does in fact remain constant, and is therefore conserved.

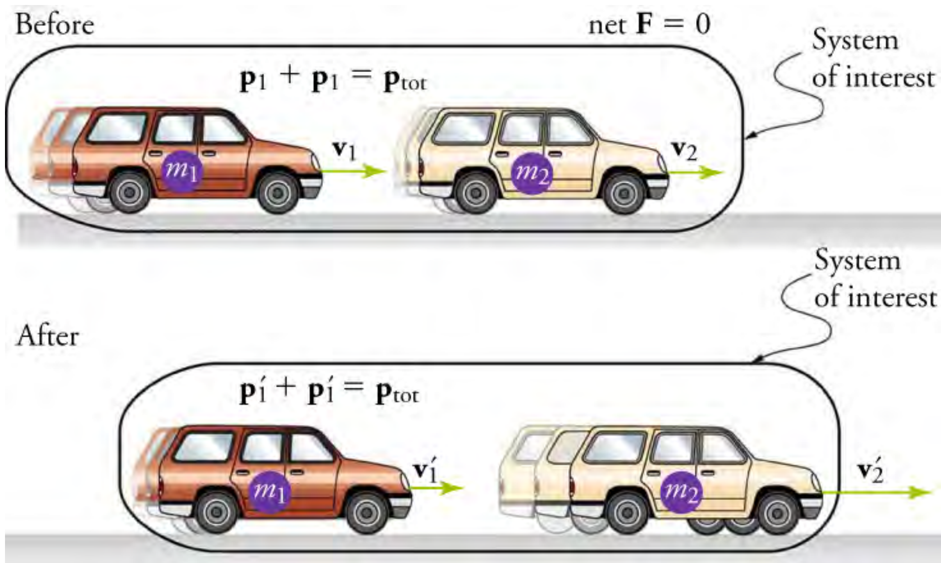


Figure 8.4 Car of mass m_1 moving with a velocity of $\mathbf{v}_1$ bumps into another car of mass m_2 and velocity $\mathbf{v}_2$. As a result, the first car slows down to a velocity of $\mathbf{v'}_1$ and the second speeds up to a velocity of $\mathbf{v'}_2$. The momentum of each car is changed, but the total momentum $\mathbf{p}_{tot}$ of the two cars is the same before and after the collision if you assume friction is negligible.

Using the impulse-momentum theorem, the change in momentum of car 1 is given by

$$\Delta \mathbf{p}_1 = \mathbf{F}_1 \Delta t,$$

where $\mathbf{F}_1$ is the force on car 1 due to car 2, and Δt is the time the force acts, or the duration of the collision.

Similarly, the change in momentum of car 2 is $\Delta \mathbf{p}_2 = \mathbf{F}_2 \Delta t$ where $\mathbf{F}_2$ is the force on car 2 due to car 1, and we assume the duration of the collision Δt is the same for both cars. We know from Newton's third law of motion that $\mathbf{F}_2 = -\mathbf{F}_1$, and so $\Delta \mathbf{p}_2 = -\mathbf{F}_1 \Delta t = -\Delta \mathbf{p}_1$.

Therefore, the changes in momentum are equal and opposite, and $\Delta \mathbf{p}_1 + \Delta \mathbf{p}_2 = 0$.

Because the changes in momentum add to zero, the total momentum of the two-car system is constant. That is,

$$\mathbf{p}_1 + \mathbf{p}_2 = \text{constant}$$

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2,$$

where $\mathbf{p}'_1$ and $\mathbf{p}'_2$ are the momenta of cars 1 and 2 after the collision.

This result that momentum is conserved is true not only for this example involving the two cars, but for any system where the net external force is zero, which is known as an **isolated system**. The **law of conservation of momentum** states that for an isolated system with any number of objects in it, the total momentum is conserved. In equation form, the law of conservation of momentum for an isolated system is written as

$$\mathbf{p}_{tot} = \text{constant}$$

or

$$\mathbf{p}_{tot} = \mathbf{p}'_{tot},$$

where $\mathbf{p}_{tot}$ is the total momentum, or the sum of the momenta of the individual objects in the system at a given time, and $\mathbf{p}'_{tot}$ is the total momentum some time later.

The conservation of momentum principle can be applied to systems as diverse as a comet striking the Earth or a gas containing huge numbers of atoms and molecules. Conservation of momentum appears to be violated only when the net external force is not zero. But another larger system can always be considered in which momentum is conserved by simply including the source of the external force. For example, in the collision of two cars considered above, the two-car system conserves momentum while each one-car system does not.

TIPS FOR SUCCESS

Momenta is the plural form of the word momentum. One object is said to have momentum, but two or more objects are said to have *momenta*.

FUN IN PHYSICS

Angular Momentum in Figure Skating

So far we have covered linear momentum, which describes the inertia of objects traveling in a straight line. But we know that many objects in nature have a curved or circular path. Just as linear motion has linear momentum to describe its tendency to move forward, circular motion has the equivalent **angular momentum** to describe how rotational motion is carried forward.

This is similar to how torque is analogous to force, angular acceleration is analogous to translational acceleration, and mr^2 is analogous to mass or inertia. You may recall learning that the quantity mr^2 is called the rotational inertia or moment of inertia of a point mass m at a distance r from the center of rotation.

We already know the equation for linear momentum, $\mathbf{p} = m\mathbf{v}$. Since angular momentum is analogous to linear momentum, the moment of inertia (I) is analogous to mass, and angular velocity ($\boldsymbol{\omega}$) is analogous to linear velocity, it makes sense that angular momentum ($\mathbf{L}$) is defined as

$$\mathbf{L} = I\boldsymbol{\omega}$$

Angular momentum is conserved when the net external torque ($\boldsymbol{\tau}$) is zero, just as linear momentum is conserved when the net external force is zero.

Figure skaters take advantage of the conservation of angular momentum, likely without even realizing it. In Figure 8.5, a figure skater is executing a spin. The net torque on her is very close to zero, because there is relatively little friction between her skates and the ice, and because the friction is exerted very close to the pivot point. Both $\mathbf{F}$ and r are small, and so $\boldsymbol{\tau}$ is negligibly small.

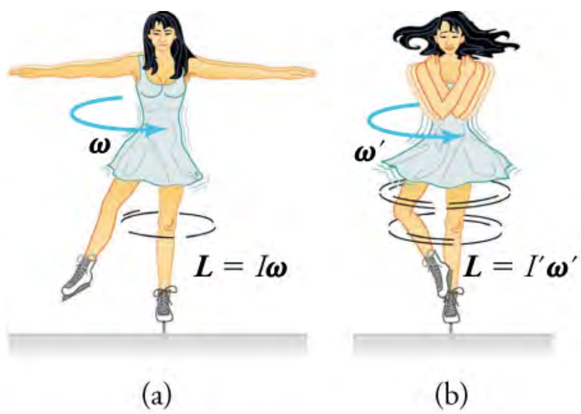


Figure 8.5 (a) An ice skater is spinning on the tip of her skate with her arms extended. In the next image, (b), her rate of spin increases greatly when she pulls in her arms.

Consequently, she can spin for quite some time. She can do something else, too. She can increase her rate of spin by pulling her arms and legs in. Why does pulling her arms and legs in increase her rate of spin? The answer is that her angular momentum is constant, so that $\mathbf{L} = \mathbf{L}'$.

Expressing this equation in terms of the moment of inertia,

$$I\boldsymbol{\omega} = I'\boldsymbol{\omega}',$$

where the primed quantities refer to conditions after she has pulled in her arms and reduced her moment of inertia. Because I' is smaller, the angular velocity $\boldsymbol{\omega}'$ must increase to keep the angular momentum constant. This allows her to spin much faster without exerting any extra torque.

A video (http://openstax.org/l/28figureskater) is also available that shows a real figure skater executing a spin. It discusses the physics of spins in figure skating.

GRASP CHECK

Based on the equation $\mathbf{L} = I\omega$, how would you expect the moment of inertia of an object to affect angular momentum? How would angular velocity affect angular momentum?

a. Large moment of inertia implies large angular momentum, and large angular velocity implies large angular momentum.
b. Large moment of inertia implies small angular momentum, and large angular velocity implies small angular momentum.
c. Large moment of inertia implies large angular momentum, and large angular velocity implies small angular momentum.
d. Large moment of inertia implies small angular momentum, and large angular velocity implies large angular momentum.

Check Your Understanding

7. When is momentum said to be conserved?

a. When momentum is changing during an event
b. When momentum is increasing during an event
c. When momentum is decreasing during an event
d. When momentum is constant throughout an event

8. A ball is hit by a racket and its momentum changes. How is momentum conserved in this case?

a. Momentum of the system can never be conserved in this case.
b. Momentum of the system is conserved if the momentum of the racket is not considered.
c. Momentum of the system is conserved if the momentum of the racket is also considered.
d. Momentum of the system is conserved if the momenta of the racket and the player are also considered.

9. State the law of conservation of momentum.

a. Momentum is conserved for an isolated system with any number of objects in it.
b. Momentum is conserved for an isolated system with an even number of objects in it.
c. Momentum is conserved for an interacting system with any number of objects in it.
d. Momentum is conserved for an interacting system with an even number of objects in it.

8.3 Elastic and Inelastic Collisions

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Distinguish between elastic and inelastic collisions
- Solve collision problems by applying the law of conservation of momentum

Section Key Terms

elastic collision	inelastic collision	point masses	recoil

Elastic and Inelastic Collisions

When objects collide, they can either stick together or bounce off one another, remaining separate. In this section, we'll cover these two different types of collisions, first in one dimension and then in two dimensions.

In an **elastic collision**, the objects separate after impact and don't lose any of their kinetic energy. Kinetic energy is the energy of motion and is covered in detail elsewhere. The law of conservation of momentum is very useful here, and it can be used whenever the net external force on a system is zero. Figure 8.6 shows an elastic collision where momentum is conserved.

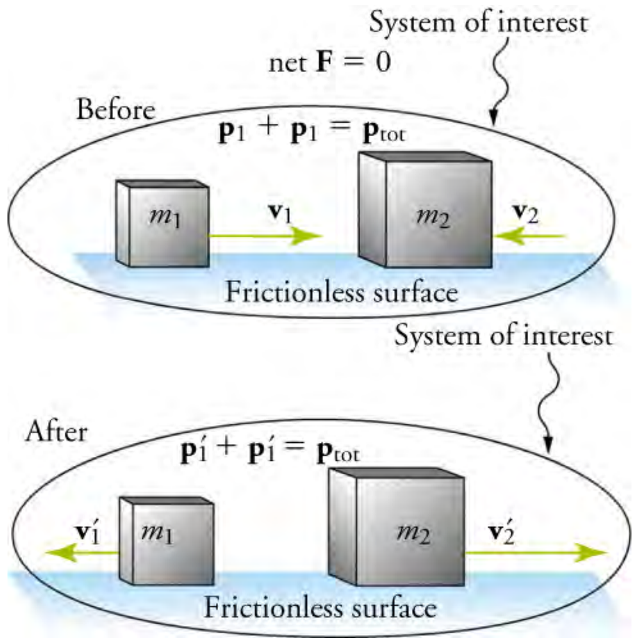


Figure 8.6 The diagram shows a one-dimensional elastic collision between two objects.

An animation of an elastic collision between balls can be seen by watching this video (http://openstax.org/l/28elasticball) . It replicates the elastic collisions between balls of varying masses.

Perfectly elastic collisions can happen only with subatomic particles. Everyday observable examples of perfectly elastic collisions don't exist—some kinetic energy is always lost, as it is converted into heat transfer due to friction. However, collisions between everyday objects are almost perfectly elastic when they occur with objects and surfaces that are nearly frictionless, such as with two steel blocks on ice.

Now, to solve problems involving one-dimensional elastic collisions between two objects, we can use the equation for conservation of momentum. First, the equation for conservation of momentum for two objects in a one-dimensional collision is

$$\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p'}_1 + \mathbf{p'}_2 (\mathbf{F}_{\text{net}} = 0).$$

Substituting the definition of momentum $\mathbf{p} = m\mathbf{v}$ for each initial and final momentum, we get

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = m_1 \mathbf{v'}_1 + m_2 \mathbf{v'}_2,$$

where the primes (') indicate values after the collision; In some texts, you may see *i* for initial (before collision) and *f* for final (after collision). The equation assumes that the mass of each object does not change during the collision.

WATCH PHYSICS

Momentum: Ice Skater Throws a Ball

This video covers an elastic collision problem in which we find the ***recoil*** *velocity* of an ice skater who throws a ball straight forward. To clarify, Sal is using the equation

$$m_{\text{ball}} \mathbf{V}_{\text{ball}} + m_{\text{skater}} \mathbf{V}_{\text{skater}} = m_{\text{ball}} \mathbf{v'}_{\text{ball}} + m_{\text{skater}} \mathbf{v'}_{\text{skater}}.$$

Click to view content (https://www.khanacademy.org/embed_video?v=vPkkCOlGND4)

GRASP CHECK

The resultant vector of the addition of vectors $\vec{a}$ and $\vec{b}$ is $\vec{r}$. The magnitudes of $\vec{a}$, $\vec{b}$, and $\vec{r}$ are A, B, and R, respectively. Which of the following is true?

a. $R_x + R_y = 0$

b. $A_x + A_y = \vec{A}$

c. $A_x + B_y = B_x + A_y$
d. $A_x + B_x = R_x$

Now, let us turn to the second type of collision. An **inelastic collision** is one in which objects stick together after impact, and kinetic energy is *not* conserved. This lack of conservation means that the forces between colliding objects may convert kinetic energy to other forms of energy, such as potential energy or thermal energy. The concepts of energy are discussed more thoroughly elsewhere. For inelastic collisions, kinetic energy may be lost in the form of heat. Figure 8.7 shows an example of an inelastic collision. Two objects that have equal masses head toward each other at equal speeds and then stick together. The two objects come to rest after sticking together, conserving momentum but not kinetic energy after they collide. Some of the energy of motion gets converted to thermal energy, or heat.

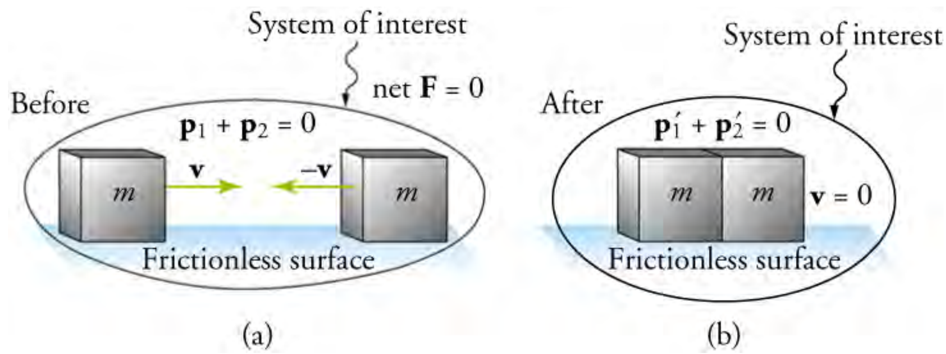


Figure 8.7 A one-dimensional inelastic collision between two objects. Momentum is conserved, but kinetic energy is not conserved. (a) Two objects of equal mass initially head directly toward each other at the same speed. (b) The objects stick together, creating a perfectly inelastic collision. In the case shown in this figure, the combined objects stop; This is not true for all inelastic collisions.

Since the two objects stick together after colliding, they move together at the same speed. This lets us simplify the conservation of momentum equation from

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2$$

to

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = (m_1 + m_2) \mathbf{v}'$$

for inelastic collisions, where **v**' is the final velocity for both objects as they are stuck together, either in motion or at rest.

WATCH PHYSICS

Introduction to Momentum

This video reviews the definitions of momentum and impulse. It also covers an example of using conservation of momentum to solve a problem involving an inelastic collision between a car with constant velocity and a stationary truck. Note that Sal accidentally gives the unit for impulse as Joules; it is actually N · s or k · gm/s.

Click to view content (https://www.khanacademy.org/embed_video?v=XFhntPxowoU)

GRASP CHECK

How would the final velocity of the car-plus-truck system change if the truck had some initial velocity moving in the same direction as the car? What if the truck were moving in the opposite direction of the car initially? Why?

a. If the truck was initially moving in the same direction as the car, the final velocity would be greater. If the truck was initially moving in the opposite direction of the car, the final velocity would be smaller.
b. If the truck was initially moving in the same direction as the car, the final velocity would be smaller. If the truck was initially moving in the opposite direction of the car, the final velocity would be greater.
c. The direction in which the truck was initially moving would not matter. If the truck was initially moving in either

direction, the final velocity would be smaller.
d. The direction in which the truck was initially moving would not matter. If the truck was initially moving in either direction, the final velocity would be greater.

Snap Lab

Ice Cubes and Elastic Collisions

In this activity, you will observe an elastic collision by sliding an ice cube into another ice cube on a smooth surface, so that a negligible amount of energy is converted to heat.

- Several ice cubes (The ice must be in the form of cubes.)
- A smooth surface

Procedure

1. Find a few ice cubes that are about the same size and a smooth kitchen tabletop or a table with a glass top.
2. Place the ice cubes on the surface several centimeters away from each other.
3. Flick one ice cube toward a stationary ice cube and observe the path and velocities of the ice cubes after the collision. Try to avoid edge-on collisions and collisions with rotating ice cubes.
4. Explain the speeds and directions of the ice cubes using momentum.

GRASP CHECK

Was the collision elastic or inelastic?

a. perfectly elastic
b. perfectly inelastic
c. Nearly perfect elastic
d. Nearly perfect inelastic

TIPS FOR SUCCESS

Here's a trick for remembering which collisions are elastic and which are inelastic: Elastic is a bouncy material, so when objects *bounce* off one another in the collision and separate, it is an elastic collision. When they don't, the collision is inelastic.

Solving Collision Problems

The Khan Academy videos referenced in this section show examples of elastic and inelastic collisions in one dimension. In one-dimensional collisions, the incoming and outgoing velocities are all along the same line. But what about collisions, such as those between billiard balls, in which objects scatter to the side? These are two-dimensional collisions, and just as we did with two-dimensional forces, we will solve these problems by first choosing a coordinate system and separating the motion into its x and y components.

One complication with two-dimensional collisions is that the objects might rotate before or after their collision. For example, if two ice skaters hook arms as they pass each other, they will spin in circles. We will not consider such rotation until later, and so for now, we arrange things so that no rotation is possible. To avoid rotation, we consider only the scattering of **point masses**—that is, structureless particles that cannot rotate or spin.

We start by assuming that $\mathbf{F}_{\text{net}} = 0$, so that momentum $\mathbf{p}$ is conserved. The simplest collision is one in which one of the particles is initially at rest. The best choice for a coordinate system is one with an axis parallel to the velocity of the incoming particle, as shown in Figure 8.8. Because momentum is conserved, the components of momentum along the x- and y-axes, displayed as $\mathbf{p}_x$ and $\mathbf{p}_y$, will also be conserved. With the chosen coordinate system, $\mathbf{p}_y$ is initially zero and $\mathbf{p}_x$ is the momentum of the incoming particle.

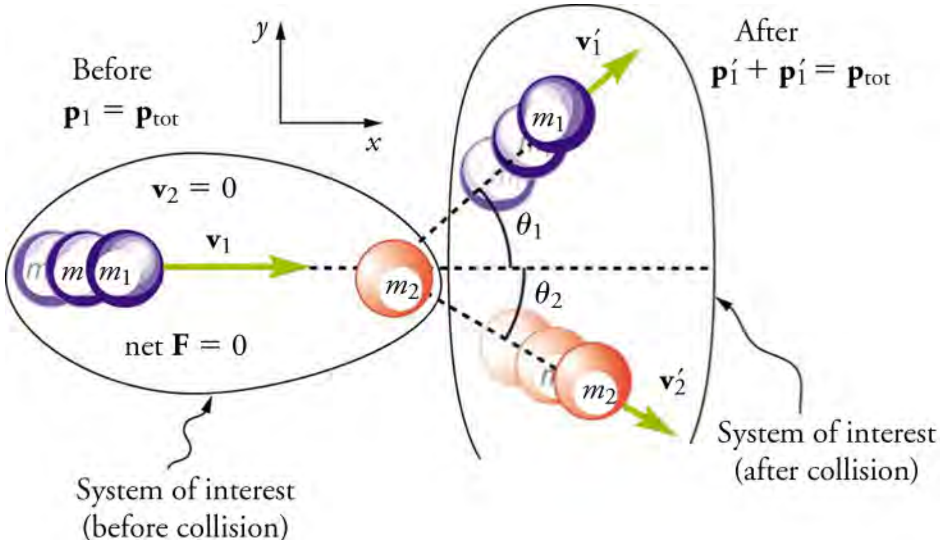


Figure 8.8 A two-dimensional collision with the coordinate system chosen so that m_2 is initially at rest and $\mathbf{v}_1$ is parallel to the x-axis.

Now, we will take the conservation of momentum equation, $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2$ and break it into its x and y components.

Along the x-axis, the equation for conservation of momentum is

$$\mathbf{p}_{1x} + \mathbf{p}_{2x} = \mathbf{p}'_{1x} + \mathbf{p}'_{2x}.$$

In terms of masses and velocities, this equation is

$$m_1 \mathbf{v}_{1\ x} + m_2 \mathbf{v}_{2\ x} = m_1 \mathbf{v}'_{1\ x} + m_2 \mathbf{v}'_{2\ x}. \tag{8.3}$$

But because particle 2 is initially at rest, this equation becomes

$$m_1 \mathbf{v}_{1\ x} = m_1 \mathbf{v}'_{1\ x} + m_2 \mathbf{v}'_{2\ x}. \tag{8.4}$$

The components of the velocities along the x-axis have the form $\mathbf{v} \cos \theta$. Because particle 1 initially moves along the x-axis, we find $\mathbf{v}_{1x} = \mathbf{v}_1$. Conservation of momentum along the x-axis gives the equation

$$m_1 \mathbf{v}_1 = m_1 \mathbf{v}'_1 \cos \theta_1 + m_2 \mathbf{v}'_2 \cos \theta_2,$$

where θ_1 and θ_2 are as shown in Figure 8.8.

Along the y-axis, the equation for conservation of momentum is

$$\mathbf{p}_{1\ y} + \mathbf{p}_{2\ y} = \mathbf{p}'_{1\ y} + \mathbf{p}'_{2\ y}, \tag{8.5}$$

or

$$m_1 \mathbf{v}_{1\ y} + m_2 \mathbf{v}_{2\ y} = m_1 \mathbf{v}'_{1\ y} + m_2 \mathbf{v}'_{2\ y}. \tag{8.6}$$

But $\mathbf{v}_1 y$ is zero, because particle 1 initially moves along the x-axis. Because particle 2 is initially at rest, $\mathbf{v}_2 y$ is also zero. The equation for conservation of momentum along the y-axis becomes

$$0 = m_1 \mathbf{v}'_1 y + m_2 \mathbf{v}'_2 y. \tag{8.7}$$

The components of the velocities along the y-axis have the form $\mathbf{v} \sin \theta$. Therefore, conservation of momentum along the y-axis gives the following equation:

$$0 = m_1 \mathbf{v}'_1 \sin \theta_1 + m_2 \mathbf{v}'_2 \sin \theta_2$$

Virtual Physics

Collision Lab

In this simulation, you will investigate collisions on an air hockey table. Place checkmarks next to the momentum vectors and momenta diagram options. Experiment with changing the masses of the balls and the initial speed of ball 1. How does this affect the momentum of each ball? What about the total momentum? Next, experiment with changing the elasticity of the collision. You will notice that collisions have varying degrees of elasticity, ranging from perfectly elastic to perfectly inelastic.

Click to view content (https://archive.cnx.org/specials/2c7acb3c-2fbd-11e5-b2d9-e7f92291703c/collision-lab/)

GRASP CHECK

If you wanted to maximize the velocity of ball 2 after impact, how would you change the settings for the masses of the balls, the initial speed of ball 1, and the elasticity setting? Why? Hint—Placing a checkmark next to the velocity vectors and removing the momentum vectors will help you visualize the velocity of ball 2, and pressing the More Data button will let you take readings.

a. Maximize the mass of ball 1 and initial speed of ball 1; minimize the mass of ball 2; and set elasticity to 50 percent.
b. Maximize the mass of ball 2 and initial speed of ball 1; minimize the mass of ball 1; and set elasticity to 100 percent.
c. Maximize the mass of ball 1 and initial speed of ball 1; minimize the mass of ball 2; and set elasticity to 100 percent.
d. Maximize the mass of ball 2 and initial speed of ball 1; minimize the mass of ball 1; and set elasticity to 50 percent.

WORKED EXAMPLE

Calculating Velocity: Inelastic Collision of a Puck and a Goalie

Find the recoil velocity of a 70 kg ice hockey goalie who catches a 0.150-kg hockey puck slapped at him at a velocity of 35 m/s. Assume that the goalie is at rest before catching the puck, and friction between the ice and the puck-goalie system is negligible (see Figure 8.9).

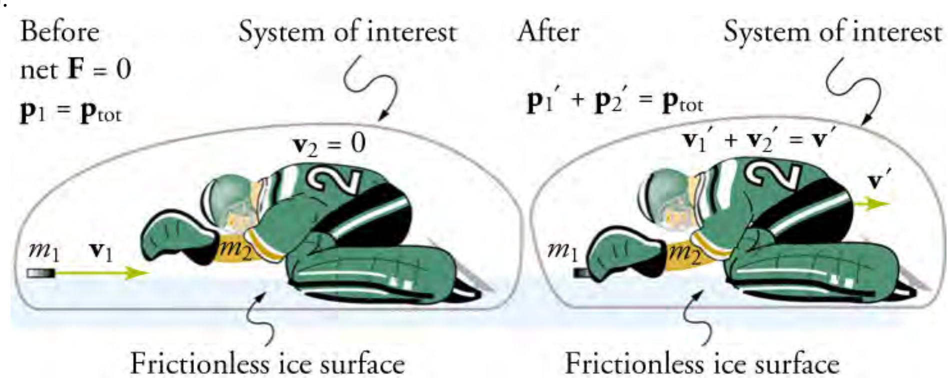


Figure 8.9 An ice hockey goalie catches a hockey puck and recoils backward in an inelastic collision.

Strategy

Momentum is conserved because the net external force on the puck-goalie system is zero. Therefore, we can use conservation of momentum to find the final velocity of the puck and goalie system. Note that the initial velocity of the goalie is zero and that the final velocity of the puck and goalie are the same.

Solution

For an inelastic collision, conservation of momentum is

$$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\mathbf{v}', \tag{8.8}$$

where $\mathbf{v}'$ is the velocity of both the goalie and the puck after impact. Because the goalie is initially at rest, we know $\mathbf{v}_2 = 0$. This simplifies the equation to

$$m_1\mathbf{v}_1 = (m_1 + m_2)\mathbf{v}'. \tag{8.9}$$

Solving for $\mathbf{v}'$ yields

$$\mathbf{v}' = \left(\frac{m_1}{m_1 + m_2}\right)\mathbf{v}_1. \tag{8.10}$$

Entering known values in this equation, we get

$$\begin{aligned} \mathbf{v}' &= \left(\frac{0.150 \text{ kg}}{70.0 \text{ kg}+0.150 \text{ kg}}\right)(35 \text{ m/s}) \\ &= 7.48 \times 10^{-2} \text{m/s}. \end{aligned} \tag{8.11}$$

Discussion

This recoil velocity is small and in the same direction as the puck's original velocity.

WORKED EXAMPLE

Calculating Final Velocity: Elastic Collision of Two Carts

Two hard, steel carts collide head-on and then ricochet off each other in opposite directions on a frictionless surface (see Figure 8.10). Cart 1 has a mass of 0.350 kg and an initial velocity of 2 m/s. Cart 2 has a mass of 0.500 kg and an initial velocity of −0.500 m/s. After the collision, cart 1 recoils with a velocity of −4 m/s. What is the final velocity of cart 2?

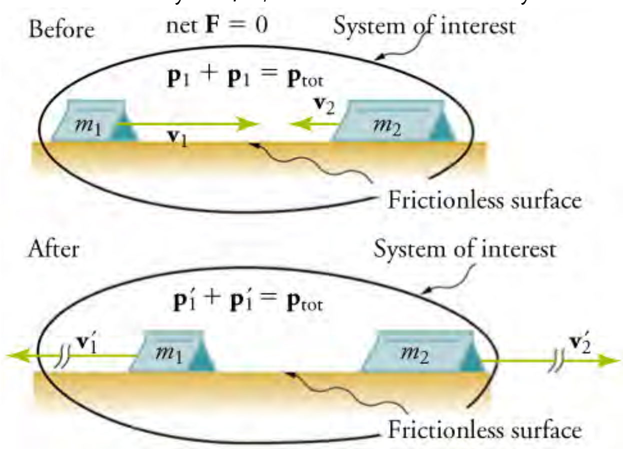


Figure 8.10 Two carts collide with each other in an elastic collision.

Strategy

Since the track is frictionless, $\mathbf{F}_{\text{net}} = 0$ and we can use conservation of momentum to find the final velocity of cart 2.

Solution

As before, the equation for conservation of momentum for a one-dimensional elastic collision in a two-object system is

$$m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 = m_1 \mathbf{v}'_1 + m_2 \mathbf{v}'_2. \tag{8.12}$$

The only unknown in this equation is $\mathbf{v}'_2$. Solving for $\mathbf{v}'_2$ and substituting known values into the previous equation yields

$$\begin{aligned} \mathbf{v}'_2 &= \frac{m_1 \mathbf{v}_1 + m_2 \mathbf{v}_2 - m_1 \mathbf{v}'_1}{m_2} \\ &= \frac{(0.350\text{kg})(2.00\text{m/s}) + (0.500 \text{ kg})(-0.500 \text{ m/s}) - (0.350\text{kg})(-4.00\text{m/s})}{0.500\text{kg}} \\ &= 3.70\text{m/s}. \end{aligned} \tag{8.13}$$

Discussion

The final velocity of cart 2 is large and positive, meaning that it is moving to the right after the collision.

WORKED EXAMPLE

Calculating Final Velocity in a Two-Dimensional Collision

Suppose the following experiment is performed (Figure 8.11). An object of mass 0.250 kg (m_1) is slid on a frictionless surface into a dark room, where it strikes an initially stationary object of mass 0.400 kg (m_2). The 0.250 kg object emerges from the room at an angle of 45° with its incoming direction. The speed of the 0.250 kg object is originally 2 m/s and is 1.50 m/s after the collision. Calculate the magnitude and direction of the velocity (v'_2 and θ_2) of the 0.400 kg object after the collision.

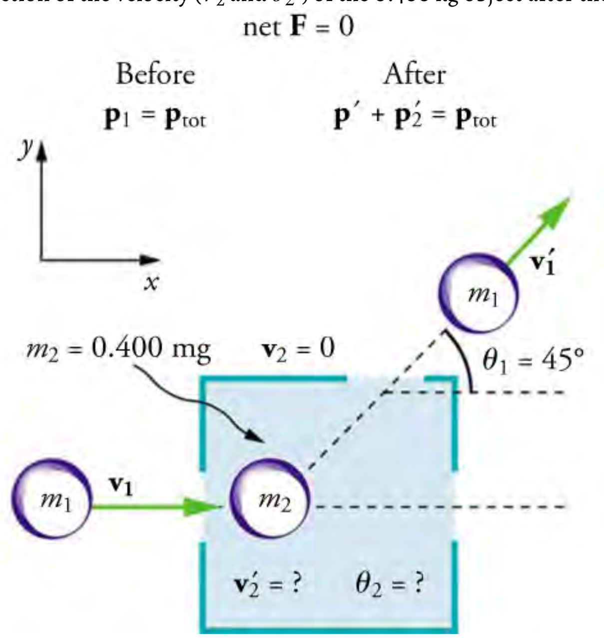


Figure 8.11 The incoming object of mass m_1 is scattered by an initially stationary object. Only the stationary object's mass m_2 is known. By measuring the angle and speed at which the object of mass m_1 emerges from the room, it is possible to calculate the magnitude and direction of the initially stationary object's velocity after the collision.

Strategy

Momentum is conserved because the surface is frictionless. We chose the coordinate system so that the initial velocity is parallel to the *x*-axis, and conservation of momentum along the *x*- and *y*-axes applies.

Everything is known in these equations except $\mathbf{v}'_2$ and θ_2, which we need to find. We can find two unknowns because we have two independent equations—the equations describing the conservation of momentum in the *x* and *y* directions.

Solution

First, we'll solve both conservation of momentum equations ($m_1\mathbf{v}_1 = m_1\mathbf{v}'_1\cos\theta_1 + m_2\mathbf{v}'_2\cos\theta_2$ and $0 = m_1\mathbf{v}'_1\sin\theta_1 + m_2\mathbf{v}'_2\sin\theta_2$) for $\mathbf{v}'_2\sin\theta_2$.

For conservation of momentum along x-axis, let's substitute $\sin\theta_2/\tan\theta_2$ for $\cos\theta_2$ so that terms may cancel out later on. This comes from rearranging the definition of the trigonometric identity $\tan\theta = \sin\theta/\cos\theta$. This gives us

$$m_1\mathbf{v}_1 = m_1\mathbf{v}'_1\cos\theta_1 + m_2\mathbf{v}'_2\frac{\sin\theta_2}{\tan\theta_2}. \tag{8.14}$$

Solving for $\mathbf{v}'_2\sin\theta_2$ yields

$$\mathbf{v}'_2\sin\theta_2 = \frac{(m_1\mathbf{v}_1 - m_1\mathbf{v}'_1\cos\theta_1)(\tan\theta_2)}{m_2}. \tag{8.15}$$

For conservation of momentum along *y*-axis, solving for $\mathbf{v}'_2\sin\theta_2$ yields

$$\mathbf{v}'_2\sin\theta_2 = \frac{-(m_1\mathbf{v}'_1\sin\theta_1)}{m_2}. \tag{8.16}$$

Since both equations equal $\mathbf{v}'_2 \sin\theta_2$, we can set them equal to one another, yielding

$$\frac{(m_1\mathbf{v}_1 - m_1\mathbf{v}'_1 \cos\theta_1)(\tan\theta_2)}{m_2} = \frac{-(m_1\mathbf{v}'_1 \sin\theta_1)}{m_2}. \tag{8.17}$$

Solving this equation for $\tan\theta_2$, we get

$$\tan\theta_2 = \frac{\mathbf{v}'_1 \sin\theta_1}{\mathbf{v}'_1 \cos\theta_1 - \mathbf{v}_1}. \tag{8.18}$$

Entering known values into the previous equation gives

$$\tan\theta_2 = \frac{(1.50)(0.707)}{(1.50)(0.707) - 2.00} = -1.129\ . \tag{8.19}$$

Therefore,

$$\theta_2 = \tan^{-1}(-1.129) = 312^0. \tag{8.20}$$

Since angles are defined as positive in the counterclockwise direction, m_2 is scattered to the right.

We'll use the conservation of momentum along the y-axis equation to solve for $\mathbf{v}'_2$.

$$\mathbf{v}'_2 = -\frac{m_1}{m_2}\mathbf{v}'_1\frac{\sin\theta_1}{\sin\theta_2} \tag{8.21}$$

Entering known values into this equation gives

$$\mathbf{v}'_2 = -\frac{(0.250)}{(0.400)}(1.50)\left(\frac{0.7071}{-0.7485}\right). \tag{8.22}$$

Therefore,

$$\mathbf{v}'_2 = 0.886\ \text{m/s}. \tag{8.23}$$

Discussion

Either equation for the *x*- or *y*-axis could have been used to solve for $\mathbf{v}'_2$, but the equation for the *y*-axis is easier because it has fewer terms.

Practice Problems

10. In an elastic collision, an object with momentum $25\ \text{kg}\cdot\text{m/s}$ collides with another object moving to the right that has a momentum $35\ \text{kg}\cdot\text{m/s}$. After the collision, both objects are still moving to the right, but the first object's momentum changes to $10\ \text{kg}\cdot\text{m/s}$. What is the final momentum of the second object?

a. $10\ \text{kg}\cdot\text{m/s}$
b. $20\ \text{kg}\cdot\text{m/s}$
c. $35\ \text{kg}\cdot\text{m/s}$
d. $50\ \text{kg}\cdot\text{m/s}$

11. In an elastic collision, an object with momentum 25 kg · m/s collides with another that has a momentum 35 kg · m/s. The first object's momentum changes to 10 kg · m/s. What is the final momentum of the second object?

a. 10 kg · m/s
b. 20 kg · m/s
c. 35 kg · m/s
d. 50 kg · m/s

Check Your Understanding

12. What is an elastic collision?

a. An elastic collision is one in which the objects after impact are deformed permanently.
b. An elastic collision is one in which the objects after impact lose some of their internal kinetic energy.

c. An elastic collision is one in which the objects after impact do not lose any of their internal kinetic energy.
d. An elastic collision is one in which the objects after impact become stuck together and move with a common velocity.

13. Are perfectly elastic collisions possible?
a. Perfectly elastic collisions are not possible.
b. Perfectly elastic collisions are possible only with subatomic particles.
c. Perfectly elastic collisions are possible only when the objects stick together after impact.
d. Perfectly elastic collisions are possible if the objects and surfaces are nearly frictionless.

14. What is the equation for conservation of momentum for two objects in a one-dimensional collision?
a. $\mathbf{p}_1 + \mathbf{p}_1' = \mathbf{p}_2 + \mathbf{p}_2'$
b. $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}_1' + \mathbf{p}_2'$
c. $\mathbf{p}_1 - \mathbf{p}_2 = \mathbf{p}_1' - \mathbf{p}_2'$
d. $\mathbf{p}_1 + \mathbf{p}_2 + \mathbf{p}_1' + \mathbf{p}_2' = 0$

KEY TERMS

angular momentum the product of the moment of inertia and angular velocity

change in momentum the difference between the final and initial values of momentum; the mass times the change in velocity

elastic collision collision in which objects separate after impact and kinetic energy is conserved

impulse average net external force multiplied by the time the force acts; equal to the change in momentum

impulse–momentum theorem the impulse, or change in momentum, is the product of the net external force and the time over which the force acts

inelastic collision collision in which objects stick together after impact and kinetic energy is not conserved

isolated system system in which the net external force is zero

law of conservation of momentum when the net external force is zero, the total momentum of the system is conserved or constant

linear momentum the product of a system's mass and velocity

point masses structureless particles that cannot rotate or spin

recoil backward movement of an object caused by the transfer of momentum from another object in a collision

SECTION SUMMARY

8.1 Linear Momentum, Force, and Impulse

- Linear momentum, often referenced as *momentum* for short, is defined as the product of a system's mass multiplied by its velocity, $\mathbf{p} = m\mathbf{v}$.
- The SI unit for momentum is kg m/s.
- Newton's second law of motion in terms of momentum states that the net external force equals the change in momentum of a system divided by the time over which it changes, $\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$.
- Impulse is the average net external force multiplied by the time this force acts, and impulse equals the change in momentum, $\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t$.
- Forces are usually not constant over a period of time, so we use the average of the force over the time it acts.

8.2 Conservation of Momentum

- The law of conservation of momentum is written $\mathbf{p}_{\text{tot}}$ = constant or $\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}}$ (isolated system), where $\mathbf{p}_{\text{tot}}$ is the initial total momentum and $\mathbf{p}'_{\text{tot}}$ is the total momentum some time later.
- In an isolated system, the net external force is zero.
- Conservation of momentum applies only when the net external force is zero, within the defined system.

8.3 Elastic and Inelastic Collisions

- If objects separate after impact, the collision is elastic; If they stick together, the collision is inelastic.
- Kinetic energy is conserved in an elastic collision, but not in an inelastic collision.
- The approach to two-dimensional collisions is to choose a convenient coordinate system and break the motion into components along perpendicular axes. Choose a coordinate system with the x-axis parallel to the velocity of the incoming particle.
- Two-dimensional collisions of point masses, where mass 2 is initially at rest, conserve momentum along the initial direction of mass 1, or the x-axis, and along the direction perpendicular to the initial direction, or the y-axis.
- Point masses are structureless particles that cannot spin.

KEY EQUATIONS

8.1 Linear Momentum, Force, and Impulse

impulse	$\mathbf{F}_{\text{net}} \Delta t$
impulse–momentum theorem	$\Delta \mathbf{p} = \mathbf{F}_{\text{net}} \Delta t$
linear momentum	$\mathbf{p} = m\mathbf{v}$
Newton's second law in terms of momentum	$\mathbf{F}_{\text{net}} = \frac{\Delta \mathbf{p}}{\Delta t}$

8.2 Conservation of Momentum

law of conservation of momentum	$\mathbf{p}_{\text{tot}}$ = constant, or $\mathbf{p}_{\text{tot}} = \mathbf{p}'_{\text{tot}}$

conservation of momentum for two objects	$\mathbf{p}_1 + \mathbf{p}_2 = \text{constant}$, or $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2$
angular momentum	$\mathbf{L} = I\boldsymbol{\omega}$

8.3 Elastic and Inelastic Collisions

conservation of momentum in an elastic collision	$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}'_1 + m_2\mathbf{v}'_2,$
conservation of momentum in an inelastic collision	$m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = (m_1 + m_2)\mathbf{v}'$
conservation of momentum along *x*-axis for 2D collisions	$m_1\mathbf{v}_1 = m_1\mathbf{v}'_1\cos\theta_1 + m_2\mathbf{v}'_2\cos\theta_2$
conservation of momentum along *y*-axis for 2D collisions	$0 = m_1\mathbf{v}'_1\sin\theta_1 + m_2\mathbf{v}'_2\sin\theta_2$

CHAPTER REVIEW

Concept Items

8.1 Linear Momentum, Force, and Impulse

1. What is impulse?
 a. Change in velocity
 b. Change in momentum
 c. Rate of change of velocity
 d. Rate of change of momentum

2. In which equation of Newton's second law is mass assumed to be constant?
 a. $\mathbf{F} = ma$
 b. $\mathbf{F} = \frac{\Delta\mathbf{p}}{\Delta t}$
 c. $\mathbf{F} = \Delta\mathbf{p}\Delta t$
 d. $\mathbf{F} = \frac{\Delta m}{\Delta a}$

3. What is the SI unit of momentum?
 a. N
 b. $\text{kg}\cdot\text{m}$
 c. $\text{kg}\cdot\text{m/s}$
 d. $\text{kg}\cdot\text{m/s}^2$

4. What is the equation for linear momentum?
 a. $\mathbf{p} = m\mathbf{v}$
 b. $\mathbf{p} = m/\mathbf{v}$
 c. $\mathbf{p} = \text{m}\mathbf{v}^2$
 d. $\mathbf{p} = \frac{1}{2}\text{m}\mathbf{v}^2$

8.2 Conservation of Momentum

5. What is angular momentum?
 a. The sum of moment of inertia and angular velocity
 b. The ratio of moment of inertia to angular velocity
 c. The product of moment of inertia and angular velocity
 d. Half the product of moment of inertia and square of angular velocity

6. What is an isolated system?
 a. A system in which the net internal force is zero
 b. A system in which the net external force is zero
 c. A system in which the net internal force is a nonzero constant
 d. A system in which the net external force is a nonzero constant

8.3 Elastic and Inelastic Collisions

7. In the equation $\mathbf{p}_1 + \mathbf{p}_2 = \mathbf{p}'_1 + \mathbf{p}'_2$ for the collision of two objects, what is the assumption made regarding the friction acting on the objects?
 a. Friction is zero.
 b. Friction is nearly zero.
 c. Friction acts constantly.
 d. Friction before and after the impact remains the same.

8. What is an inelastic collision?

a. when objects stick together after impact, and their internal energy is not conserved
b. when objects stick together after impact, and their internal energy is conserved
c. when objects stick together after impact, and always come to rest instantaneously after collision
d. when objects stick together after impact, and their internal energy increases

Critical Thinking Items

8.1 Linear Momentum, Force, and Impulse

9. Consider two objects of the same mass. If a force of 100 N acts on the first for a duration of 1 s and on the other for a duration of 2 s, which of the following statements is true?
a. The first object will acquire more momentum.
b. The second object will acquire more momentum.
c. Both objects will acquire the same momentum.
d. Neither object will experience a change in momentum.

10. Cars these days have parts that can crumple or collapse in the event of an accident. How does this help protect the passengers?
a. It reduces injury to the passengers by increasing the time of impact.
b. It reduces injury to the passengers by decreasing the time of impact.
c. It reduces injury to the passengers by increasing the change in momentum.
d. It reduces injury to the passengers by decreasing the change in momentum.

11. How much force would be needed to cause a 17 kg · m/s change in the momentum of an object, if the force acted for 5 seconds?
a. 3.4 N
b. 12 N
c. 22 N
d. 85 N

8.2 Conservation of Momentum

12. A billiards ball rolling on the table has momentum $\mathbf{p}_1$. It hits another stationary ball, which then starts rolling. Considering friction to be negligible, what will happen to the momentum of the first ball?
a. It will decrease.
b. It will increase.
c. It will become zero.
d. It will remain the same.

13. A ball rolling on the floor with momentum $\mathbf{p}_1$ collides with a stationary ball and sets it in motion. The momentum of the first ball becomes $\mathbf{p}'_1$, and that of the second becomes $\mathbf{p}'_2$. Compare the magnitudes of $\mathbf{p}_1$ and $\mathbf{p}'_2$.
a. Momenta $\mathbf{p}_1$ and $\mathbf{p}'_2$ are the same in magnitude.
b. The sum of the magnitudes of $\mathbf{p}_1$ and $\mathbf{p}'_2$ is zero.
c. The magnitude of $\mathbf{p}_1$ is greater than that of $\mathbf{p}'_2$.
d. The magnitude of $\mathbf{p}'_2$ is greater than that of $\mathbf{p}_1$.

14. Two cars are moving in the same direction. One car with momentum $\mathbf{p}_1$ collides with another, which has momentum $\mathbf{p}_2$. Their momenta become $\mathbf{p}'_1$ and $\mathbf{p}'_2$ respectively. Considering frictional losses, compare $(\mathbf{p}'_1 + \mathbf{p}'_2)$ with $(\mathbf{p}_1 + \mathbf{p}_2)$.
a. The value of $(\mathbf{p}'_1 + \mathbf{p}'_2)$ is zero.
b. The values of $(\mathbf{p}_1 + \mathbf{p}_2)$ and $(\mathbf{p}'_1 + \mathbf{p}'_2)$ are equal.
c. The value of $(\mathbf{p}_1 + \mathbf{p}_2)$ will be greater than $(\mathbf{p}'_1 + \mathbf{p}'_2)$.
d. The value of $(\mathbf{p}'_1 + \mathbf{p}'_2)$ will be greater than $(\mathbf{p}_1 + \mathbf{p}_2)$.

8.3 Elastic and Inelastic Collisions

15. Two people, who have the same mass, throw two different objects at the same velocity. If the first object is heavier than the second, compare the velocities gained by the two people as a result of recoil.
a. The first person will gain more velocity as a result of recoil.
b. The second person will gain more velocity as a result of recoil.
c. Both people will gain the same velocity as a result of recoil.
d. The velocity of both people will be zero as a result of recoil.

Problems

8.1 Linear Momentum, Force, and Impulse

16. If a force of 50 N is applied to an object for 0.2 s, and it changes its velocity by 10 m/s, what could be the mass of the object?
a. 1 kg
b. 2 kg
c. 5 kg
d. 250 kg

17. For how long should a force of 130 N be applied to an object of mass 50 kg to change its speed from 20 m/s to 60 m/s?
a. 0.031 s
b. 0.065 s
c. 15.4 s

d. 40 s

8.3 Elastic and Inelastic Collisions

18. If a man with mass 70 kg, standing still, throws an object with mass 5 kg at 50 m/s, what will be the recoil velocity of the man, assuming he is standing on a frictionless surface?

a. −3.6 m/s
b. 0 m/s
c. 3.6 m/s
d. 50.0 m/s

19. Find the recoil velocity of a $65\ \text{kg}$ ice hockey goalie who catches a $0.15\ \text{kg}$ hockey puck slapped at him at a velocity of $50\ \text{m/s}$. Assume that the goalie is at rest before catching the puck, and friction between the ice and the puck-goalie system is negligible.

a. $-0.12\ \text{m/s}$
b. $0\ \text{m/s}$
c. $0.12\ \text{m/s}$
d. $7.5\ \text{m/s}$

Performance Task

8.3 Elastic and Inelastic Collisions

20. You will need the following:

- balls of different weights
- a ruler or wooden strip
- some books
- a paper cup

Make an inclined plane by resting one end of a ruler on a stack of books. Place a paper cup on the other end. Roll a ball from the top of the ruler so that it hits the paper cup. Measure the displacement of the paper cup due to the collision. Now use increasingly heavier balls for this activity and see how that affects the displacement of the cup. Plot a graph of mass vs. displacement. Now repeat the same activity, but this time, instead of using different balls, change the incline of the ruler by varying the height of the stack of books. This will give you different velocities of the ball. See how this affects the displacement of the paper cup.

TEST PREP

Multiple Choice

8.1 Linear Momentum, Force, and Impulse

21. What kind of quantity is momentum?

a. Scalar
b. Vector

22. When does the net force on an object increase?

a. When $\Delta \mathbf{p}$ decreases
b. When Δt increases
c. When Δt decreases

23. In the equation $\Delta \mathbf{p} = m(\mathbf{v}_f - \mathbf{v}_i)$, which quantity is considered to be constant?

a. Initial velocity
b. Final velocity
c. Mass
d. Momentum

24. For how long should a force of $50\ \text{N}$ be applied to change the momentum of an object by $12\ \text{kg} \cdot \text{m/s}$?

a. $0.24\ \text{s}$
b. $4.15\ \text{s}$
c. $62\ \text{s}$
d. $600\ \text{s}$

8.2 Conservation of Momentum

25. In the equation $\mathbf{L} = I\boldsymbol{\omega}$, what is I?

a. Linear momentum
b. Angular momentum
c. Torque
d. Moment of inertia

26. Give an example of an isolated system.

a. A cyclist moving along a rough road
b. A figure skater gliding in a straight line on an ice rink
c. A baseball player hitting a home run
d. A man drawing water from a well

8.3 Elastic and Inelastic Collisions

27. In which type of collision is kinetic energy conserved?

a. Elastic
b. Inelastic

28. In physics, what are structureless particles that cannot rotate or spin called?

a. Elastic particles
b. Point masses
c. Rigid masses

29. Two objects having equal masses and velocities collide with each other and come to a rest. What type of a collision is this and why?

a. Elastic collision, because internal kinetic energy is conserved

b. Inelastic collision, because internal kinetic energy is not conserved
c. Elastic collision, because internal kinetic energy is not conserved
d. Inelastic collision, because internal kinetic energy is conserved

30. Two objects having equal masses and velocities collide with each other and come to a rest. Is momentum conserved in this case?
a. Yes
b. No

Short Answer

8.1 Linear Momentum, Force, and Impulse

31. If an object's velocity is constant, what is its momentum proportional to?
a. Its shape
b. Its mass
c. Its length
d. Its breadth

32. If both mass and velocity of an object are constant, what can you tell about its impulse?
a. Its impulse would be constant.
b. Its impulse would be zero.
c. Its impulse would be increasing.
d. Its impulse would be decreasing.

33. When the momentum of an object increases with respect to time, what is true of the net force acting on it?
a. It is zero, because the net force is equal to the rate of change of the momentum.
b. It is zero, because the net force is equal to the product of the momentum and the time interval.
c. It is nonzero, because the net force is equal to the rate of change of the momentum.
d. It is nonzero, because the net force is equal to the product of the momentum and the time interval.

34. How can you express impulse in terms of mass and velocity when neither of those are constant?
a. $\Delta\mathbf{p} = \Delta(m\mathbf{v})$
b. $\frac{\Delta\mathbf{p}}{\Delta t} = \frac{\Delta(m\mathbf{v})}{\Delta t}$
c. $\Delta\mathbf{p} = \Delta(\frac{m}{\mathbf{v}})$
d. $\frac{\Delta\mathbf{p}}{\Delta t} = \frac{1}{\Delta t}\cdot\Delta(m\mathbf{v})$

35. How can you express impulse in terms of mass and initial and final velocities?
a. $\Delta\mathbf{p} = m(\mathbf{v}_\mathrm{f} - \mathbf{v}_\mathrm{i})$
b. $\frac{\Delta\mathbf{p}}{\Delta t} = \frac{m(\mathbf{v}_\mathrm{f}-\mathbf{v}_\mathrm{i})}{\Delta t}$
c. $\Delta\mathbf{p} = \frac{(\mathbf{v}_\mathrm{f}-\mathbf{v}_\mathrm{i})}{m}$
d. $\frac{\Delta\mathbf{p}}{\Delta t} = \frac{1}{m}\frac{(\mathbf{v}_\mathrm{f}-\mathbf{v}_\mathrm{i})}{\Delta t}$

36. Why do we use average force while solving momentum problems? How is net force related to the momentum of the object?
a. Forces are usually constant over a period of time, and net force acting on the object is equal to the rate of change of the momentum.
b. Forces are usually not constant over a period of time, and net force acting on the object is equal to the product of the momentum and the time interval.
c. Forces are usually constant over a period of time, and net force acting on the object is equal to the product of the momentum and the time interval.
d. Forces are usually not constant over a period of time, and net force acting on the object is equal to the rate of change of the momentum.

8.2 Conservation of Momentum

37. Under what condition(s) is the angular momentum of a system conserved?
a. When net torque is zero
b. When net torque is not zero
c. When moment of inertia is constant
d. When both moment of inertia and angular momentum are constant

38. If the moment of inertia of an isolated system increases, what happens to its angular velocity?
a. It increases.
b. It decreases.
c. It stays constant.
d. It becomes zero.

39. If both the moment of inertia and the angular velocity of a system increase, what must be true of the force acting on the system?
a. Force is zero.
b. Force is not zero.
c. Force is constant.
d. Force is decreasing.

8.3 Elastic and Inelastic Collisions

40. Two objects collide with each other and come to a rest. How can you use the equation of conservation of momentum to describe this situation?
a. $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = 0$
b. $m_1\mathbf{v}_1 - m_2\mathbf{v}_2 = 0$
c. $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_1'$
d. $m_1\mathbf{v}_1 + m_2\mathbf{v}_2 = m_1\mathbf{v}_2$

41. What is the difference between momentum and impulse?
a. Momentum is the sum of mass and velocity. Impulse is the change in momentum.
b. Momentum is the sum of mass and velocity. Impulse is the rate of change in momentum.
c. Momentum is the product of mass and velocity. Impulse is the change in momentum.
d. Momentum is the product of mass and velocity. Impulse is the rate of change in momentum.

42. What is the equation for conservation of momentum along the *x*-axis for 2D collisions in terms of mass and velocity, where one of the particles is initially at rest?
a. $m_1\mathbf{v}_1 = m_1\mathbf{v}_1'\cos\theta_1$
b. $m_1\mathbf{v}_1 = m_1\mathbf{v}_1'\cos\theta_1 + m_2\mathbf{v}_2'\cos\theta_2$
c. $m_1\mathbf{v}_1 = m_1\mathbf{v}_1'\cos\theta_1 - m_2\mathbf{v}_2'\cos\theta_2$
d. $m_1\mathbf{v}_1 = m_1\mathbf{v}_1'\sin\theta_1 + m_2\mathbf{v}_2'\sin\theta_2$

43. What is the equation for conservation of momentum along the *y*-axis for 2D collisions in terms of mass and velocity, where one of the particles is initially at rest?
a. $0 = m_1\mathbf{v}_1'\sin\theta_1$
b. $0 = m_1\mathbf{v}_1'\sin\theta_1 + m_2\mathbf{v}_2'\sin\theta_2$
c. $0 = m_1\mathbf{v}_1'\sin\theta_1 - m_2\mathbf{v}_2'\sin\theta_2$
d. $0 = m_1\mathbf{v}_1'\cos\theta_1 + m_2\mathbf{v}_2'\cos\theta_2$

Extended Response

8.1 Linear Momentum, Force, and Impulse

44. Can a lighter object have more momentum than a heavier one? How?
a. No, because momentum is independent of the velocity of the object.
b. No, because momentum is independent of the mass of the object.
c. Yes, if the lighter object's velocity is considerably high.
d. Yes, if the lighter object's velocity is considerably low.

45. Why does it hurt less when you fall on a softer surface?
a. The softer surface increases the duration of the impact, thereby reducing the effect of the force.
b. The softer surface decreases the duration of the impact, thereby reducing the effect of the force.
c. The softer surface increases the duration of the impact, thereby increasing the effect of the force.
d. The softer surface decreases the duration of the impact, thereby increasing the effect of the force.

46. Can we use the equation $\mathrm{F}_{\text{net}} = \frac{\Delta p}{\Delta t}$ when the mass is constant?
a. No, because the given equation is applicable for the variable mass only.
b. No, because the given equation is not applicable for the constant mass.
c. Yes, and the resultant equation is $\mathrm{F} = m\mathbf{v}$
d. Yes, and the resultant equation is $\mathrm{F} = ma$

8.2 Conservation of Momentum

47. Why does a figure skater spin faster if he pulls his arms and legs in?
a. Due to an increase in moment of inertia
b. Due to an increase in angular momentum
c. Due to conservation of linear momentum
d. Due to conservation of angular momentum

8.3 Elastic and Inelastic Collisions

48. A driver sees another car approaching him from behind. He fears it is going to collide with his car. Should he speed up or slow down in order to reduce damage?
a. He should speed up.
b. He should slow down.
c. He should speed up and then slow down just before the collision.
d. He should slow down and then speed up just before the collision.

49. What approach would you use to solve problems involving 2D collisions?
a. Break the momenta into components and then choose a coordinate system.
b. Choose a coordinate system and then break the momenta into components.
c. Find the total momenta in the x and y directions, and then equate them to solve for the unknown.
d. Find the sum of the momenta in the x and y directions, and then equate it to zero to solve for the unknown.

CHAPTER 9
Work, Energy, and Simple Machines

Figure 9.1 People on a roller coaster experience thrills caused by changes in types of energy. (Jonrev, Wikimedia Commons)

Chapter Outline

INTRODUCTION Roller coasters have provided thrills for daring riders around the world since the nineteenth century. Inventors of roller coasters used simple physics to build the earliest examples using railroad tracks on mountainsides and old mines. Modern roller coaster designers use the same basic laws of physics to create the latest amusement park favorites. Physics principles are used to engineer the machines that do the work to lift a roller coaster car up its first big incline before it is set loose to roll. Engineers also have to understand the changes in the car's energy that keep it speeding over hills, through twists, turns, and even loops.

What exactly is energy? How can changes in force, energy, and simple machines move objects like roller coaster cars? How can machines help us do work? In this chapter, you will discover the answer to this question and many more, as you learn about work, energy, and simple machines.

9.1 Work, Power, and the Work–Energy Theorem

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe and apply the work–energy theorem
- Describe and calculate work and power

Section Key Terms

energy	gravitational potential energy	joule	kinetic energy	mechanical energy
potential energy	power	watt	work	work–energy theorem

The Work–Energy Theorem

In physics, the term **work** has a very specific definition. Work is application of force, $\mathbf{f}$, to move an object over a distance, *d*, in the direction that the force is applied. Work, *W*, is described by the equation

$$W = \mathbf{f}d.$$

Some things that we typically consider to be work are not work in the scientific sense of the term. Let's consider a few examples. Think about why each of the following statements is true.

- Homework *is not* work.
- Lifting a rock upwards off the ground *is* work.
- Carrying a rock in a straight path across the lawn at a constant speed *is not* work.

The first two examples are fairly simple. Homework is not work because objects are not being moved over a distance. Lifting a rock up off the ground is work because the rock is moving in the direction that force is applied. The last example is less obvious. Recall from the laws of motion that force is *not* required to move an object at constant velocity. Therefore, while some force may be applied to keep the rock up off the ground, no net force is applied to keep the rock moving forward at constant velocity.

Work and **energy** are closely related. When you do work to move an object, you change the object's energy. You (or an object) also expend energy to do work. In fact, energy can be defined as the ability to do work. Energy can take a variety of different forms, and one form of energy can transform to another. In this chapter we will be concerned with **mechanical energy**, which comes in two forms: **kinetic energy** and **potential energy**.

- Kinetic energy is also called energy of motion. A moving object has kinetic energy.
- Potential energy, sometimes called stored energy, comes in several forms. **Gravitational potential energy** is the stored energy an object has as a result of its position above Earth's surface (or another object in space). A roller coaster car at the top of a hill has gravitational potential energy.

Let's examine how doing work on an object changes the object's energy. If we apply force to lift a rock off the ground, we increase the rock's potential energy, *PE*. If we drop the rock, the force of gravity increases the rock's kinetic energy as the rock moves downward until it hits the ground.

The force we exert to lift the rock is equal to its weight, *w*, which is equal to its mass, *m*, multiplied by acceleration due to gravity, $\mathbf{g}$.

$$\mathbf{f} = w = m\mathbf{g}$$

The work we do on the rock equals the force we exert multiplied by the distance, *d*, that we lift the rock. The work we do on the rock also equals the rock's gain in gravitational potential energy, PE_e.

$$W = PE_e = \mathbf{f}m\mathbf{g}$$

Kinetic energy depends on the mass of an object and its velocity, $\mathbf{v}$.

$$KE = \frac{1}{2}m\mathbf{v}^2$$

When we drop the rock the force of gravity causes the rock to fall, giving the rock kinetic energy. When work done on an object increases only its kinetic energy, then the net work equals the change in the value of the quantity $\frac{1}{2}m\mathbf{v}^2$. This is a statement of the **work–energy theorem**, which is expressed mathematically as

$$W = \Delta KE = \frac{1}{2}m\mathbf{v}_2^2 - \frac{1}{2}m\mathbf{v}_1^2.$$

The subscripts $_2$ and $_1$ indicate the final and initial velocity, respectively. This theorem was proposed and successfully tested by James Joule, shown in Figure 9.2.

Does the name Joule sound familiar? The **joule** (J) is the metric unit of measurement for both work and energy. The measurement of work and energy with the same unit reinforces the idea that work and energy are related and can be converted into one another. 1.0 J = 1.0 N•m, the units of force multiplied by distance. 1.0 N = 1.0 k•m/s^2, so 1.0 J = 1.0 k•m^2/s^2. Analyzing the units of the term (1/2)$m\mathbf{v}^2$ will produce the same units for joules.

Figure 9.2 The joule is named after physicist James Joule (1818–1889). (C. H. Jeens, Wikimedia Commons)

WATCH PHYSICS

Work and Energy

This video explains the work energy theorem and discusses how work done on an object increases the object's KE.

Click to view content (https://www.khanacademy.org/embed_video?v=2WS1sG9fhOk)

GRASP CHECK

True or false—The energy increase of an object acted on only by a gravitational force is equal to the product of the object's weight and the distance the object falls.

a. True
b. False

Calculations Involving Work and Power

In applications that involve work, we are often interested in how fast the work is done. For example, in roller coaster design, the amount of time it takes to lift a roller coaster car to the top of the first hill is an important consideration. Taking a half hour on the ascent will surely irritate riders and decrease ticket sales. Let's take a look at how to calculate the time it takes to do work.

Recall that a rate can be used to describe a quantity, such as work, over a period of time. **Power** is the rate at which work is done. In this case, rate means *per unit of time*. Power is calculated by dividing the work done by the time it took to do the work.

$$P = \frac{W}{t}$$

Let's consider an example that can help illustrate the differences among work, force, and power. Suppose the woman in Figure 9.3 lifting the TV with a pulley gets the TV to the fourth floor in two minutes, and the man carrying the TV up the stairs takes five minutes to arrive at the same place. They have done the same amount of work $(\mathbf{f}d)$ on the TV, because they have moved the same mass over the same vertical distance, which requires the same amount of upward force. However, the woman using the pulley has generated more power. This is because she did the work in a shorter amount of time, so the denominator of the power formula, t, is smaller. (For simplicity's sake, we will leave aside for now the fact that the man climbing the stairs has also done work on himself.)

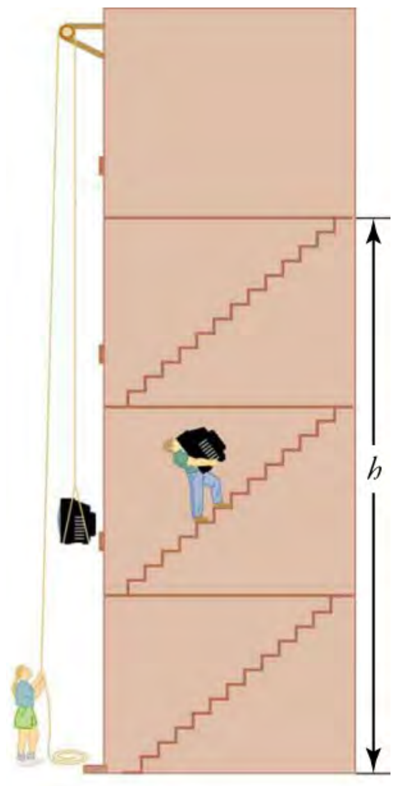


Figure 9.3 No matter how you move a TV to the fourth floor, the amount of work performed and the potential energy gain are the same.

Power can be expressed in units of **watts** (W). This unit can be used to measure power related to any form of energy or work. You have most likely heard the term used in relation to electrical devices, especially light bulbs. Multiplying power by time gives the amount of energy. Electricity is sold in kilowatt-hours because that equals the amount of electrical energy consumed.

The watt unit was named after James Watt (1736–1819) (see Figure 9.4). He was a Scottish engineer and inventor who discovered how to coax more power out of steam engines.

Figure 9.4 Is James Watt thinking about watts? (Carl Frederik von Breda, Wikimedia Commons)

LINKS TO PHYSICS

Watt's Steam Engine

James Watt did not invent the steam engine, but by the time he was finished tinkering with it, it was more useful. The first steam engines were not only inefficient, they only produced a back and forth, or reciprocal, motion. This was natural because pistons move in and out as the pressure in the chamber changes. This limitation was okay for simple tasks like pumping water or mashing potatoes, but did not work so well for moving a train. Watt was able build a steam engine that converted reciprocal motion to circular motion. With that one innovation, the industrial revolution was off and running. The world would never be the same. One of Watt's steam engines is shown in Figure 9.5. The video that follows the figure explains the importance of the steam engine in the industrial revolution.

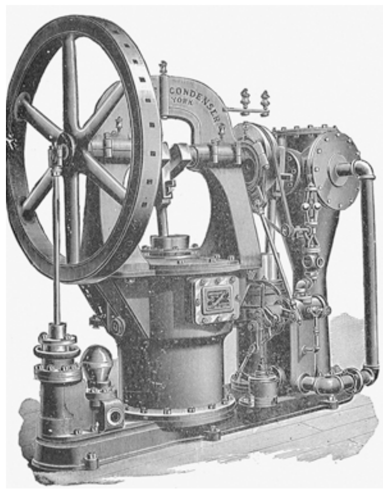


Figure 9.5 A late version of the Watt steam engine. (Nehemiah Hawkins, Wikimedia Commons)

WATCH PHYSICS

Watt's Role in the Industrial Revolution

This video demonstrates how the watts that resulted from Watt's inventions helped make the industrial revolution possible and allowed England to enter a new historical era.

Click to view content (https://www.youtube.com/embed/zhL5DCizj5c)

GRASP CHECK

Which form of mechanical energy does the steam engine generate?

a. Potential energy
b. Kinetic energy
c. Nuclear energy
d. Solar energy

Before proceeding, be sure you understand the distinctions among force, work, energy, and power. Force exerted on an object over a distance does work. Work can increase energy, and energy can do work. Power is the rate at which work is done.

WORKED EXAMPLE

Applying the Work–Energy Theorem

An ice skater with a mass of 50 kg is gliding across the ice at a speed of 8 m/s when her friend comes up from behind and gives her a push, causing her speed to increase to 12 m/s. How much work did the friend do on the skater?

Strategy

The work–energy theorem can be applied to the problem. Write the equation for the theorem and simplify it if possible.

$$W = \Delta\text{KE} = \frac{1}{2}m\mathbf{v}_2^2 - \frac{1}{2}m\mathbf{v}_1^2$$

$$\text{Simplify to } W = \frac{1}{2}m(\mathbf{v}_2^2 - \mathbf{v}_1^2)$$

Solution

Identify the variables. $m = 50$ kg,

$$\mathbf{v}_2 = 12\frac{\text{m}}{\text{s}}, \text{ and } \mathbf{v}_1 = 8\frac{\text{m}}{\text{s}} \tag{9.1}$$

Substitute.

$$W = \frac{1}{2}50(12^2 - 8^2) = 2{,}000 \text{ J} \tag{9.2}$$

Discussion

Work done on an object or system increases its energy. In this case, the increase is to the skater's kinetic energy. It follows that the increase in energy must be the difference in KE before and after the push.

TIPS FOR SUCCESS

This problem illustrates a general technique for approaching problems that require you to apply formulas: Identify the unknown and the known variables, express the unknown variables in terms of the known variables, and then enter all the known values.

Practice Problems

1. How much work is done when a weightlifter lifts a 200 N barbell from the floor to a height of 2 m?
   a. 0 J
   b. 100 J
   c. 200 J
   d. 400 J

2. Identify which of the following actions generates more power. Show your work.
   - carrying a 100 N TV to the second floor in 50 s or
   - carrying a 24 N watermelon to the second floor in 10 s?

   a. Carrying a 100 N TV generates more power than carrying a 24 N watermelon to the same height because power is defined as work done times the time interval.
   b. Carrying a 100 N TV generates more power than carrying a 24 N watermelon to the same height because power is defined as the ratio of work done to the time interval.
   c. Carrying a 24 N watermelon generates more power than carrying a 100 N TV to the same height because power is defined as work done times the time interval.
   d. Carrying a 24 N watermelon generates more power than carrying a 100 N TV to the same height because power is defined as the ratio of work done and the time interval.

Check Your Understanding

3. Identify two properties that are expressed in units of joules.
   a. work and force
   b. energy and weight
   c. work and energy
   d. weight and force

4. When a coconut falls from a tree, work W is done on it as it falls to the beach. This work is described by the equation

$$W = Fd = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2. \tag{9.3}$$

Identify the quantities F, d, m, v_1, and v_2 in this event.

a. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the earth, v_1 is the initial velocity, and v_2 is the velocity with which it hits the beach.
b. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the coconut, v_1 is the initial velocity, and v_2 is the velocity with which it hits the beach.
c. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the earth, v_1 is the velocity with which it hits the beach, and v_2 is the initial velocity.
d. F is the force of gravity, which is equal to the weight of the coconut, d is the distance the nut falls, m is the mass of the coconut, v_1 is the velocity with which it hits the beach, and v_2 is the initial velocity.

9.2 Mechanical Energy and Conservation of Energy

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the law of conservation of energy in terms of kinetic and potential energy
- Perform calculations related to kinetic and potential energy. Apply the law of conservation of energy

Section Key Terms

law of conservation of energy

Mechanical Energy and Conservation of Energy

We saw earlier that mechanical energy can be either potential or kinetic. In this section we will see how energy is transformed from one of these forms to the other. We will also see that, in a closed system, the sum of these forms of energy remains constant.

Quite a bit of potential energy is gained by a roller coaster car and its passengers when they are raised to the top of the first hill. Remember that the *potential* part of the term means that energy has been stored and can be used at another time. You will see that this stored energy can either be used to do work or can be transformed into kinetic energy. For example, when an object that has gravitational potential energy falls, its energy is converted to kinetic energy. Remember that both work and energy are expressed in joules.

Refer back to . The amount of work required to raise the TV from point A to point B is equal to the amount of gravitational potential energy the TV gains from its height above the ground. This is generally true for any object raised above the ground. If all the work done on an object is used to raise the object above the ground, the amount work equals the object's gain in gravitational potential energy. However, note that because of the work done by friction, these energy–work transformations are never perfect. Friction causes the loss of some useful energy. In the discussions to follow, we will use the approximation that transformations are frictionless.

Now, let's look at the roller coaster in Figure 9.6. Work was done on the roller coaster to get it to the top of the first rise; at this point, the roller coaster has gravitational potential energy. It is moving slowly, so it also has a small amount of kinetic energy. As the car descends the first slope, its PE is converted to KE. At the low point much of the original PE has been transformed to KE, and speed is at a maximum. As the car moves up the next slope, some of the KE is transformed back into PE and the car slows down.

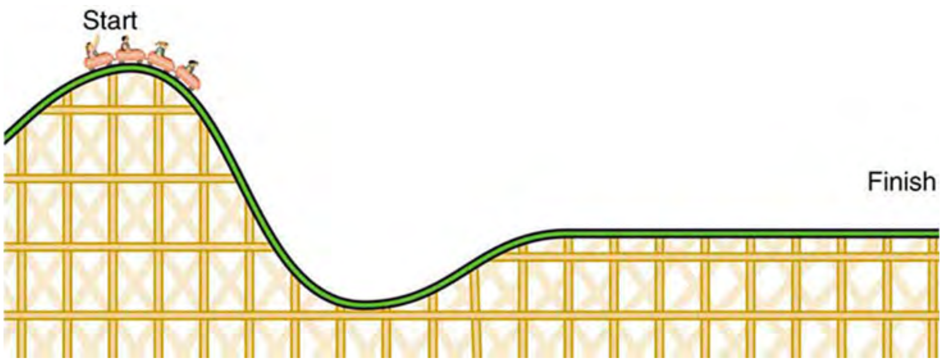


Figure 9.6 During this roller coaster ride, there are conversions between potential and kinetic energy.

Virtual Physics

Energy Skate Park Basics

This simulation shows how kinetic and potential energy are related, in a scenario similar to the roller coaster. Observe the changes in *KE* and *PE* by clicking on the bar graph boxes. Also try the three differently shaped skate parks. Drag the skater to the track to start the animation.

Click to view content (http://phet.colorado.edu/sims/html/energy-skate-park-basics/latest/energy-skate-park-basics_en.html)

GRASP CHECK

This simulation (http://phet.colorado.edu/en/simulation/energy-skate-park-basics (http://phet.colorado.edu/en/simulation/energy-skate-park-basics)) shows how kinetic and potential energy are related, in a scenario similar to the roller coaster. Observe the changes in KE and PE by clicking on the bar graph boxes. Also try the three differently shaped skate parks. Drag the skater to the track to start the animation. The bar graphs show how KE and PE are transformed back and forth. Which statement best explains what happens to the mechanical energy of the system as speed is increasing?

a. The mechanical energy of the system increases, provided there is no loss of energy due to friction. The energy would transform to kinetic energy when the speed is increasing.
b. The mechanical energy of the system remains constant provided there is no loss of energy due to friction. The energy would transform to kinetic energy when the speed is increasing.
c. The mechanical energy of the system increases provided there is no loss of energy due to friction. The energy would transform to potential energy when the speed is increasing.
d. The mechanical energy of the system remains constant provided there is no loss of energy due to friction. The energy would transform to potential energy when the speed is increasing.

On an actual roller coaster, there are many ups and downs, and each of these is accompanied by transitions between kinetic and potential energy. Assume that no energy is lost to friction. At any point in the ride, the total mechanical energy is the same, and it is equal to the energy the car had at the top of the first rise. This is a result of the **law of conservation of energy**, which says that, in a closed system, total energy is conserved—that is, it is constant. Using subscripts 1 and 2 to represent initial and final energy, this law is expressed as

$$KE_1 + PE_1 = KE_2 + PE_2.$$

Either side equals the total mechanical energy. The phrase *in a closed system* means we are assuming no energy is lost to the surroundings due to friction and air resistance. If we are making calculations on dense falling objects, this is a good assumption. For the roller coaster, this assumption introduces some inaccuracy to the calculation.

Calculations involving Mechanical Energy and Conservation of Energy

TIPS FOR SUCCESS

When calculating work or energy, use units of meters for distance, newtons for force, kilograms for mass, and seconds for time. This will assure that the result is expressed in joules.

WATCH PHYSICS

Conservation of Energy

This video discusses conversion of *PE* to *KE* and conservation of energy. The scenario is very similar to the roller coaster and the skate park. It is also a good explanation of the energy changes studied in the snap lab.

Click to view content (https://www.khanacademy.org/embed_video?v=kw_4Loo1HR4)

GRASP CHECK

Did you expect the speed at the bottom of the slope to be the same as when the object fell straight down? Which statement best explains why this is not exactly the case in real-life situations?

a. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is large amount of friction. In real life, much of the mechanical energy is lost as heat caused by friction.
b. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is small amount of friction. In real life, much of the mechanical energy is lost as heat caused by friction.
c. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is large amount of friction. In real life, no mechanical energy is lost due to conservation of the mechanical energy.
d. The speed was the same in the scenario in the animation because the object was sliding on the ice, where there is small amount of friction. In real life, no mechanical energy is lost due to conservation of the mechanical energy.

WORKED EXAMPLE

Applying the Law of Conservation of Energy

A 10 kg rock falls from a 20 m cliff. What is the kinetic and potential energy when the rock has fallen 10 m?

Strategy

Choose the equation.

$$KE_1 + PE_1 = KE_2 + PE_2 \tag{9.4}$$

$$KE = \frac{1}{2}m\mathbf{v}^2; \quad PE = m\mathbf{g}h \tag{9.5}$$

$$\frac{1}{2}m\mathbf{v}_1^2 + m\mathbf{g}h_1 = \frac{1}{2}m\mathbf{v}_2^2 + m\mathbf{g}h_2 \tag{9.6}$$

List the knowns.

$m = 10$ kg, $\mathbf{v}_1 = 0$, $\mathbf{g} = 9.80$

$$\frac{\text{m}}{\text{s}^2}, \tag{9.7}$$

$h_1 = 20$ m, $h_2 = 10$ m

Identify the unknowns.

KE_2 and PE_2

Substitute the known values into the equation and solve for the unknown variables.

Solution

$$PE_2 = mgh_2 = 10\,(9.80)\,10 = 980\ \text{J} \tag{9.8}$$

$$KE_2 = PE_2 - (KE_1 + PE_1) = 980 - \{[0 - [10\,(9.80)\,20]]\} = 980\ \text{J} \tag{9.9}$$

Discussion

Alternatively, conservation of energy equation could be solved for $\mathbf{v}_2$ and KE_2 could be calculated. Note that m could also be eliminated.

TIPS FOR SUCCESS

Note that we can solve many problems involving conversion between KE and PE without knowing the mass of the object in question. This is because kinetic and potential energy are both proportional to the mass of the object. In a situation where $KE = PE$, we know that $m\mathbf{g}h = (1/2)m\mathbf{v}^2$.
Dividing both sides by m and rearranging, we have the relationship

$2\mathbf{g}h = \mathbf{v}^2$.

Practice Problems

5. A child slides down a playground slide. If the slide is 3 m high and the child weighs 300 N, how much potential energy does the child have at the top of the slide? (Round g to $10\ \ m/s^2$.)
   a. 0 J
   b. 100 J
   c. 300 J
   d. 900 J

6. A 0.2 kg apple on an apple tree has a potential energy of 10 J. It falls to the ground, converting all of its PE to kinetic energy. What is the velocity of the apple just before it hits the ground?
   a. 0 m/s
   b. 2 m/s
   c. 10 m/s
   d. 50 m/s

Snap Lab

Converting Potential Energy to Kinetic Energy

In this activity, you will calculate the potential energy of an object and predict the object’s speed when all that potential energy has been converted to kinetic energy. You will then check your prediction.

You will be dropping objects from a height. Be sure to stay a safe distance from the edge. Don’t lean over the railing too far. Make sure that you do not drop objects into an area where people or vehicles pass by. Make sure that dropping objects will not cause damage.

You will need the following:

Materials for each pair of students:

- Four marbles (or similar small, dense objects)
- Stopwatch

Materials for class:

- Metric measuring tape long enough to measure the chosen height
- A scale

Instructions

Procedure

1. Work with a partner. Find and record the mass of four small, dense objects per group.
2. Choose a location where the objects can be safely dropped from a height of at least 15 meters. A bridge over water with a safe pedestrian walkway will work well.
3. Measure the distance the object will fall.
4. Calculate the potential energy of the object before you drop it using $PE = m\mathbf{g}h = (9.80)mh$.
5. Predict the kinetic energy and velocity of the object when it lands using $PE = KE$ and so,
   $m\mathbf{g}h = \frac{m\mathbf{v}^2}{2};\ \mathbf{v} = \sqrt{2(9.80)h} = 4.43\sqrt{h}.$
6. One partner drops the object while the other measures the time it takes to fall.
7. Take turns being the dropper and the timer until you have made four measurements.
8. Average your drop multiplied by and calculate the velocity of the object when it landed using $\mathbf{v} = \mathbf{a}t = \mathbf{g}t = (9.80)t$.
9. Compare your results to your prediction.

GRASP CHECK

Galileo's experiments proved that, contrary to popular belief, heavy objects do not fall faster than light objects. How do the equations you used support this fact?

a. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy of the system, the mass term gets cancelled and the velocity is independent of the mass. In real life, the variation in the velocity of the different objects is observed because of the non-zero air resistance.
b. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy of the system, the mass term does not get cancelled and the velocity is dependent on the mass. In real life, the variation in the velocity of the different objects is observed because of the non-zero air resistance.
c. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy the system, the mass term gets cancelled and the velocity is independent of the mass. In real life, the variation in the velocity of the different objects is observed because of zero air resistance.
d. Heavy objects do not fall faster than the light objects because while conserving the mechanical energy of the system, the mass term does not get cancelled and the velocity is dependent on the mass. In real life, the variation in the velocity of the different objects is observed because of zero air resistance.

Check Your Understanding

7. Describe the transformation between forms of mechanical energy that is happening to a falling skydiver before his parachute opens.
   a. Kinetic energy is being transformed into potential energy.
   b. Potential energy is being transformed into kinetic energy.
   c. Work is being transformed into kinetic energy.
   d. Kinetic energy is being transformed into work.

8. True or false—If a rock is thrown into the air, the increase in the height would increase the rock's kinetic energy, and then the increase in the velocity as it falls to the ground would increase its potential energy.
   a. True
   b. False

9. Identify equivalent terms for *stored energy* and *energy of motion*.
   a. Stored energy is potential energy, and energy of motion is kinetic energy.
   b. Energy of motion is potential energy, and stored energy is kinetic energy.
   c. Stored energy is the potential as well as the kinetic energy of the system.
   d. Energy of motion is the potential as well as the kinetic energy of the system.

9.3 Simple Machines

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe simple and complex machines
- Calculate mechanical advantage and efficiency of simple and complex machines

Section Key Terms

complex machine	efficiency output	ideal mechanical advantage	inclined plane	input work
lever	mechanical advantage	output work	pulley	screw
simple machine	wedge	wheel and axle		

Simple Machines

Simple machines make work easier, but they do not decrease the amount of work you have to do. Why can't simple machines change the amount of work that you do? Recall that in closed systems the total amount of energy is conserved. A machine cannot increase the amount of energy you put into it. So, why is a simple machine useful? Although it cannot change the amount of work you do, a simple machine can change the amount of force you must apply to an object, and the distance over which you apply the force. In most cases, a simple machine is used to reduce the amount of force you must exert to do work. The down side is that you must exert the force over a greater distance, because the product of force and distance, $\mathbf{f}d$, (which equals work) does not change.

Let's examine how this works in practice. In Figure 9.7(a), the worker uses a type of **lever** to exert a small force over a large distance, while the pry bar pulls up on the nail with a large force over a small distance. Figure 9.7(b) shows the how a lever works mathematically. The effort force, applied at $\mathbf{F_e}$, lifts the load (the resistance force) which is pushing down at $\mathbf{F}_r$. The triangular pivot is called the fulcrum; the part of the lever between the fulcrum and $\mathbf{F}_e$ is the effort arm, L_e; and the part to the left is the resistance arm, L_r. The **mechanical advantage** is a number that tells us how many times a simple machine multiplies the effort force. The **ideal mechanical advantage**, *IMA*, is the mechanical advantage of a perfect machine with no loss of useful work caused by friction between moving parts. The equation for *IMA* is shown in Figure 9.7(b).

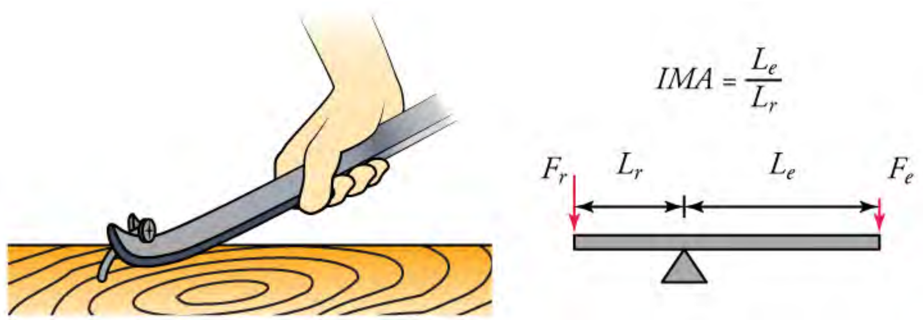


Figure 9.7 (a) A pry bar is a type of lever. (b) The ideal mechanical advantage equals the length of the effort arm divided by the length of the resistance arm of a lever.

In general, the *IMA* = the resistance force, $\mathbf{F}_r$, divided by the effort force, $\mathbf{F}_e$. *IMA* also equals the distance over which the effort is applied, d_e, divided by the distance the load travels, d_r.

$$IMA = \frac{\mathbf{F}_r}{\mathbf{F}_e} = \frac{d_e}{d_r}$$

Getting back to conservation of energy, for any simple machine, the work put into the machine, W_i, equals the work the machine puts out, W_o. Combining this with the information in the paragraphs above, we can write

$$W_i = W_o$$
$$\mathbf{F}_e d_e = \mathbf{F}_r d_r$$
$$\text{If } \mathbf{F}_e < \mathbf{F}_r \text{, then } d_e > d_r.$$

The equations show how a simple machine can output the same amount of work while reducing the amount of effort force by increasing the distance over which the effort force is applied.

WATCH PHYSICS

Introduction to Mechanical Advantage

This video shows how to calculate the *IMA* of a lever by three different methods: (1) from effort force and resistance force; (2) from the lengths of the lever arms, and; (3) from the distance over which the force is applied and the distance the load moves.

Click to view content (https://www.youtube.com/embed/pfzJ-z5Ij48)

GRASP CHECK

Two children of different weights are riding a seesaw. How do they position themselves with respect to the pivot point (the fulcrum) so that they are balanced?

a. The heavier child sits closer to the fulcrum.
b. The heavier child sits farther from the fulcrum.
c. Both children sit at equal distance from the fulcrum.
d. Since both have different weights, they will never be in balance.

Some levers exert a large force to a short effort arm. This results in a smaller force acting over a greater distance at the end of the resistance arm. Examples of this type of lever are baseball bats, hammers, and golf clubs. In another type of lever, the fulcrum is at the end of the lever and the load is in the middle, as in the design of a wheelbarrow.

The simple machine shown in Figure 9.8 is called a **wheel and axle.** It is actually a form of lever. The difference is that the effort arm can rotate in a complete circle around the fulcrum, which is the center of the axle. Force applied to the outside of the wheel causes a greater force to be applied to the rope that is wrapped around the axle. As shown in the figure, the ideal mechanical advantage is calculated by dividing the radius of the wheel by the radius of the axle. Any crank-operated device is an example of a wheel and axle.

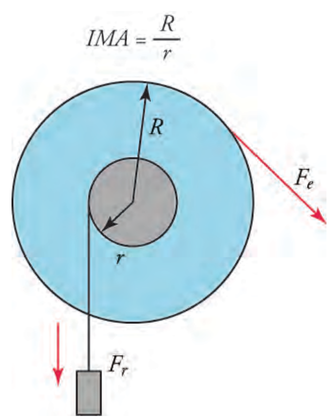


Figure 9.8 Force applied to a wheel exerts a force on its axle.

An **inclined plane** and a **wedge** are two forms of the same simple machine. A wedge is simply two inclined planes back to back. Figure 9.9 shows the simple formulas for calculating the *IMA*s of these machines. All sloping, paved surfaces for walking or driving are inclined planes. Knives and axe heads are examples of wedges.

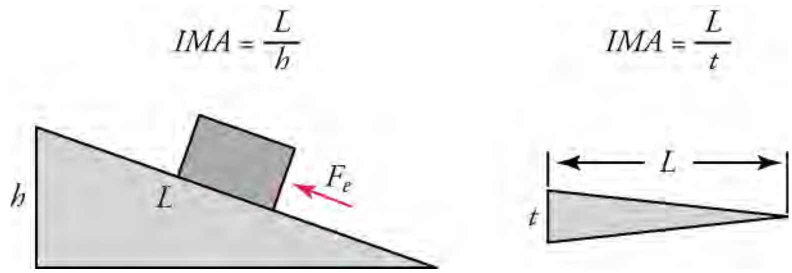


Figure 9.9 An inclined plane is shown on the left, and a wedge is shown on the right.

The **screw** shown in Figure 9.10 is actually a lever attached to a circular inclined plane. Wood screws (of course) are also examples of screws. The lever part of these screws is a screw driver. In the formula for *IMA*, the distance between screw threads is called *pitch* and has the symbol *P*.

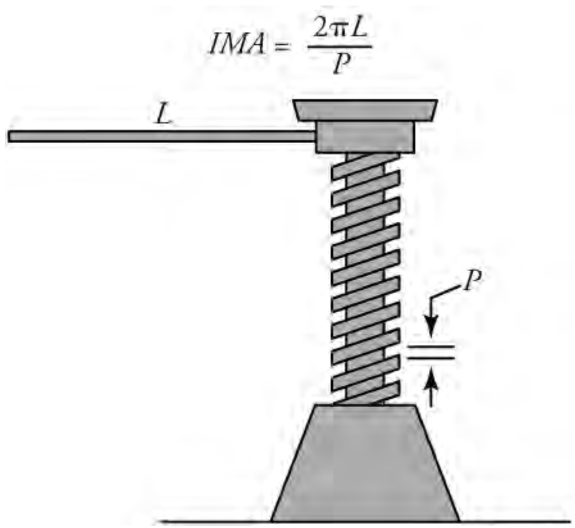


Figure 9.10 The screw shown here is used to lift very heavy objects, like the corner of a car or a house a short distance.

Figure 9.11 shows three different **pulley** systems. Of all simple machines, mechanical advantage is easiest to calculate for pulleys. Simply count the number of ropes supporting the load. That is the *IMA*. Once again we have to exert force over a longer distance to multiply force. To raise a load 1 meter with a pulley system you have to pull *N* meters of rope. Pulley systems are often used to raise flags and window blinds and are part of the mechanism of construction cranes.

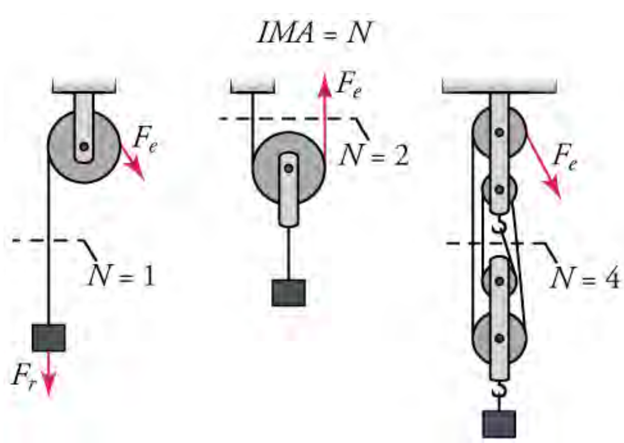


Figure 9.11 Three pulley systems are shown here.

WATCH PHYSICS

Mechanical Advantage of Inclined Planes and Pulleys

The first part of this video shows how to calculate the *IMA* of pulley systems. The last part shows how to calculate the *IMA* of an inclined plane.

Click to view content (https://www.khanacademy.org/embed_video?v=vSsK7Rfa3yA)

GRASP CHECK

How could you use a pulley system to lift a light load to great height?

a. Reduce the radius of the pulley.
b. Increase the number of pulleys.
c. Decrease the number of ropes supporting the load.
d. Increase the number of ropes supporting the load.

A **complex machine** is a combination of two or more simple machines. The wire cutters in Figure 9.12 combine two levers and two wedges. Bicycles include wheel and axles, levers, screws, and pulleys. Cars and other vehicles are combinations of many machines.

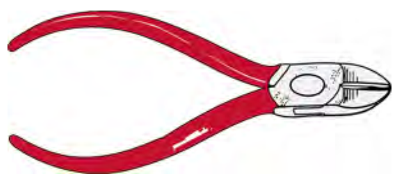


Figure 9.12 Wire cutters are a common complex machine.

Calculating Mechanical Advantage and Efficiency of Simple Machines

In general, the *IMA* = the resistance force, $\mathbf{F}_r$, divided by the effort force, $\mathbf{F}_e$. *IMA* also equals the distance over which the effort is applied, d_e, divided by the distance the load travels, d_r.

$$IMA = \frac{\mathbf{F}_r}{\mathbf{F}_e} = \frac{d_e}{d_r}$$

Refer back to the discussions of each simple machine for the specific equations for the *IMA* for each type of machine.

No simple or complex machines have the actual mechanical advantages calculated by the *IMA* equations. In real life, some of the applied work always ends up as wasted heat due to friction between moving parts. Both the **input work** (W_i) and **output work** (W_o) are the result of a force, **F**, acting over a distance, *d*.

$$W_i = \mathbf{F}_i d_i \text{and} W_o = \mathbf{F}_o d_o$$

The **efficiency output** of a machine is simply the output work divided by the input work, and is usually multiplied by 100 so that it is expressed as a percent.

$$\%\text{ efficiency} = \frac{W_o}{W_i} \times 100$$

Look back at the pictures of the simple machines and think about which would have the highest efficiency. Efficiency is related to friction, and friction depends on the smoothness of surfaces and on the area of the surfaces in contact. How would lubrication affect the efficiency of a simple machine?

WORKED EXAMPLE

Efficiency of a Lever

The input force of 11 N acting on the effort arm of a lever moves 0.4 m, which lifts a 40 N weight resting on the resistance arm a distance of 0.1 m. What is the efficiency of the machine?

Strategy

State the equation for efficiency of a simple machine, $\% \text{ efficiency} = \frac{W_o}{W_i} \times 100$, and calculate W_o and W_i. Both work values are the product *Fd*.

Solution

$W_i = \mathbf{F}_i d_i = (11)(0.4) = 4.4$ J and $W_o = \mathbf{F}_o d_o = (40)(0.1) = 4.0$ J, then $\% \text{ efficiency} = \frac{W_o}{W_i} \times 100 = \frac{4.0}{4.4} \times 100 = 91\%$

Discussion

Efficiency in real machines will always be less than 100 percent because of work that is converted to unavailable heat by friction and air resistance. W_o and W_i can always be calculated as a force multiplied by a distance, although these quantities are not always as obvious as they are in the case of a lever.

Practice Problems

10. What is the IMA of an inclined plane that is 5 m long and 2 m high?

a. 0.4
b. 2.5
c. 0.4 m
d. 2.5 m

11. If a pulley system can lift a 200N load with an effort force of 52 N and has an efficiency of almost 100 percent, how many ropes are supporting the load?

a. 1 rope is required because the actual mechanical advantage is 0.26.
b. 1 rope is required because the actual mechanical advantage is 3.80.
c. 4 ropes are required because the actual mechanical advantage is 0.26.
d. 4 ropes are required because the actual mechanical advantage is 3.80.

Check Your Understanding

12. True or false—The efficiency of a simple machine is always less than 100 percent because some small fraction of the input work is always converted to heat energy due to friction.

a. True
b. False

13. The circular handle of a faucet is attached to a rod that opens and closes a valve when the handle is turned. If the rod has a diameter of 1 cm and the IMA of the machine is 6, what is the radius of the handle?

A. 0.08 cm
B. 0.17 cm
C. 3.0 cm
D. 6.0 cm

KEY TERMS

complex machine a machine that combines two or more simple machines

efficiency output work divided by input work

energy the ability to do work

gravitational potential energy energy acquired by doing work against gravity

ideal mechanical advantage the mechanical advantage of an idealized machine that loses no energy to friction

inclined plane a simple machine consisting of a slope

input work effort force multiplied by the distance over which it is applied

joule the metric unit for work and energy; equal to 1 newton meter (N•m)

kinetic energy energy of motion

law of conservation of energy states that energy is neither created nor destroyed

lever a simple machine consisting of a rigid arm that pivots on a fulcrum

mechanical advantage the number of times the input force is multiplied

mechanical energy kinetic or potential energy

output work output force multiplied by the distance over which it acts

potential energy stored energy

power the rate at which work is done

pulley a simple machine consisting of a rope that passes over one or more grooved wheels

screw a simple machine consisting of a spiral inclined plane

simple machine a machine that makes work easier by changing the amount or direction of force required to move an object

watt the metric unit of power; equivalent to joules per second

wedge a simple machine consisting of two back-to-back inclined planes

wheel and axle a simple machine consisting of a rod fixed to the center of a wheel

work force multiplied by distance

work–energy theorem states that the net work done on a system equals the change in kinetic energy

SECTION SUMMARY

9.1 Work, Power, and the Work–Energy Theorem

- Doing work on a system or object changes its energy.
- The work–energy theorem states that an amount of work that changes the velocity of an object is equal to the change in kinetic energy of that object.The work–energy theorem states that an amount of work that changes the velocity of an object is equal to the change in kinetic energy of that object.
- Power is the rate at which work is done.

9.2 Mechanical Energy and Conservation of Energy

- Mechanical energy may be either kinetic (energy of motion) or potential (stored energy).
- Doing work on an object or system changes its energy.
- Total energy in a closed, isolated system is constant.

9.3 Simple Machines

- The six types of simple machines make work easier by changing the **f***d* term so that force is reduced at the expense of increased distance.
- The ratio of output force to input force is a machine's mechanical advantage.
- Combinations of two or more simple machines are called complex machines.
- The ratio of output work to input work is a machine's efficiency.

KEY EQUATIONS

9.1 Work, Power, and the Work–Energy Theorem

equation for work	$W = \mathbf{f}d$
force	$\mathbf{f} = w = m\mathbf{g}$
work equivalencies	$W = PE_e = \mathbf{f}m\mathbf{g}$
kinetic energy	$KE = \frac{1}{2}m\mathbf{v}^2$
work–energy theorem	$W = \Delta\text{KE} = \frac{1}{2}m\mathbf{v}_2^2 - \frac{1}{2}m\mathbf{v}_1^2$
power	$P = \frac{W}{t}$

9.2 Mechanical Energy and Conservation of Energy

conservation of energy	$KE_1 + PE_1 = KE_2 + PE_2$

9.3 Simple Machines

ideal mechanical advantage (general)	$IMA = \frac{\mathbf{F}_r}{\mathbf{F}_e} = \frac{d_e}{d_r}$
ideal mechanical advantage (lever)	$IMA = \frac{L_e}{L_r}$
ideal mechanical advantage (wheel and axle)	$IMA = \frac{R}{r}$
ideal mechanical advantage (inclined plane)	$IMA = \frac{L}{h}$
ideal mechanical advantage (wedge)	$IMA = \frac{L}{t}$
ideal mechanical advantage (pulley)	$IMA = N$
ideal mechanical advantage (screw)	$IMA = \frac{2\pi L}{P}$
input work	$W_i = \mathbf{F}_i d_i$
output work	$W_o = \mathbf{F}_o d_o$
efficiency output	$\%\ \text{efficiency} = \frac{W_o}{W_i} \times 100$

CHAPTER REVIEW

Concept Items

9.1 Work, Power, and the Work–Energy Theorem

1. Is it possible for the sum of kinetic energy and potential energy of an object to change without work having been done on the object? Explain.

a. No, because the work-energy theorem states that work done on an object is equal to the change in kinetic energy, and change in KE requires a change in velocity. It is assumed that mass is constant.

b. No, because the work-energy theorem states that work done on an object is equal to the sum of kinetic energy, and the change in KE requires a change in displacement. It is assumed that mass is constant.

c. Yes, because the work-energy theorem states that work done on an object is equal to the change in kinetic energy, and change in KE requires a change in velocity. It is assumed that mass is constant.

d. Yes, because the work-energy theorem states that work done on an object is equal to the sum of kinetic energy, and the change in KE requires a change in displacement. It is assumed that mass is constant.

2. Define work for one-dimensional motion.

a. Work is defined as the ratio of the force over the distance.

b. Work is defined as the sum of the force and the distance.

c. Work is defined as the square of the force over the distance.

d. Work is defined as the product of the force and the distance.

3. A book with a mass of 0.30 kg falls 2 m from a shelf to the floor. This event is described by the work–energy theorem: $W = fd = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$ Explain why this is enough information to calculate the speed with which the book hits the floor.

a. The mass of the book, m, and distance, d, are stated. F is the weight of the book mg . v_1 is the initial velocity and v_2 is the final velocity. The final velocity is the only unknown quantity.

b. The mass of the book, m, and distance, d, are stated. F is the weight of the book mg . v_1 is the final velocity and v_2 is the initial velocity. The final velocity is the only unknown quantity.

c. The mass of the book, m, and distance, d, are stated. F is the weight of the book mg . v_1 is the initial velocity and v_2 is the final velocity. The final velocity and the initial velocities are the only unknown quantities.

d. The mass of the book, m, and distance, d, are stated. F is the weight of the book mg . v_1 is the final velocity and v_2 is the initial velocity. The final velocity and the initial velocities are the only unknown quantities.

9.2 Mechanical Energy and Conservation of Energy

4. Describe the changes in KE and PE of a person jumping up and down on a trampoline.
   a. While going up, the person's KE would change to PE. While coming down, the person's PE would change to KE.
   b. While going up, the person's PE would change to KE. While coming down, the person's KE would change to PE.
   c. While going up, the person's KE would not change, but while coming down, the person's PE would change to KE.
   d. While going up, the person's PE would change to KE, but while coming down, the person's KE would not change.

5. You know the height from which an object is dropped. Which equation could you use to calculate the velocity as the object hits the ground?
   a. $v = h$
   b. $v = \sqrt{2h}$
   c. $v = gh$
   d. $v = \sqrt{2gh}$

6. The starting line of a cross country foot race is at the bottom of a hill. Which form(s) of mechanical energy of the runners will change when the starting gun is fired?
   a. Kinetic energy only
   b. Potential energy only
   c. Both kinetic and potential energy
   d. Neither kinetic nor potential energy

9.3 Simple Machines

7. How does a simple machine make work easier?
   a. It reduces the input force and the output force.
   b. It reduces the input force and increases the output force.
   c. It increases the input force and reduces the output force.
   d. It increases the input force and the output force.

8. Which type of simple machine is a knife?
   a. A ramp
   b. A wedge
   c. A pulley
   d. A screw

Critical Thinking Items

9.1 Work, Power, and the Work–Energy Theorem

9. Which activity requires a person to exert force on an object that causes the object to move but does not change the kinetic or potential energy of the object?
   a. Moving an object to a greater height with acceleration
   b. Moving an object to a greater height without acceleration
   c. Carrying an object with acceleration at the same height
   d. Carrying an object without acceleration at the same height

10. Which statement explains how it is possible to carry books to school without changing the kinetic or potential energy of the books or doing any work?
   a. By moving the book without acceleration and keeping the height of the book constant
   b. By moving the book with acceleration and keeping the height of the book constant
   c. By moving the book without acceleration and changing the height of the book
   d. By moving the book with acceleration and changing the height of the book

9.2 Mechanical Energy and Conservation of Energy

11. True or false—A cyclist coasts down one hill and up another hill until she comes to a stop. The point at which the bicycle stops is lower than the point at which it started coasting because part of the original potential energy has been converted to a quantity of heat and this makes the tires of the bicycle warm.
   a. True
   b. False

9.3 Simple Machines

12. We think of levers being used to decrease effort force. Which of the following describes a lever that requires a large effort force which causes a smaller force to act over a large distance and explains how it works?
   a. Anything that is swung by a handle, such as a hammer or racket. Force is applied near the fulcrum over a short distance, which makes the other end move rapidly over a long distance.
   b. Anything that is swung by a handle, such as a hammer or racket. Force is applied far from the fulcrum over a large distance, which makes the other end move rapidly over a long distance.
   c. A lever used to lift a heavy stone. Force is applied near the fulcrum over a short distance, which

makes the other end lift a heavy object easily.
   d. A lever used to lift a heavy stone. Force is applied far from the fulcrum over a large distance, which makes the other end lift a heavy object easily

13. A baseball bat is a lever. Which of the following explains how a baseball bat differs from a lever like a pry bar?
   a. In a baseball bat, effort force is smaller and is applied over a large distance, while the resistance force is smaller and is applied over a long distance.
   b. In a baseball bat, effort force is smaller and is applied over a large distance, while the resistance force is smaller and is applied over a short distance.
   c. In a baseball bat, effort force is larger and is applied over a short distance, while the resistance force is smaller and is applied over a long distance.
   d. In a baseball bat, effort force is larger and is applied over a short distance, while the resistance force is smaller and is applied over a short distance.

Problems

9.1 Work, Power, and the Work–Energy Theorem

14. A baseball player exerts a force of $100\ \text{N}$ on a ball for a distance of $0.5\ \text{m}$ as he throws it. If the ball has a mass of $0.15\ \text{kg}$, what is its velocity as it leaves his hand?
   a. $-36.5\ \text{m/s}$
   b. $-25.8\ \text{m/s}$
   c. $25.8\ \text{m/s}$
   d. $36.5\ \text{m/s}$

15. A boy pushes his little sister on a sled. The sled accelerates from 0 to 3.2 m/s . If the combined mass of his sister and the sled is 40.0 kg and 18 W of power were generated, how long did the boy push the sled?
   a. 205 s
   b. 128 s
   c. 23 s
   d. 11 s

9.2 Mechanical Energy and Conservation of Energy

16. What is the kinetic energy of a $0.01\ \text{kg}$ bullet traveling at a velocity of $700\ \text{m/s}$?
   a. $3.5\ \text{J}$
   b. $7\ \text{J}$
   c. $2.45 \times 10^3\ \text{J}$
   d. $2.45 \times 10^5\ \text{J}$

17. A marble rolling across a flat, hard surface at $2\ \text{m/s}$ rolls up a ramp. Assuming that $g = 10\ \text{m/s}^2$ and no energy is lost to friction, what will be the vertical height of the marble when it comes to a stop before rolling back down? Ignore effects due to the rotational kinetic energy.
   a. $0.1\ \text{m}$
   b. $0.2\ \text{m}$
   c. $0.4\ \text{m}$
   d. $2\ \text{m}$

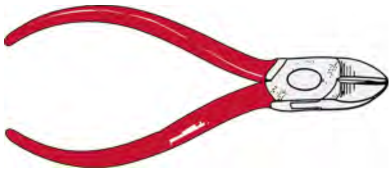
18. The potential energy stored in a compressed spring is $U = \frac{1}{2}kx^2$, where k is the force constant and x is the distance the spring is compressed from the equilibrium position. Four experimental setups described below can be used to determine the force constant of a spring. Which one(s) require measurement of the fewest number of variables to determine k? Assume the acceleration due to gravity is known.
   I. An object is propelled vertically by a compressed spring.
   II. An object is propelled horizontally on a frictionless surface by a compressed spring.
   III. An object is statically suspended from a spring.
   IV. An object suspended from a spring is set into oscillatory motion.

   a. I only
   b. III only
   c. I and II only
   d. III and IV only

9.3 Simple Machines

19. A man is using a wedge to split a block of wood by hitting the wedge with a hammer. This drives the wedge into the wood creating a crack in the wood. When he hits the wedge with a force of 400 N it travels 4 cm into the wood. This caused the wedge to exert a force of 1,400 N sideways increasing the width of the crack by 1 cm . What is the efficiency of the wedge?
   a. 0.875 percent
   b. 0.14
   c. 0.751
   d. 87.5 percent

20. An electrician grips the handles of a wire cutter, like the one shown, 10 cm from the pivot and places a wire between the jaws 2 cm from the pivot. If the cutter blades are 2 cm wide and 0.3 cm thick, what is the overall IMA of this complex machine?

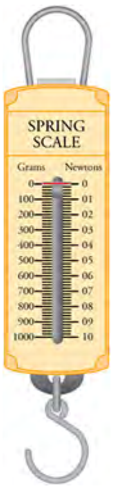


a. 1.34
b. 1.53
c. 33.3
d. 33.5

Performance Task

9.3 Simple Machines

21. Conservation of Energy and Energy Transfer; Cause and Effect; and S&EP, Planning and Carrying Out Investigations
Plan an investigation to measure the mechanical advantage of simple machines and compare to the *IMA* of the machine. Also measure the efficiency of each machine studied. Design an investigation to make these measurements for these simple machines: lever, inclined plane, wheel and axle and a pulley system. In addition to these machines, include a spring scale, a tape measure, and a weight with a loop on top that can be attached to the hook on the spring scale. A spring scale is shown in the image.



A spring scale measures weight, not mass.

LEVER: Beginning with the lever, explain how you would measure input force, output force, effort arm, and resistance arm. Also explain how you would find the distance the load travels and the distance over which the effort force is applied. Explain how you would use this data to determine *IMA* and efficiency.
INCLINED PLANE: Make measurements to determine *IMA* and efficiency of an inclined plane. Explain how you would use the data to calculate these values. Which property do you already know? Note that there are no effort and resistance arm measurements, but there are height and length measurements.
WHEEL AND AXLE: Again, you will need two force measurements and four distance measurements. Explain how you would use these to calculate *IMA* and efficiency.
SCREW: You will need two force measurements, two distance traveled measurements, and two length measurements. You may describe a screw like the one shown in Figure 9.10 or you could use a screw and screw driver. (Measurements would be easier for the former). Explain how you would use these to calculate *IMA* and efficiency.
PULLEY SYSTEM: Explain how you would determine the *IMA* and efficiency of the four-pulley system shown in Figure 9.11. Why do you only need two distance measurements for this machine?
Design a table that compares the efficiency of the five simple machines. Make predictions as to the most and least efficient machines.

TEST PREP

Multiple Choice

9.1 Work, Power, and the Work–Energy Theorem

22. Which expression represents power?
a. fd
b. mgh
c. $\frac{mv^2}{2}$
d. $\frac{W}{t}$

23. The work–energy theorem states that the change in the kinetic energy of an object is equal to what?
a. The work done on the object
b. The force applied to the object
c. The loss of the object's potential energy
d. The object's total mechanical energy minus its kinetic energy

24. A runner at the start of a race generates 250 W of power as he accelerates to 5 m/s . If the runner has a mass of 60 kg, how long did it take him to reach that speed?
a. 0.33 s
b. 0.83 s
c. 1.2 s
d. 3.0 s

25. A car's engine generates 100,000 W of power as it exerts a force of 10,000 N. How long does it take the car to travel 100 m?

a. 0.001 s
b. 0.01 s
c. 10 s
d. 1,000 s

9.2 Mechanical Energy and Conservation of Energy

26. Why is this expression for kinetic energy incorrect? $\mathrm{KE} = (m)(v)^2$.

a. The constant g is missing.
b. The term v should not be squared.
c. The expression should be divided by 2.
d. The energy lost to friction has not been subtracted.

27. What is the kinetic energy of a $10 \mathrm{kg}$ object moving at $2.0 \mathrm{~m/s}$?

a. $10 \mathrm{~J}$
b. $20 \mathrm{~J}$
c. $40 \mathrm{~J}$
d. $100 \mathrm{~J}$

28. Which statement best describes the PE-KE transformations for a javelin, starting from the instant the javelin leaves the thrower's hand until it hits the ground.

a. Initial PE is transformed to KE until the javelin reaches the high point of its arc. On the way back down, KE is transformed into PE. At every point in the flight, mechanical energy is being transformed into heat energy.
b. Initial KE is transformed to PE until the javelin reaches the high point of its arc. On the way back down, PE is transformed into KE. At every point in the flight, mechanical energy is being transformed into heat energy.
c. Initial PE is transformed to KE until the javelin reaches the high point of its arc. On the way back down, there is no transformation of mechanical energy. At every point in the flight, mechanical energy is being transformed into heat energy.
d. Initial KE is transformed to PE until the javelin reaches the high point of its arc. On the way back down, there is no transformation of mechanical energy. At every point in the flight, mechanical energy is being transformed into heat energy.

29. At the beginning of a roller coaster ride, the roller coaster car has an initial energy mostly in the form of PE. Which statement explains why the fastest speeds of the car will be at the lowest points in the ride?

a. At the bottom of the slope kinetic energy is at its maximum value and potential energy is at its minimum value.
b. At the bottom of the slope potential energy is at its maximum value and kinetic energy is at its minimum value.
c. At the bottom of the slope both kinetic and potential energy reach their maximum values
d. At the bottom of the slope both kinetic and potential energy reach their minimum values.

9.3 Simple Machines

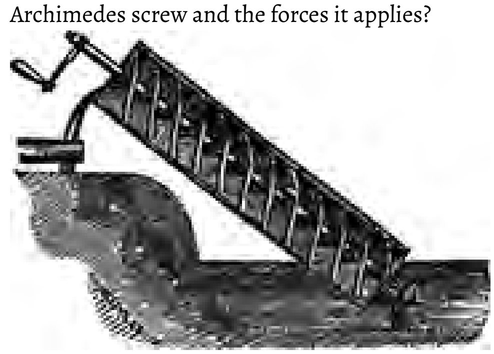
30. A large radius divided by a small radius is the expression used to calculate the IMA of what?

a. A screw
b. A pulley
c. A wheel and axle
d. An inclined plane.

31. What is the IMA of a wedge that is $12 \mathrm{~cm}$ long and $3 \mathrm{~cm}$ thick?

a. 2
b. 3
c. 4
d. 9

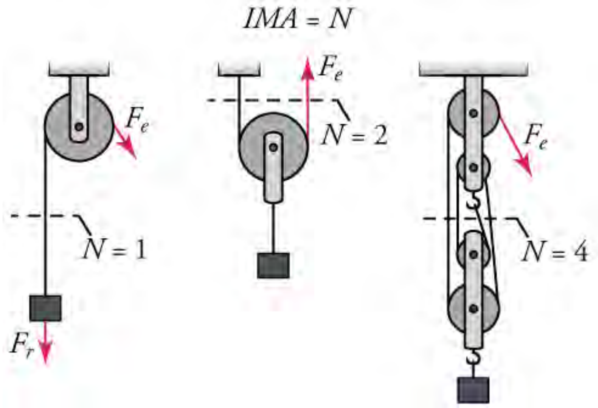
32. Which statement correctly describes the simple machines, like the crank in the image, that make up an Archimedes screw and the forces it applies?



a. The crank is a wedge in which the IMA is the length of the tube divided by the radius of the tube. The applied force is the effort force and the weight of the water is the resistance force.
b. The crank is an inclined plane in which the IMA is the length of the tube divided by the radius of the tube. The applied force is the effort force and the weight of the water is the resistance force.
c. The crank is a wheel and axle. The effort force of the crank becomes the resistance force of the screw.
d. The crank is a wheel and axle. The resistance force of the crank becomes the effort force of the screw.

33. Refer to the pulley system on right in the image. Assume this pulley system is an ideal machine.
How hard would you have to pull on the rope to lift a 120 N

load?
How many meters of rope would you have to pull out of the system to lift the load 1 m?



a. 480 N
4 m
b. 480 N
$\frac{1}{4}$ m
c. 30 N
4 m
d. 30 N
$\frac{1}{4}$ m

Short Answer

9.1 Work, Power, and the Work–Energy Theorem

34. Describe two ways in which doing work on an object can increase its mechanical energy.
  a. Raising an object to a higher elevation does work as it increases its PE; increasing the speed of an object does work as it increases its KE.
  b. Raising an object to a higher elevation does work as it increases its KE; increasing the speed of an object does work as it increases its PE.
  c. Raising an object to a higher elevation does work as it increases its PE; decreasing the speed of an object does work as it increases its KE.
  d. Raising an object to a higher elevation does work as it increases its KE; decreasing the speed of an object does work as it increases its PE.

35. True or false—While riding a bicycle up a gentle hill, it is fairly easy to increase your potential energy, but to increase your kinetic energy would make you feel exhausted.
  a. True
  b. False

36. Which statement best explains why running on a track with constant speed at 3 m/s is not work, but climbing a mountain at 1 m/s is work?
  a. At constant speed, change in the kinetic energy is zero but climbing a mountain produces change in the potential energy.
  b. At constant speed, change in the potential energy is zero, but climbing a mountain produces change in the kinetic energy.
  c. At constant speed, change in the kinetic energy is finite, but climbing a mountain produces no change in the potential energy.
  d. At constant speed, change in the potential energy is finite, but climbing a mountain produces no change in the kinetic energy.

37. You start at the top of a hill on a bicycle and coast to the bottom without applying the brakes. By the time you reach the bottom of the hill, work has been done on you and your bicycle, according to the equation: $W = \frac{1}{2}m(v_2^2 - v_1^2)$ If m is the mass of you and your bike, what are v_1 and v_2?
  a. v_1 is your speed at the top of the hill, and v_2 is your speed at the bottom.
  b. v_1 is your speed at the bottom of the hill, and v_2 is your speed at the top.
  c. v_1 is your displacement at the top of the hill, and v_2 is your displacement at the bottom.
  d. v_1 is your displacement at the bottom of the hill, and v_2 is your displacement at the top.

9.2 Mechanical Energy and Conservation of Energy

38. True or false—The formula for gravitational potential energy can be used to explain why joules, J, are equivalent to kg × mg² / s² . Show your work.
  a. True
  b. False

39. Which statement best explains why accelerating a car from 20 mph to 40 mph quadruples its kinetic energy?
  a. Because kinetic energy is directly proportional to the square of the velocity.
  b. Because kinetic energy is inversely proportional to the square of the velocity.
  c. Because kinetic energy is directly proportional to the fourth power of the velocity.
  d. Because kinetic energy is inversely proportional to

the fourth power of the velocity.

40. A coin falling through a vacuum loses no energy to friction, and yet, after it hits the ground, it has lost all its potential and kinetic energy. Which statement best explains why the law of conservation of energy is still valid in this case?

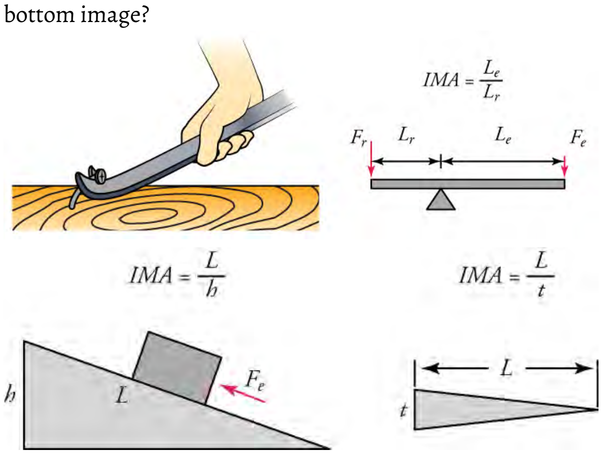
a. When the coin hits the ground, the ground gains potential energy that quickly changes to thermal energy.
b. When the coin hits the ground, the ground gains kinetic energy that quickly changes to thermal energy.
c. When the coin hits the ground, the ground gains thermal energy that quickly changes to kinetic energy.
d. When the coin hits the ground, the ground gains thermal energy that quickly changes to potential energy.

41. True or false—A marble rolls down a slope from height h_1 and up another slope to height h_2, where ($h_2 < h_1$). The difference $mg(h_1 - h_2)$ is equal to the heat lost due to the friction.

a. True
b. False

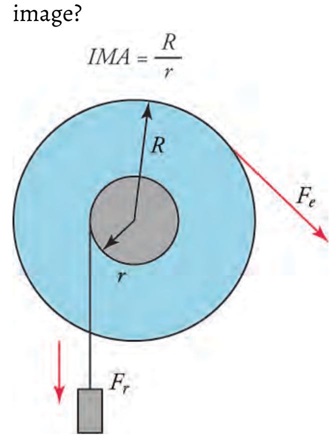
9.3 Simple Machines

42. Why would you expect the lever shown in the top image to have a greater efficiency than the inclined plane shown in the bottom image?



a. The resistance arm is shorter in case of the inclined plane.
b. The effort arm is shorter in case of the inclined plane.
c. The area of contact is greater in case of the inclined plane.

43. Why is the wheel on a wheelbarrow **not** a simple machine in the same sense as the simple machine in the image?



a. The wheel on the wheelbarrow has no fulcrum.
b. The center of the axle is not the fulcrum for the wheels of a wheelbarrow.
c. The wheelbarrow differs in the way in which load is attached to the axle.
d. The wheelbarrow has less resistance force than a wheel and axle design.

44. A worker pulls down on one end of the rope of a pulley system with a force of 75 N to raise a hay bale tied to the other end of the rope. If she pulls the rope down 2.0 m and the bale raises 1.0 m, what else would you have to know to calculate the efficiency of the pulley system?

a. the weight of the worker
b. the weight of the hay bale
c. the radius of the pulley
d. the height of the pulley from ground

45. True or false—A boy pushed a box with a weight of 300 N up a ramp. He said that, because the ramp was 1.0 m high and 3.0 m long, he must have been pushing with force of exactly 100 N.

a. True
b. False

Extended Response

9.1 Work, Power, and the Work–Energy Theorem

46. Work can be negative as well as positive because an object or system can do work on its surroundings as well as have work done on it. Which of the following statements describes:
a situation in which an object does work on its surroundings by decreasing its velocity and
a situation in which an object can do work on its surroundings by decreasing its altitude?

a. A gasoline engine burns less fuel at a slower speed. Solar cells capture sunlight to generate electricity.

b. A hybrid car charges its batteries as it decelerates.
   Falling water turns a turbine to generate electricity.
c. Airplane flaps use air resistance to slow down for landing.
   Rising steam turns a turbine to generate electricity.
d. An electric train requires less electrical energy as it decelerates.
   A parachute captures air to slow a skydiver's fall.

47. A boy is pulling a girl in a child's wagon at a constant speed. He begins to pull harder, which increases the speed of the wagon. Which of the following describes two ways you could calculate the change in energy of the wagon and girl if you had all the information you needed?

a. Calculate work done from the force and the velocity.
   Calculate work done from the change in the potential energy of the system.
b. Calculate work done from the force and the displacement.
   Calculate work done from the change in the potential energy of the system.
c. Calculate work done from the force and the velocity.
   Calculate work done from the change in the kinetic energy of the system.
d. Calculate work done from the force and the displacement.
   Calculate work done from the change in the kinetic energy of the system.

9.2 Mechanical Energy and Conservation of Energy

48. Acceleration due to gravity on the moon is 1.6 m/s^2 or about 16% of the value of g on Earth.
If an astronaut on the moon threw a moon rock to a height of 7.8 m, what would be its velocity as it struck the moon's surface?
How would the fact that the moon has no atmosphere affect the velocity of the falling moon rock? Explain your answer.

a. The velocity of the rock as it hits the ground would be 5.0 m/s. Due to the lack of air friction, there would be complete transformation of the potential energy into the kinetic energy as the rock hits the moon's surface.
b. The velocity of the rock as it hits the ground would be 5.0 m/s. Due to the lack of air friction, there would be incomplete transformation of the potential energy into the kinetic energy as the rock hits the moon's surface.
c. The velocity of the rock as it hits the ground would be 12 m/s. Due to the lack of air friction, there would be complete transformation of the potential energy into the kinetic energy as the rock hits the moon's surface.
d. The velocity of the rock as it hits the ground would be 12 m/s. Due to the lack of air friction, there would be incomplete transformation of the potential energy into the kinetic energy as the rock hits the moon's surface.

49. A boulder rolls from the top of a mountain, travels across a valley below, and rolls part way up the ridge on the opposite side. Describe all the energy transformations taking place during these events and identify when they happen.

a. As the boulder rolls down the mountainside, KE is converted to PE. As the boulder rolls up the opposite slope, PE is converted to KE. The boulder rolls only partway up the ridge because some of the PE has been converted to thermal energy due to friction.
b. As the boulder rolls down the mountainside, KE is converted to PE. As the boulder rolls up the opposite slope, KE is converted to PE. The boulder rolls only partway up the ridge because some of the PE has been converted to thermal energy due to friction.
c. As the boulder rolls down the mountainside, PE is converted to KE. As the boulder rolls up the opposite slope, PE is converted to KE. The boulder rolls only partway up the ridge because some of the PE has been converted to thermal energy due to friction.
d. As the boulder rolls down the mountainside, PE is converted to KE. As the boulder rolls up the opposite slope, KE is converted to PE. The boulder rolls only partway up the ridge because some of the PE has been converted to thermal energy due to friction.

9.3 Simple Machines

50. To dig a hole, one holds the handles together and thrusts the blades of a posthole digger, like the one in the image, into the ground. Next, the handles are pulled apart, which squeezes the dirt between them, making it possible to remove the dirt from the hole. This complex machine is composed of two pairs of two different simple machines. Identify and describe the parts that are simple machines and explain how you would find the IMA of each type of simple machine.

a. Each handle and its attached blade is a lever with the

fulcrum at the hinge. Each blade is a wedge.
The IMA of a lever would be the length of the handle divided by the length of the blade. The IMA of the wedges would be the length of the blade divided by its width.

b. Each handle and its attached to blade is a lever with the fulcrum at the end. Each blade is a wedge.
The IMA of a lever would be the length of the handle divided by the length of the blade. The IMA of the wedges would be the length of the blade divided by its width.

c. Each handle and its attached blade is a lever with the fulcrum at the hinge. Each blade is a wedge.
The IMA of a lever would be the length of the handle multiplied by the length of the blade. The IMA of the wedges would be the length of the blade multiplied by its width.

d. Each handle and its attached blade is a lever with the fulcrum at the end. Each blade is a wedge.
The IMA of a lever would be the length of the handle multiplied by the length of the blade. The IMA of the wedges would be the length of the blade multiplied by its width.

51. A wooden crate is pulled up a ramp that is 1.0 m high and 6.0 m long. The crate is attached to a rope that is wound around an axle with a radius of 0.020 m . The axle is turned by a 0.20 m long handle. What is the overall IMA of the complex machine?

A. 6
B. 10
C. 16
D. 60

CHAPTER 10
Special Relativity

Figure 10.1 Special relativity explains why travel to other star systems, such as these in the Orion Nebula, is unlikely using our current level of technology. (s58y, Flickr)

Chapter Outline

INTRODUCTION Have you ever dreamed of traveling to other planets in faraway star systems? The trip might seem possible by traveling fast enough, but you will read in this chapter why it is not. In 1905, Albert Einstein developed the theory of **special relativity**. Einstein developed the theory to help explain inconsistencies between the equations describing electromagnetism and Newtonian mechanics, and to explain why the ether did not exist. This theory explains the limit on an object's speed among other implications.

Relativity is the study of how different observers moving with respect to one another measure the same events. Galileo and Newton developed the first correct version of classical relativity. Einstein developed the modern theory of relativity. Modern relativity is divided into two parts. Special relativity deals with observers moving at constant velocity. **General relativity** deals with observers moving at constant acceleration. Einstein's theories of relativity made revolutionary predictions. Most importantly, his predictions have been verified by experiments.

In this chapter, you learn how experiments and puzzling contradictions in existing theories led to the development of the theory of special relativity. You will also learn the simple postulates on which the theory was based; a postulate is a statement that is assumed to be true for the purposes of reasoning in a scientific or mathematic argument.

10.1 Postulates of Special Relativity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the experiments and scientific problems that led Albert Einstein to develop the special theory of relativity
- Understand the postulates on which the special theory of relativity was based

Section Key Terms

ether	frame of reference	inertial reference frame
general relativity	postulate	relativity
simultaneity	special relativity	

Scientific Experiments and Problems

Relativity is not new. Way back around the year 1600, Galileo explained that motion is relative. Wherever you happen to be, it seems like you are at a fixed point and that everything moves with respect to you. Everyone else feels the same way. Motion is always measured with respect to a fixed point. This is called establishing a **frame of reference**. But the choice of the point is arbitrary, and all frames of reference are equally valid. A passenger in a moving car is not moving with respect to the driver, but they are both moving from the point of view of a person on the sidewalk waiting for a bus. They are moving even faster as seen by a person in a car coming toward them. It is all relative.

TIPS FOR SUCCESS

A frame of reference is not a complicated concept. It is just something you decide is a fixed point or group of connected points. It is completely up to you. For example, when you look up at celestial objects in the sky, you choose the earth as your frame of reference, and the sun, moon, etc., seem to move across the sky.

Light is involved in the discussion of relativity because theories related to electromagnetism are inconsistent with Galileo's and Newton's explanation of relativity. The true nature of light was a hot topic of discussion and controversy in the late 19th century. At the time, it was not generally believed that light could travel across empty space. It was known to travel as waves, and all other types of energy that propagated as waves needed to travel though a material medium. It was believed that space was filled with an invisible medium that light waves traveled through. This imaginary (as it turned out) material was called the **ether** (also spelled aether). It was thought that everything moved through this mysterious fluid. In other words, ether was the one fixed frame of reference. The Michelson–Morley experiment proved it was not.

In 1887, Albert Michelson and Edward Morley designed the interferometer shown in Figure 10.2 to measure the speed of Earth through the ether. A light beam is split into two perpendicular paths and then recombined. Recombining the waves produces an inference pattern, with a bright fringe at the locations where the two waves arrive in phase; that is, with the crests of both waves arriving together and the troughs arriving together. A dark fringe appears where the crest of one wave coincides with a trough of the other, so that the two cancel. If Earth is traveling through the ether as it orbits the sun, the peaks in one arm would take longer than in the other to reach the same location. The places where the two waves arrive in phase would change, and the interference pattern would shift. But, using the interferometer, there was no shift seen! This result led to two conclusions: that there is no ether and that the speed of light is the same regardless of the relative motion of source and observer. The Michelson–Morley investigation has been called the most famous failed experiment in history.

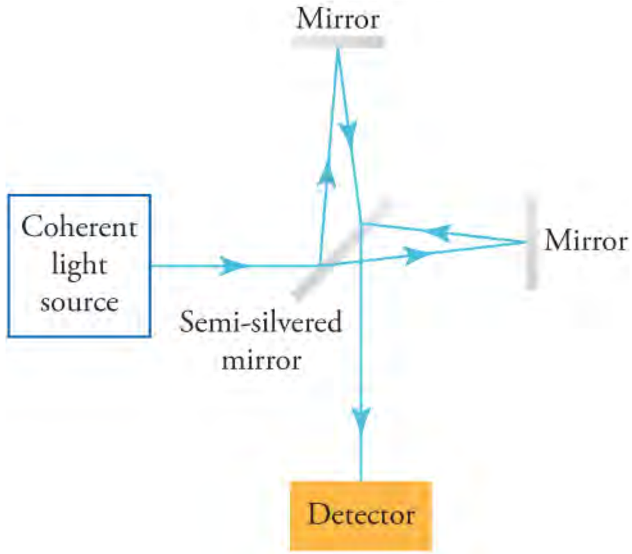


Figure 10.2 This is a diagram of the instrument used in the Michelson–Morley experiment.

To see what Michelson and Morley expected to find when they measured the speed of light in two directions, watch this animation (http://openstax.org/l/28MMexperiment) . In the video, two people swimming in a lake are represented as an analogy to light beams leaving Earth as it moves through the ether (if there were any ether). The swimmers swim away from and back to a platform that is moving through the water. The swimmers swim in different directions with respect to the motion of the platform. Even though they swim equal distances at the same speed, the motion of the platform causes them to arrive at different times.

Einstein's Postulates

The results described above left physicists with some puzzling and unsettling questions such as, why doesn't light emitted by a fast-moving object travel faster than light from a street lamp? A radical new theory was needed, and Albert Einstein, shown in Figure 10.3, was about to become everyone's favorite genius. Einstein began with two simple **postulates** based on the two things we have discussed so far in this chapter.

1. The laws of physics are the same in all inertial reference frames.
2. The speed of light is the same in all inertial reference frames and is not affected by the speed of its source.

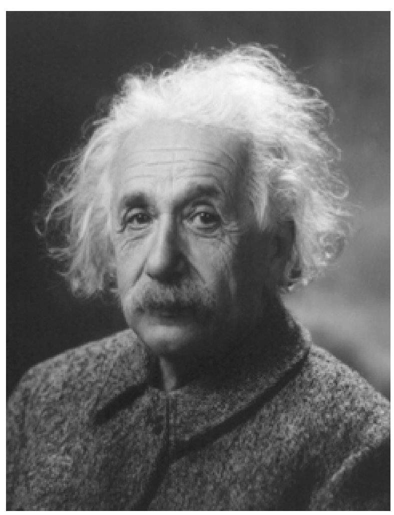


Figure 10.3 Albert Einstein (1879–1955) developed modern relativity and also made fundamental contributions to the foundations of quantum mechanics. (The Library of Congress)

The speed of light is given the symbol *c* and is equal to exactly 299,792,458 m/s. This is the speed of light in vacuum; that is, in the absence of air. For most purposes, we round this number off to 3.00×10^8 m/s The term **inertial reference frame** simply refers to a frame of reference where all objects follow Newton's first law of motion: Objects at rest remain at rest, and objects in motion remain in motion at a constant velocity in a straight line, unless acted upon by an external force. The inside of a car moving along a road at constant velocity and the inside of a stationary house are inertial reference frames.

WATCH PHYSICS

The Speed of Light

This lecture on light summarizes the most important facts about the speed of light. If you are interested, you can watch the whole video, but the parts relevant to this chapter are found between 3:25 and 5:10, which you find by running your cursor along the bottom of the video.

Click to view content (https://www.youtube.com/embed/rLNM8zI4Q_M)

GRASP CHECK

An airliner traveling at 200 m/s emits light from the front of the plane. Which statement describes the speed of the light?

a. It travels at a speed of $c + 200$ m/s.
b. It travels at a speed of $c - 200$ m/s.
c. It travels at a speed c, like all light.
d. It travels at a speed slightly less than c.

Snap Lab

Measure the Speed of Light

In this experiment, you will measure the speed of light using a microwave oven and a slice of bread. The waves generated by a microwave oven are not part of the visible spectrum, but they are still electromagnetic radiation, so they travel at the speed of light. If we know the wavelength, λ, and frequency, f, of a wave, we can calculate its speed, v, using the equation $v = \lambda f$. You can measure the wavelength. You will find the frequency on a label on the back of a microwave oven. The wave in a microwave is a standing wave with areas of high and low intensity. The high intensity sections are one-half wavelength apart.

- High temperature: Very hot temperatures are encountered in this lab. These can cause burns.

- a microwave oven
- one slice of plain white bread
- a centimeter ruler
- a calculator

1. Work with a partner.
2. Turn off the revolving feature of the microwave oven or remove the wheels under the microwave dish that make it turn. It is important that the dish does not turn.
3. Place the slice of bread on the dish, set the microwave on high, close the door, run the microwave for about 15 seconds.
4. A row of brown or black marks should appear on the bread. Stop the microwave as soon as they appear. Measure the distance between two adjacent burn marks and multiply the result by 2. This is the wavelength.
5. The frequency of the waves is written on the back of the microwave. Look for something like "2,450 MHz." Hz is the unit hertz, which means *per second*. The M represents mega, which stands for million, so multiply the number by 10^6.
6. Express the wavelength in meters and multiply it times the frequency. If you did everything correctly, you will get a number very close to the speed of light. Do not eat the bread. It is a general laboratory safety rule never to eat anything in the lab.

GRASP CHECK

How does your measured value of the speed of light compare to the accepted value (% error)?

a. The measured value of speed will be equal to c.
b. The measured value of speed will be slightly less than c.
c. The measured value of speed will be slightly greater than c.
d. The measured value of speed will depend on the frequency of the microwave.

Einstein's postulates were carefully chosen, and they both seemed very likely to be true. Einstein proceeded despite realizing that these two ideas taken together and applied to extreme conditions led to results that contradict Newtonian mechanics. He just took the ball and ran with it.

In the traditional view, velocities are additive. If you are running at 3 m/s and you throw a ball forward at a speed of 10 m/s, the ball should have a net speed of 13 m/s. However, according to relativity theory, the speed of a moving light source is not added to the speed of the emitted light.

In addition, Einstein's theory shows that if you were moving forward relative to Earth at nearly c (the speed of light) and could throw a ball forward at c, an observer at rest on the earth would not see the ball moving at nearly twice the speed of light. The observer would see it moving at a speed that is still less than c. This result conforms to both of Einstein's postulates: The speed of light has a fixed maximum and neither reference frame is privileged.

Consider how we measure elapsed time. If we use a stopwatch, for example, how do we know when to start and stop the watch? One method is to use the arrival of light from the event, such as observing a light turn green to start a drag race. The timing will be more accurate if some sort of electronic detection is used, avoiding human reaction times and other complications.

Now suppose we use this method to measure the time interval between two flashes of light produced by flash lamps on a moving train. (See Figure 10.4)

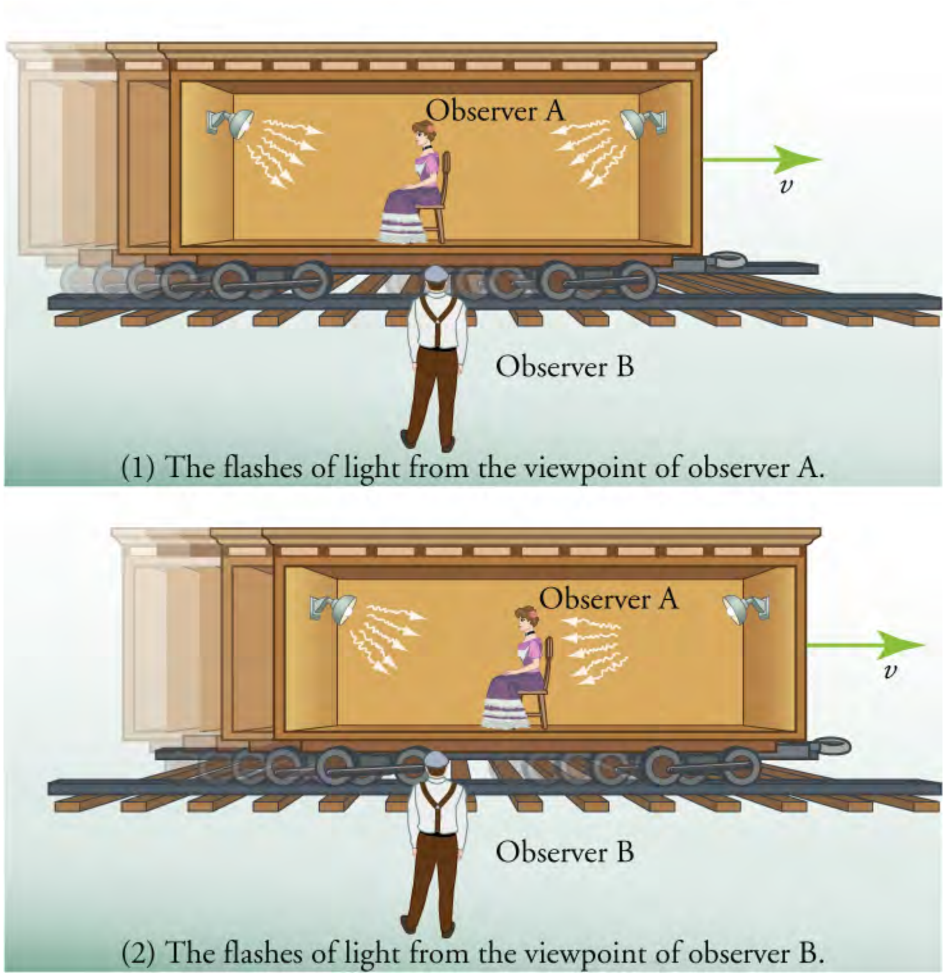


Figure 10.4 Light arriving to observer A as seen by two different observers.

A woman (observer A) is seated in the center of a rail car, with two flash lamps at opposite sides equidistant from her. Multiple light rays that are emitted from the flash lamps move towards observer A, as shown with arrows. A velocity vector arrow for the rail car is shown towards the right. A man (observer B) standing on the platform is facing the woman and also observes the flashes of light.

Observer A moves with the lamps on the rail car as the rail car moves towards the right of observer B. Observer B receives the light flashes simultaneously, and sees the bulbs as both having flashed at the same time. However, he sees observer A receive the flash from the right first. Because the pulse from the right reaches her first, in her frame of reference she sees the bulbs as not having flashed simultaneously. Here, a relative velocity between observers affects whether two events at well-separated locations are observed to be simultaneous. **Simultaneity**, or whether different events occur at the same instant, depends on the frame of reference of the observer. Remember that velocity equals distance divided by time, so $t = d/\mathbf{v}$. If velocity appears to be different, then duration of time appears to be different.

This illustrates the power of clear thinking. We might have guessed incorrectly that, if light is emitted simultaneously, then two observers halfway between the sources would see the flashes simultaneously. But careful analysis shows this not to be the case. Einstein was brilliant at this type of thought experiment (in German, *Gedankenexperiment*). He very carefully considered how an observation is made and disregarded what might seem obvious. The validity of thought experiments, of course, is determined by actual observation. The genius of Einstein is evidenced by the fact that experiments have repeatedly confirmed his theory of relativity. No experiments after that of Michelson and Morley were able to detect any ether medium. We will describe later how experiments also confirmed other predictions of **special relativity**, such as the distance between two objects and the time interval of two events being different for two observers moving with respect to each other.

In summary: Two events are defined to be simultaneous if an observer measures them as occurring at the same time (such as by receiving light from the events). Two events are not necessarily simultaneous to all observers.

The discrepancies between Newtonian mechanics and relativity theory illustrate an important point about how science advances. Einstein's theory did not replace Newton's but rather extended it. It is not unusual that a new theory must be developed to account for new information. In most cases, the new theory is built on the foundation of older theory. It is rare that old theories are completely replaced.

In this chapter, you will learn about the theory of special relativity, but, as mentioned in the introduction, Einstein developed two relativity theories: special and general. Table 10.1 summarizes the differences between the two theories.

Special Relativity	General Relativity
Published in 1905	Final form published in 1916
A theory of space-time	A theory of gravity
Applies to observers moving at constant speed	Applies to observers that are accelerating
Most useful in the field of nuclear physics	Most useful in the field of astrophysics
Accepted quickly and put to practical use by nuclear physicists and quantum chemists	Largely ignored until 1960 when new mathematical techniques made the theory more accessible and astronomers found some important applications
Also note that the theory of general relativity includes the theory of special relativity.	

Table 10.1 Comparing Special Relativity and General Relativity

WORKED EXAMPLE

Calculating the Time it Takes Light to Travel a Given Distance

The sun is 1.50×10^8 km from Earth. How long does it take light to travel from the sun to Earth in minutes and seconds?

Strategy

Identify knowns.

$$\begin{aligned} \text{Distance} &= 1.50 \times 10^8 \text{ km} \\ \text{Speed} &= 3.00 \times 10^8 \text{ km} \end{aligned} \tag{10.1}$$

Identify unknowns.

Time

Find the equation that relates knowns and unknowns.

$$v = \frac{d}{t};\quad t = \frac{d}{v} \tag{10.2}$$

Be sure to use consistent units.

Solution

$$t = \frac{d}{v} = \frac{(1.50 \times 10^8 km) \times \frac{10^3 \text{m}}{\text{km}}}{3.00 \times 10^8 \frac{\text{m}}{\text{s}}} = 5.00 \times 10^2 s$$

$$500 \text{ s} = 8 \text{ min and } 20 \text{ s}$$

Discussion

The answer is written as 5.00×10^2 rather than 500 in order to show that there are three significant figures. When astronomers witness an event on the sun, such as a sunspot, it actually happened minutes earlier. Compare 8 light *minutes* to the distance to stars, which are light *years* away. Any events on other stars happened years ago.

Practice Problems

1. Light travels through 1.00 m of water in 4.42×10^{-9} s. What is the speed of light in water?
   a. 4.42×10^{-9} m/s
   b. 4.42×10^{9} m/s
   c. 2.26×10^{8} m/s
   d. 226×10^{8} m/s
2. An astronaut on the moon receives a message from mission control on Earth. The signal is sent by a form of electromagnetic radiation and takes 1.28 s to travel the distance between Earth and the moon. What is the distance from Earth to the moon?
   a. 2.34×10^{5} km
   b. 2.34×10^{8} km
   c. 3.84×10^{5} km
   d. 3.84×10^{8} km

Check Your Understanding

3. Explain what is meant by a frame of reference.
   a. A frame of reference is a graph plotted between distance and time.
   b. A frame of reference is a graph plotted between speed and time.
   c. A frame of reference is the velocity of an object through empty space without regard to its surroundings.
   d. A frame of reference is an arbitrarily fixed point with respect to which motion of other points is measured.
4. Two people swim away from a raft that is floating downstream. One swims upstream and returns, and the other swims across the current and back. If this scenario represents the Michelson–Morley experiment, what do (i) the water, (ii) the swimmers, and (iii) the raft represent?
   a. the ether rays of light Earth
   b. rays of light the ether Earth
   c. the ether Earth rays of light
   d. Earth rays of light the ether
5. If Michelson and Morley had observed the interference pattern shift in their interferometer, what would that have indicated?
   a. The speed of light is the same in all frames of reference.
   b. The speed of light depends on the motion relative to the ether.
   c. The speed of light changes upon reflection from a surface.
   d. The speed of light in vacuum is less than 3.00×10^{8} m/s.
6. If you designate a point as being fixed and use that point to measure the motion of surrounding objects, what is the point called?
   a. An origin
   b. A frame of reference
   c. A moving frame
   d. A coordinate system

10.2 Consequences of Special Relativity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the relativistic effects seen in time dilation, length contraction, and conservation of relativistic momentum
- Explain and perform calculations involving mass-energy equivalence

Section Key Terms

binding energy	length contraction	mass defect	time dilation

proper length	relativistic	relativistic momentum
relativistic energy	relativistic factor	rest mass

Relativistic Effects on Time, Distance, and Momentum

Consideration of the measurement of elapsed time and simultaneity leads to an important relativistic effect. **Time dilation** is the phenomenon of time passing more slowly for an observer who is moving relative to another observer.

For example, suppose an astronaut measures the time it takes for light to travel from the light source, cross her ship, bounce off a mirror, and return. (See Figure 10.5.) How does the elapsed time the astronaut measures compare with the elapsed time measured for the same event by a person on the earth? Asking this question (another thought experiment) produces a profound result. We find that the elapsed time for a process depends on who is measuring it. In this case, the time measured by the astronaut is smaller than the time measured by the earth bound observer. The passage of time is different for the two observers because the distance the light travels in the astronaut's frame is smaller than in the earth bound frame. Light travels at the same speed in each frame, and so it will take longer to travel the greater distance in the earth bound frame.

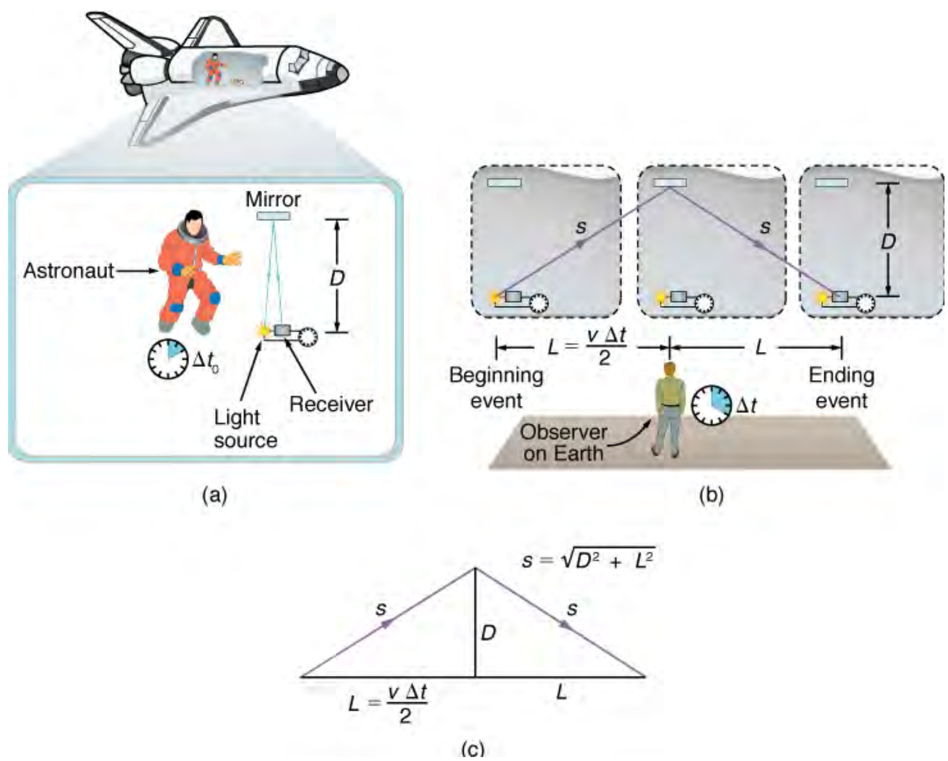


Figure 10.5 (a) An astronaut measures the time Δt_0 for light to cross her ship using an electronic timer. Light travels a distance $2D$ in the astronaut's frame. (b) A person on the earth sees the light follow the longer path $2s$ and take a longer time Δt.

The relationship between Δt and Δt_0 is given by

$$\Delta t = \gamma \Delta t_0,$$

where γ is the **relativistic factor** given by

$$\gamma = \frac{1}{\sqrt{1 - \frac{v^2}{c^2}}},$$

and v and c are the speeds of the moving observer and light, respectively.

TIPS FOR SUCCESS

Try putting some values for v into the expression for the relativistic factor (γ). Observe at which speeds this factor will make a difference and when γ is so close to 1 that it can be ignored. Try 225 m/s, the speed of an airliner; 2.98×10^4 m/s, the speed of Earth in its orbit; and 2.990×10^8 m/s, the speed of a particle in an accelerator.

Notice that when the velocity v is small compared to the speed of light c, then v/c becomes small, and γ becomes close to 1. When this happens, time measurements are the same in both frames of reference. **Relativistic** effects, meaning those that have to do with special relativity, usually become significant when speeds become comparable to the speed of light. This is seen to be the case for time dilation.

You may have seen science fiction movies in which space travelers return to Earth after a long trip to find that the planet and everyone on it has aged much more than they have. This type of scenario is a based on a thought experiment, known as the twin paradox, which imagines a pair of twins, one of whom goes on a trip into space while the other stays home. When the space traveler returns, she finds her twin has aged much more than she. This happens because the traveling twin has been in two frames of reference, one leaving Earth and one returning.

Time dilation has been confirmed by comparing the time recorded by an atomic clock sent into orbit to the time recorded by a clock that remained on Earth. GPS satellites must also be adjusted to compensate for time dilation in order to give accurate positioning.

Have you ever driven on a road, like that shown in Figure 10.6, that seems like it goes on forever? If you look ahead, you might say you have about 10 km left to go. Another traveler might say the road ahead looks like it is about 15 km long. If you both measured the road, however, you would agree. Traveling at everyday speeds, the distance you both measure would be the same. You will read in this section, however, that this is not true at relativistic speeds. Close to the speed of light, distances measured are not the same when measured by different observers moving with respect to one other.

Figure 10.6 People might describe distances differently, but at relativistic speeds, the distances really are different. (Corey Leopold, Flickr)

One thing all observers agree upon is their relative speed. When one observer is traveling away from another, they both see the other receding at the same speed, regardless of whose frame of reference is chosen. Remember that speed equals distance divided by time: $v = d/t$. If the observers experience a difference in elapsed time, they must also observe a difference in distance traversed. This is because the ratio d/t must be the same for both observers.

The shortening of distance experienced by an observer moving with respect to the points whose distance apart is measured is called **length contraction**. **Proper length**, L_0, is the distance between two points measured in the reference frame where the observer and the points are at rest. The observer in motion with respect to the points measures L. These two lengths are related by the equation

$$L = \frac{L_0}{\gamma}.$$

Because γ is the same expression used in the time dilation equation above, the equation becomes

$$L = L_0\sqrt{1 - \frac{v^2}{c^2}}.$$

To see how length contraction is seen by a moving observer, go to this simulation (http://openstax.org/l/28simultaneity) . Here you can also see that simultaneity, time dilation, and length contraction are interrelated phenomena.

This link is to a simulation that illustrates the relativity of simultaneous events.

In classical physics, momentum is a simple product of mass and velocity. When special relativity is taken into account, objects that have mass have a speed limit. What effect do you think mass and velocity have on the momentum of objects moving at relativistic speeds; i.e., speeds close to the speed of light?

Momentum is one of the most important concepts in physics. The broadest form of Newton's second law is stated in terms of momentum. Momentum is conserved in classical mechanics whenever the net external force on a system is zero. This makes momentum conservation a fundamental tool for analyzing collisions. We will see that momentum has the same importance in modern physics. **Relativistic momentum** is conserved, and much of what we know about subatomic structure comes from the analysis of collisions of accelerator-produced relativistic particles.

One of the postulates of special relativity states that the laws of physics are the same in all inertial frames. Does the law of conservation of momentum survive this requirement at high velocities? The answer is yes, provided that the momentum is defined as follows.

Relativistic momentum, $\mathbf{p}$, is classical momentum multiplied by the relativistic factor γ.

$$\mathbf{p} = \gamma m\mathbf{u}, \tag{10.3}$$

where m is the **rest mass** of the object (that is, the mass measured at rest, without any γ factor involved), $\mathbf{u}$ is its velocity relative to an observer, and γ, as before, is the relativistic factor. We use the mass of the object as measured at rest because we cannot determine its mass while it is moving.

Note that we use $\mathbf{u}$ for velocity here to distinguish it from relative velocity $\mathbf{v}$ between observers. Only one observer is being considered here. With $\mathbf{p}$ defined in this way, $\mathbf{p}_{\text{tot}}$ is conserved whenever the net external force is zero, just as in classical physics. Again we see that the relativistic quantity becomes virtually the same as the classical at low velocities. That is, relativistic momentum $\gamma m\mathbf{u}$ becomes the classical $m\mathbf{u}$ at low velocities, because γ is very nearly equal to 1 at low velocities.

Relativistic momentum has the same intuitive feel as classical momentum. It is greatest for large masses moving at high velocities. Because of the factor γ, however, relativistic momentum behaves differently from classical momentum by approaching infinity as $\mathbf{u}$ approaches c. (See Figure 10.7.) This is another indication that an object with mass cannot reach the speed of light. If it did, its momentum would become infinite, which is an unreasonable value.

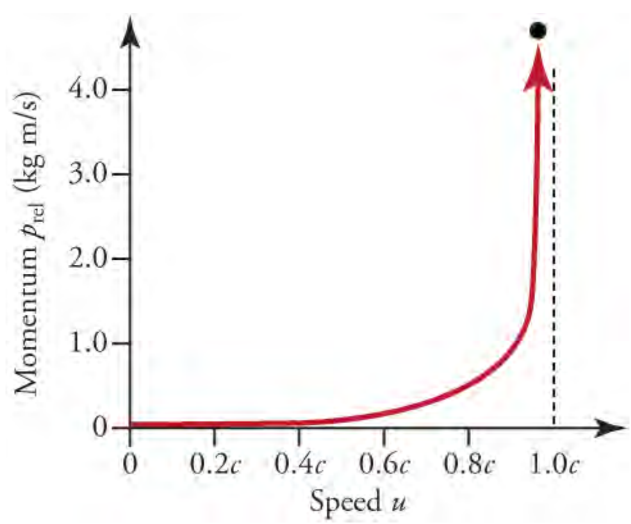


Figure 10.7 Relativistic momentum approaches infinity as the velocity of an object approaches the speed of light.

Relativistic momentum is defined in such a way that the conservation of momentum will hold in all inertial frames. Whenever the net external force on a system is zero, relativistic momentum is conserved, just as is the case for classical momentum. This has been verified in numerous experiments.

Mass-Energy Equivalence

Let us summarize the calculation of relativistic effects on objects moving at speeds near the speed of light. In each case we will need to calculate the relativistic factor, given by

$$\gamma = \frac{1}{\sqrt{1 - \frac{\mathbf{v}^2}{c^2}}},$$

where **v** and c are as defined earlier. We use **u** as the velocity of a particle or an object in one frame of reference, and **v** for the velocity of one frame of reference with respect to another.

Time Dilation

Elapsed time on a moving object, Δt_0, as seen by a stationary observer is given by $\Delta t = \gamma \Delta t_0$, where Δt_0 is the time observed on the moving object when it is taken to be the frame or reference.

Length Contraction

Length measured by a person at rest with respect to a moving object, L, is given by

$$L = \frac{L_0}{\gamma},$$

where L_0 is the length measured on the moving object.

Relativistic Momentum

Momentum, $\mathbf{p}$, of an object of mass, m, traveling at relativistic speeds is given by $\mathbf{p} = \gamma m \mathbf{u}$, where **u** is velocity of a moving object as seen by a stationary observer.

Relativistic Energy

The original source of all the energy we use is the conversion of mass into energy. Most of this energy is generated by nuclear reactions in the sun and radiated to Earth in the form of electromagnetic radiation, where it is then transformed into all the forms with which we are familiar. The remaining energy from nuclear reactions is produced in nuclear power plants and in Earth's interior. In each of these cases, the source of the energy is the conversion of a small amount of mass into a large amount of energy. These sources are shown in Figure 10.8.

Figure 10.8 The sun (a) and the Susquehanna Steam Electric Station (b) both convert mass into energy. ((a) NASA/Goddard Space Flight Center, Scientific Visualization Studio; (b) U.S. government)

The first postulate of relativity states that the laws of physics are the same in all inertial frames. Einstein showed that the law of conservation of energy is valid relativistically, if we define energy to include a relativistic factor. The result of his analysis is that a particle or object of mass m moving at velocity **u** has **relativistic energy** given by

$$E = \gamma mc^2.$$

This is the expression for the total energy of an object of mass m at any speed **u** and includes both kinetic and potential energy. Look back at the equation for γ and you will see that it is equal to 1 when **u** is 0; that is, when an object is at rest. Then the rest energy, E_0, is simply

$$E_0 = mc^2.$$

This is the correct form of Einstein's famous equation.

This equation is very useful to nuclear physicists because it can be used to calculate the energy released by a nuclear reaction. This is done simply by subtracting the mass of the products of such a reaction from the mass of the reactants. The difference is the m in $E_0 = mc^2$. Here is a simple example:

A positron is a type of antimatter that is just like an electron, except that it has a positive charge. When a positron and an electron collide, their masses are completely annihilated and converted to energy in the form of gamma rays. Because both particles have a rest mass of 9.11×10^{-31} kg, we multiply the mc^2 term by 2. So the energy of the gamma rays is

$$\begin{aligned} E_0 &= 2(9.11 \times 10^{-31}\ \text{kg})\left(3.00 \times 10^8\ \tfrac{\text{m}}{\text{s}}\right)^2 \\ &= 1.64 \times 10^{-13}\ \tfrac{\text{kg}\cdot\text{m}^2}{\text{s}^2} \\ &= 1.64 \times 10^{-13}\ \text{J} \end{aligned} \tag{10.4}$$

where we have the expression for the joule (J) in terms of its SI base units of kg, m, and s. In general, the nuclei of stable isotopes have less mass then their constituent subatomic particles. The energy equivalent of this difference is called the **binding energy** of the nucleus. This energy is released during the formation of the isotope from its constituent particles because the product is more stable than the reactants. Expressed as mass, it is called the **mass defect**. For example, a helium nucleus is made of two neutrons and two protons and has a mass of 4.0003 atomic mass units (u). The sum of the masses of two protons and two neutrons is 4.0330 u. The mass defect then is 0.0327 u. Converted to kg, the mass defect is 5.0442×10^{-30} kg. Multiplying this mass times c^2 gives a binding energy of 4.540×10^{-12} J. This does not sound like much because it is only one atom. If you were to make one gram of helium out of neutrons and protons, it would release 683,000,000,000 J. By comparison, burning one gram of coal releases about 24 J.

BOUNDLESS PHYSICS

The RHIC Collider

Figure 10.9 shows the Brookhaven National Laboratory in Upton, NY. The circular structure houses a particle accelerator called the RHIC, which stands for Relativistic Heavy Ion Collider. The heavy ions in the name are gold nuclei that have been stripped of their electrons. Streams of ions are accelerated in several stages before entering the big ring seen in the figure. Here, they are accelerated to their final speed, which is about 99.7 percent the speed of light. Such high speeds are called relativistic. All the relativistic phenomena we have been discussing in this chapter are very pronounced in this case. At this speed $\gamma = 12.9$, so that relativistic time dilates by a factor of about 13, and relativistic length contracts by the same factor.

Figure 10.9 Brookhaven National Laboratory. The circular structure houses the RHIC. (energy.gov, Wikimedia Commons)

Two ion beams circle the 2.4-mile long track around the big ring in opposite directions. The paths can then be made to cross, thereby causing ions to collide. The collision event is very short-lived but amazingly intense. The temperatures and pressures produced are greater than those in the hottest suns. At 4 trillion degrees Celsius, this is the hottest material ever created in a laboratory

But what is the point of creating such an extreme event? Under these conditions, the neutrons and protons that make up the gold nuclei are smashed apart into their components, which are called quarks and gluons. The goal is to recreate the conditions that theorists believe existed at the very beginning of the universe. It is thought that, at that time, matter was a sort of soup of quarks and gluons. When things cooled down after the initial bang, these particles condensed to form protons and neutrons.

Some of the results have been surprising and unexpected. It was thought the quark-gluon soup would resemble a gas or plasma. Instead, it behaves more like a liquid. It has been called a *perfect* liquid because it has virtually no viscosity, meaning that it has no resistance to flow.

GRASP CHECK

Calculate the relativistic factor γ, for a particle traveling at 99.7 percent of the speed of light.

a. 0.08
b. 0.71
c. 1.41
d. 12.9

WORKED EXAMPLE

The Speed of Light

One night you are out looking up at the stars and an extraterrestrial spaceship flashes across the sky. The ship is 50 meters long and is travelling at 95 percent of the speed of light. What would the ship's length be when measured from your earthbound frame of reference?

Strategy

List the knowns and unknowns.

Knowns: proper length of the ship, $L_0 = 50$ m; velocity, $\mathbf{v}$, = 0.95c

Unknowns: observed length of the ship accounting for relativistic length contraction, L.

Choose the relevant equation.

$$L = \frac{L_0}{\gamma} = L_0\sqrt{1 - \frac{\mathbf{u}^2}{c^2}}$$

Solution

$$L = 50\text{ m}\sqrt{1 - \frac{(0.95)^2 c^2}{c^2}} = 50\text{ m}\sqrt{1 - (0.95)^2} = 16\text{ m}$$

Discussion

Calculations of γ can usually be simplified in this way when v is expressed as a percentage of c because the c^2 terms cancel. Be sure to also square the decimal representing the percentage before subtracting from 1. Note that the aliens will still see the length as L_0 because they are moving with the frame of reference that is the ship.

Practice Problems

7. Calculate the relativistic factor, γ, for an object traveling at 2.00×10^8 m/s.
   a. 0.74
   b. 0.83
   c. 1.2
   d. 1.34

8. The distance between two points, called the proper length, L0, is 1.00 km. An observer in motion with respect to the frame of

reference of the two points measures 0.800 km, which is L. What is the relative speed of the frame of reference with respect to the observer?

a. 1.80×10^8 m/s
b. 2.34×10^8 m/s
c. 3.84×10^8 m/s
d. 5.00×10^8 m/s

9. Consider the nuclear fission reaction $n + {}^{235}_{92}U \rightarrow {}^{137}_{55}Cs + {}^{97}_{37}Rb + 2n + E$. If a neutron has a rest mass of 1.009u, ${}^{235}_{92}U$ has a rest mass of 235.044u, ${}^{137}_{55}Cs$ has rest mass of 136.907u, and ${}^{97}_{37}Rb$ has a rest mass of 96.937u, what is the value of E in joules?
   a. 1.8×10^{-11} J
   b. 2.9×10^{-11} J
   c. 1.8×10^{-10} J
   d. 2.9×10^{-10} J

Solution

The correct answer is (b). The mass deficit in the reaction is $235.044\text{ u} - (136.907 + 96.937 + 1.009)\text{ u}$, or 0.191u. Converting that mass to kg and applying $E = mc^2$ to find the energy equivalent of the mass deficit gives $(0.191\text{ u})\left(1.66 \times 10^{-27}\text{kg/u}\right)\left(3.00 \times 10^8\text{m/s}\right)^2 \cong 2.85 \times 10^{-11}\text{J}$.

10. Consider the nuclear fusion reaction ${}^{2}_{1}H + {}^{2}_{1}H \rightarrow {}^{3}_{1}H + {}^{1}_{1}H + E$. If ${}^{2}_{1}H$ has a rest mass of 2.014u, ${}^{3}_{1}H$ has a rest mass of 3.016u, and ${}^{1}_{1}H$ has a rest mass of 1.008u, what is the value of E in joules?
    a. 6×10^{-13} J
    b. 6×10^{-12} J
    c. 6×10^{-11} J
    d. 6×10^{-10} J

Solution

The correct answer is (a). The mass deficit in the reaction is $2\,(2.014\text{ u}) - (3.016 + 1.008)\text{ u}$, or 0.004u. Converting that mass to kg and applying $E = mc^2$ to find the energy equivalent of the mass deficit gives $(0.004\text{ u})\left(1.66 \times 10^{-27}\text{kg/u}\right)\left(3.00 \times 10^8\text{m/s}\right)^2 \cong 5.98 \times 10^{-13}\text{J}$.

Check Your Understanding

11. Describe time dilation and state under what conditions it becomes significant.
    a. When the speed of one frame of reference past another reaches the speed of light, a time interval between two events at the same location in one frame appears longer when measured from the second frame.
    b. When the speed of one frame of reference past another becomes comparable to the speed of light, a time interval between two events at the same location in one frame appears longer when measured from the second frame.
    c. When the speed of one frame of reference past another reaches the speed of light, a time interval between two events at the same location in one frame appears shorter when measured from the second frame.
    d. When the speed of one frame of reference past another becomes comparable to the speed of light, a time interval between two events at the same location in one frame appears shorter when measured from the second frame.

12. The equation used to calculate relativistic momentum is $p = \gamma \cdot m \cdot u$. Define the terms to the right of the equal sign and state how m and u are measured.
    a. γ is the relativistic factor, m is the rest mass measured when the object is at rest in the frame of reference, and u is the velocity of the frame.
    b. γ is the relativistic factor, m is the rest mass measured when the object is at rest in the frame of reference, and u is the velocity relative to an observer.
    c. γ is the relativistic factor, m is the relativistic mass $\left(\text{i.e.,} \frac{m}{\sqrt{1-\frac{u^2}{c^2}}}\right)$ measured when the object is moving in the frame of reference, and u is the velocity of the frame.

d. γ is the relativistic factor, m is the relativistic mass $\left(\text{i.e.,} \frac{m}{\sqrt{1-\frac{u^2}{c^2}}}\right)$ measured when the object is moving in the frame of reference, and u is the velocity relative to an observer.

13. Describe length contraction and state when it occurs.

a. When the speed of an object becomes the speed of light, its length appears to shorten when viewed by a stationary observer.
b. When the speed of an object approaches the speed of light, its length appears to shorten when viewed by a stationary observer.
c. When the speed of an object becomes the speed of light, its length appears to increase when viewed by a stationary observer.
d. When the speed of an object approaches the speed of light, its length appears to increase when viewed by a stationary observer.

KEY TERMS

binding energy the energy equivalent of the difference between the mass of a nucleus and the masses of its nucleons

ether scientists once believed there was a medium that carried light waves; eventually, experiments proved that ether does not exist

frame of reference the point or collection of points arbitrarily chosen, which motion is measured in relation to

general relativity the theory proposed to explain gravity and acceleration

inertial reference frame a frame of reference where all objects follow Newton's first law of motion

length contraction the shortening of an object as seen by an observer who is moving relative to the frame of reference of the object

mass defect the difference between the mass of a nucleus and the masses of its nucleons

postulate a statement that is assumed to be true for the purposes of reasoning in a scientific or mathematic argument

proper length the length of an object within its own frame of reference, as opposed to the length observed by an observer moving relative to that frame of reference

relativistic having to do with modern relativity, such as the effects that become significant only when an object is moving close enough to the speed of light for γ to be significantly greater than 1

relativistic energy the total energy of a moving object or particle $E = \gamma mc^2$, which includes both its rest energy mc^2 and its kinetic energy

relativistic factor $\gamma = \frac{1}{\sqrt{1-\frac{\mathbf{u}^2}{c^2}}}$, where **u** is the velocity of a moving object and c is the speed of light

relativistic momentum $\mathbf{p} = \gamma m\mathbf{u}$, where γ is the relativistic factor, m is rest mass of an object, and **u** is the velocity relative to an observer

relativity the explanation of how objects move relative to one another

rest mass the mass of an object that is motionless with respect to its frame of reference

simultaneity the property of events that occur at the same time

special relativity the theory proposed to explain the consequences of requiring the speed of light and the laws of physics to be the same in all inertial frames

time dilation the contraction of time as seen by an observer in a frame of reference that is moving relative to the observer

SECTION SUMMARY

10.1 Postulates of Special Relativity

- One postulate of special relativity theory is that the laws of physics are the same in all inertial frames of reference.
- The other postulate is that the speed of light in a vacuum is the same in all inertial frames.
- Einstein showed that simultaneity, or lack of it, depends on the frame of reference of the observer.

10.2 Consequences of Special Relativity

- Time dilates, length contracts, and momentum increases as an object approaches the speed of light.
- Energy and mass are interchangeable, according to the relationship $E = mc2$. The laws of conservation of mass and energy are combined into the law of conservation of mass-energy.

KEY EQUATIONS

10.1 Postulates of Special Relativity

speed of light	$v = \lambda f$
constant value for the speed of light	$c = 3.00 \times 10^8 \mathrm{m/s}$

10.2 Consequences of Special Relativity

elapsed time	$\Delta t = \gamma \Delta t_0$
relativistic factor	$\gamma = \frac{1}{\sqrt{1-\frac{\mathbf{u}^2}{c^2}}}$
length contraction	$L = \frac{L_0}{\gamma}$
relativistic momentum	$\mathbf{p} = \gamma m\mathbf{u}$

relativistic energy	$E = \gamma mc^2$	rest energy	$E_0 = mc^2$

CHAPTER REVIEW

Concept Items

10.1 Postulates of Special Relativity

1. Why was it once believed that light must travel through a medium and could not propagate across empty space?
   a. The longitudinal nature of light waves implies this.
   b. Light shows the phenomenon of diffraction.
   c. The speed of light is the maximum possible speed.
   d. All other wave energy needs a medium to travel.

2. Describe the relative motion of Earth and the sun:
   1. if Earth is taken as the inertial frame of reference and
   2. if the sun is taken as the inertial frame of reference.
      a. 1. Earth is at rest and the sun orbits Earth.
         2. The sun is at rest and Earth orbits the sun.
      b. 1. The sun is at rest and Earth orbits the sun.
         2. Earth is at rest and the sun orbits Earth.
      c. 1. The sun is at rest and Earth orbits the sun.
         2. The sun is at rest and Earth orbits the sun.
      d. 1. Earth is at rest and the sun orbits Earth.
         2. Earth is at rest and the sun orbits Earth.

10.2 Consequences of Special Relativity

3. A β particle (a free electron) is speeding around the track in a cyclotron, rapidly gaining speed. How will the particle's momentum change as its speed approaches the speed of light? Explain.
   a. The particle's momentum will rapidly decrease.
   b. The particle's momentum will rapidly increase.
   c. The particle's momentum will remain constant.
   d. The particle's momentum will approach zero.

4. An astronaut goes on a long space voyage at near the speed of light. When she returns home, how will her age compare to the age of her twin who stayed on Earth?
   a. Both of them will be the same age.
   b. This is a paradox and hence the ages cannot be compared.
   c. The age of the twin who traveled will be less than the age of her twin.
   d. The age of the twin who traveled will be greater than the age of her twin.

5. A comet reaches its greatest speed as it travels near the sun. True or false— Relativistic effects make the comet's tail look longer to an observer on Earth.
   a. True
   b. False

Critical Thinking Items

10.1 Postulates of Special Relativity

6. Explain how the two postulates of Einstein's theory of special relativity, when taken together, could lead to a situation that seems to contradict the mechanics and laws of motion as described by Newton.
   a. In Newtonian mechanics, velocities are multiplicative but the speed of a moving light source cannot be multiplied to the speed of light because, according to special relativity, the speed of light is the maximum speed possible.
   b. In Newtonian mechanics, velocities are additive but the speed of a moving light source cannot be added to the speed of light because the speed of light is the maximum speed possible.
   c. An object that is at rest in one frame of reference may appear to be in motion in another frame of reference, while in Newtonian mechanics such a situation is not possible.
   d. The postulates of Einstein's theory of special relativity do not contradict any situation that Newtonian mechanics explains.

7. It takes light 6.0 minutes to travel from the sun to the planet Venus. How far is Venus from the sun?
   a. $18 \times 10^6 \text{ km}$
   b. $18 \times 10^8 \text{ km}$
   c. $1.08 \times 10^{11} \text{ km}$
   d. $1.08 \times 10^8 \text{ km}$

8. In 2003, Earth and Mars were the closest they had been in 50,000 years. The two planets were aligned so that Earth was between Mars and the sun. At that time it took light from the sun 500 s to reach Earth and 687 s to get to Mars. What was the distance from Mars to Earth?
   a. 5.6×10^7 km
   b. 5.6×10^{10} km
   c. 6.2×10^6 km
   d. 6.2×10^{12} km

9. Describe two ways in which light differs from all other forms of wave energy.

a. 1. Light travels as a longitudinal wave.
   2. Light travels through a medium that fills up the empty space in the universe.

b. 1. Light travels as a transverse wave.
   2. Light travels through a medium that fills up the empty space in the universe.

c. 1. Light travels at the maximum possible speed in the universe.
   2. Light travels through a medium that fills up the empty space in the universe.

d. 1. Light travels at the maximum possible speed in the universe.
   2. Light does not require any material medium to travel.

10. Use the postulates of the special relativity theory to explain why the speed of light emitted from a fast-moving light source cannot exceed 3.00×10^8 m/s.

a. The speed of light is maximum in the frame of reference of the moving object.
b. The speed of light is minimum in the frame of reference of the moving object.
c. The speed of light is the same in all frames of reference, including in the rest frame of its source.
d. Light always travels in a vacuum with a speed less than 3.00×10^8 m/s, regardless of the speed of the source.

10.2 Consequences of Special Relativity

11. Halley's Comet comes near Earth every 75 years as it travels around its 22 billion km orbit at a speed of up to 700, 000 m/s. If it were possible to put a clock on the comet and read it each time the comet passed, which part of special relativity theory could be tested? What would be the expected result? Explain.

a. It would test time dilation. The clock would appear to be slightly slower.
b. It would test time dilation. The clock would appear to be slightly faster.
c. It would test length contraction. The length of the orbit would appear to be shortened from Earth's frame of reference.
d. It would test length contraction. The length of the orbit would appear to be shortened from the comet's frame of reference.

12. The nucleus of the isotope fluorine-18 (^{18} F) has mass defect of 2.44×10^{-28} kg. What is the binding energy of ^{18}F?

a. 2.2×10^{-11} J
b. 7.3×10^{-20} J
c. 2.2×10^{-20} J
d. 2.4×10^{-28} J

Problems

10.2 Consequences of Special Relativity

13. Deuterium (2 H) is an isotope of hydrogen that has one proton and one neutron in its nucleus. The binding energy of deuterium is 3.56×10^{-13} J. What is the mass defect of deuterium?

a. 3.20×10^{-4} kg
b. 1.68×10^{-6} kg
c. 1.19×10^{-21} kg
d. 3.96×10^{-30} kg

14. The sun orbits the center of the galaxy at a speed of 2.3×10^5 m/s. The diameter of the sun is 1.391684×109 m. An observer is in a frame of reference that is stationary with respect to the center of the galaxy. True or false—The sun is moving fast enough for the observer to notice length contraction of the sun's diameter.

a. True
b. False

15. Consider the nuclear fission reaction $n + {}^{235}_{92}U \rightarrow {}^{144}_{56}Ba + {}^{89}_{36}Kr + 3n + E$. If a neutron has a rest mass of 1.009u, ${}^{235}_{92}U$ has a rest mass of 235.044u, ${}^{144}_{56}Ba$ has rest mass of 143.923u, and ${}^{89}_{36}Kr$ has a rest mass of 88.918u, what is the value of E in joules?

a. 1.8×10^{-11} J
b. 2.8×10^{-11} J
c. 1.8×10^{-10} J
d. 3.3×10^{-10} J

16. Consider the nuclear fusion reaction ${}^{2}_{1}H + {}^{3}_{1}H \rightarrow {}^{4}_{2}He + n + E$. If ${}^{2}_{1}H$ has a rest mass of 2.014u, ${}^{3}_{1}H$ has a rest mass of 3.016u, ${}^{4}_{2}He$ has a rest mass of 4.003u, and a neutron has a rest mass of 1.009u, what is the value of E in joules?

a. 2.7×10^{-14} J
b. 2.7×10^{-13} J
c. 2.7×10^{-12} J
d. 2.7×10^{-11} J

Performance Task

10.2 Consequences of Special Relativity

17. People are fascinated by the possibility of traveling across the universe to discover intelligent life on other planets. To do this, we would have to travel enormous distances. Suppose we could somehow travel at up to 90 percent of the speed of light. The closest star is Alpha Centauri, which is 4.37 light years away. (A light year is the distance light travels in one year.)

a. How long, from the point of view of people on Earth, would it take a space ship to travel to Alpha Centauri and back at 0.9*c*?
b. How much would the astronauts on the spaceship have aged by the time they got back to Earth?
c. Discuss the problems related to travel to stars that are 20 or 30 light years away. Assume travel speeds near the speed of light.

TEST PREP

Multiple Choice

10.1 Postulates of Special Relativity

18. What was the purpose of the Michelson–Morley experiment?
a. To determine the exact speed of light
b. To analyze the electromagnetic spectrum
c. To establish that Earth is the true frame of reference
d. To learn how the ether affected the propagation of light

19. What is the speed of light in a vacuum to three significant figures?
a. 1.86×10^5 m/s
b. 3.00×10^8 m/s
c. 6.71×10^8 m/s
d. 1.50×10^{11} m/s

20. How far does light travel in 1.00 min?
a. 1.80×10^7 km
b. 1.80×10^{13} km
c. 5.00×10^6 m
d. 5.00×10^8 m

21. Describe what is meant by the sentence, “Simultaneity is not absolute.”
a. Events may appear simultaneous in all frames of reference.
b. Events may not appear simultaneous in all frames of reference.
c. The speed of light is not the same in all frames of reference.
d. The laws of physics may be different in different inertial frames of reference.

22. In 2003, Earth and Mars were aligned so that Earth was between Mars and the sun. Earth and Mars were 5.6×10^7 km from each other, which was the closest they had been in 50,000 years. People looking up saw Mars as a very bright red light on the horizon. If Mars was 2.06×10^8 km from the sun, how long did the reflected light people saw take to travel from the sun to Earth?
a. 14 min and 33 s
b. 12 min and 15 s
c. 11 min and 27 s
d. 3 min and 7 s

10.2 Consequences of Special Relativity

23. What does this expression represent: $\frac{1}{\sqrt{1-\frac{u^2}{c^2}}}$
a. time dilation
b. relativistic factor
c. relativistic energy
d. length contraction

24. What is the rest energy, E_0, of an object with a mass of 1.00 g ?
a. 3.00×10^5 J
b. 3.00×10^{11} J
c. 9.00×10^{13} J
d. 9.00×10^{16} J

25. The fuel rods in a nuclear reactor must be replaced from time to time because so much of the radioactive material has reacted that they can no longer produce energy. How would the mass of the spent fuel rods compare to their mass when they were new? Explain your answer.
a. The mass of the spent fuel rods would decrease.
b. The mass of the spent fuel rods would increase.
c. The mass of the spent fuel rods would remain the same.
d. The mass of the spent fuel rods would become close to zero.

Short Answer

10.1 Postulates of Special Relativity

26. What is the postulate having to do with the speed of light on which the theory of special relativity is based?
   a. The speed of light remains the same in all inertial frames of reference.
   b. The speed of light depends on the speed of the source emitting the light.
   c. The speed of light changes with change in medium through which it travels.
   d. The speed of light does not change with change in medium through which it travels.

27. What is the postulate having to do with reference frames on which the theory of special relativity is based?
   a. The frame of reference chosen is arbitrary as long as it is inertial.
   b. The frame of reference is chosen to have constant nonzero acceleration.
   c. The frame of reference is chosen in such a way that the object under observation is at rest.
   d. The frame of reference is chosen in such a way that the object under observation is moving with a constant speed.

28. If you look out the window of a moving car at houses going past, you sense that you are moving. What have you chosen as your frame of reference?
   a. the car
   b. the sun
   c. a house

29. Why did Michelson and Morley orient light beams at right angles to each other?
   a. To observe the particle nature of light
   b. To observe the effect of the passing ether on the speed of light
   c. To obtain a diffraction pattern by combination of light
   d. To obtain a constant path difference for interference of light

10.2 Consequences of Special Relativity

30. What is the relationship between the binding energy and the mass defect of an atomic nucleus?
   a. The binding energy is the energy equivalent of the mass defect, as given by $E_0 = mc$.
   b. The binding energy is the energy equivalent of the mass defect, as given by $E_0 = mc^2$.
   c. The binding energy is the energy equivalent of the mass defect, as given by $E_0 = \frac{m}{c}$
   d. The binding energy is the energy equivalent of the mass defect, as given by $E_0 = \frac{m}{c^2}$.

31. True or false—It is possible to just use the relationships $F = ma$ and $E = Fd$ to show that both sides of the equation $E_0 = mc^2$ have the same units.
   a. True
   b. False

32. Explain why the special theory of relativity caused the law of conservation of energy to be modified.
   a. The law of conservation of energy is not valid in relativistic mechanics.
   b. The law of conservation of energy has to be modified because of time dilation.
   c. The law of conservation of energy has to be modified because of length contraction.
   d. The law of conservation of energy has to be modified because of mass-energy equivalence.

33. The sun loses about 4×10^9 kg of mass every second. Explain in terms of special relativity why this is happening.
   a. The sun loses mass because of its high temperature.
   b. The sun loses mass because it is continuously releasing energy.
   c. The Sun loses mass because the diameter of the sun is contracted.
   d. The sun loses mass because the speed of the sun is very high and close to the speed of light.

Extended Response

10.1 Postulates of Special Relativity

34. Explain how Einstein's conclusion that nothing can travel faster than the speed of light contradicts an older concept about the speed of an object propelled from another, already moving, object.
   a. The older concept is that speeds are subtractive. For example, if a person throws a ball while running, the speed of the ball relative to the ground is the speed at which the person was running minus the speed of the throw. A relativistic example is when light is emitted from car headlights, it moves faster than the speed of light emitted from a stationary source.
   b. The older concept is that speeds are additive. For example, if a person throws a ball while running, the speed of the ball relative to the ground is the speed at which the person was running plus the speed of the throw. A relativistic example is when light is emitted from car headlights, it moves no

faster than the speed of light emitted from a stationary source. The car's speed does not affect the speed of light.

c. The older concept is that speeds are multiplicative. For example, if a person throws a ball while running, the speed of the ball relative to the ground is the speed at which the person was running multiplied by the speed of the throw. A relativistic example is when light is emitted from car headlights, it moves no faster than the speed of light emitted from a stationary source. The car's speed does not affect the speed of light.

d. The older concept is that speeds are frame independent. For example, if a person throws a ball while running, the speed of the ball relative to the ground has nothing to do with the speed at which the person was running. A relativistic example is when light is emitted from car headlights, it moves no faster than the speed of light emitted from a stationary source. The car's speed does not affect the speed of light.

35. A rowboat is drifting downstream. One person swims 20 m toward the shore and back, and another, leaving at the same time, swims upstream 20 m and back to the boat. The swimmer who swam toward the shore gets back first. Explain how this outcome is similar to the outcome expected in the Michelson–Morley experiment.

a. The rowboat represents Earth, the swimmers are beams of light, and the water is acting as the ether. Light going against the current of the ether would get back later because, by then, Earth would have moved on.

b. The rowboat represents the beam of light, the swimmers are the ether, and water is acting as Earth. Light going against the current of the ether would get back later because, by then, Earth would have moved on.

c. The rowboat represents the ether, the swimmers are ray of light, and the water is acting as the earth. Light going against the current of the ether would get back later because, by then, Earth would have moved on.

d. The rowboat represents the Earth, the swimmers are the ether, and the water is acting as the rays of light. Light going against the current of the ether would get back later because, by then, Earth would have moved on.

10.2 Consequences of Special Relativity

36. A helium-4 nucleus is made up of two neutrons and two protons. The binding energy of helium-4 is 4.53×10^{-12} J. What is the difference in the mass of this helium nucleus and the sum of the masses of two neutrons and two protons? Which weighs more, the nucleus or its constituents?

a. 1.51×10^{-20} kg; the constituents weigh more
b. 5.03×10^{-29} kg; the constituents weigh more
c. 1.51×10^{-29} kg; the nucleus weighs more
d. 5.03×10^{-29} kg; the nucleus weighs more

37. Use the equation for length contraction to explain the relationship between the length of an object perceived by a stationary observer who sees the object as moving, and the proper length of the object as measured in the frame of reference where it is at rest.

a. As the speed v of an object moving with respect to a stationary observer approaches c, the length perceived by the observer approaches zero. For other speeds, the length perceived is always less than the proper length.

b. As the speed v of an object moving with respect to a stationary observer approaches c, the length perceived by the observer approaches zero. For other speeds, the length perceived is always greater than the proper length.

c. As the speed v of an object moving with respect to a stationary observer approaches c, the length perceived by the observer approaches infinity. For other speeds, the length perceived is always less than the proper length.

d. As the speed v of an object moving with respect to a stationary observer approaches c, the length perceived by the observer approaches infinity. For other speeds, the length perceived is always greater than the proper length.

CHAPTER 11
Thermal Energy, Heat, and Work

Figure 11.1 The welder's gloves and helmet protect the welder from the electric arc, which transfers enough thermal energy to melt the rod, spray sparks, and emit high-energy electromagnetic radiation that can burn the retina of an unprotected eye. The thermal energy can be felt on exposed skin a few meters away, and its light can be seen for kilometers (Kevin S. O'Brien, U.S. Navy)

Chapter Outline

INTRODUCTION Heat is something familiar to all of us. We feel the warmth of the summer sun, the hot vapor rising up out of a cup of hot cocoa, and the cooling effect of our sweat. When we feel warmth, it means that heat is transferring energy *to* our bodies; when we feel cold, that means heat is transferring energy *away from* our bodies. Heat transfer is the movement of thermal energy from one place or material to another, and is caused by temperature differences. For example, much of our weather is caused by Earth evening out the temperature across the planet through wind and violent storms, which are driven by heat transferring energy away from the equator towards the cold poles. In this chapter, we'll explore the precise meaning of heat, how it relates to temperature as well as to other forms of energy, and its connection to work.

11.1 Temperature and Thermal Energy

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain that temperature is a measure of internal kinetic energy
- Interconvert temperatures between Celsius, Kelvin, and Fahrenheit scales

Section Key Terms

absolute zero	Celsius scale	degree Celsius ($^\circ\text{C}$)	thermal energy
degree Fahrenheit ($^\circ\text{F}$)	Fahrenheit scale	heat	
kelvin (K)	Kelvin scale	temperature	

Temperature

What is **temperature**? It's one of those concepts so ingrained in our everyday lives that, although we know what it means intuitively, it can be hard to define. It is tempting to say that temperature measures heat, but this is not strictly true. **Heat** is the transfer of energy due to a temperature difference. Temperature is defined in terms of the instrument we use to tell us how hot or cold an object is, based on a mechanism and scale invented by people. Temperature is literally defined as what we measure on a thermometer.

Heat is often confused with temperature. For example, we may say that the heat was unbearable, when we actually mean that the temperature was high. This is because we are sensitive to the flow of energy by heat, rather than the temperature. Since heat, like work, transfers energy, it has the SI unit of joule (J).

Atoms and molecules are constantly in motion, bouncing off one another in random directions. Recall that kinetic energy is the energy of motion, and that it increases in proportion to velocity squared. Without going into mathematical detail, we can say that **thermal energy**—the energy associated with heat—is the average kinetic energy of the particles (molecules or atoms) in a substance. Faster moving molecules have greater kinetic energies, and so the substance has greater thermal energy, and thus a higher temperature. The total internal energy of a system is the sum of the kinetic and potential energies of its atoms and molecules. Thermal energy is one of the subcategories of internal energy, as is chemical energy.

To measure temperature, some scale must be used as a standard of measurement. The three most commonly used temperature scales are the Fahrenheit, Celsius, and Kelvin scales. Both the **Fahrenheit scale** and **Celsius scale** are relative temperature scales, meaning that they are made around a reference point. For example, the Celsius scale uses the freezing point of water as its reference point; all measurements are either lower than the freezing point of water by a given number of degrees (and have a negative sign), or higher than the freezing point of water by a given number of degrees (and have a positive sign). The boiling point of water is 100 $^\circ\text{C}$ for the Celsius scale, and its unit is the degree Celsius ($^\circ\text{C}$).

On the Fahrenheit scale, the freezing point of water is at 32 $^\circ\text{F}$, and the boiling point is at 212 $^\circ\text{F}$. The unit of temperature on this scale is the degree Fahrenheit ($^\circ\text{F}$). Note that the difference in degrees between the freezing and boiling points is greater for the Fahrenheit scale than for the Celsius scale. Therefore, a temperature difference of one degree Celsius is greater than a temperature difference of one degree Fahrenheit. Since 100 Celsius degrees span the same range as 180 Fahrenheit degrees, one degree on the Celsius scale is 1.8 times larger than one degree on the Fahrenheit scale (because $\frac{180}{100} = \frac{9}{5} = 1.8$). This relationship can be used to convert between temperatures in Fahrenheit and Celsius (see Figure 11.2).

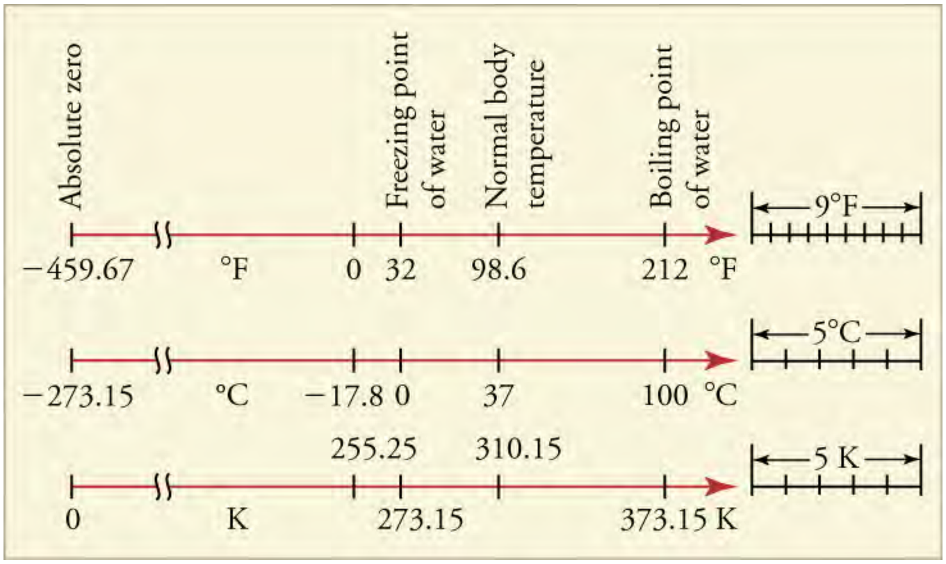


Figure 11.2 Relationships between the Fahrenheit, Celsius, and Kelvin temperature scales, rounded to the nearest degree. The relative sizes of the scales are also shown.

The **Kelvin scale** is the temperature scale that is commonly used in science because it is an absolute temperature scale. This means that the theoretically lowest-possible temperature is assigned the value of zero. Zero degrees on the Kelvin scale is known as **absolute zero**; it is theoretically the point at which there is no molecular motion to produce thermal energy. On the original Kelvin scale first created by Lord Kelvin, all temperatures have positive values, making it useful for scientific work. The official temperature unit on this scale is the kelvin, which is abbreviated as K. The freezing point of water is 273.15 K, and the boiling point of water is 373.15 K.

Although absolute zero is possible in theory, it cannot be reached in practice. The lowest temperature ever created and measured during a laboratory experiment was 1.0×10^{-10} K, at Helsinki University of Technology in Finland. In comparison, the coldest recorded temperature for a place on Earth's surface was 183 K (–89 °C), at Vostok, Antarctica, and the coldest known place (outside the lab) in the universe is the Boomerang Nebula, with a temperature of 1 K. Luckily, most of us humans will never have to experience such extremes.

The average normal body temperature is 98.6 °F (37.0 °C), but people have been known to survive with body temperatures ranging from 75 °F to 111 °F (24 °C to 44 °C).

WATCH PHYSICS

Comparing Celsius and Fahrenheit Temperature Scales

This video shows how the Fahrenheit and Celsius temperature scales compare to one another.

Click to view content (https://www.openstax.org/l/02celfahtemp)

GRASP CHECK

Even without the number labels on the thermometer, you could tell which side is marked Fahrenheit and which is Celsius by how the degree marks are spaced. Why?

a. The separation between two consecutive divisions on the Fahrenheit scale is greater than a similar separation on the Celsius scale, because each degree Fahrenheit is equal to 1.8 degrees Celsius.
b. The separation between two consecutive divisions on the Fahrenheit scale is smaller than the similar separation on the Celsius scale, because each degree Celsius is equal to 1.8 degrees Fahrenheit.
c. The separation between two consecutive divisions on the Fahrenheit scale is greater than a similar separation on the Celsius scale, because each degree Fahrenheit is equal to 3.6 degrees Celsius.
d. The separation between two consecutive divisions on the Fahrenheit scale is smaller than a similar separation on the Celsius scale, because each degree Celsius is equal to 3.6 degrees Fahrenheit.

Converting Between Celsius, Kelvin, and Fahrenheit Scales

While the Fahrenheit scale is still the most commonly used scale in the United States, the majority of the world uses Celsius, and scientists prefer Kelvin. It's often necessary to convert between these scales. For instance, if the TV meteorologist gave the local weather report in kelvins, there would likely be some confused viewers! Table 11.1 gives the equations for conversion between the three temperature scales.

To Convert From...	Use This Equation
Celsius to Fahrenheit	$T_{°F} = \frac{9}{5}T_{°C} + 32$
Fahrenheit to Celsius	$T_{°C} = \frac{5}{9}(T_{°F}-32)$
Celsius to Kelvin	$T_{K} = T_{°C} + 273.15$
Kelvin to Celsius	$T_{°C} = T_{K} - 273.15$
Fahrenheit to Kelvin	$T_{K} = \frac{5}{9}(T_{°F} - 32) + 273.15$
Kelvin to Fahrenheit	$T_{°F} = \frac{9}{5}(T_{K} - 273.15) + 32$

Table 11.1 Temperature Conversions

WORKED EXAMPLE

Room temperature is generally defined to be 25 °C. (a) What is room temperature in °F? (b) What is it in K?

STRATEGY

To answer these questions, all we need to do is choose the correct conversion equations and plug in the known values.

Solution for (a)

1. Choose the right equation. To convert from °C to °F, use the equation

$$T_{°F} = \frac{9}{5}T_{°C} + 32. \tag{11.1}$$

2. Plug the known value into the equation and solve.

$$T_{°F} = \frac{9}{5}25\ °C + 32 = 77\ °F \tag{11.2}$$

Solution for (b)

1. Choose the right equation. To convert from °C to K, use the equation

$$T_{K} = T_{°C} + 273.15. \tag{11.3}$$

2. Plug the known value into the equation and solve.

$$T_{K} = 25\ °C + 273.15 = 298 K \tag{11.4}$$

Discussion

Living in the United States, you are likely to have more of a sense of what the temperature feels like if it's described as 77 °F than as 25 °C (or 298 K, for that matter).

WORKED EXAMPLE

Converting Between Temperature Scales: The Reaumur Scale

The Reaumur scale is a temperature scale that was used widely in Europe in the 18th and 19th centuries. On the Reaumur temperature scale, the freezing point of water is 0 °R and the boiling temperature is 80 °R. If "room temperature" is 25 °C on the Celsius scale, what is it on the Reaumur scale?

STRATEGY

To answer this question, we must compare the Reaumur scale to the Celsius scale. The difference between the freezing point and boiling point of water on the Reaumur scale is 80 °R . On the Celsius scale, it is 100 °C . Therefore, $100\ °C = 80\ °R$. Both scales start at 0 ° for freezing, so we can create a simple formula to convert between temperatures on the two scales.

Solution

1. Derive a formula to convert from one scale to the other.

$$T_{°R} = \frac{0.80°R}{°C} \times T_{°C} \tag{11.5}$$

2. Plug the known value into the equation and solve.

$$T_{°R} = \frac{0.80°R}{°C} \times 25\ °C = 20°R \tag{11.6}$$

Discussion

As this example shows, relative temperature scales are somewhat arbitrary. If you wanted, you could create your own temperature scale!

Practice Problems

1. What is 12.0 °C in kelvins?
   a. 112.0 K
   b. 273.2 K
   c. 12.0 K
   d. 285.2 K

2. What is 32.0 °C in degrees Fahrenheit?
   a. 57.6 °F
   b. 25.6 °F
   c. 305.2 °F
   d. 89.6 °F

TIPS FOR SUCCESS

Sometimes it is not so easy to guess the temperature of the air accurately. Why is this? Factors such as humidity and wind speed affect how hot or cold we feel. Wind removes thermal energy from our bodies at a faster rate than usual, making us feel colder than we otherwise would; on a cold day, you may have heard the TV weather person refer to the *wind chill*.
On humid summer days, people tend to feel hotter because sweat doesn't evaporate from the skin as efficiently as it does on dry days, when the evaporation of sweat cools us off.

Check Your Understanding

3. What is thermal energy?
   a. The thermal energy is the average potential energy of the particles in a system.
   b. The thermal energy is the total sum of the potential energies of the particles in a system.
   c. The thermal energy is the average kinetic energy of the particles due to the interaction among the particles in a system.
   d. The thermal energy is the average kinetic energy of the particles in a system.

4. What is used to measure temperature?

a. a galvanometer
b. a manometer
c. a thermometer
d. a voltmeter

11.2 Heat, Specific Heat, and Heat Transfer

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain heat, heat capacity, and specific heat
- Distinguish between conduction, convection, and radiation
- Solve problems involving specific heat and heat transfer

Section Key Terms

conduction	convection	heat capacity	radiation	specific heat

Heat Transfer, Specific Heat, and Heat Capacity

We learned in the previous section that temperature is proportional to the average kinetic energy of atoms and molecules in a substance, and that the average internal kinetic energy of a substance is higher when the substance's temperature is higher.

If two objects at different temperatures are brought in contact with each other, energy is transferred from the hotter object (that is, the object with the greater temperature) to the colder (lower temperature) object, until both objects are at the same temperature. There is no net heat transfer once the temperatures are equal because the amount of heat transferred from one object to the other is the same as the amount of heat returned. One of the major effects of heat transfer is temperature change: Heating increases the temperature while cooling decreases it. Experiments show that the heat transferred to or from a substance depends on three factors—the change in the substance's temperature, the mass of the substance, and certain physical properties related to the phase of the substance.

The equation for heat transfer Q is

$$Q = mc\Delta T, \tag{11.7}$$

where m is the mass of the substance and ΔT is the change in its temperature, in units of Celsius or Kelvin. The symbol c stands for **specific heat**, and depends on the material and phase. The specific heat is the amount of heat necessary to change the temperature of 1.00 kg of mass by 1.00 °C. The specific heat c is a property of the substance; its SI unit is J/(kg ⋅ K) or J/(kg ⋅ $°\mathrm{C}$). The temperature change (ΔT) is the same in units of kelvins and degrees Celsius (but not degrees Fahrenheit). Specific heat is closely related to the concept of **heat capacity**. Heat capacity is the amount of heat necessary to change the temperature of a substance by 1.00 $°\mathrm{C}$. In equation form, heat capacity C is $C = mc$, where m is mass and c is specific heat. Note that heat capacity is the same as specific heat, but without any dependence on mass. Consequently, two objects made up of the same material but with different masses will have different heat capacities. This is because the heat capacity is a property of an object, but specific heat is a property of *any* object made of the same material.

Values of specific heat must be looked up in tables, because there is no simple way to calculate them. Table 11.2 gives the values of specific heat for a few substances as a handy reference. We see from this table that the specific heat of water is five times that of glass, which means that it takes five times as much heat to raise the temperature of 1 kg of water than to raise the temperature of 1 kg of glass by the same number of degrees.

Substances	Specific Heat (c)
Solids	J/(kg ⋅ $°\mathrm{C}$)
Aluminum	900

Table 11.2 Specific Heats of Various Substances.

Substances	Specific Heat (c)
Asbestos	800
Concrete, granite (average)	840
Copper	387
Glass	840
Gold	129
Human body (average)	3500
Ice (average)	2090
Iron, steel	452
Lead	128
Silver	235
Wood	1700
Liquids	
Benzene	1740
Ethanol	2450
Glycerin	2410
Mercury	139
Water	4186
Gases (at 1 atm constant pressure)	
Air (dry)	1015
Ammonia	2190
Carbon dioxide	833
Nitrogen	1040
Oxygen	913
Steam	2020

Table 11.2 Specific Heats of Various Substances.

Snap Lab

Temperature Change of Land and Water

What heats faster, land or water? You will answer this question by taking measurements to study differences in specific heat capacity.

- Open flame—Tie back all loose hair and clothing before igniting an open flame. Follow all of your teacher's instructions on how to ignite the flame. Never leave an open flame unattended. Know the location of fire safety equipment in the laboratory.

- Sand or soil
- Water
- Oven or heat lamp
- Two small jars
- Two thermometers

Instructions

Procedure

1. Place equal masses of dry sand (or soil) and water at the same temperature into two small jars. (The average density of soil or sand is about 1.6 times that of water, so you can get equal masses by using 50 percent more water by volume.)
2. Heat both substances (using an oven or a heat lamp) for the same amount of time.
3. Record the final temperatures of the two masses.
4. Now bring both jars to the same temperature by heating for a longer period of time.
5. Remove the jars from the heat source and measure their temperature every 5 minutes for about 30 minutes.

GRASP CHECK

Did it take longer to heat the water or the sand/soil to the same temperature? Which sample took longer to cool? What does this experiment tell us about how the specific heat of water compared to the specific heat of land?

a. The sand/soil will take longer to heat as well as to cool. This tells us that the specific heat of land is greater than that of water.
b. The sand/soil will take longer to heat as well as to cool. This tells us that the specific heat of water is greater than that of land.
c. The water will take longer to heat as well as to cool. This tells us that the specific heat of land is greater than that of water.
d. The water will take longer to heat as well as to cool. This tells us that the specific heat of water is greater than that of land.

Conduction, Convection, and Radiation

Whenever there is a temperature difference, heat transfer occurs. Heat transfer may happen rapidly, such as through a cooking pan, or slowly, such as through the walls of an insulated cooler.

There are three different heat transfer methods: **conduction**, **convection**, and **radiation**. At times, all three may happen simultaneously. See Figure 11.3.

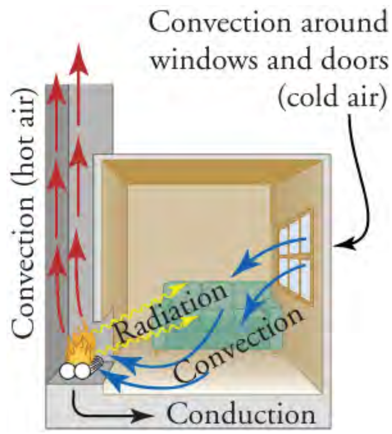


Figure 11.3 In a fireplace, heat transfer occurs by all three methods: conduction, convection, and radiation. Radiation is responsible for most of the heat transferred into the room. Heat transfer also occurs through conduction into the room, but at a much slower rate. Heat transfer by convection also occurs through cold air entering the room around windows and hot air leaving the room by rising up the chimney.

Conduction is heat transfer through direct physical contact. Heat transferred between the electric burner of a stove and the bottom of a pan is transferred by conduction. Sometimes, we try to control the conduction of heat to make ourselves more comfortable. Since the rate of heat transfer is different for different materials, we choose fabrics, such as a thick wool sweater, that slow down the transfer of heat away from our bodies in winter.

As you walk barefoot across the living room carpet, your feet feel relatively comfortable…until you step onto the kitchen's tile floor. Since the carpet and tile floor are both at the same temperature, why does one feel colder than the other? This is explained by different rates of heat transfer: The tile material removes heat from your skin at a greater rate than the carpeting, which makes it *feel* colder.

Some materials simply conduct thermal energy faster than others. In general, metals (like copper, aluminum, gold, and silver) are good heat conductors, whereas materials like wood, plastic, and rubber are poor heat conductors.

Figure 11.4 shows particles (either atoms or molecules) in two bodies at different temperatures. The (average) kinetic energy of a particle in the hot body is higher than in the colder body. If two particles collide, energy transfers from the particle with greater kinetic energy to the particle with less kinetic energy. When two bodies are in contact, many particle collisions occur, resulting in a net flux of heat from the higher-temperature body to the lower-temperature body. The heat flux depends on the temperature difference $\Delta T = T_{\text{hot}} - T_{\text{cold}}$. Therefore, you will get a more severe burn from boiling water than from hot tap water.

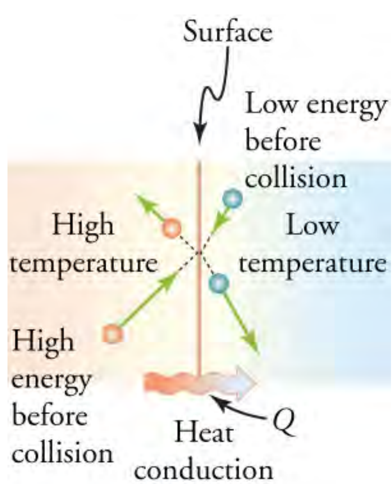


Figure 11.4 The particles in two bodies at different temperatures have different average kinetic energies. Collisions occurring at the contact surface tend to transfer energy from high-temperature regions to low-temperature regions. In this illustration, a particle in the lower-temperature region (right side) has low kinetic energy before collision, but its kinetic energy increases after colliding with the contact

surface. In contrast, a particle in the higher-temperature region (left side) has more kinetic energy before collision, but its energy decreases after colliding with the contact surface.

Convection is heat transfer by the movement of a fluid. This type of heat transfer happens, for example, in a pot boiling on the stove, or in thunderstorms, where hot air rises up to the base of the clouds.

TIPS FOR SUCCESS

In everyday language, the term *fluid* is usually taken to mean liquid. For example, when you are sick and the doctor tells you to "push fluids," that only means to drink more beverages—not to breath more air. However, in physics, fluid means a liquid *or a gas*. Fluids move differently than solid material, and even have their own branch of physics, known as *fluid dynamics*, that studies how they move.

As the temperature of fluids increase, they expand and become less dense. For example, Figure 11.4 could represent the wall of a balloon with different temperature gases inside the balloon than outside in the environment. The hotter and thus faster moving gas particles inside the balloon strike the surface with more force than the cooler air outside, causing the balloon to expand. This decrease in density relative to its environment creates buoyancy (the tendency to rise). Convection is driven by buoyancy—hot air rises because it is less dense than the surrounding air.

Sometimes, we control the temperature of our homes or ourselves by controlling air movement. Sealing leaks around doors with weather stripping keeps out the cold wind in winter. The house in Figure 11.5 and the pot of water on the stove in Figure 11.6 are both examples of convection and buoyancy by human design. Ocean currents and large-scale atmospheric circulation transfer energy from one part of the globe to another, and are examples of natural convection.

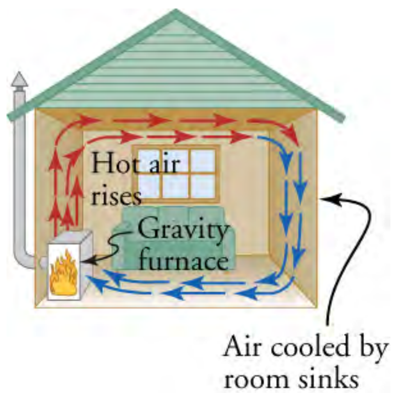


Figure 11.5 Air heated by the so-called gravity furnace expands and rises, forming a convective loop that transfers energy to other parts of the room. As the air is cooled at the ceiling and outside walls, it contracts, eventually becoming denser than room air and sinking to the floor. A properly designed heating system like this one, which uses natural convection, can be quite efficient in uniformly heating a home.

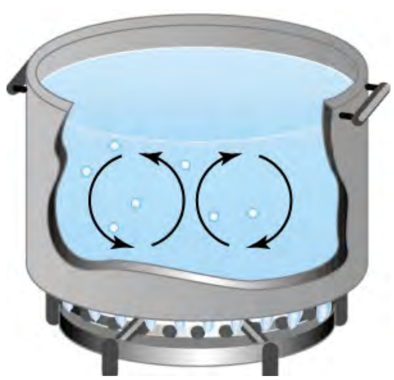


Figure 11.6 Convection plays an important role in heat transfer inside this pot of water. Once conducted to the inside fluid, heat transfer to other parts of the pot is mostly by convection. The hotter water expands, decreases in density, and rises to transfer heat to other regions of the water, while colder water sinks to the bottom. This process repeats as long as there is water in the pot.

Radiation is a form of heat transfer that occurs when electromagnetic radiation is emitted or absorbed. Electromagnetic radiation includes radio waves, microwaves, infrared radiation, visible light, ultraviolet radiation, X-rays, and gamma rays, all of which have different wavelengths and amounts of energy (shorter wavelengths have higher frequency and more energy).

You can feel the heat transfer from a fire and from the sun. Similarly, you can sometimes tell that the oven is hot without touching its door or looking inside—it may just warm you as you walk by. Another example is thermal radiation from the human body; people are constantly emitting infrared radiation, which is not visible to the human eye, but is felt as heat.

Radiation is the only method of heat transfer where no medium is required, meaning that the heat doesn't need to come into direct contact with or be transported by any matter. The space between Earth and the sun is largely empty, without any possibility of heat transfer by convection or conduction. Instead, heat is transferred by radiation, and Earth is warmed as it absorbs electromagnetic radiation emitted by the sun.

Figure 11.7 Most of the heat transfer from this fire to the observers is through infrared radiation. The visible light transfers relatively little thermal energy. Since skin is very sensitive to infrared radiation, you can sense the presence of a fire without looking at it directly. (Daniel X. O'Neil)

All objects absorb and emit electromagnetic radiation (see Figure 11.7). The rate of heat transfer by radiation depends mainly on the color of the object. Black is the most effective absorber and radiator, and white is the least effective. People living in hot climates generally avoid wearing black clothing, for instance. Similarly, black asphalt in a parking lot will be hotter than adjacent patches of grass on a summer day, because black absorbs better than green. The reverse is also true—black radiates better than green. On a clear summer night, the black asphalt will be colder than the green patch of grass, because black radiates energy faster than green. In contrast, white is a poor absorber and also a poor radiator. A white object reflects nearly all radiation, like a mirror.

Virtual Physics

Energy Forms and Changes

Click to view content (http://www.openstax.org/l/28energyForms)

In this animation, you will explore heat transfer with different materials. Experiment with heating and cooling the iron, brick, and water. This is done by dragging and dropping the object onto the pedestal and then holding the lever either to Heat or Cool. Drag a thermometer beside each object to measure its temperature—you can watch how quickly it heats or cools in real time.

Now let's try transferring heat between objects. Heat the brick and then place it in the cool water. Now heat the brick again, but then place it on top of the iron. What do you notice?

Selecting the fast forward option lets you speed up the heat transfers, to save time.

GRASP CHECK

Compare how quickly the different materials are heated or cooled. Based on these results, what material do you think has the greatest specific heat? Why? Which has the smallest specific heat? Can you think of a real-world situation where you would want to use an object with large specific heat?

a. Water will take the longest, and iron will take the shortest time to heat, as well as to cool. Objects with greater specific heat would be desirable for insulation. For instance, woolen clothes with large specific heat would prevent heat loss from the body.

b. Water will take the shortest, and iron will take the longest time to heat, as well as to cool. Objects with greater specific heat would be desirable for insulation. For instance, woolen clothes with large specific heat would prevent heat loss from the body.
c. Brick will take shortest and iron will take longest time to heat up as well as to cool down. Objects with greater specific heat would be desirable for insulation. For instance, woolen clothes with large specific heat would prevent heat loss from the body.
d. Water will take shortest and brick will take longest time to heat up as well as to cool down. Objects with greater specific heat would be desirable for insulation. For instance, woolen clothes with large specific heat would prevent heat loss from the body.

Solving Heat Transfer Problems

WORKED EXAMPLE

Calculating the Required Heat: Heating Water in an Aluminum Pan

A 0.500 kg aluminum pan on a stove is used to heat 0.250 L of water from 20.0 $^\circ\mathrm{C}$ to 80.0 $^\circ\mathrm{C}$. (a) How much heat is required? What percentage of the heat is used to raise the temperature of (b) the pan and (c) the water?

STRATEGY

The pan and the water are always at the same temperature. When you put the pan on the stove, the temperature of the water and the pan is increased by the same amount. We use the equation for heat transfer for the given temperature change and masses of water and aluminum. The specific heat values for water and aluminum are given in the previous table.

Solution to (a)

Because the water is in thermal contact with the aluminum, the pan and the water are at the same temperature.

1. Calculate the temperature difference.

$$\Delta T = T_f - T_i = 60.0\ ^\circ\mathrm{C} \tag{11.8}$$

2. Calculate the mass of water using the relationship between density, mass, and volume. Density is mass per unit volume, or $\rho = \frac{m}{V}$. Rearranging this equation, solve for the mass of water.

$$m_w = \rho \cdot V = 1000\ \mathrm{kg/m^3} \times \left(0.250\ \mathrm{L} \times \frac{0.001\ \mathrm{m^3}}{1\ \mathrm{L}}\right) = 0.250\ \mathrm{kg} \tag{11.9}$$

3. Calculate the heat transferred to the water. Use the specific heat of water in the previous table.

$$Q_w = m_w c_w \Delta T = (0.250\ \mathrm{kg})\,(4186\ \mathrm{J/kg^\circ C})\,(60.0^\circ\mathrm{C}) = 62.8\ \mathrm{kJ} \tag{11.10}$$

4. Calculate the heat transferred to the aluminum. Use the specific heat for aluminum in the previous table.

$$Q_{Al} = m_{Al} c_{Al} \Delta T = (0.500\ \mathrm{kg})\,(900\ \mathrm{J/kg^\circ C})\,(60.0^\circ\mathrm{C}) = 27.0 \times 10^3\ \mathrm{J} = 27.0\ \mathrm{kJ} \tag{11.11}$$

5. Find the total transferred heat.

$$Q_{Total} = Q_w + Q_{Al} = 62.8\ \mathrm{kJ} + 27.0\ \mathrm{kJ} = 89.8\ \mathrm{kJ} \tag{11.12}$$

Solution to (b)

The percentage of heat going into heating the pan is

$$\frac{27.0\ \mathrm{kJ}}{89.8\ \mathrm{kJ}} \times 100\% = 30.1\% \tag{11.13}$$

Solution to (c)

The percentage of heat going into heating the water is

$$\frac{62.8\ \mathrm{kJ}}{89.8\ \mathrm{kJ}} \times 100\% = 69.9\% \tag{11.14}$$

Discussion

In this example, most of the total heat transferred is used to heat the water, even though the pan has twice as much mass. This is because the specific heat of water is over four times greater than the specific heat of aluminum. Therefore, it takes a bit more than twice as much heat to achieve the given temperature change for the water than for the aluminum pan.

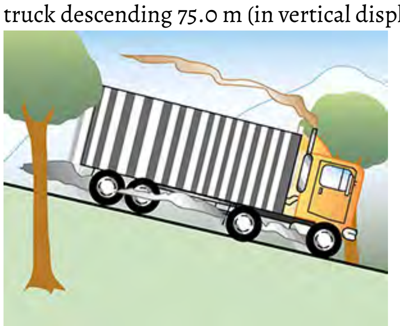
Water can absorb a tremendous amount of energy with very little resulting temperature change. This property of water allows for life on Earth because it stabilizes temperatures. Other planets are less habitable because wild temperature swings make for a harsh environment. You may have noticed that climates closer to large bodies of water, such as oceans, are milder than climates landlocked in the middle of a large continent. This is due to the climate-moderating effect of water's large heat capacity—water stores large amounts of heat during hot weather and releases heat gradually when it's cold outside.

WORKED EXAMPLE

Calculating Temperature Increase: Truck Brakes Overheat on Downhill Runs

When a truck headed downhill brakes, the brakes must do work to convert the gravitational potential energy of the truck to internal energy of the brakes. This conversion prevents the gravitational potential energy from being converted into kinetic energy of the truck, and keeps the truck from speeding up and losing control. The increased internal energy of the brakes raises their temperature. When the hill is especially steep, the temperature increase may happen too quickly and cause the brakes to overheat.

Calculate the temperature increase of 100 kg of brake material with an average specific heat of 800 J/kg $\cdot$°C from a 10,000 kg truck descending 75.0 m (in vertical displacement) at a constant speed.



STRATEGY

We first calculate the gravitational potential energy (*Mgh*) of the truck, and then find the temperature increase produced in the brakes.

Solution

1. Calculate the change in gravitational potential energy as the truck goes downhill.

$$Mgh = (10{,}000\ \text{kg})\,(9.80\ \text{m/s}^2)\,(75.0\ \text{m}) = 7.35 \times 10^6\,\text{J} \tag{11.15}$$

2. Calculate the temperature change from the heat transferred by rearranging the equation $Q = mc\Delta T$ to solve for ΔT.

$$\Delta T = \frac{Q}{mc}, \tag{11.16}$$

where m is the mass of the brake material (not the entire truck). Insert the values $Q = 7.35 \times 10^6$ J (since the heat transfer is equal to the change in gravitational potential energy), $m = 100$ kg, and $c = 800$ J/kg $\cdot$ °C to find

$$\Delta T = \frac{7.35 \times 10^6\,\text{J}}{(100\ \text{kg})(800\ \text{J/kg} \cdot {}^\circ\text{C})} = 91.9\ {}^\circ\text{C}. \tag{11.17}$$

Discussion

This temperature is close to the boiling point of water. If the truck had been traveling for some time, then just before the descent, the brake temperature would likely be higher than the ambient temperature. The temperature increase in the descent would likely raise the temperature of the brake material above the boiling point of water, which would be hard on the brakes. This is why truck drivers sometimes use a different technique for called "engine braking" to avoid burning their brakes during steep descents. Engine braking is using the slowing forces of an engine in low gear rather than brakes to slow down.

Practice Problems

5. How much heat does it take to raise the temperature of 10.0 kg of water by 1.0 °C ?

a. 84 J
b. 42 J
c. 84 kJ
d. 42 kJ

6. Calculate the change in temperature of 1.0 kg of water that is initially at room temperature if 3.0 kJ of heat is added.

a. 358 °C
b. 716 °C
c. 0.36 °C
d. 0.72 °C

Check Your Understanding

7. What causes heat transfer?

a. The mass difference between two objects causes heat transfer.
b. The density difference between two objects causes heat transfer.
c. The temperature difference between two systems causes heat transfer.
d. The pressure difference between two objects causes heat transfer.

8. When two bodies of different temperatures are in contact, what is the overall direction of heat transfer?

a. The overall direction of heat transfer is from the higher-temperature object to the lower-temperature object.
b. The overall direction of heat transfer is from the lower-temperature object to the higher-temperature object.
c. The direction of heat transfer is first from the lower-temperature object to the higher-temperature object, then back again to the lower-temperature object, and so-forth, until the objects are in thermal equilibrium.
d. The direction of heat transfer is first from the higher-temperature object to the lower-temperature object, then back again to the higher-temperature object, and so-forth, until the objects are in thermal equilibrium.

9. What are the different methods of heat transfer?

a. conduction, radiation, and reflection
b. conduction, reflection, and convection
c. convection, radiation, and reflection
d. conduction, radiation, and convection

10. True or false—Conduction and convection cannot happen simultaneously

a. True
b. False

11.3 Phase Change and Latent Heat

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain changes in heat during changes of state, and describe latent heats of fusion and vaporization
- Solve problems involving thermal energy changes when heating and cooling substances with phase changes

Section Key Terms

condensation	freezing	latent heat	sublimation
latent heat of fusion	latent heat of vaporization	melting	vaporization
phase change	phase diagram	plasma	

Phase Changes

So far, we have learned that adding thermal energy by heat increases the temperature of a substance. But surprisingly, there are situations where adding energy does not change the temperature of a substance at all! Instead, the additional thermal energy acts to loosen bonds between molecules or atoms and causes a **phase change**. Because this energy enters or leaves a system during a phase change without causing a temperature change in the system, it is known as **latent heat** (latent means *hidden*).

The three phases of matter that you frequently encounter are solid, liquid and gas (see Figure 11.8). Solid has the least energetic state; atoms in solids are in close contact, with forces between them that allow the particles to vibrate but not change position with neighboring particles. (These forces can be thought of as springs that can be stretched or compressed, but not easily broken.)

Liquid has a more energetic state, in which particles can slide smoothly past one another and change neighbors, although they are still held together by their mutual attraction.

Gas has a more energetic state than liquid, in which particles are broken free of their bonds. Particles in gases are separated by distances that are large compared with the size of the particles.

The most energetic state of all is **plasma**. Although you may not have heard much about plasma, it is actually the most common state of matter in the universe—stars are made up of plasma, as is lightning. The plasma state is reached by heating a gas to the point where particles are pulled apart, separating the electrons from the rest of the particle. This produces an ionized gas that is a combination of the negatively charged free electrons and positively charged ions, known as plasma.

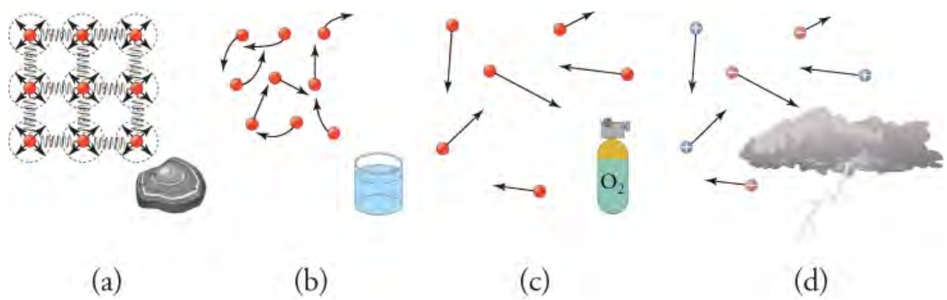


Figure 11.8 (a) Particles in a solid always have the same neighbors, held close by forces represented here by springs. These particles are essentially in contact with one another. A rock is an example of a solid. This rock retains its shape because of the forces holding its atoms or molecules together. (b) Particles in a liquid are also in close contact but can slide over one another. Forces between them strongly resist attempts to push them closer together and also hold them in close contact. Water is an example of a liquid. Water can flow, but it also remains in an open container because of the forces between its molecules. (c) Particles in a gas are separated by distances that are considerably larger than the size of the particles themselves, and they move about freely. A gas must be held in a closed container to prevent it from moving out into its surroundings. (d) The atmosphere is ionized in the extreme heat of a lightning strike.

During a phase change, matter changes from one phase to another, either through the addition of energy by heat and the transition to a more energetic state, or from the removal of energy by heat and the transition to a less energetic state.

Phase changes to a more energetic state include the following:

- **Melting**—Solid to liquid
- **Vaporization**—Liquid to gas (included boiling and evaporation)
- **Sublimation**—Solid to gas

Phase changes to a less energetic state are as follows:

- **Condensation**—Gas to liquid
- **Freezing**—Liquid to solid

Energy is required to melt a solid because the bonds between the particles in the solid must be broken. Since the energy involved in a phase changes is used to break bonds, there is no increase in the kinetic energies of the particles, and therefore no rise in temperature. Similarly, energy is needed to vaporize a liquid to overcome the attractive forces between particles in the liquid. There is no temperature change until a phase change is completed. The temperature of a cup of soda and ice that is initially at 0 °C stays at 0 °C until all of the ice has melted. In the reverse of these processes—freezing and condensation—energy is released from the latent heat (see Figure 11.9).

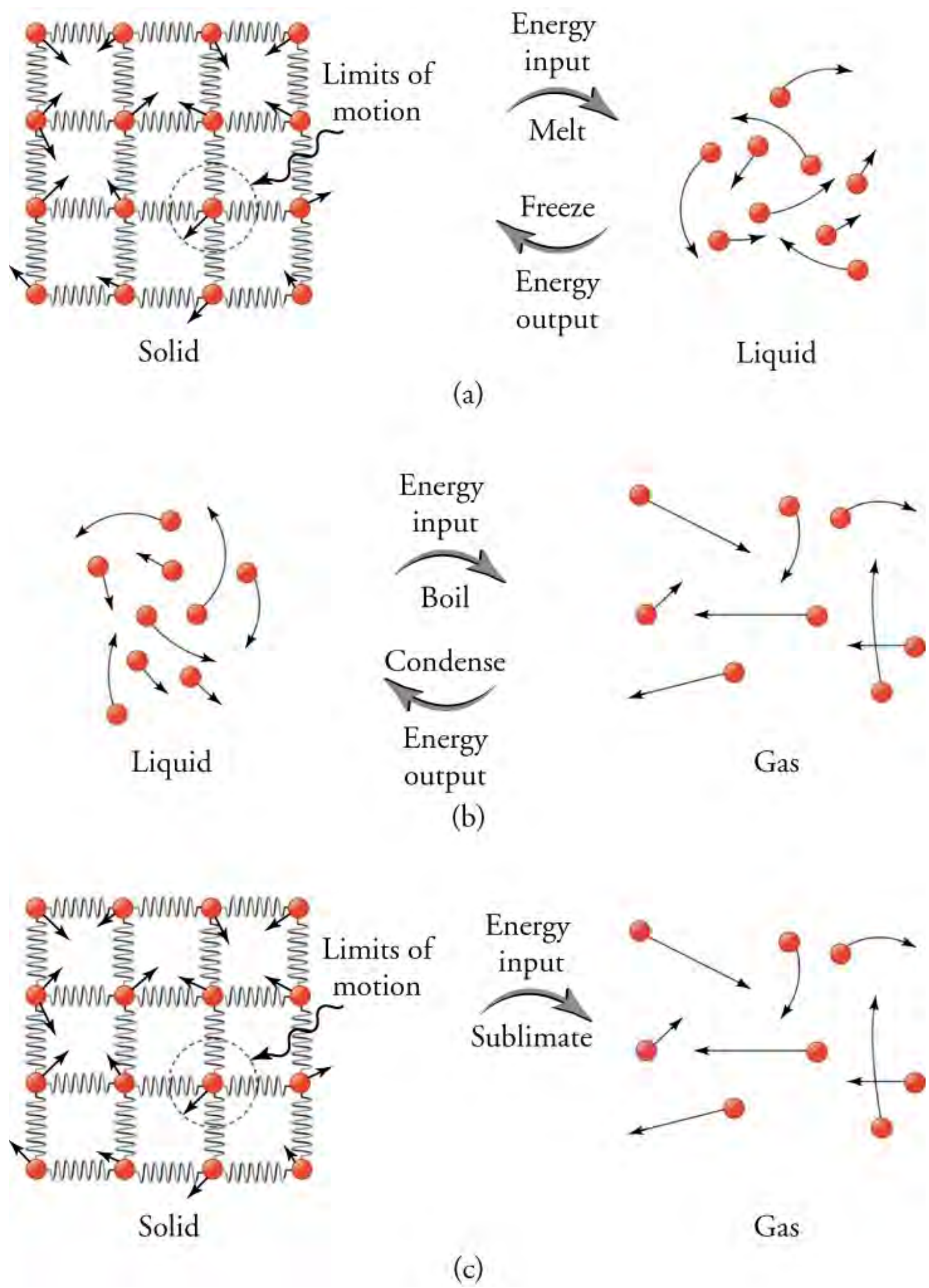


Figure 11.9 (a) Energy is required to partially overcome the attractive forces between particles in a solid to form a liquid. That same energy must be removed for freezing to take place. (b) Particles are separated by large distances when changing from liquid to vapor, requiring significant energy to overcome molecular attraction. The same energy must be removed for condensation to take place. There is no temperature change until a phase change is completed. (c) Enough energy is added that the liquid state is skipped over completely as a substance undergoes sublimation.

The heat, Q, required to change the phase of a sample of mass m is

$Q = mL_f$ (for melting/freezing),

$Q = mL_v$ (for vaporization/condensation),

where L_f is the **latent heat of fusion**, and L_v is the **latent heat of vaporization**. The latent heat of fusion is the amount of heat needed to cause a phase change between solid and liquid. The latent heat of vaporization is the amount of heat needed to cause a phase change between liquid and gas. L_f and L_v are coefficients that vary from substance to substance, depending on the strength of intermolecular forces, and both have standard units of J/kg. See Table 11.3 for values of L_f and L_v of different substances.

Substance	Melting Point ($°C$)	*Lf* (kJ/kg)	Boiling Point ($°C$)	*Lv* (kJ/kg)
Helium	−269.7	5.23	−268.9	20.9
Hydrogen	−259.3	58.6	−252.9	452
Nitrogen	−210.0	25.5	−195.8	201
Oxygen	−218.8	13.8	−183.0	213
Ethanol	−114	104	78.3	854
Ammonia	−78	332	−33.4	1370
Mercury	−38.9	11.8	357	272
Water	0.00	334	100.0	2256
Sulfur	119	38.1	444.6	326
Lead	327	24.5	1750	871
Antimony	631	165	1440	561
Aluminum	660	380	2520	11400
Silver	961	88.3	2193	2336
Gold	1063	64.5	2660	1578
Copper	1083	134	2595	5069
Uranium	1133	84	3900	1900
Tungsten	3410	184	5900	4810

Table 11.3 Latent Heats of Fusion and Vaporization, along with Melting and Boiling Points

Let's consider the example of adding heat to ice to examine its transitions through all three phases—solid to liquid to gas. A phase diagram indicating the temperature changes of water as energy is added is shown in Figure 11.10. The ice starts out at −20 $°C$, and its temperature rises linearly, absorbing heat at a constant rate until it reaches 0 $°$. Once at this temperature, the ice gradually melts, absorbing 334 kJ/kg. The temperature remains constant at 0 $°C$ during this phase change. Once all the ice has melted, the temperature of the liquid water rises, absorbing heat at a new constant rate. At 100 $°C$, the water begins to boil and the temperature again remains constant while the water absorbs 2256 kJ/kg during this phase change. When all the liquid has become steam, the temperature rises again at a constant rate.

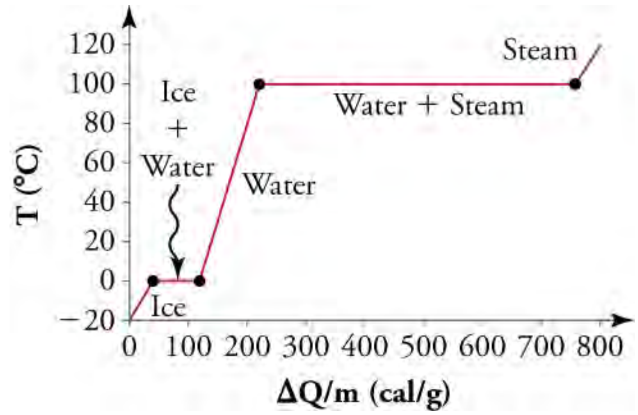


Figure 11.10 A graph of temperature versus added energy. The system is constructed so that no vapor forms while ice warms to become liquid water, and so when vaporization occurs, the vapor remains in the system. The long stretches of constant temperature values at 0 °C and 100 °C reflect the large latent heats of melting and vaporization, respectively.

We have seen that vaporization requires heat transfer to a substance from its surroundings. Condensation is the reverse process, where heat in transferred *away from* a substance *to* its surroundings. This release of latent heat increases the temperature of the surroundings. Energy must be removed from the condensing particles to make a vapor condense. This is why condensation occurs on cold surfaces: the heat transfers energy away from the warm vapor to the cold surface. The energy is exactly the same as that required to cause the phase change in the other direction, from liquid to vapor, and so it can be calculated from $Q = mL_v$. Latent heat is also released into the environment when a liquid freezes, and can be calculated from $Q = mL_f$.

FUN IN PHYSICS

Making Ice Cream

Figure 11.11 With the proper ingredients, some ice and a couple of plastic bags, you could make your own ice cream in five minutes. (ElinorD, Wikimedia Commons)

Ice cream is certainly easy enough to buy at the supermarket, but for the hardcore ice cream enthusiast, that may not be satisfying enough. Going through the process of making your own ice cream lets you invent your own flavors and marvel at the physics firsthand (Figure 11.11).

The first step to making homemade ice cream is to mix heavy cream, whole milk, sugar, and your flavor of choice; it could be as simple as cocoa powder or vanilla extract, or as fancy as pomegranates or pistachios.

The next step is to pour the mixture into a container that is deep enough that you will be able to churn the mixture without it spilling over, and that is also freezer-safe. After placing it in the freezer, the ice cream has to be stirred vigorously every 45 minutes for four to five hours. This slows the freezing process and prevents the ice cream from turning into a solid block of ice. Most people prefer a soft creamy texture instead of one giant popsicle.

As it freezes, the cream undergoes a phase change from liquid to solid. By now, we're experienced enough to know that this means that the cream must experience a loss of heat. Where does that heat go? Due to the temperature difference between the freezer and the ice cream mixture, heat transfers thermal energy from the ice cream to the air in the freezer. Once the temperature in the freezer rises enough, the freezer is cooled by pumping excess heat outside into the kitchen.

A faster way to make ice cream is to chill it by placing the mixture in a plastic bag, surrounded by another plastic bag half full of ice. (You can also add a teaspoon of salt to the outer bag to lower the temperature of the ice/salt mixture.) Shaking the bag for five minutes churns the ice cream while cooling it evenly. In this case, the heat transfers energy out of the ice cream mixture and into the ice during the phase change.

This video (http://www.openstax.org/l/28icecream) gives a demonstration of how to make home-made ice cream using ice and plastic bags.

GRASP CHECK

Why does the ice bag method work so much faster than the freezer method for making ice cream?

a. Ice has a smaller specific heat than the surrounding air in a freezer. Hence, it absorbs more energy from the ice-cream mixture.
b. Ice has a smaller specific heat than the surrounding air in a freezer. Hence, it absorbs less energy from the ice-cream mixture.
c. Ice has a greater specific heat than the surrounding air in a freezer. Hence, it absorbs more energy from the ice-cream mixture.
d. Ice has a greater specific heat than the surrounding air in a freezer. Hence, it absorbs less energy from the ice-cream mixture.

Solving Thermal Energy Problems with Phase Changes

WORKED EXAMPLE

Calculating Heat Required for a Phase Change

Calculate a) how much energy is needed to melt 1.000 kg of ice at 0 °C (freezing point), and b) how much energy is required to vaporize 1.000 kg of water at 100 °C (boiling point).

STRATEGY FOR (A)

Using the equation for the heat required for melting, and the value of the latent heat of fusion of water from the previous table, we can solve for part (a).

Solution to (a)

The energy to melt 1.000 kg of ice is

$$Q = mL_f = (1.000 \text{ kg})(334 \text{ kJ/kg}) = 334 \text{ kJ}. \tag{11.18}$$

STRATEGY FOR (B)

To solve part (b), we use the equation for heat required for vaporization, along with the latent heat of vaporization of water from the previous table.

Solution to (b)

The energy to vaporize 1.000 kg of liquid water is

$$Q = mL_v = (1.000 \text{ kg})(2256 \text{ kJ/kg}) = 2256 \text{ kJ}. \tag{11.19}$$

Discussion

The amount of energy need to melt a kilogram of ice (334 kJ) is the same amount of energy needed to raise the temperature of 1.000 kg of liquid water from 0 °C to 79.8 °C. This example shows that the energy for a phase change is enormous compared to energy associated with temperature changes. It also demonstrates that the amount of energy needed for vaporization is even greater.

WORKED EXAMPLE

Calculating Final Temperature from Phase Change: Cooling Soda with Ice Cubes

Ice cubes are used to chill a soda at 20 °C and with a mass of $m_{soda} = 0.25\ \text{kg}$. The ice is at 0 °C and the total mass of the ice cubes is 0.018 kg. Assume that the soda is kept in a foam container so that heat loss can be ignored, and that the soda has the same specific heat as water. Find the final temperature when all of the ice has melted.

STRATEGY

The ice cubes are at the melting temperature of 0 °C. Heat is transferred from the soda to the ice for melting. Melting of ice occurs in two steps: first, the phase change occurs and solid (ice) transforms into liquid water at the melting temperature; then, the temperature of this water rises. Melting yields water at 0 °C, so more heat is transferred from the soda to this water until they are the same temperature. Since the amount of heat leaving the soda is the same as the amount of heat transferred to the ice.

$$Q_{ice} = -Q_{soda} \tag{11.20}$$

The heat transferred to the ice goes partly toward the phase change (melting), and partly toward raising the temperature after melting. Recall from the last section that the relationship between heat and temperature change is $Q = mc\Delta T$. For the ice, the temperature change is $T_f - 0\ °\text{C}$. The total heat transferred to the ice is therefore

$$Q_{ice} = m_{ice}L_f + m_{ice}c_w(T_f - 0\ °\text{C}). \tag{11.21}$$

Since the soda doesn't change phase, but only temperature, the heat given off by the soda is

$$Q_{soda} = m_{soda}c_w(T_f - 20\ °\text{C}). \tag{11.22}$$

Since $Q_{ice} = -Q_{soda}$,

$$m_{ice}L_f + m_{ice}c_w(T_f - 0\ °\text{C}) = -m_{soda}c_w(T_f - 20\ °\text{C}). \tag{11.23}$$

Bringing all terms involving T_f to the left-hand-side of the equation, and all other terms to the right-hand-side, we can solve for T_f.

$$T_f = \frac{m_{soda}c_w(20\ °\text{C}) - m_{ice}L_f}{(m_{soda} + m_{ice})c_w} \tag{11.24}$$

Substituting the known quantities

$$T_f = \frac{(0.25\ \text{kg})(4186\ \text{J/kg}\cdot°\text{C})(20\ °\text{C}) - (0.018\ \text{kg})(334{,}000\ \text{J/kg})}{(0.25\ \text{kg} + 0.018\ \text{kg})(4186\ \text{K/kg}\cdot°\text{C})} = 13\ °\text{C} \tag{11.25}$$

Discussion

This example shows the enormous energies involved during a phase change. The mass of the ice is about 7 percent the mass of the soda, yet it causes a noticeable change in the soda's temperature.

TIPS FOR SUCCESS

If the ice were not already at the freezing point, we would also have to factor in how much energy would go into raising its temperature up to 0 °C, before the phase change occurs. This would be a realistic scenario, because the temperature of ice is often below 0 °C.

Practice Problems

11. How much energy is needed to melt 2.00 kg of ice at 0 °C ?

a. 334 kJ
b. 336 kJ
c. 167 kJ
d. 668 kJ

12. If $2500\ \text{kJ}$ of energy is just enough to melt $3.0\ \text{kg}$ of a substance, what is the substance's latent heat of fusion?

a. $7500\ \text{kJ} \cdot \text{kg}$
b. $7500\ \text{kJ/kg}$
c. $830\ \text{kJ} \cdot \text{kg}$
d. $830\ \text{kJ/kg}$

Check Your Understanding

13. What is latent heat?

a. It is the heat that must transfer energy to or from a system in order to cause a mass change with a slight change in the temperature of the system.
b. It is the heat that must transfer energy to or from a system in order to cause a mass change without a temperature change in the system.
c. It is the heat that must transfer energy to or from a system in order to cause a phase change with a slight change in the temperature of the system.
d. It is the heat that must transfer energy to or from a system in order to cause a phase change without a temperature change in the system.

14. In which phases of matter are molecules capable of changing their positions?

a. gas, liquid, solid
b. liquid, plasma, solid
c. liquid, gas, plasma
d. plasma, gas, solid

KEY TERMS

absolute zero lowest possible temperature; the temperature at which all molecular motion ceases

Celsius scale temperature scale in which the freezing point of water is 0 $°C$ and the boiling point of water is 100 $°C$ at 1 atm of pressure

condensation phase change from gas to liquid

conduction heat transfer through stationary matter by physical contact

convection heat transfer by the movement of fluid

degree Celsius unit on the Celsius temperature scale

degree Fahrenheit unit on the Fahrenheit temperature scale

Fahrenheit scale temperature scale in which the freezing point of water is 32 $°F$ and the boiling point of water is 212 $°F$

freezing phase change from liquid to solid

heat transfer of thermal (or internal) energy due to a temperature difference

heat capacity amount of heat necessary to change the temperature of a substance by 1.00 $°C$

Kelvin unit on the Kelvin temperature scale; note that it is never referred to in terms of "degrees" Kelvin

Kelvin scale temperature scale in which 0 K is the lowest possible temperature, representing absolute zero

latent heat heat related to the phase change of a substance rather than a change of temperature

latent heat of fusion amount of heat needed to cause a phase change between solid and liquid

latent heat of vaporization amount of heat needed to cause a phase change between liquid and gas

melting phase change from solid to liquid

phase change transition between solid, liquid, or gas states of a substance

plasma ionized gas that is a combination of the negatively charged free electrons and positively charged ions

radiation energy transferred by electromagnetic waves

specific heat amount of heat necessary to change the temperature of 1.00 kg of a substance by 1.00 $°C$

sublimation phase change from solid to gas

temperature quantity measured by a thermometer

thermal energy average random kinetic energy of a molecule or an atom

vaporization phase change from liquid to gas

SECTION SUMMARY

11.1 Temperature and Thermal Energy

- Temperature is the quantity measured by a thermometer.
- Temperature is related to the average kinetic energy of atoms and molecules in a system.
- Absolute zero is the temperature at which there is no molecular motion.
- There are three main temperature scales: Celsius, Fahrenheit, and Kelvin.
- Temperatures on one scale can be converted into temperatures on another scale.

11.2 Heat, Specific Heat, and Heat Transfer

- Heat is thermal (internal) energy transferred due to a temperature difference.
- The transfer of heat Q that leads to a change ΔT in the temperature of a body with mass m is $Q = mc\Delta T$, where c is the specific heat of the material.
- Heat is transferred by three different methods: conduction, convection, and radiation.
- Heat conduction is the transfer of heat between two objects in direct contact with each other.
- Convection is heat transfer by the movement of mass.
- Radiation is heat transfer by electromagnetic waves.

11.3 Phase Change and Latent Heat

- Most substances have four distinct phases: solid, liquid, gas, and plasma.
- Gas is the most energetic state and solid is the least.
- During a phase change, a substance undergoes transition to a higher energy state when heat is added, or to a lower energy state when heat is removed.
- Heat is added to a substance during melting and vaporization.
- Latent heat is released by a substance during condensation and freezing.
- Phase changes occur at fixed temperatures called boiling and freezing (or melting) points for a given substance.

KEY EQUATIONS

11.1 Temperature and Thermal Energy

Celsius to Fahrenheit conversion	$T_{^\circ F} = \frac{9}{5} T_{^\circ C} + 32$
Fahrenheit to Celsius conversion	$T_{^\circ C} = \frac{5}{9}(T_{^\circ F} - 32)$
Celsius to Kelvin conversion	$T_K = T_{^\circ C} + 273.15$
Kelvin to Celsius conversion	$T_{^\circ C} = T_K - 273.15$
Fahrenheit to Kelvin conversion	$T_K = \frac{5}{9}(T_{^\circ F} - 32) + 273.15$
Kelvin to Fahrenheit conversion	$T_{^\circ F} = \frac{9}{5}(T_K - 273.15) + 32$

11.2 Heat, Specific Heat, and Heat Transfer

heat transfer	$Q = mc\Delta T$
density	$\rho = \frac{m}{V}$

11.3 Phase Change and Latent Heat

heat transfer for melting/freezing phase change	$Q = mL_f$
heat transfer for vaporization/condensation phase change	$Q = mL_v$

CHAPTER REVIEW

Concept Items

11.1 Temperature and Thermal Energy

1. A glass of water has a temperature of 31 degrees Celsius. What state of matter is it in?
  a. solid
  b. liquid
  c. gas
  d. plasma

2. What is the difference between thermal energy and internal energy?
  a. The thermal energy of the system is the average kinetic energy of the system's constituent particles due to their motion. The total internal energy of the system is the sum of the kinetic energies and the potential energies of its constituent particles.
  b. The thermal energy of the system is the average potential energy of the system's constituent particles due to their motion. The total internal energy of the system is the sum of the kinetic energies and the potential energies of its constituent particles.
  c. The thermal energy of the system is the average kinetic energy of the system's constituent particles due to their motion. The total internal energy of the system is the sum of the kinetic energies of its constituent particles.
  d. The thermal energy of the system is the average potential energy of the systems' constituent particles due to their motion. The total internal energy of the system is the sum of the kinetic energies of its constituent particles.

3. What does the Celsius scale use as a reference point?
  a. The boiling point of mercury
  b. The boiling point of wax
  c. The freezing point of water
  d. The freezing point of wax

11.2 Heat, Specific Heat, and Heat Transfer

4. What are the SI units of specific heat?
  a. $J/kg^2 \cdot {}^\circ C$
  b. $J \cdot kg^2/{}^\circ C$
  c. $J \cdot kg/{}^\circ C$
  d. $J/kg \cdot {}^\circ C$

5. What is radiation?
  a. The transfer of energy through emission and absorption of the electromagnetic waves is known as radiation.
  b. The transfer of energy without any direct physical

contact between any two substances.

c. The transfer of energy through direct physical contact between any two substances.

d. The transfer of energy by means of the motion of fluids at different temperatures and with different densities.

11.3 Phase Change and Latent Heat

6. Why is there no change in temperature during a phase change, even if energy is absorbed by the system?

a. The energy is used to break bonds between particles, and so does not increase the potential energy of the system's particles.

b. The energy is used to break bonds between particles, and so increases the potential energy of the system's particles.

c. The energy is used to break bonds between particles, and so does not increase the kinetic energy of the system's particles.

d. The energy is used to break bonds between particles, and so increases the kinetic energy of the system's particles.

7. In which two phases of matter do atoms and molecules have the most distance between them?

a. gas and solid

b. gas and liquid

c. gas and plasma

d. liquid and plasma

Critical Thinking Items

11.1 Temperature and Thermal Energy

8. The temperature of two equal quantities of water needs to be raised - the first container by 5 degrees Celsius and the second by 5 degrees Fahrenheit. Which one would require more heat?

a. The heat required by the first container is more than the second because each degree Celsius is equal to 1.8 degrees Fahrenheit.

b. The heat required by the first container is less than the second because each degree Fahrenheit is equal to 1.8 degrees Celsius.

c. The heat required by the first container is more than the second because each degree Celsius is equal to 3.6 degrees Fahrenheit.

d. The heat required by the first container is less than the second because each degree Fahrenheit is equal to 3.6 degrees Celsius.

9. What is 100.00 °C in kelvins?

a. 212.00 K

b. 100.00 K

c. 473.15 K

d. 373.15 K

11.2 Heat, Specific Heat, and Heat Transfer

10. The value of specific heat is the same whether the units are J/kg·K or J/kg·°C. How?

a. Temperature difference is dependent on the chosen temperature scale.

b. Temperature change is different in units of kelvins and degrees Celsius.

c. Reading of temperatures in kelvins and degree Celsius are the same.

d. The temperature change is the same in units of kelvins and degrees Celsius.

11. If the thermal energy of a perfectly black object is increased by conduction, will the object remain black in appearance? Why or why not?

a. No, the energy of the radiation increases as the temperature increases, and the radiation becomes visible at certain temperatures.

b. Yes, the energy of the radiation decreases as the temperature increases, and the radiation remains invisible at those energies.

c. No, the energy of the radiation decreases as the temperature increases, until the frequencies of the radiation are the same as those of visible light.

d. Yes, as the temperature increases, and the energy is transferred from the object by other mechanisms besides radiation, so that the energy of the radiation does not increase.

12. What is the specific heat of a substance that requires 5.00 kJ of heat to raise the temperature of 3.00 kg by 5.00 °F?

a. 3.33×103 J/kg ·° C

b. 6.00×103 J/kg ·° C

c. 3.33×102 J/kg · ° C

d. 6.00×102 J/kg · ° C

11.3 Phase Change and Latent Heat

13. Assume 1.0 kg of ice at 0 °C starts to melt. It absorbs 300 kJ of energy by heat. What is the temperature of the water afterwards?

a. 10 °C

b. 20 °C

c. 5 °C

d. 0 °C

Problems

11.1 Temperature and Thermal Energy

14. What is 35.0 °F in kelvins?
   a. 1.67 K
   b. 35.0 K
   c. -271.5 K
   d. 274.8 K

15. Design a temperature scale where the freezing point of water is 0 degrees and its boiling point is 70 degrees. What would be the room temperature on this scale?
   a. If room temperature is 25.0 °C, the temperature on the new scale will be 17.5 °.
   b. If room temperature is 25.0 °C, the temperature on the new scale will be 25.0°.
   c. If the room temperature is 25.0 °C, the temperature on the new scale will be 35.7°.
   d. If the room temperature is 25.0 °C, the temperature on the new scale will be 50.0°.

11.2 Heat, Specific Heat, and Heat Transfer

16. A certain quantity of water is given 4.0 kJ of heat. This raises its temperature by 30.0 °F. What is the mass of the water in grams?
   a. 5.7 g
   b. 570 g
   c. 5700 g
   d. 57 g

17. 5290 J of heat is given to 0.500 kg water at 15.00 °C. What will its final temperature be?
   a. 15.25° C
   b. 12.47 ° C
   c. 40.3° C
   d. 17.53° C

11.3 Phase Change and Latent Heat

18. How much energy would it take to heat 1.00 kg of ice at 0 °C to water at 15.0 °C?
   a. 271 kJ
   b. 334 kJ
   c. 62.8 kJ
   d. 397 kJ

19. Ice cubes are used to chill a soda with a mass m_{soda} = 0.300 kg at 15.0 °C. The ice is at 0 °C, and the total mass of the ice cubes is 0.020 kg. Assume that the soda is kept in a foam container so that heat loss can be ignored, and that the soda has the same specific heat as water. Find the final temperature when all ice has melted.
   a. 19.02 °C
   b. 90.3 °C
   c. 0.11 °C
   d. 9.03 °C

Performance Task

11.3 Phase Change and Latent Heat

20. You have been tasked with designing a baking pan that will bake batter the fastest. There are four materials available for you to test.
   - Four pans of similar design, consisting of aluminum, iron (steel), copper, and glass
   - Oven or similar heating source
   - Device for measuring high temperatures
   - Balance for measuring mass

Instructions

Procedure

1. Design a safe experiment to test the specific heat of each material (i.e., no extreme temperatures should be used)
2. Write down the materials needed for your experiment and the procedure you will follow. Make sure that you include every detail, so that the experiment can be repeated by others.
3. Carry out the experiment and record any data collected.
4. Review your results and make a recommendation as to which metal should be used for the pan.
   a. What physical quantities do you need to measure to determine the specific heats for the different materials?
   b. How does the glass differ from the metals in terms of thermal properties?
   c. What are your sources of error?

TEST PREP

Multiple Choice

11.1 Temperature and Thermal Energy

21. The temperature difference of 1 K is the same as
   a. 1 degree Celsius
   b. 1 degree Fahrenheit
   c. 273.15 degrees Celsius
   d. 273.15 degrees Fahrenheit

22. What is the preferred temperature scale used in scientific laboratories?
a. celsius
b. fahrenheit
c. kelvin
d. rankine

11.2 Heat, Specific Heat, and Heat Transfer

23. Which phase of water has the largest specific heat?
a. solid
b. liquid
c. gas

24. What kind of heat transfer requires no medium?
a. conduction
b. convection
c. reflection
d. radiation

25. Which of these substances has the greatest specific heat?
a. copper
b. mercury
c. aluminum
d. wood

26. Give an example of heat transfer through convection.
a. The energy emitted from the filament of an electric bulb
b. The energy coming from the sun
c. A pan on a hot burner
d. Water boiling in a pot

11.3 Phase Change and Latent Heat

27. What are the SI units of latent heat?
a. J/kg
b. J.kg
c. J/cal
d. cal/kg

28. Which substance has the largest latent heat of fusion?
a. gold
b. water
c. mercury
d. tungsten

29. In which phase changes does matter undergo a transition to a more energetic state?
a. freezing and vaporization
b. melting and sublimation
c. melting and vaporization
d. melting and freezing

30. A room has a window made from thin glass. The room is colder than the air outside. There is some condensation on the glass window. On which side of the glass would the condensation most likely be found?
a. Condensation is on the outside of the glass when the cool, dry air outside the room comes in contact with the cold pane of glass.
b. Condensation is on the outside of the glass when the warm, moist air outside the room comes in contact with the cold pane of glass.
c. Condensation is on the inside of the glass when the cool, dry air inside the room comes in contact with the cold pane of glass.
d. Condensation is on the inside of the glass when the warm, moist air inside the room comes in contact with the cold pane of glass.

Short Answer

11.1 Temperature and Thermal Energy

31. What is *absolute zero* on the Fahrenheit scale?
a. 0 °F
b. 32 °F
c. -273.15 °F
d. -459.67 °F

32. What is *absolute zero* on the Celsius scale?
a. 0 °C
b. 273.15 °C
c. -459.67 °C
d. -273.15 °C

33. A planet's atmospheric pressure is such that water there boils at a lower temperature than it does at sea level on Earth. If a Celsius scale is derived on this planet, will it be the same as that on Earth?
a. The Celsius scale derived on the planet will be the same as that on Earth, because the Celsius scale is independent of the freezing and boiling points of water.
b. The Celsius scale derived on that planet will not be the same as that on Earth, because the Celsius scale is dependent and derived by using the freezing and boiling points of water.
c. The Celsius scale derived on the planet will be the same as that on Earth, because the Celsius scale is an absolute temperature scale based on molecular motion, which is independent of pressure.
d. The Celsius scale derived on the planet will not be the same as that on Earth, but the Fahrenheit scale

will be the same, because its reference temperatures are not based on the freezing and boiling points of water.

34. What is the difference between the freezing point and boiling point of water on the Reaumur scale?
   a. The boiling point of water is 80° on the Reaumur scale.
   b. Reaumur scale is less than 120°.
   c. 100°
   d. 80°

11.2 Heat, Specific Heat, and Heat Transfer

35. In the specific heat equation what does *c* stand for?
   a. Total heat
   b. Specific heat
   c. Specific temperature
   d. Specific mass

36. Specific heat may be measured in J/kg · K, J/kg · °C. What other units can it be measured in?
   a. kg/kcal · °C
   b. kcal · °C/kg
   c. kg · °C/kcal
   d. kcal/kg · °C

37. What is buoyancy?
   a. Buoyancy is a downward force exerted by a solid that opposes the weight of an object.
   b. Buoyancy is a downward force exerted by a fluid that opposes the weight of an immersed object.
   c. Buoyancy is an upward force exerted by a solid that opposes the weight an object.
   d. Buoyancy is an upward force exerted by a fluid that opposes the weight of an immersed object.

38. Give an example of convection found in nature.
   a. heat transfer through metallic rod
   b. heat transfer from the sun to Earth
   c. heat transfer through ocean currents
   d. heat emitted by a light bulb into its environment

39. Calculate the temperature change in a substance with specific heat 735 J/kg · °C when 14 kJ of heat is given to a 3.0-kg sample of that substance.
   a. 57 °C
   b. 63 °C
   c. 1.8×10^{-2} °C
   d. 6.3 °C

40. Aluminum has a specific heat of 900 J/kg·°C. How much energy would it take to change the temperature of 2 kg aluminum by 3 °C?
   a. 1.3 kJ
   b. 0.60 kJ
   c. 54 kJ
   d. 5.4 kJ

11.3 Phase Change and Latent Heat

41. Upon what does the required amount of heat removed to freeze a sample of a substance depend?
   a. The mass of the substance and its latent heat of vaporization
   b. The mass of the substance and its latent heat of fusion
   c. The mass of the substance and its latent heat of sublimation
   d. The mass of the substance only

42. What do latent heats, L_f and L_v, depend on?
   a. L_f and L_v depend on the forces between the particles in the substance.
   b. L_f and L_v depend on the mass of the substance.
   c. L_f and L_v depend on the volume of the substance.
   d. L_f and L_v depend on the temperature of the substance.

43. How much energy is required to melt 7.00 kg a block of aluminum that is at its melting point? (Latent heat of fusion of aluminum is 380 kJ/kg.)
   a. 54.3 kJ
   b. 2.66 kJ
   c. 0.0184 kJ
   d. 2.66×10^3 kJ

44. A 3.00 kg sample of a substance is at its boiling point. If 5,360 kJ of energy are enough to boil away the entire substance, what is its latent heat of vaporization?
   a. 2,685 kJ/kg
   b. 3,580 kJ/kg
   c. 895 kJ/kg
   d. 1,790 kJ/kg

Extended Response

11.1 Temperature and Thermal Energy

45. What is the meaning of absolute zero?
   a. It is the temperature at which the internal energy of the system is maximum, because the speed of its constituent particles increases to maximum at this point.
   b. It is the temperature at which the internal energy of the system is maximum, because the speed of its constituent particles decreases to zero at this point.

c. It is the temperature at which the internal energy of the system approaches zero, because the speed of its constituent particles increases to a maximum at this point.
d. It is that temperature at which the internal energy of the system approaches zero, because the speed of its constituent particles decreases to zero at this point.

46. Why does it feel hotter on more humid days, even though there is no difference in temperature?
a. On hot, dry days, the evaporation of the sweat from the skin cools the body, whereas on humid days the concentration of water in the atmosphere is lower, which reduces the evaporation rate from the skin's surface.
b. On hot, dry days, the evaporation of the sweat from the skin cools the body, whereas on humid days the concentration of water in the atmosphere is higher, which reduces the evaporation rate from the skin's surface.
c. On hot, dry days, the evaporation of the sweat from the skin cools the body, whereas on humid days the concentration of water in the atmosphere is lower, which increases the evaporation rate from the skin's surface.
d. On hot, dry days, the evaporation of the sweat from the skin cools the body, whereas on humid days the concentration of water in the atmosphere is higher, which increases the evaporation rate from the skin's surface.

11.2 Heat, Specific Heat, and Heat Transfer

47. A hot piece of metal needs to be cooled. If you were to put the metal in ice or in cold water, such that the ice did not melt and the temperature of either changed by the same amount, which would reduce the metal's temperature more? Why?
a. Water would reduce the metal's temperature more, because water has a greater specific heat than ice.
b. Water would reduce the metal's temperature more, because water has a smaller specific heat than ice.
c. Ice would reduce the metal's temperature more, because ice has a smaller specific heat than water.
d. Ice would reduce the metal's temperature more, because ice has a greater specific heat than water.

48. On a summer night, why does a black object seem colder than a white one?
a. The black object radiates energy faster than the white one, and hence reaches a lower temperature in less time.
b. The black object radiates energy slower than the white one, and hence reaches a lower temperature in less time.
c. The black object absorbs energy faster than the white one, and hence reaches a lower temperature in less time.
d. The black object absorbs energy slower than the white one, and hence reaches a lower temperature in less time.

49. Calculate the difference in heat required to raise the temperatures of 1.00 kg of gold and 1.00 kg of aluminum by 1.00 °C. (The specific heat of aluminum equals 900 J/kg · °C; the specific heat of gold equals 129 J/kg · °C.)
a. 771 J
b. 129 J
c. 90 J
d. 900 J

11.3 Phase Change and Latent Heat

50. True or false—You have an ice cube floating in a glass of water with a thin thread resting across the cube. If you cover the ice cube and thread with a layer of salt, they will stick together, so that you are able to lift the ice-cube when you pick up the thread.
a. True
b. False

51. Suppose the energy required to freeze 0.250 kg of water were added to the same mass of water at an initial temperature of 1.0 °C. What would be the final temperature of the water?
a. -69.8 °C
b. 79.8 °C
c. -78.8 °C
d. 80.8 °C

CHAPTER 12
Thermodynamics

Figure 12.1 A steam engine uses energy transfer by heat to do work. (Modification of work by Gerald Friedrich, Pixabay)

Chapter Outline

INTRODUCTION Energy can be transferred to or from a system, either through a temperature difference between it and another system (i.e., by heat) or by exerting a force through a distance (work). In these ways, energy can be converted into other forms of energy in other systems. For example, a car engine burns fuel for heat transfer into a gas. Work is done by the gas as it exerts a force through a distance by pushing a piston outward. This work converts the energy into a variety of other forms—into an increase in the car's kinetic or gravitational potential energy; into electrical energy to run the spark plugs, radio, and lights; and back into stored energy in the car's battery. But most of the thermal energy transferred by heat from the fuel burning in the engine does not do work on the gas. Instead, much of this energy is released into the surroundings at lower temperature (i.e., lost through heat), which is quite inefficient. Car engines are only about 25 to 30 percent efficient. This inefficiency leads to increased fuel costs, so there is great interest in improving fuel efficiency. However, it is common knowledge that modern gasoline engines cannot be made much more efficient. The same is true about the conversion to electrical energy in large power stations, whether they are coal, oil, natural gas, or nuclear powered. Why is this the case?

The answer lies in the nature of heat. Basic physical laws govern how heat transfer for doing work takes place and limit the maximum possible efficiency of the process. This chapter will explore these laws as well their applications to everyday machines. These topics are part of *thermodynamics*—the study of heat and its relationship to doing work.

12.1 Zeroth Law of Thermodynamics: Thermal Equilibrium

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the zeroth law of thermodynamics

Section Key Terms

thermal equilibrium	zeroth law of thermodynamics

We learned in the previous chapter that when two objects (or *systems*) are in contact with one another, heat will transfer thermal energy from the object at higher temperature to the one at lower temperature until they both reach the same temperature. The objects are then in **thermal equilibrium**, and no further temperature changes will occur if they are isolated from other systems. The systems interact and change because their temperatures are different, and the changes stop once their temperatures are the same. Thermal equilibrium is established when two bodies are in *thermal contact* with each other—meaning heat transfer (i.e., the transfer of energy by heat) can occur between them. If two systems cannot freely exchange energy, they will not reach thermal equilibrium. (It is fortunate that empty space stands between Earth and the sun, because a state of thermal equilibrium with the sun would be too toasty for life on this planet!)

If two systems, A and B, are in thermal equilibrium with each another, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C. This statement may seem obvious, because all three have the same temperature, but it is basic to thermodynamics. It is called the **zeroth law of thermodynamics**.

TIPS FOR SUCCESS

The zeroth law of thermodynamics is very similar to the transitive property of equality in mathematics: If a = b and b = c, then a = c.

You may be wondering at this point, why the wacky name? Shouldn't this be called the *first* law of thermodynamics rather than the *zeroth*? The explanation is that this law was discovered after the first and second laws of thermodynamics but is so fundamental that scientists decided it should logically come first.

As an example of the zeroth law in action, consider newborn babies in neonatal intensive-care units in hospitals. Prematurely born or sick newborns are placed in special incubators. These babies have very little covering while in the incubators, so to an observer, they look as though they may not be warm enough. However, inside the incubator, the temperature of the air, the cot, and the baby are all the same—that is, they are in thermal equilibrium. The ambient temperature is just high enough to keep the baby safe and comfortable.

WORK IN PHYSICS

Thermodynamics Engineer

Thermodynamics engineers apply the principles of thermodynamics to mechanical systems so as to create or test products that rely on the interactions between heat, work, pressure, temperature, and volume. This type of work typically takes place in the aerospace industry, chemical manufacturing companies, industrial manufacturing plants, power plants (Figure 12.2), engine manufacturers, or electronics companies.

Figure 12.2 An engineer makes a site visit to the Baghdad South power plant.

The need for energy creates quite a bit of demand for thermodynamics engineers, because both traditional energy companies and alternative (*green*) energy startups rely on interactions between heat and work and so require the expertise of thermodynamics engineers. Traditional energy companies use mainly nuclear energy and energy from burning fossil fuels, such as coal. Alternative energy is finding new ways to harness renewable and, often, more readily available energy sources, such as solar, water, wind, and bio-energy.

A thermodynamics engineer in the energy industry can find the most efficient way to turn the burning of a biofuel or fossil fuel into energy, store that energy for times when it's needed most, or figure out how to best deliver that energy from where it's produced to where it's used: in homes, factories, and businesses. Additionally, he or she might also design pollution-control equipment to remove harmful pollutants from the smoke produced as a by-product of burning fuel. For example, a thermodynamics engineer may develop a way to remove mercury from burning coal in a coal-fired power plant.

Thermodynamics engineering is an expanding field, where employment opportunities are expected to grow by as much as 27 percent between 2012 and 2022, according to the U.S. Bureau of Labor Statistics. To become a thermodynamics engineer, you must have a college degree in chemical engineering, mechanical engineering, environmental engineering, aerospace engineering, civil engineering, or biological engineering (depending on which type of career you wish to pursue), with coursework in physics and physical chemistry that focuses on thermodynamics.

GRASP CHECK

What would be an example of something a thermodynamics engineer would do in the aeronautics industry?

a. Test the fuel efficiency of a jet engine
b. Test the functioning of landing gear
c. Test the functioning of a lift control device
d. Test the autopilot functions

Check Your Understanding

1. What is thermal equilibrium?
   a. When two objects in contact with each other are at the same pressure, they are said to be in thermal equilibrium.
   b. When two objects in contact with each other are at different temperatures, they are said to be in thermal equilibrium.
   c. When two objects in contact with each other are at the same temperature, they are said to be in thermal equilibrium.
   d. When two objects not in contact with each other are at the same pressure, they are said to be in thermal equilibrium.

2. What is the zeroth law of thermodynamics?

a. Energy can neither be created nor destroyed in a chemical reaction.
b. If two systems, A and B, are in thermal equilibrium with each another, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.
c. Entropy of any isolated system not in thermal equilibrium always increases.
d. Entropy of a system approaches a constant value as temperature approaches absolute zero.

12.2 First law of Thermodynamics: Thermal Energy and Work

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe how pressure, volume, and temperature relate to one another and to work, based on the ideal gas law
- Describe pressure–volume work
- Describe the first law of thermodynamics verbally and mathematically
- Solve problems involving the first law of thermodynamics

Section Key Terms

Boltzmann constant	first law of thermodynamics	ideal gas law	internal energy	pressure

Pressure, Volume, Temperature, and the Ideal Gas Law

Before covering the first law of thermodynamics, it is first important to understand the relationship between **pressure**, volume, and temperature. Pressure, P, is defined as

$$P = \frac{F}{A}, \tag{12.1}$$

where F is a force applied to an area, A, that is perpendicular to the force.

Depending on the area over which it is exerted, a given force can have a significantly different effect, as shown in Figure 12.3.

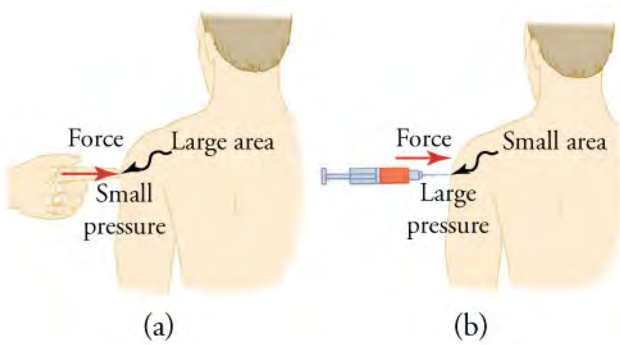


Figure 12.3 (a) Although the person being poked with the finger might be irritated, the force has little lasting effect. (b) In contrast, the same force applied to an area the size of the sharp end of a needle is great enough to break the skin.

The SI unit for pressure is the *pascal*, where $1\ \text{Pa} = 1\ \text{N/m}^2$.

Pressure is defined for all states of matter but is particularly important when discussing fluids (such as air). You have probably heard the word *pressure* being used in relation to blood (high or low blood pressure) and in relation to the weather (high- and low-pressure weather systems). These are only two of many examples of pressures in fluids.

The relationship between the pressure, volume, and temperature for an ideal gas is given by the **ideal gas law**. A gas is considered ideal at low pressure and fairly high temperature, and forces between its component particles can be ignored. The ideal gas law states that

$$PV = NkT. \tag{12.2}$$

where P is the pressure of a gas, V is the volume it occupies, N is the number of particles (atoms or molecules) in the gas, and T is its absolute temperature. The constant k is called the **Boltzmann constant** and has the value $k = 1.38 \times 10^{-23}$ J/K, For the purposes of this chapter, we will not go into calculations using the ideal gas law. Instead, it is important for us to notice from the equation that the following are true for a given mass of gas:

- When volume is constant, pressure is directly proportional to temperature.
- When temperature is constant, pressure is inversely proportional to volume.
- When pressure is constant, volume is directly proportional to temperature.

This last point describes *thermal expansion*—the change in size or volume of a given mass with temperature. What is the underlying cause of thermal expansion? An increase in temperature means that there's an increase in the kinetic energy of the individual atoms. Gases are especially affected by thermal expansion, although liquids expand to a lesser extent with similar increases in temperature, and even solids have minor expansions at higher temperatures. This is why railroad tracks and bridges have expansion joints that allow them to freely expand and contract with temperature changes.

To get some idea of how pressure, temperature, and volume of a gas are related to one another, consider what happens when you pump air into a deflated tire. The tire's volume first increases in direct proportion to the amount of air injected, without much increase in the tire pressure. Once the tire has expanded to nearly its full size, the walls limit volume expansion. If you continue to pump air into tire (which now has a nearly constant volume), the pressure increases with increasing temperature (see Figure 12.4).

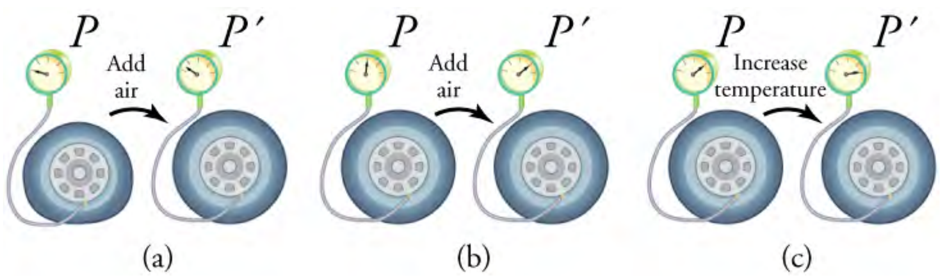


Figure 12.4 (a) When air is pumped into a deflated tire, its volume first increases without much increase in pressure. (b) When the tire is filled to a certain point, the tire walls resist further expansion, and the pressure increases as more air is added. (c) Once the tire is inflated fully, its pressure increases with temperature.

Pressure–Volume Work

Pressure–volume work is the work that is done by the compression or expansion of a fluid. Whenever there is a change in volume and external pressure remains constant, pressure–volume work is taking place. During a compression, a decrease in volume increases the internal pressure of a system as work is done *on* the system. During an expansion (Figure 12.5), an increase in volume decreases the internal pressure of a system as the system *does* work.

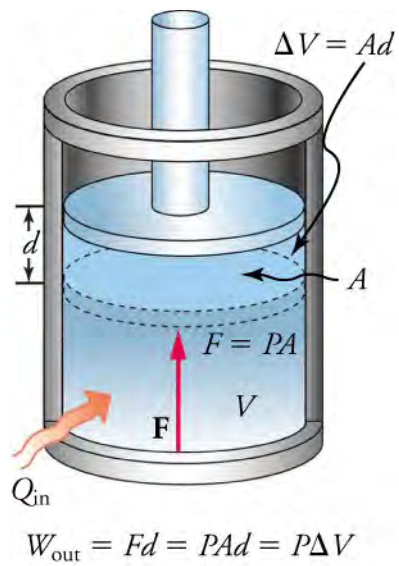


Figure 12.5 An expansion of a gas requires energy transfer to keep the pressure constant. Because pressure is constant, the work done is $P\Delta V$.

Recall that the formula for work is $W = Fd$. We can rearrange the definition of pressure, $P = \frac{F}{A}$, to get an expression for force in terms of pressure.

$$F = PA \tag{12.3}$$

Substituting this expression for force into the definition of work, we get

$$W = PAd. \tag{12.4}$$

Because area multiplied by displacement is the change in volume, $W = P\Delta V$, the mathematical expression for pressure–volume work is

$$W = P\Delta V. \tag{12.5}$$

Just as we say that work is force acting over a distance, for fluids, we can say that work is the pressure acting through the change in volume. For pressure–volume work, pressure is analogous to force, and volume is analogous to distance in the traditional definition of work.

WATCH PHYSICS

Work from Expansion

This video describes work from expansion (or pressure–volume work). Sal combines the equations $W = P\Delta V$ and $\Delta U = Q - W$ to get $\Delta U = Q - P\Delta V$.

[Click to view content (https://www.openstax.org/l/28expansionWork)](https://www.openstax.org/l/28expansionWork)

GRASP CHECK

If the volume of a system increases while pressure remains constant, is the value of work done by the system W positive or negative? Will this increase or decrease the internal energy of the system?

a. Positive; internal energy will decrease
b. Positive; internal energy will increase
c. Negative; internal energy will decrease
d. Negative; internal energy will increase

The First Law of Thermodynamics

Heat (Q) and work (W) are the two ways to add or remove energy from a system. The processes are very different. Heat is driven by temperature differences, while work involves a force exerted through a distance. Nevertheless, heat and work can produce identical results. For example, both can cause a temperature increase. Heat transfers energy into a system, such as when the sun warms the air in a bicycle tire and increases the air's temperature. Similarly, work can be done on the system, as when the bicyclist pumps air into the tire. Once the temperature increase has occurred, it is impossible to tell whether it was caused by heat or work. Heat and work are both energy in transit—neither is stored as such in a system. However, both can change the internal energy, U, of a system.

Internal energy is the sum of the kinetic and potential energies of a system's atoms and molecules. It can be divided into many subcategories, such as thermal and chemical energy, and depends only on the state of a system (that is, P, V, and T), not on how the energy enters or leaves the system.

In order to understand the relationship between heat, work, and internal energy, we use the **first law of thermodynamics**. The first law of thermodynamics applies the *conservation of energy* principle to systems where heat and work are the methods of transferring energy into and out of the systems. It can also be used to describe how energy transferred by heat is converted and transferred again by work.

TIPS FOR SUCCESS

Recall that the principle of conservation of energy states that energy cannot be created or destroyed, but it can be altered from one form to another.

The first law of thermodynamics states that the change in internal energy of a closed system equals the net heat transfer *into* the system minus the net work done *by* the system. In equation form, the first law of thermodynamics is

$$\Delta U = Q - W. \tag{12.6}$$

Here, ΔU is the *change in internal energy*, U, of the system. As shown in Figure 12.6, Q is the *net heat transferred into the system*—that is, Q is the sum of all heat transfers into and out of the system. W is the *net work done by the system*—that is, W is the sum of all work done on or by the system. By convention, if Q is positive, then there is a net heat transfer into the system; if W is positive, then there is net work done by the system. So positive Q adds energy to the system by heat, and positive W takes energy from the system by work. Note that if heat transfers more energy into the system than that which is done by work, the difference is stored as internal energy.

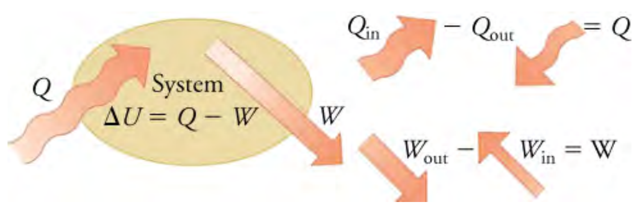


Figure 12.6 The first law of thermodynamics is the *conservation of energy* principle stated for a system, where heat and work are the methods of transferring energy to and from a system. Q represents the net heat transfer—it is the sum of all transfers of energy by heat into and out of the system. Q is positive for net heat transfer *into* the system. W_{out} is the work done *by* the system, and W_{in} is the work done *on* the system. W is the total work done on or *by* the system. W is positive when more work is done *by* the system than *on* it. The change in the internal energy of the system, ΔU, is related to heat and work by the first law of thermodynamics: $\Delta U = Q - W$.

It follows also that negative Q indicates that energy is transferred *away* from the system by heat and so decreases the system's internal energy, whereas negative W is work done *on* the system, which increases the internal energy.

WATCH PHYSICS

First Law of Thermodynamics/Internal Energy

This video explains the first law of thermodynamics, conservation of energy, and internal energy. It goes over an example of energy transforming between kinetic energy, potential energy, and heat transfer due to air resistance.

Click to view content (https://www.openstax.org/l/28FirstThermo)

GRASP CHECK

Consider the example of tossing a ball when there's air resistance. As air resistance increases, what would you expect to happen to the final velocity and final kinetic energy of the ball? Why?

a. Both will decrease. Energy is transferred to the air by heat due to air resistance.
b. Both will increase. Energy is transferred from the air to the ball due to air resistance.
c. Final velocity will increase, but final kinetic energy will decrease. Energy is transferred by heat to the air from the ball through air resistance.
d. Final velocity will decrease, but final kinetic energy will increase. Energy is transferred by heat from the air to the ball through air resistance.

WATCH PHYSICS

More on Internal Energy

This video goes into further detail, explaining internal energy and how to use the equation $\Delta U = Q - W$. Note that Sal uses the equation $\Delta U = Q + W$, where W is the work done *on* the system, whereas we use W to represent work done *by* the system.

Click to view content (https://www.openstax.org/l/28IntrnEnergy)

GRASP CHECK

If 5 J are taken away by heat from the system, and the system does 5 J of work, what is the change in internal energy of the system?

a. -10 J
b. 0 J
c. 10 J
d. 25 J

LINKS TO PHYSICS

Biology: Biological Thermodynamics

We often think about thermodynamics as being useful for inventing or testing machinery, such as engines or steam turbines. However, thermodynamics also applies to living systems, such as our own bodies. This forms the basis of the biological thermodynamics (Figure 12.7).

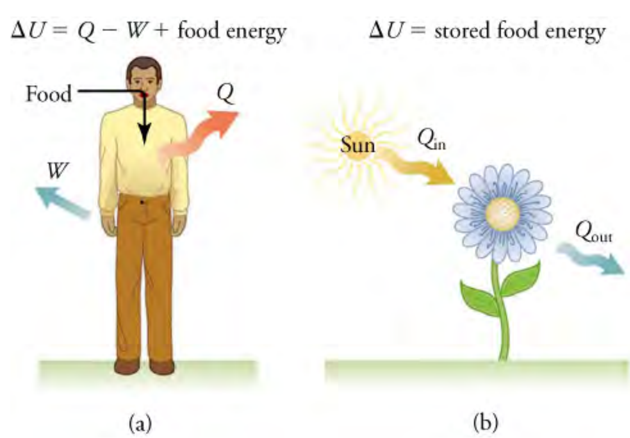


Figure 12.7 (a) The first law of thermodynamics applies to metabolism. Heat transferred out of the body (Q) and work done by the body (W) remove internal energy, whereas food intake replaces it. (Food intake may be considered work done on the body.) (b) Plants convert part of

the radiant energy in sunlight into stored chemical energy, a process called *photosynthesis*.

Life itself depends on the biological transfer of energy. Through photosynthesis, plants absorb solar energy from the sun and use this energy to convert carbon dioxide and water into glucose and oxygen. Photosynthesis takes in one form of energy—light—and converts it into another form—chemical potential energy (glucose and other carbohydrates).

Human metabolism is the conversion of food into energy given off by heat, work done by the body's cells, and stored fat. Metabolism is an interesting example of the first law of thermodynamics in action. Eating increases the internal energy of the body by adding chemical potential energy; this is an unromantic view of a good burrito.

The body metabolizes all the food we consume. Basically, metabolism is an oxidation process in which the chemical potential energy of food is released. This implies that food input is in the form of work. Exercise helps you lose weight, because it provides energy transfer from your body by both heat and work and raises your metabolic rate even when you are at rest.

Biological thermodynamics also involves the study of transductions between cells and living organisms. *Transduction* is a process where genetic material—DNA—is transferred from one cell to another. This often occurs during a viral infection (e.g., influenza) and is how the virus spreads, namely, by transferring its genetic material to an increasing number of previously healthy cells. Once enough cells become infected, you begin to feel the effects of the virus (flu symptoms—muscle weakness, coughing, and congestion).

Energy is transferred along with the genetic material and so obeys the first law of thermodynamics. Energy is transferred—not created or destroyed—in the process. When work is done on a cell or heat transfers energy to a cell, the cell's internal energy increases. When a cell does work or loses heat, its internal energy decreases. If the amount of work done by a cell is the same as the amount of energy transferred in by heat, or the amount of work performed on a cell matches the amount of energy transferred out by heat, there will be no net change in internal energy.

GRASP CHECK

Based on what you know about heat transfer and the first law of thermodynamics, do you need to eat more or less to maintain a constant weight in colder weather? Explain why.

a. more; as more energy is lost by the body in colder weather, the need to eat increases so as to maintain a constant weight
b. more; eating more food means accumulating more fat, which will insulate the body from colder weather and will reduce the energy loss
c. less; as less energy is lost by the body in colder weather, the need to eat decreases so as to maintain a constant weight
d. less; eating less food means accumulating less fat, so less energy will be required to burn the fat, and, as a result, weight will remain constant

Solving Problems Involving the First Law of Thermodynamics

WORKED EXAMPLE

Calculating Change in Internal Energy

Suppose 40.00 J of energy is transferred by heat to a system, while the system does 10.00 J of work. Later, heat transfers 25.00 J out of the system, while 4.00 J is done by work on the system. What is the net change in the system's internal energy?

STRATEGY

You must first calculate the net heat and net work. Then, using the first law of thermodynamics, $\Delta U = Q - W$, find the change in internal energy.

Solution

The net heat is the transfer into the system by heat minus the transfer out of the system by heat, or

$$Q = 40.00\text{ J} - 25.00\text{ J} = 15.00\text{ J}. \tag{12.7}$$

The total work is the work done by the system minus the work done on the system, or

$$W = 10.00\text{ J} - 4.00\text{ J} = 6.00\text{ J}. \tag{12.8}$$

The change in internal energy is given by the first law of thermodynamics.

$$\Delta U = Q - W = 15.00\text{ J} - 6.00\text{ J} = 9.00\text{ J} \tag{12.9}$$

Discussion

A different way to solve this problem is to find the change in internal energy for each of the two steps separately and then add the two changes to get the total change in internal energy. This approach would look as follows:

For 40.00 J of heat in and 10.00 J of work out, the change in internal energy is

$$\Delta U_1 = Q_1 - W_1 = 40.00\text{ J} - 10.00\text{ J} = 30.00\text{ J}. \tag{12.10}$$

For 25.00 J of heat out and 4.00 J of work in, the change in internal energy is

$$\Delta U_2 = Q_2 - W_2 = -25.00\text{ J} - (-4.00\text{ J}) = -21.00\text{ J}. \tag{12.11}$$

The total change is

$$\Delta U = \Delta U_1 + \Delta U_2 = 30.00\text{ J} + (-21.00\text{ J}) = 9.00\text{ J}. \tag{12.12}$$

No matter whether you look at the overall process or break it into steps, the change in internal energy is the same.

WORKED EXAMPLE

Calculating Change in Internal Energy: The Same Change in *U* is Produced by Two Different Processes

What is the change in the internal energy of a system when a total of 150.00 J is transferred by heat from the system and 159.00 J is done by work on the system?

STRATEGY

The net heat and work are already given, so simply use these values in the equation $\Delta U = Q - W$.

Solution

Here, the net heat and total work are given directly as $Q = -150.00\text{ J}$ and $W = -159.00\text{ J}$, so that

$$\Delta U = Q - W = -150.00\text{ J} - (-159.00\text{ J}) = 9.00\text{ J}. \tag{12.13}$$

Discussion

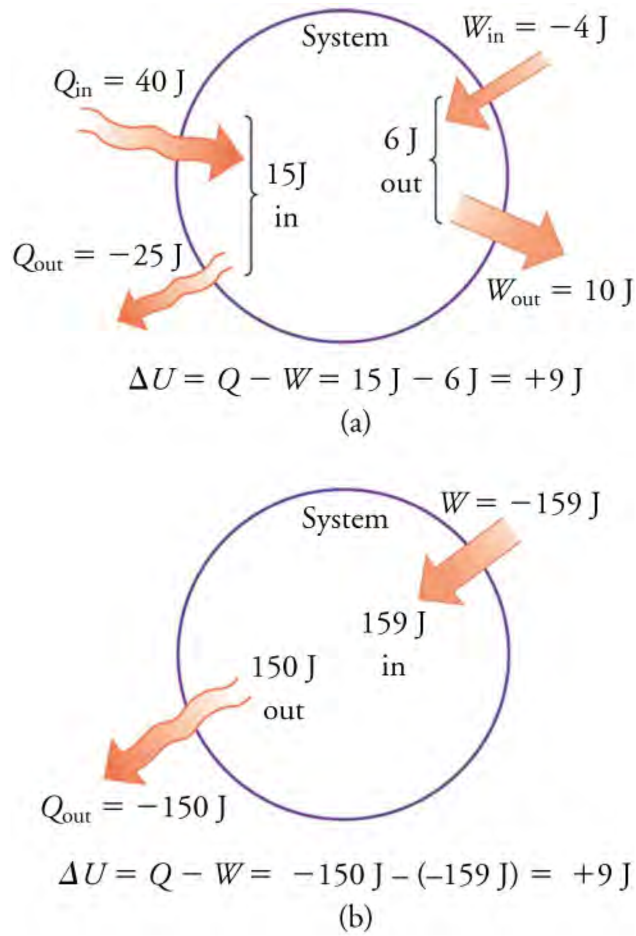


Figure 12.8 Two different processes produce the same change in a system. (a) A total of 15.00 J of heat transfer occurs into the system, while work takes out a total of 6.00 J. The change in internal energy is ΔU = Q – W = 9.00 J. (b) Heat transfer removes 150.00 J from the system while work puts 159.00 J into it, producing an increase of 9.00 J in internal energy. If the system starts out in the same state in (a) and (b), it will end up in the same final state in either case—its final state is related to internal energy, not how that energy was acquired.

A very different process in this second worked example produces the same 9.00 J change in internal energy as in the first worked example. Note that the change in the system in both parts is related to ΔU and not to the individual Q's or W's involved. The system ends up in the *same* state in both problems. Note that, as usual, in Figure 12.8 above, W_{out} is work done *by* the system, and W_{in} is work done *on* the system.

Practice Problems

3. What is the pressure-volume work done by a system if a pressure of $20\ \text{Pa}$ causes a change in volume of $3.0\ \text{m}^3$?
   a. $0.15\ \text{J}$
   b. $6.7\ \text{J}$
   c. $23\ \text{J}$
   d. $60\ \text{J}$

4. What is the net heat out of the system when $25\ \text{J}$ is transferred by heat into the system and $45\ \text{J}$ is transferred out of it?
   a. $-70\ \text{J}$
   b. $-20\ \text{J}$
   c. $20\ \text{J}$
   d. $70\ \text{J}$

Check Your Understanding

5. What is pressure?
    a. Pressure is force divided by length.
    b. Pressure is force divided by area.
    c. Pressure is force divided by volume.
    d. Pressure is force divided by mass.

6. What is the SI unit for pressure?
    a. pascal, or N/m^3
    b. coulomb
    c. newton
    d. pascal, or N/m^2

7. What is pressure-volume work?
    a. It is the work that is done by the compression or expansion of a fluid.
    b. It is the work that is done by a force on an object to produce a certain displacement.
    c. It is the work that is done by the surface molecules of a fluid.
    d. It is the work that is done by the high-energy molecules of a fluid.

8. When is pressure-volume work said to be done ON a system?
    a. When there is an increase in both volume and internal pressure.
    b. When there is a decrease in both volume and internal pressure.
    c. When there is a decrease in volume and an increase in internal pressure.
    d. When there is an increase in volume and a decrease in internal pressure.

9. What are the ways to add energy to or remove energy from a system?
    a. Transferring energy by heat is the only way to add energy to or remove energy from a system.
    b. Doing compression work is the only way to add energy to or remove energy from a system.
    c. Doing expansion work is the only way to add energy to or remove energy from a system.
    d. Transferring energy by heat or by doing work are the ways to add energy to or remove energy from a system.

10. What is internal energy?
    a. It is the sum of the kinetic energies of a system's atoms and molecules.
    b. It is the sum of the potential energies of a system's atoms and molecules.
    c. It is the sum of the kinetic and potential energies of a system's atoms and molecules.
    d. It is the difference between the magnitudes of the kinetic and potential energies of a system's atoms and molecules.

12.3 Second Law of Thermodynamics: Entropy

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe entropy
- Describe the second law of thermodynamics
- Solve problems involving the second law of thermodynamics

Section Key Terms

entropy	second law of thermodynamics

Entropy

Recall from the chapter introduction that it is not even theoretically possible for engines to be 100 percent efficient. This phenomenon is explained by the **second law of thermodynamics**, which relies on a concept known as **entropy**. Entropy is a measure of the disorder of a system. Entropy also describes how much energy is *not* available to do work. The more disordered a system and higher the entropy, the less of a system's energy is available to do work.

Although all forms of energy can be used to do work, it is not possible to use the entire available energy for work. Consequently, not all energy transferred by heat can be converted into work, and some of it is lost in the form of waste heat—that is, heat that does not go toward doing work. The unavailability of energy is important in thermodynamics; in fact, the field originated from efforts to convert heat to work, as is done by engines.

The equation for the change in entropy, ΔS, is

$$\Delta S = \frac{Q}{T},$$

where Q is the heat that transfers energy during a process, and T is the absolute temperature at which the process takes place.

Q is positive for energy transferred *into* the system by heat and negative for energy transferred *out of* the system by heat. In SI, entropy is expressed in units of joules per kelvin (J/K). If temperature changes during the process, then it is usually a good approximation (for small changes in temperature) to take T to be the average temperature in order to avoid trickier math (calculus).

TIPS FOR SUCCESS

Absolute temperature is the temperature measured in Kelvins. The Kelvin scale is an absolute temperature scale that is measured in terms of the number of degrees above absolute zero. All temperatures are therefore positive. Using temperatures from another, nonabsolute scale, such as Fahrenheit or Celsius, will give the wrong answer.

Second Law of Thermodynamics

Have you ever played the card game 52 pickup? If so, you have been on the receiving end of a practical joke and, in the process, learned a valuable lesson about the nature of the universe as described by the second law of thermodynamics. In the game of 52 pickup, the prankster tosses an entire deck of playing cards onto the floor, and you get to pick them up. In the process of picking up the cards, you may have noticed that the amount of work required to restore the cards to an orderly state in the deck is much greater than the amount of work required to toss the cards and create the disorder.

The second law of thermodynamics states that *the total entropy of a system either increases or remains constant in any spontaneous process; it never decreases.* An important implication of this law is that heat transfers energy spontaneously from higher- to lower-temperature objects, but never spontaneously in the reverse direction. This is because entropy increases for heat transfer of energy from hot to cold (Figure 12.9). Because the change in entropy is Q/T, there is a larger change in ΔS at lower temperatures (smaller T). The decrease in entropy of the hot (larger T) object is therefore less than the increase in entropy of the cold (smaller T) object, producing an overall increase in entropy for the system.

Figure 12.9 The ice in this drink is slowly melting. Eventually, the components of the liquid will reach thermal equilibrium, as predicted by the second law of thermodynamics—that is, after heat transfers energy from the warmer liquid to the colder ice. (Jon Sullivan, PDPhoto.org)

Another way of thinking about this is that it is impossible for any process to have, as its sole result, heat transferring energy from a cooler to a hotter object. Heat cannot transfer energy spontaneously from colder to hotter, because the entropy of the overall system would decrease.

Suppose we mix equal masses of water that are originally at two different temperatures, say $20.0\ {}^\circ\mathrm{C}$ and $40.0\ {}^\circ\mathrm{C}$. The result will be water at an intermediate temperature of $30.0\ {}^\circ\mathrm{C}$. Three outcomes have resulted: entropy has increased, some energy has become unavailable to do work, and the system has become less orderly. Let us think about each of these results.

First, why has entropy increased? Mixing the two bodies of water has the same effect as the heat transfer of energy from the higher-temperature substance to the lower-temperature substance. The mixing decreases the entropy of the hotter water but increases the entropy of the colder water by a greater amount, producing an overall increase in entropy.

Second, once the two masses of water are mixed, there is no more temperature difference left to drive energy transfer by heat and therefore to do work. The energy is still in the water, but it is now *unavailable* to do work.

Third, the mixture is less orderly, or to use another term, less structured. Rather than having two masses at different temperatures and with different distributions of molecular speeds, we now have a single mass with a broad distribution of molecular speeds, the average of which yields an intermediate temperature.

These three results—entropy, unavailability of energy, and disorder—not only are related but are, in fact, essentially equivalent. Heat transfer of energy from hot to cold is related to the tendency in nature for systems to become disordered and for less energy to be available for use as work.

Based on this law, what cannot happen? A cold object in contact with a hot one never spontaneously transfers energy by heat to the hot object, getting colder while the hot object gets hotter. Nor does a hot, stationary automobile ever spontaneously cool off and start moving.

Another example is the expansion of a puff of gas introduced into one corner of a vacuum chamber. The gas expands to fill the chamber, but it never regroups on its own in the corner. The random motion of the gas molecules could take them all back to the corner, but this is never observed to happen (Figure 12.10).

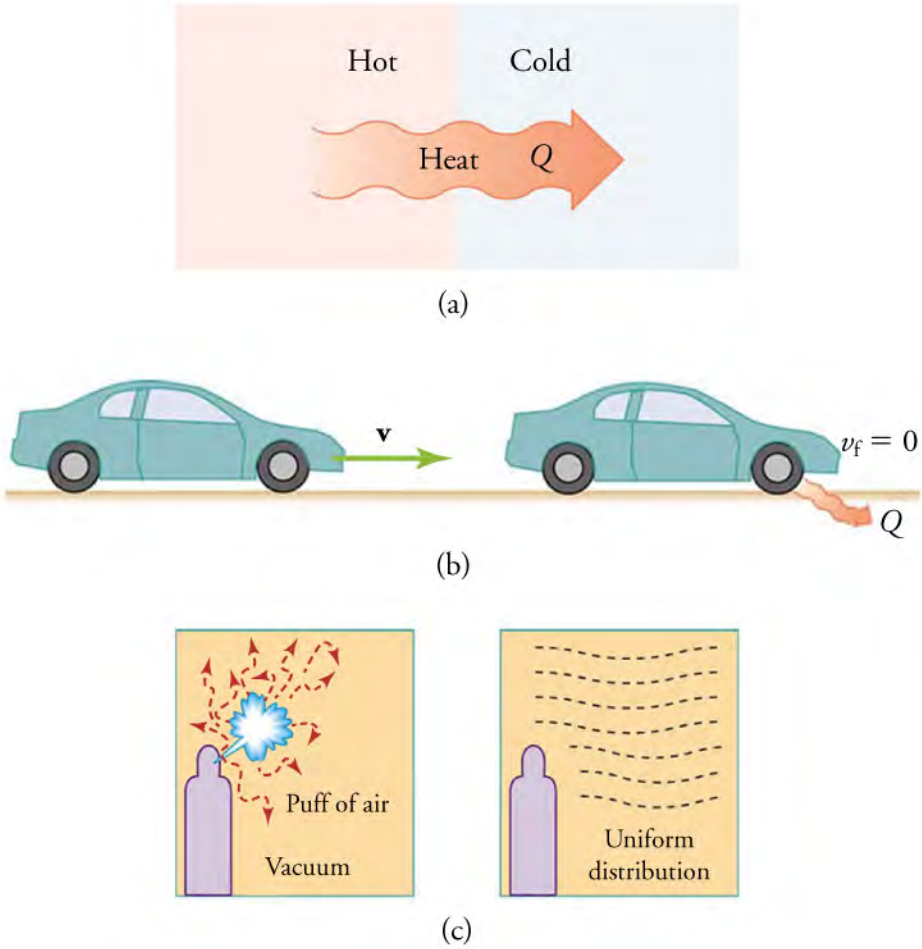


Figure 12.10 Examples of one-way processes in nature. (a) Heat transfer occurs spontaneously from hot to cold, but not from cold to hot. (b) The brakes of this car convert its kinetic energy to increase their internal energy (temperature), and heat transfers this energy to the environment. The reverse process is impossible. (c) The burst of gas released into this vacuum chamber quickly expands to uniformly fill every part of the chamber. The random motions of the gas molecules will prevent them from returning altogether to the corner.

We've explained that heat never transfers energy spontaneously from a colder to a hotter object. The key word here is *spontaneously*. If we *do work* on a system, it *is* possible to transfer energy by heat from a colder to hotter object. We'll learn more about this in the next section, covering refrigerators as one of the applications of the laws of thermodynamics.

Sometimes people misunderstand the second law of thermodynamics, thinking that based on this law, it is impossible for entropy to decrease at any particular location. But, it actually *is* possible for the entropy of *one part* of the universe to decrease, as long as the total change in entropy of the universe increases. In equation form, we can write this as

$$\Delta S_{\text{tot}} = \Delta S_{\text{syst}} + \Delta S_{\text{envir}} > 0.$$

Based on this equation, we see that ΔS_{syst} can be negative as long as ΔS_{envir} is positive and greater in magnitude.

How is it possible for the entropy of a system to decrease? Energy transfer is necessary. If you pick up marbles that are scattered about the room and put them into a cup, your work has decreased the entropy of that system. If you gather iron ore from the ground and convert it into steel and build a bridge, your work has decreased the entropy of that system. Energy coming from the sun can decrease the entropy of local systems on Earth—that is, ΔS_{syst} is negative. But the overall entropy of the rest of the universe increases by a greater amount—that is, ΔS_{envir} is positive and greater in magnitude. In the case of the iron ore, although you made the system of the bridge and steel more structured, you did so at the expense of the universe. Altogether, the entropy of the universe is increased by the disorder created by digging up the ore and converting it to steel. Therefore,

$$\Delta S_{\text{tot}} = \Delta S_{\text{syst}} + \Delta S_{\text{envir}} > 0, \tag{12.14}$$

and the second law of thermodynamics is *not* violated.

Every time a plant stores some solar energy in the form of chemical potential energy, or an updraft of warm air lifts a soaring bird, Earth experiences local decreases in entropy as it uses part of the energy transfer from the sun into deep space to do work. There is a large total increase in entropy resulting from this massive energy transfer. A small part of this energy transfer by heat is stored in structured systems on Earth, resulting in much smaller, local decreases in entropy.

Solving Problems Involving the Second Law of Thermodynamics

Entropy is related not only to the unavailability of energy to do work; it is also a measure of disorder. For example, in the case of a melting block of ice, a highly structured and orderly system of water molecules changes into a disorderly liquid, in which molecules have no fixed positions (Figure 12.11). There is a large increase in entropy for this process, as we'll see in the following worked example.

Figure 12.11 These ice floes melt during the Arctic summer. Some of them refreeze in the winter, but the second law of thermodynamics predicts that it would be extremely unlikely for the water molecules contained in these particular floes to reform in the distinctive alligator-like shape they possessed when this picture was taken in the summer of 2009. (Patrick Kelley, U.S. Coast Guard, U.S. Geological Survey)

WORKED EXAMPLE

Entropy Associated with Disorder

Find the increase in entropy of 1.00 kg of ice that is originally at $0\ °\text{C}$ and melts to form water at $0\ °\text{C}$.

STRATEGY

The change in entropy can be calculated from the definition of ΔS once we find the energy, Q, needed to melt the ice.

Solution

The change in entropy is defined as

$$\Delta S = \frac{Q}{T}. \tag{12.15}$$

Here, Q is the heat necessary to melt 1.00 kg of ice and is given by

$$Q = mL_f, \tag{12.16}$$

where m is the mass and L_f is the latent heat of fusion. $L_f = 334\ \text{kJ/kg}$ for water, so

$$Q = (1.00\ \text{kg})(334\ \text{kJ/kg}) = 3.34 \times 10^5\,\text{J}. \tag{12.17}$$

Because Q is the amount of energy heat adds to the ice, its value is positive, and T is the melting temperature of ice, $T = 273\ \text{K}$ So the change in entropy is

$$\Delta S = \frac{Q}{T} = \frac{3.34 \times 10^5\,\text{J}}{273\ \text{K}} = 1.22 \times 10^3\,\text{J/K}. \tag{12.18}$$

Discussion

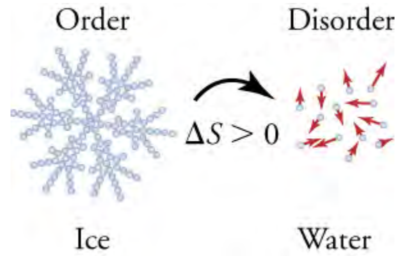


Figure 12.12 When ice melts, it becomes more disordered and less structured. The systematic arrangement of molecules in a crystal structure is replaced by a more random and less orderly movement of molecules without fixed locations or orientations. Its **entropy** increases because heat transfer occurs into it. Entropy is a measure of disorder.

The change in entropy is positive, because heat transfers energy *into* the ice to cause the phase change. This is a significant increase in entropy, because it takes place at a relatively low temperature. It is accompanied by an increase in the disorder of the water molecules.

Practice Problems

11. If $30.0\ \text{J}$ are added by heat to water at 12°C, what is the change in entropy?
    a. $0.105\ \text{J/K}$
    b. $2.5\ \text{J/K}$
    c. $0.45\ \text{J/K}$
    d. $9.50\ \text{J/K}$

12. What is the increase in entropy when $3.00\ \text{kg}$ of ice at 0°C melt to form water at 0°C?
    a. $1.84 \times 10^3\ \text{J/K}$
    b. $3.67 \times 10^3\ \text{J/K}$
    c. $1.84 \times 10^8\ \text{J/K}$
    d. $3.67 \times 10^8\ \text{J/K}$

Check Your Understanding

13. What is entropy?
    a. Entropy is a measure of the potential energy of a system.
    b. Entropy is a measure of the net work done by a system.
    c. Entropy is a measure of the disorder of a system.
    d. Entropy is a measure of the heat transfer of energy into a system.

14. Which forms of energy can be used to do work?
    a. Only work is able to do work.
    b. Only heat is able to do work.
    c. Only internal energy is able to do work.
    d. Heat, work, and internal energy are all able to do work.

15. What is the statement for the second law of thermodynamics?
    a. All the spontaneous processes result in decreased total entropy of a system.
    b. All the spontaneous processes result in increased total entropy of a system.
    c. All the spontaneous processes result in decreased or constant total entropy of a system.
    d. All the spontaneous processes result in increased or constant total entropy of a system.

16. For heat transferring energy from a high to a low temperature, what usually happens to the entropy of the whole system?
    a. It decreases.
    b. It must remain constant.
    c. The entropy of the system cannot be predicted without specific values for the temperatures.

d. It increases.

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain how heat engines, heat pumps, and refrigerators work in terms of the laws of thermodynamics
- Describe thermal efficiency
- Solve problems involving thermal efficiency

Section Key Terms

cyclical process	heat engine	heat pump
thermal efficiency		

Heat Engines, Heat Pumps, and Refrigerators

In this section, we'll explore how heat engines, heat pumps, and refrigerators operate in terms of the laws of thermodynamics.

One of the most important things we can do with heat is to use it to do work for us. A **heat engine** does exactly this—it makes use of the properties of thermodynamics to transform heat into work. Gasoline and diesel engines, jet engines, and steam turbines that generate electricity are all examples of heat engines.

Figure 12.13 illustrates one of the ways in which heat transfers energy to do work. Fuel combustion releases chemical energy that heat transfers throughout the gas in a cylinder. This increases the gas temperature, which in turn increases the pressure of the gas and, therefore, the force it exerts on a movable piston. The gas does work on the outside world, as this force moves the piston through some distance. Thus, heat transfer of energy to the gas in the cylinder results in work being done.

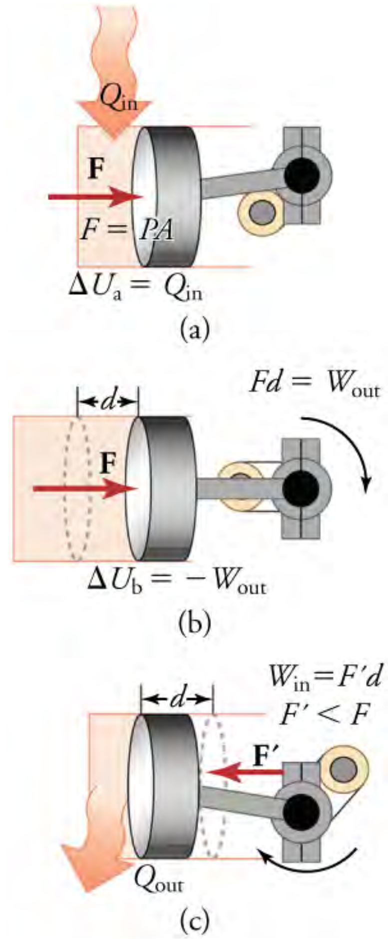


Figure 12.13 (a) Heat transfer to the gas in a cylinder increases the internal energy of the gas, creating higher pressure and temperature. (b) The force exerted on the movable cylinder does work as the gas expands. Gas pressure and temperature decrease during expansion, indicating that the gas's internal energy has decreased as it does work. (c) Heat transfer of energy to the environment further reduces pressure in the gas, so that the piston can more easily return to its starting position.

To repeat this process, the piston needs to be returned to its starting point. Heat now transfers energy from the gas to the surroundings, so that the gas's pressure decreases, and a force is exerted by the surroundings to push the piston back through some distance.

A **cyclical process** brings a system, such as the gas in a cylinder, back to its original state at the end of every cycle. All heat engines use cyclical processes.

Heat engines do work by using part of the energy transferred by heat from some source. As shown in Figure 12.14, heat transfers energy, Q_h, from the high-temperature object (or hot reservoir), whereas heat transfers unused energy, Q_c, into the low-temperature object (or cold reservoir), and the work done by the engine is W. In physics, a *reservoir* is defined as an infinitely large mass that can take in or put out an unlimited amount of heat, depending upon the needs of the system. The temperature of the hot reservoir is T_h, and the temperature of the cold reservoir is T_c .

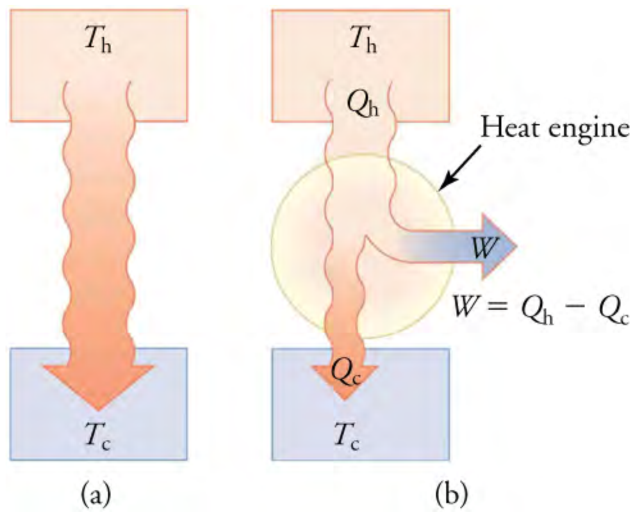


Figure 12.14 (a) Heat transfers energy spontaneously from a hot object to a cold one, as is consistent with the second law of thermodynamics. (b) A heat engine, represented here by a circle, uses part of the energy transferred by heat to do work. The hot and cold objects are called the hot and cold reservoirs. Q_h is the heat out of the hot reservoir, W is the work output, and Q_c is the unused heat into the cold reservoir.

As noted, a cyclical process brings the system back to its original condition at the end of every cycle. Such a system's internal energy, U, is the same at the beginning and end of every cycle—that is, $\Delta U = 0$. The first law of thermodynamics states that $\Delta U = Q - W$, where Q is the *net* heat transfer during the cycle, and W is the *net* work done by the system. The net heat transfer is the energy transferred in by heat from the hot reservoir minus the amount that is transferred out to the cold reservoir ($Q = Q_h - Q_c$). Because there is no change in internal energy for a complete cycle ($\Delta U = 0$), we have

$$0 = Q - W, \tag{12.19}$$

so that

$$W = Q. \tag{12.20}$$

Therefore, the net work done by the system equals the net heat into the system, or

$$W = Q_h - Q_c \tag{12.21}$$

for a cyclical process.

Because the hot reservoir is heated externally, which is an energy-intensive process, it is important that the work be done as efficiently as possible. In fact, we want W to equal Q_h , and for there to be no heat to the environment (that is, $Q_c = 0$). Unfortunately, this is impossible. According to the second law of thermodynamics, heat engines cannot have perfect conversion of heat into work. Recall that entropy is a measure of the disorder of a system, which is also how much energy is unavailable to do work. The second law of thermodynamics requires that the total entropy of a system either increases or remains constant in any process. Therefore, there is a minimum amount of Q_h that cannot be used for work. The amount of heat rejected to the cold reservoir, Q_c , depends upon the efficiency of the heat engine. The smaller the increase in entropy, ΔS , the smaller the value of Q_c , and the more heat energy is available to do work.

Heat pumps, air conditioners, and refrigerators utilize heat transfer of energy from low to high temperatures, which is the opposite of what heat engines do. Heat transfers energy Q_c from a cold reservoir and delivers energy Q_h into a hot one. This requires work input, W, which produces a transfer of energy by heat. Therefore, the total heat transfer to the hot reservoir is

$$Q_h = Q_c + W. \tag{12.22}$$

The purpose of a heat pump is to transfer energy by heat to a warm environment, such as a home in the winter. The great advantage of using a heat pump to keep your home warm rather than just burning fuel in a fireplace or furnace is that a heat pump supplies $Q_h = Q_c + W$. Heat Q_c comes from the outside air, even at a temperature below freezing, to the indoor space. You only pay for W, and you get an additional heat transfer of Q_c from the outside at no cost. In many cases, at least twice as much energy is transferred to the heated space as is used to run the heat pump. When you burn fuel to keep warm, you pay for all of it. The disadvantage to a heat pump is that the work input (required by the second law of thermodynamics) is sometimes more expensive than simply burning fuel, especially if the work is provided by electrical energy.

The basic components of a heat pump are shown in Figure 12.15. A working fluid, such as a refrigerant, is used. In the outdoor coils (the evaporator), heat Q_c enters the working fluid from the cold outdoor air, turning it into a gas.

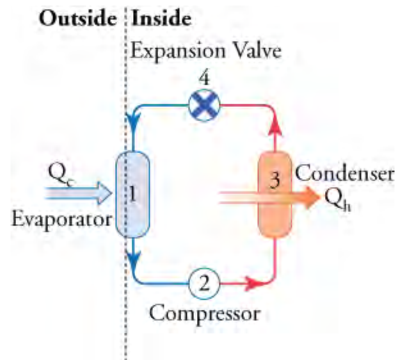


Figure 12.15 A simple heat pump has four basic components: (1) an evaporator, (2) a compressor, (3) a condenser, and (4) an expansion valve. In the heating mode, heat transfers Q_c to the working fluid in the evaporator (1) from the colder, outdoor air, turning it into a gas. The electrically driven compressor (2) increases the temperature and pressure of the gas and forces it into the condenser coils (3) inside the heated space. Because the temperature of the gas is higher than the temperature in the room, heat transfers energy from the gas to the room as the gas condenses into a liquid. The working fluid is then cooled as it flows back through an expansion valve (4) to the outdoor evaporator coils.

The electrically driven compressor (work input W) raises the temperature and pressure of the gas and forces it into the condenser coils that are inside the heated space. Because the temperature of the gas is higher than the temperature inside the room, heat transfers energy to the room, and the gas condenses into a liquid. The liquid then flows back through an expansion (pressure-reducing) valve. The liquid, having been cooled through expansion, returns to the outdoor evaporator coils to resume the cycle.

The quality of a heat pump is judged by how much energy is transferred by heat into the warm space (Q_h) compared with how much input work (W) is required.

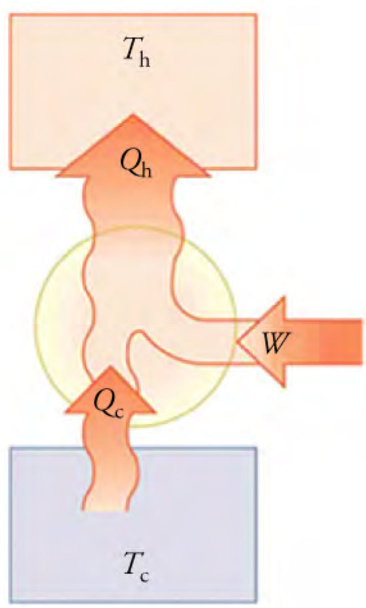


Figure 12.16 Heat pumps, air conditioners, and refrigerators are heat engines operated backward. Almost every home contains a refrigerator. Most people don't realize that they are also sharing their homes with a heat pump.

Air conditioners and refrigerators are designed to cool substances by transferring energy by heat Q_c out of a cool environment to a warmer one, where heat Q_h is given up. In the case of a refrigerator, heat is moved out of the inside of the fridge into the surrounding room. For an air conditioner, heat is transferred outdoors from inside a home. Heat pumps are also often used in a reverse setting to cool rooms in the summer.

As with heat pumps, work input is required for heat transfer of energy from cold to hot. The quality of air conditioners and refrigerators is judged by how much energy is removed by heat Q_c from a cold environment, compared with how much work, W, is required. So, what is considered the energy benefit in a heat pump, is considered waste heat in a refrigerator.

Thermal Efficiency

In the conversion of energy into work, we are always faced with the problem of getting less out than we put in. The problem is that, in all processes, there is some heat Q_c that transfers energy to the environment—and usually a very significant amount at that. A way to quantify how efficiently a machine runs is through a quantity called **thermal efficiency**.

We define thermal efficiency, *Eff*, to be the ratio of useful energy output to the energy input (or, in other words, the ratio of what we get to what we spend). The efficiency of a heat engine is the output of net work, W, divided by heat-transferred energy, Q_h, into the engine; that is

$$Eff = \frac{W}{Q_h}.$$

An efficiency of 1, or 100 percent, would be possible only if there were no heat to the environment ($Q_c = 0$).

TIPS FOR SUCCESS

All values of heat (Q_h and Q_c) are positive; there is no such thing as negative heat. The *direction* of heat is indicated by a plus or minus sign. For example, Q_c is out of the system, so it is preceded by a minus sign in the equation for net heat.

$$Q = Q_h - Q_c \tag{12.23}$$

Solving Thermal Efficiency Problems

WORKED EXAMPLE

Daily Work Done by a Coal-Fired Power Station and Its Efficiency

A coal-fired power station is a huge heat engine. It uses heat to transfer energy from burning coal to do work to turn turbines, which are used then to generate electricity. In a single day, a large coal power station transfers $2.50 \times 10^{14}\,\mathrm{J}$ by heat from burning coal and transfers $1.48 \times 10^{14}\,\mathrm{J}$ by heat into the environment. (a) What is the work done by the power station? (b) What is the efficiency of the power station?

STRATEGY

We can use $W = Q_h - Q_c$ to find the work output, W, assuming a cyclical process is used in the power station. In this process, water is boiled under pressure to form high-temperature steam, which is used to run steam turbine-generators and then condensed back to water to start the cycle again.

Solution

Work output is given by

$$W = Q_h - Q_c. \tag{12.24}$$

Substituting the given values,

$$W = 2.50 \times 10^{14}\,\mathrm{J} - 1.48 \times 10^{14}\,\mathrm{J} = 1.02 \times 10^{14}\,\mathrm{J}. \tag{12.25}$$

STRATEGY

The efficiency can be calculated with $Eff = \frac{W}{Q_h}$, because Q_h is given, and work, W, was calculated in the first part of this example.

Solution

Efficiency is given by

$$Eff = \frac{W}{Q_\mathrm{h}}. \quad (12.26)$$

The work, W, is found to be $1.02 \times 10^{14}\,\mathrm{J}$, and Q_h is given ($2.50 \times 10^{14}\,\mathrm{J}$), so the efficiency is

$$Eff = \frac{1.02 \times 10^{14}\,\mathrm{J}}{2.50 \times 10^{14}\,\mathrm{J}} = 0.408,\text{ or } 40.8\%. \quad (12.27)$$

Discussion

The efficiency found is close to the usual value of 42 percent for coal-burning power stations. It means that fully 59.2 percent of the energy is transferred by heat to the environment, which usually results in warming lakes, rivers, or the ocean near the power station and is implicated in a warming planet generally. While the laws of thermodynamics limit the efficiency of such plants—including plants fired by nuclear fuel, oil, and natural gas—the energy transferred by heat to the environment could be, and sometimes is, used for heating homes or for industrial processes.

Practice Problems

17. A heat engine is given $120\,\mathrm{J}$ by heat and releases $20\,\mathrm{J}$ by heat to the environment. What is the amount of work done by the system?

a. $-100\,\mathrm{J}$
b. $-60\,\mathrm{J}$
c. $60\,\mathrm{J}$
d. $100\,\mathrm{J}$

18. A heat engine takes in 6.0 kJ from heat and produces waste heat of 4.8 kJ. What is its efficiency?

a. 25 percent
b. 2.50 percent
c. 2.00 percent
d. 20 percent

Check Your Understanding

19. What is a heat engine?

a. A heat engine converts mechanical energy into thermal energy.
b. A heat engine converts thermal energy into mechanical energy.
c. A heat engine converts thermal energy into electrical energy.
d. A heat engine converts electrical energy into thermal energy.

20. Give an example of a heat engine.

a. A generator
b. A battery
c. A water pump
d. A car engine

21. What is thermal efficiency?

a. Thermal efficiency is the ratio of work input to the energy input.
b. Thermal efficiency is the ratio of work output to the energy input.
c. Thermal efficiency is the ratio of work input to the energy output.
d. Thermal efficiency is the ratio of work output to the energy output.

22. What is the mathematical expression for thermal efficiency?

a. $Eff = \frac{Q_\mathrm{h}}{Q_\mathrm{h} - Q_c}$
b. $Eff = \frac{Q_\mathrm{h}}{Q_c}$
c. $Eff = \frac{Q_\mathrm{c}}{Q_\mathrm{h}}$
d. $Eff = \frac{Q_\mathrm{h} - Q_\mathrm{c}}{Q_\mathrm{h}}$

KEY TERMS

Boltzmann constant constant with the value $k = 1.38 \times 10^{-23}$ J/K, which is used in the ideal gas law

cyclical process process in which a system is brought back to its original state at the end of every cycle

entropy measurement of a system's disorder and how much energy is not available to do work in a system

first law of thermodynamics states that the change in internal energy of a system equals the net energy transfer by heat *into* the system minus the net work done *by* the system

heat engine machine that uses energy transfer by heat to do work

heat pump machine that generates the heat transfer of energy from cold to hot

ideal gas law physical law that relates the pressure and volume of a gas to the number of gas molecules or atoms, or number of moles of gas, and the absolute temperature of the gas

internal energy sum of the kinetic and potential energies of a system's constituent particles (atoms or molecules)

pressure force per unit area perpendicular to the force, over which the force acts

second law of thermodynamics states that the total entropy of a system either increases or remains constant in any spontaneous process; it never decreases

thermal efficiency ratio of useful energy output to the energy input

thermal equilibrium condition in which heat no longer transfers energy between two objects that are in contact; the two objects have the same temperature

zeroth law of thermodynamics states that if two objects are in thermal equilibrium, and a third object is in thermal equilibrium with one of those objects, it is also in thermal equilibrium with the other object

SECTION SUMMARY

12.1 Zeroth Law of Thermodynamics: Thermal Equilibrium

- Systems are in thermal equilibrium when they have the same temperature.
- Thermal equilibrium occurs when two bodies are in contact with each other and can freely exchange energy.
- The zeroth law of thermodynamics states that when two systems, A and B, are in thermal equilibrium with each other, and B is in thermal equilibrium with a third system, C, then A is also in thermal equilibrium with C.

12.2 First law of Thermodynamics: Thermal Energy and Work

- Pressure is the force per unit area over which the force is applied perpendicular to the area.
- Thermal expansion is the increase, or decrease, of the size (length, area, or volume) of a body due to a change in temperature.
- The ideal gas law relates the pressure and volume of a gas to the number of gas particles (atoms or molecules) and the absolute temperature of the gas.
- Heat and work are the two distinct methods of energy transfer.
- Heat is energy transferred solely due to a temperature difference.
- The first law of thermodynamics is given as $\Delta U = Q - W$, where ΔU is the change in internal energy of a system, Q is the net energy transfer into the system by heat (the sum of all transfers by heat into and out of the system), and W is the net work done by the system (the sum of all energy transfers by work out of or into the system).
- Both Q and W represent energy in transit; only ΔU represents an independent quantity of energy capable of being stored.
- The internal energy U of a system depends only on the state of the system, and not how it reached that state.

12.3 Second Law of Thermodynamics: Entropy

- Entropy is a measure of a system's disorder: the greater the disorder, the larger the entropy.
- Entropy is also the reduced availability of energy to do work.
- The second law of thermodynamics states that, for any spontaneous process, the total entropy of a system either increases or remains constant; it never decreases.
- Heat transfers energy spontaneously from higher- to lower-temperature bodies, but never spontaneously in the reverse direction.

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

- Heat engines use the heat transfer of energy to do work.
- Cyclical processes are processes that return to their original state at the end of every cycle.
- The thermal efficiency of a heat engine is the ratio of work output divided by the amount of energy input.
- The amount of work a heat engine can do is determined by the net heat transfer of energy during a cycle; more waste heat leads to less work output.

- Heat pumps draw energy by heat from cold outside air and use it to heat an interior room.
- A refrigerator is a type of heat pump; it takes energy from the warm air from the inside compartment and transfers it to warmer exterior air.

KEY EQUATIONS

12.2 First law of Thermodynamics: Thermal Energy and Work

ideal gas law	$PV = NkT$
first law of thermodynamics	$\Delta U = Q - W$
pressure	$P = \frac{F}{A}$
pressure–volume work	$W = P\Delta V$

12.3 Second Law of Thermodynamics: Entropy

change in entropy	$\Delta S = \frac{Q}{T}$

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

thermal efficiency of a heat engine	$Eff = \frac{W}{Q_{\mathrm{h}}}$
work output for a cyclical process	$W = Q_{\mathrm{h}} - Q_{\mathrm{c}}$

CHAPTER REVIEW

Concept Items

12.1 Zeroth Law of Thermodynamics: Thermal Equilibrium

1. When are two bodies in thermal equilibrium?
   a. When they are in thermal contact and are at different pressures
   b. When they are not in thermal contact but are at the same pressure
   c. When they are not in thermal contact but are at different temperatures
   d. When they are in thermal contact and are at the same temperature

2. What is thermal contact?
   a. Two objects are said to be in thermal contact when they are in contact with each other in such a way that the transfer of energy by heat can occur between them.
   b. Two objects are said to be in thermal contact when they are in contact with each other in such a way that the transfer of energy by mass can occur between them.
   c. Two objects are said to be in thermal contact when they neither lose nor gain energy by heat. There is no transfer of energy between them.
   d. Two objects are said to be in thermal contact when they only gain energy by heat. There is transfer of energy between them.

3. To which mathematical property is the zeroth law of thermodynamics similar?
   a. Associative property
   b. Commutative property
   c. Distributive property
   d. Transitive property

12.2 First law of Thermodynamics: Thermal Energy and Work

4. Why does thermal expansion occur?
   a. An increase in temperature causes intermolecular distances to increase.
   b. An increase in temperature causes intermolecular distances to decrease.
   c. An increase in temperature causes an increase in the work done on the system.
   d. An increase in temperature causes an increase in the work done by the system.

5. How does pressure-volume work relate to heat and internal energy of a system?
   a. The energy added to a system by heat minus the change in the internal energy of that system is equal to the pressure-volume work done by the system.
   b. The sum of the energy released by a system by heat and the change in the internal energy of that system is equal to the pressure-volume work done by the system.
   c. The product of the energy added to a system by heat and the change in the internal energy of that system

is equal to the pressure-volume work done by the system.
   d. If the energy added to a system by heat is divided by the change in the internal energy of that system, the quotient is equal to the pressure-volume work done by the system.

6. On what does internal energy depend?
   a. The path of energy changes in the system
   b. The state of the system
   c. The size of the system
   d. The shape of the system

7. The first law of thermodynamics helps us understand the relationships among which three quantities?
   a. Heat, work, and internal energy
   b. Heat, work, and external energy
   c. Heat, work, and enthalpy
   d. Heat, work, and entropy

12.3 Second Law of Thermodynamics: Entropy

8. Air freshener is sprayed from a bottle. The molecules spread throughout the room and cannot make their way back into the bottle. Why is this the case?
   a. The entropy of the molecules increases.
   b. The entropy of the molecules decreases.
   c. The heat content (*enthalpy*, or total energy available for heat) of the molecules increases.
   d. The heat content (*enthalpy*, or total energy available for heat) of the molecules decreases.

9. Give an example of entropy as experienced in everyday life.
   a. rotation of Earth
   b. formation of a solar eclipse
   c. filling a car tire with air
   d. motion of a pendulum bob

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

10. What is the quality by which air conditioners are judged?
   a. The amount of energy generated by heat from a hot environment, compared with the required work input
   b. The amount of energy transferred by heat from a cold environment, compared with the required work input
   c. The amount of energy transferred by heat from a hot environment, compared with the required work output
   d. The amount of energy transferred by heat from a cold environment, compared with the required work output

11. Why is the efficiency of a heat engine never 100 percent?
   a. Some energy is always gained by heat from the environment.
   b. Some energy is always lost by heat to the environment.
   c. Work output is always greater than energy input.
   d. Work output is infinite for any energy input.

12. What is a cyclic process?
   a. A process in which the system returns to its original state at the end of the cycle
   b. A process in which the system does not return to its original state at the end of the cycle
   c. A process in which the system follows the same path for every cycle
   d. A process in which the system follows a different path for every cycle

Critical Thinking Items

12.1 Zeroth Law of Thermodynamics: Thermal Equilibrium

13. What are the necessary conditions for energy transfer by heat to occur between two bodies through the process of conduction?
   a. They should be at the same temperature, and they should be in thermal contact.
   b. They should be at the same temperature, and they should not be in thermal contact.
   c. They should be at different temperatures, and they should be in thermal contact.
   d. They should be at different temperatures, and they should not be in thermal contact.

14. Oil is heated in a pan on a hot plate. The pan is in thermal equilibrium with the hot plate and also with the oil. The temperature of the hot plate is 150 °C . What is the temperature of the oil?
   a. 160 °C
   b. 150 °C
   c. 140 °C
   d. 130 °C

12.2 First law of Thermodynamics: Thermal Energy and Work

15. When an inflated balloon experiences a decrease in size,

the air pressure inside the balloon remains nearly constant. If there is no transfer of energy by heat to or from the balloon, what physical change takes place in the balloon?

a. The average kinetic energy of the gas particles decreases, so the balloon becomes colder.
b. The average kinetic energy of the gas particles increases, so the balloon becomes hotter.
c. The average potential energy of the gas particles decreases, so the balloon becomes colder.
d. The average potential energy of the gas particles increases, so the balloon becomes hotter.

16. When heat adds energy to a system, what is likely to happen to the pressure and volume of the system?

a. Pressure and volume may both decrease with added energy.
b. Pressure and volume may both increase with added energy.
c. Pressure must increase with added energy, while volume must remain constant.
d. Volume must decrease with added energy, while pressure must remain constant.

17. If more energy is transferred into the system by net heat as compared to the net work done by the system, what happens to the difference in energy?

a. It is transferred back to its surroundings.
b. It is stored in the system as internal energy.
c. It is stored in the system as potential energy.
d. It is stored in the system as entropy.

18. Air is pumped into a car tire, causing its temperature to increase. In another tire, the temperature increase is due to exposure to the sun. Is it possible to tell what caused the temperature increase in each tire? Explain your answer.

a. No, because it is a chemical change, and the cause of that change does not matter; the final state of both systems are the same.
b. Although the final state of each system is identical, the source is different in each case.
c. No, because the changes in energy for both systems are the same, and the cause of that change does not matter; the state of each system is identical.
d. Yes, the changes in the energy for both systems are the same, but the causes of that change are different, so the states of each system are not identical.

19. How does the transfer of energy from the sun help plants?

a. Plants absorb solar energy from the sun and utilize it during the fertilization process.
b. Plants absorb solar energy from the sun and utilize it during the process of photosynthesis to turn it into plant matter.
c. Plants absorb solar energy from the sun and utilize it to increase the temperature inside them.
d. Plants absorb solar energy from the sun and utilize it during the shedding of their leaves and fruits.

12.3 Second Law of Thermodynamics: Entropy

20. If an engine were constructed to perform such that there would be no losses due to friction, what would be its efficiency?

a. It would be 0 percent.
b. It would be less than 100 percent.
c. It would be 100 percent.
d. It would be greater than 100 percent.

21. Entropy never decreases in a spontaneous process. Give an example to support this statement.

a. The transfer of energy by heat from colder bodies to hotter bodies is a spontaneous process in which the entropy of the system of bodies increases.
b. The melting of an ice cube placed in a room causes an increase in the entropy of the room.
c. The dissolution of salt in water is a spontaneous process in which the entropy of the system increases.
d. A plant uses energy from the sun and converts it into sugar molecules by the process of photosynthesis.

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

22. What is the advantage of a heat pump as opposed to burning fuel (as in a fireplace) for keeping warm?

a. A heat pump supplies energy by heat from the cold, outside air.
b. A heat pump supplies energy generated by the work done.
c. A heat pump supplies energy by heat from the cold, outside air and also from the energy generated by the work done.
d. A heat pump supplies energy not by heat from the cold, outside air, nor from the energy generated by the work done, but from more accessible sources.

23. What is thermal efficiency of an engine? Can it ever be 100 percent? Why or why not?

a. Thermal efficiency is the ratio of the output (work) to the input (heat). It is always 100 percent.
b. Thermal efficiency is the ratio of the output (heat)

to the input (work). It is always 100 percent.

c. Thermal efficiency is the ratio of the output (heat) to the input (work). It is never 100 percent.

d. Thermal efficiency is the ratio of the output (work) to the input (heat). It is never 100 percent.

24. When would 100 percent thermal efficiency be possible?

a. When all energy is transferred by heat to the environment

b. When mass transferred to the environment is zero

c. When mass transferred to the environment is at a maximum

d. When no energy is transferred by heat to the environment

Problems

12.2 First law of Thermodynamics: Thermal Energy and Work

25. Some amount of energy is transferred by heat into a system. The net work done by the system is $50\,\text{J}$, while the increase in its internal energy is $30\,\text{J}$. What is the amount of net heat?

a. $-80\,\text{J}$

b. $-20\,\text{J}$

c. $20\,\text{J}$

d. $80\,\text{J}$

26. Eighty joules are added by heat to a system, while it does $20\,\text{J}$ of work. Later, $30\,\text{J}$ are added by heat to the system, and it does $40\,\text{J}$ of work. What is the change in the system's internal energy?

a. $30\,\text{J}$

b. $50\,\text{J}$

c. $60\,\text{J}$

d. $110\,\text{J}$

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

27. A coal power station functions at 40.0 percent efficiency. What is the amount of work it does if it takes in 1.20×10^{12} J by heat?

a. 3×10^{10} J

b. 4.8×10^{11} J

c. 3×10^{12} J

d. 4.8×10^{13} J

28. A heat engine functions with 70.7 percent thermal efficiency and consumes 12.0 kJ from heat daily. If its efficiency were raised to 75.0 percent, how much energy from heat would be saved daily, while providing the same output?

a. –10.8 kJ

b. –1.08 kJ

c. 0.7 kJ

d. 7 kJ

Performance Task

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

29. You have been tasked to design and construct a thermometer that works on the principle of thermal expansion. There are four materials available for you to test, each of which will find use under different sets of conditions and temperature ranges:

Materials

- Four sample materials with similar mass or volume: copper, steel, water, and alcohol (ethanol or isopropanol)
- Oven or similar heating source
- Instrument (e.g., meter ruler, Vernier calipers, or micrometer) for measuring changes in dimension
- Balance for measuring mass

Procedure

1. Design a safe experiment to analyze the thermal expansion properties of each material.
2. Write down the materials needed for your experiment and the procedure you will follow. Make sure that you include every detail so that the experiment can be repeated by others.
3. Select an appropriate material to measure temperature over a predecided temperature range, and give reasons for your choice.
4. Calibrate your instrument to measure temperature changes accurately.

a. Which physical quantities are affected by temperature change and thermal expansion?

b. How do such properties as specific heat and thermal conductivity affect the use of each material as a thermometer?

c. Does a change of phase take place for any of the tested materials over the temperature range to be examined?

d. What are your independent and dependent variables for this series of tests? Which variables need to be controlled in the experiment?

e. What are your sources of error?
f. Can all the tested materials be used effectively in the same ranges of temperature? Which applications might be suitable for one or more of the tested substances but not the others?

TEST PREP

Multiple Choice

12.1 Zeroth Law of Thermodynamics: Thermal Equilibrium

30. Which law of thermodynamics describes thermal equilibrium?
    a. zeroth
    b. first
    c. second
    d. third

31. Name any two industries in which the principles of thermodynamics are used.
    a. aerospace and information technology (IT) industries
    b. industrial manufacturing and aerospace
    c. mining and textile industries
    d. mining and agriculture industries

12.2 First law of Thermodynamics: Thermal Energy and Work

32. What is the value of the Boltzmann constant?
    a. $k = 1.23 \times 10^{-38}$ J/K
    b. $k = 1.38 \times 10^{-23}$ J/K
    c. $k = 1.38 \times 10^{23}$ J/K
    d. $k = 1.23 \times 10^{38}$ J/K

33. Which of the following involves work done BY a system?
    a. increasing internal energy
    b. compression
    c. expansion
    d. cooling

34. Which principle does the first law of thermodynamics state?
    a. the ideal gas law
    b. the transitive property of equality
    c. the law of conservation of energy
    d. the principle of thermal equilibrium

35. What is the change in internal energy of a system when $Q_{\text{in}} = 50\text{ J}$ and $Q_{\text{out}} = 50\text{ J}$?
    a. 20 J
    b. 30 J
    c. 50 J
    d. 70 J

36. When does a real gas behave like an ideal gas?
    a. A real gas behaves like an ideal gas at high temperature and low pressure.
    b. A real gas behaves like an ideal gas at high temperature and high pressure.
    c. A real gas behaves like an ideal gas at low temperature and low pressure.
    d. A real gas behaves like an ideal gas at low temperature and high pressure.

12.3 Second Law of Thermodynamics: Entropy

37. In an engine, what is the unused energy converted into?
    a. internal energy
    b. pressure
    c. work
    d. heat

38. It is natural for systems in the universe to _____ spontaneously.
    a. become disordered
    b. become ordered
    c. produce heat
    d. do work

39. If Q is 120 J and T is 350 K, what is the change in entropy?
    a. 0.343 J/K
    b. 1.51 J/K
    c. 2.92 J/K
    d. 34.3 J/K

40. Why does entropy increase during a spontaneous process?
    a. Entropy increases because energy always transfers spontaneously from a dispersed state to a concentrated state.
    b. Entropy increases because energy always transfers spontaneously from a concentrated state to a dispersed state.
    c. Entropy increases because pressure always increases spontaneously.
    d. Entropy increases because temperature of any system always increases spontaneously.

41. A system consists of ice melting in a glass of water. What happens to the entropy of this system?
    a. The entropy of the ice decreases, while the entropy of the water cannot be predicted without more

specific information.
b. The entropy of the system remains constant.
c. The entropy of the system decreases.
d. The entropy of the system increases.

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

42. Which equation represents the net work done by a system in a cyclic process?
a. $W = \frac{Q_c}{Q_h}$
b. $W = Q_h + Q_c$
c. $W = Eff\,(Q_c - Q_h)$
d. $W = Q_h - Q_c$

43. Which of these quantities needs to be zero for efficiency to be 100 percent?
a. ΔU
b. W
c. Q_h
d. Q_c

44. Which of the following always has the greatest value in a system having 80 percent thermal efficiency?
a. ΔU
b. W
c. Q_h
d. Q_c

45. In the equation $Q = Q_h - Q_c$, what does the negative sign indicate?
a. Heat transfer of energy is always negative.
b. Heat transfer can only occur in one direction.
c. Heat is directed into the system from the surroundings outside the system.
d. Heat is directed out of the system.

46. What is the purpose of a heat pump?
a. A heat pump uses work to transfer energy by heat from a colder environment to a warmer environment.
b. A heat pump uses work to transfer energy by heat from a warmer environment to a colder environment.
c. A heat pump does work by using heat to convey energy from a colder environment to a warmer environment.
d. A heat pump does work by using heat to convey energy from a warmer environment to a colder environment.

Short Answer

12.1 Zeroth Law of Thermodynamics: Thermal Equilibrium

47. What does *green* energy development entail?
a. Green energy involves finding new ways to harness clean and renewable alternative energy sources.
b. Green energy involves finding new ways to conserve alternative energy sources.
c. Green energy involves decreasing the efficiency of nonrenewable energy resources.
d. Green energy involves finding new ways to harness nonrenewable energy resources.

48. Why are the sun and Earth not in thermal equilibrium?
a. The mass of the sun is much greater than the mass of Earth.
b. There is a vast amount of empty space between the sun and Earth.
c. The diameter of the sun is much greater than the diameter of Earth.
d. The sun is in thermal contact with Earth.

12.2 First law of Thermodynamics: Thermal Energy and Work

49. If a fixed quantity of an ideal gas is held at a constant volume, which variable relates to pressure, and what is that relation?
a. Temperature; inverse proportionality $\left(P \propto \frac{1}{T}\right)$
b. Temperature, direct proportionality to square root $\left(P \propto \sqrt{T}\right)$
c. Temperature; direct proportionality $(P \propto T)$
d. Temperature; direct proportionality to square $\left(P \propto T^2\right)$

50. When is volume directly proportional to temperature?
a. when the pressure of the gas is variable
b. when the pressure of the gas is constant
c. when the mass of the gas is variable
d. when the mass of the gas is constant

51. For fluids, what can work be defined as?
a. pressure acting over the change in depth
b. pressure acting over the change in temperature
c. temperature acting over the change in volume
d. pressure acting over the change in volume

52. In the equation $\Delta U = Q - P\Delta V$, what does $P\Delta V$ indicate?
a. the work done on the system
b. the work done by the system
c. the heat into the system
d. the heat out of the system

53. By convention, if Q is positive, what is the direction in which heat transfers energy with regard to the system?

a. The direction of the heat transfer of energy depends on the changes in W, regardless of the sign of Q.
b. The direction of Q cannot be determined from just the sign of Q.
c. The direction of net heat transfer of energy will be out of the system.
d. The direction of net heat transfer of energy will be into the system.

54. What is net transfer of energy by heat?

a. It is the sum of all energy transfers by heat into the system.
b. It is the product of all energy transfers by heat into the system.
c. It is the sum of all energy transfers by heat into and out of the system.
d. It is the product of all energy transfers by heat into and out of the system.

55. Three hundred ten joules of heat enter a system, after which the system does 120 J of work. What is the change in its internal energy? Would this amount change if the energy transferred by heat were added after the work was done instead of before?

a. -190 J ; this would change if heat added energy after the work was done
b. 190 J; this would change if heat added energy after the work was done
c. -190 J; this would not change even if heat added energy after the work was done
d. 190 J; this would not change even if heat added energy after the work was done

56. Ten joules are transferred by heat into a system, followed by another 20 J. What is the change in the system's internal energy? What would be the difference in this change if 30 J of energy were added by heat to the system at once?

a. 10 J; the change in internal energy would be same even if the heat added the energy at once
b. 30 J; the change in internal energy would be same even if the heat added the energy at once
c. 10 J; the change in internal energy would be more if the heat added the energy at once
d. 30 J; the change in internal energy would be more if the heat added the energy at once

12.3 Second Law of Thermodynamics: Entropy

57. How does the entropy of a system depend on how the system reaches a given state?

a. Entropy depends on the change of phase of a system, but not on any other state conditions.
b. Entropy does not depend on how the final state is reached from the initial state.
c. Entropy is least when the path between the initial state and the final state is the shortest.
d. Entropy is least when the path between the initial state and the final state is the longest.

58. Which sort of thermal energy do molecules in a solid possess?

a. electric potential energy
b. gravitational potential energy
c. translational kinetic energy
d. vibrational kinetic energy

59. A cold object in contact with a hot one never spontaneously transfers energy by heat to the hot object. Which law describes this phenomenon?

a. the first law of thermodynamics
b. the second law of thermodynamics
c. the third law of thermodynamics
d. the zeroth law of thermodynamics

60. How is it possible for us to transfer energy by heat from cold objects to hot ones?

a. by doing work on the system
b. by having work done by the system
c. by increasing the specific heat of the cold body
d. by increasing the specific heat of the hot body

61. What is the change in entropy caused by melting 5.00 kg of ice at 0 °C ?

a. 0 J/K
b. 6.11×10^3 J/K
c. 6.11×10^4 J/K
d. ∞ J/K

62. What is the amount of heat required to cause a change of 35 J/K in the entropy of a system at 400 K?

a. $1.1\times10^1\text{ J}$
b. $1.1\times10^2\text{ J}$
c. $1.4\times10^3\text{ J}$
d. $1.4\times10^4\text{ J}$

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

63. In a refrigerator, what is the function of an evaporator?

a. The evaporator converts gaseous refrigerant into liquid.
b. The evaporator converts solid refrigerant into liquid.
c. The evaporator converts solid refrigerant into gas.

d. The evaporator converts liquid refrigerant into gas.

64. Which component of an air conditioner converts gas into liquid?
a. the condenser
b. the compressor
c. the evaporator
d. the thermostat

65. What is one example for which calculating thermal efficiency is of interest?
a. A wind turbine
b. An electric pump
c. A bicycle
d. A car engine

66. How is the efficiency of a refrigerator or heat pump expressed?
a. $Eff = W\sqrt{Q_\mathrm{c}}$
b. $Eff = \frac{W}{Q_\mathrm{c}}$
c. $Eff = Q_\mathrm{c} \times W$
d. $Eff = \frac{Q_\mathrm{c}}{W}$

67. How can you mathematically express thermal efficiency in terms of Q_h and Q_c?
a. $Eff = \frac{Q_\mathrm{h}}{Q_\mathrm{h}-Q_\mathrm{c}}$
b. $Eff = \frac{Q_\mathrm{h}}{Q_\mathrm{c}}$
c. $Eff = \frac{Q_\mathrm{c}}{Q_\mathrm{h}}$
d. $Eff = \frac{Q_\mathrm{h}-Q_\mathrm{c}}{Q_\mathrm{h}}$

68. How can you calculate percentage efficiency?
a. percentage efficiency $= (Eff + 100)\ \%$
b. percentage efficiency $= \frac{Eff}{100}\%$
c. percentage efficiency $= (Eff - 100)\,\%$
d. percentage efficiency $= Eff \times 100\%$

Extended Response

12.1 Zeroth Law of Thermodynamics: Thermal Equilibrium

69. What is the meaning of efficiency in terms of a car engine?
a. An engine's efficiency equals the sum of useful energy (work) and the input energy.
b. An engine's efficiency equals the proportion of useful energy (work) to the input energy.
c. An engine's efficiency equals the product of useful energy (work) and the input energy.
d. An engine's efficiency equals the difference between the useful energy (work) and the input energy.

12.2 First law of Thermodynamics: Thermal Energy and Work

70. Why does a bridge have expansion joints?
a. because the bridge expands and contracts with the change in temperature
b. because the bridge expands and contracts with the change in motion of objects moving on the bridge
c. because the bridge expands and contracts with the change in total load on the bridge
d. because the bridge expands and contracts with the change in magnitude of wind blowing

71. Under which conditions will the work done by the gas in a system increase?
a. It will increase when a large amount of energy is added to the system, and that energy causes an increase in the gas's volume, its pressure, or both.
b. It will increase when a large amount of energy is extracted from the system, and that energy causes an increase in the gas's volume, its pressure, or both.
c. It will increase when a large amount of energy is added to the system, and that energy causes a decrease in the gas's volume, its pressure, or both.
d. It will increase when a large amount of energy is extracted from the system, and that energy causes a decrease in the gas's volume, its pressure, or both.

72. How does energy transfer by heat aid in body metabolism?
a. The energy is given to the body through the work done by the body (W) and through the intake of food, which may also be considered as the work done on the body. The transfer of energy out of the body is by heat ($-Q$) .
b. The energy given to the body is by the intake of food, which may also be considered as the work done on the body. The transfer of energy out of the body is by heat ($-Q$) and the work done by the body (W) .
c. The energy given to the body is by the transfer of energy by heat (Q) into the body, which may also be considered as the work done on the body. The transfer of energy out of the body is the work done by the body (W) .
d. The energy given to the body is by the transfer of energy by heat (Q) inside the body. The transfer of energy out of the body is by the intake of food and the work done by the body (W) .

73. Two distinct systems have the same amount of stored internal energy. Five hundred joules are added by heat to the first system, and 300 J are added by heat to the second system. What will be the change in internal energy of the first system if it does 200 J of work? How much work will the second system have to do in order to have the same internal energy?

a. 700 J; 0 J
b. 300 J; 300 J
c. 700 J; 300 J
d. 300 J; 0 J

12.3 Second Law of Thermodynamics: Entropy

74. Why is it not possible to convert all available energy into work?

a. Due to the entropy of a system, some energy is always unavailable for work.
b. Due to the entropy of a system, some energy is always available for work.
c. Due to the decrease in internal energy of a system, some energy is always made unavailable for work.
d. Due to the increase in internal energy of a system, some energy is always made unavailable for work.

75. Why does entropy increase when ice melts into water?

a. Melting converts the highly ordered solid structure into a disorderly liquid, thereby increasing entropy.
b. Melting converts the highly ordered liquid into a disorderly solid structure, thereby increasing entropy.
c. Melting converts the highly ordered solid structure into a disorderly solid structure, thereby increasing entropy.
d. Melting converts the highly ordered liquid into a disorderly liquid, thereby increasing entropy.

76. Why is change in entropy lower for higher temperatures?

a. Increase in the disorder in the substance is low for high temperature.
b. Increase in the disorder in the substance is high for high temperature.
c. Decrease in the disorder in the substance is low for high temperature.
d. Decrease in the disorder in the substance is high for high temperature.

12.4 Applications of Thermodynamics: Heat Engines, Heat Pumps, and Refrigerators

77. In the equation $W = Q_h - Q_c$, if the value of Q_c were equal to zero, what would it signify?

a. The efficiency of the engine is 75 percent.
b. The efficiency of the engine is 25 percent.
c. The efficiency of the engine is 0 percent.
d. The efficiency of the engine is 100 percent.

78. Can the value of thermal efficiency be greater than 1? Why or why not?

a. No, according to the first law of thermodynamics, energy output can never be more than the energy input.
b. No, according to the second law of thermodynamics, energy output can never be more than the energy input.
c. Yes, according to the first law of thermodynamics, energy output can be more than the energy input.
d. Yes, according to the second law of thermodynamics, energy output can be more than the energy input.

79. A coal power station transfers 3.0×10^{12} J by heat from burning coal and transfers 1.5×10^{12} J by heat into the environment. What is the efficiency of the power station?

a. 0.33
b. 0.5
c. 0.66
d. 1

CHAPTER 13
Waves and Their Properties

Figure 13.1 Waves in the ocean behave similarly to all other types of waves. (Steve Jurveston, Flickr)

Chapter Outline

INTRODUCTION Recall from the chapter on Motion in Two Dimensions that oscillations—the back-and-forth movement between two points—involve force and energy. Some oscillations create waves, such as the sound waves created by plucking a guitar string. Other examples of waves include earthquakes and visible light. Even subatomic particles, such as electrons, can behave like waves. You can make water waves in a swimming pool by slapping the water with your hand. Some of these waves, such as water waves, are visible; others, such as sound waves, are not. But every wave is a disturbance that moves from its source and carries energy. In this chapter, we will learn about the different types of waves, their properties, and how they interact with one another.

13.1 Types of Waves

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Define mechanical waves and medium, and relate the two
- Distinguish a pulse wave from a periodic wave
- Distinguish a longitudinal wave from a transverse wave and give examples of such waves

Section Key Terms

longitudinal wave	mechanical wave	medium	wave
periodic wave	pulse wave	transverse wave	

Mechanical Waves

What do we mean when we say something is a wave? A **wave** is a disturbance that travels or *propagates* from the place where it was created. Waves transfer energy from one place to another, but they do not necessarily transfer any mass. Light, sound, and waves in the ocean are common examples of waves. Sound and water waves are **mechanical waves**; meaning, they require a medium to travel through. The **medium** may be a solid, a liquid, or a gas, and the speed of the wave depends on the material properties of the medium through which it is traveling. However, light is not a mechanical wave; it can travel through a vacuum such as the empty parts of outer space.

A familiar wave that you can easily imagine is the water wave. For water waves, the disturbance is in the surface of the water, an example of which is the disturbance created by a rock thrown into a pond or by a swimmer splashing the water surface repeatedly. For sound waves, the disturbance is caused by a change in air pressure, an example of which is when the oscillating cone inside a speaker creates a disturbance. For earthquakes, there are several types of disturbances, which include the disturbance of Earth's surface itself and the pressure disturbances under the surface. Even radio waves are most easily understood using an analogy with water waves. Because water waves are common and visible, visualizing water waves may help you in studying other types of waves, especially those that are not visible.

Water waves have characteristics common to all waves, such as amplitude, period, frequency, and energy, which we will discuss in the next section.

Pulse Waves and Periodic Waves

If you drop a pebble into the water, only a few waves may be generated before the disturbance dies down, whereas in a wave pool, the waves are continuous. A **pulse wave** is a sudden disturbance in which only one wave or a few waves are generated, such as in the example of the pebble. Thunder and explosions also create pulse waves. A **periodic wave** repeats the same oscillation for several cycles, such as in the case of the wave pool, and is associated with simple harmonic motion. Each particle in the medium experiences simple harmonic motion in periodic waves by moving back and forth periodically through the same positions.

Consider the simplified water wave in Figure 13.2. This wave is an up-and-down disturbance of the water surface, characterized by a sine wave pattern. The uppermost position is called the *crest* and the lowest is the *trough*. It causes a seagull to move up and down in simple harmonic motion as the wave crests and troughs pass under the bird.

Figure 13.2 An idealized ocean wave passes under a seagull that bobs up and down in simple harmonic motion.

Longitudinal Waves and Transverse Waves

Mechanical waves are categorized by their type of motion and fall into any of two categories: transverse or longitudinal. Note that both transverse and longitudinal waves can be periodic. A **transverse wave** propagates so that the disturbance is perpendicular to the direction of propagation. An example of a transverse wave is shown in Figure 13.3, where a woman moves a toy spring up and down, generating waves that propagate away from herself in the horizontal direction while disturbing the toy spring in the vertical direction.

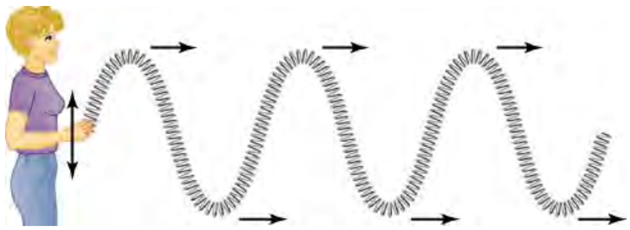


Figure 13.3 In this example of a transverse wave, the wave propagates horizontally and the disturbance in the toy spring is in the vertical direction.

In contrast, in a **longitudinal wave**, the disturbance is parallel to the direction of propagation. Figure 13.4 shows an example of a longitudinal wave, where the woman now creates a disturbance in the horizontal direction—which is the same direction as the wave propagation—by stretching and then compressing the toy spring.

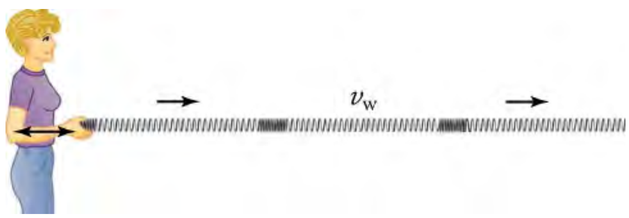


Figure 13.4 In this example of a longitudinal wave, the wave propagates horizontally and the disturbance in the toy spring is also in the horizontal direction.

TIPS FOR SUCCESS

Longitudinal waves are sometimes called *compression waves* or *compressional waves*, and transverse waves are sometimes called *shear waves*.

Waves may be transverse, longitudinal, or *a combination of the two*. The waves on the strings of musical instruments are transverse (as shown in Figure 13.5), and so are electromagnetic waves, such as visible light. Sound waves in air and water are longitudinal. Their disturbances are periodic variations in pressure that are transmitted in fluids.

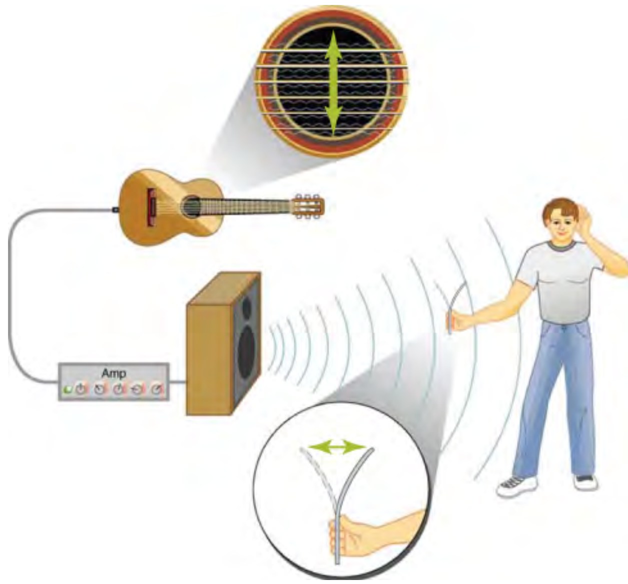


Figure 13.5 The wave on a guitar string is transverse. However, the sound wave coming out of a speaker rattles a sheet of paper in a direction that shows that such sound wave is longitudinal.

Sound in solids can be both longitudinal and transverse. Essentially, water waves are also a combination of transverse and longitudinal components, although the simplified water wave illustrated in Figure 13.2 does not show the longitudinal motion of the bird.

Earthquake waves under Earth's surface have both longitudinal and transverse components as well. The longitudinal waves in an earthquake are called pressure or P-waves, and the transverse waves are called shear or S-waves. These components have important individual characteristics; for example, they propagate at different speeds. Earthquakes also have surface waves that are similar to surface waves on water.

WATCH PHYSICS

Introduction to Waves

This video explains wave propagation in terms of momentum using an example of a wave moving along a rope. It also covers the differences between transverse and longitudinal waves, and between pulse and periodic waves.

Click to view content (https://openstax.org/l/02introtowaves)

GRASP CHECK

In a longitudinal sound wave, after a compression wave moves through a region, the density of molecules briefly decreases. Why is this?

a. After a compression wave, some molecules move forward temporarily.
b. After a compression wave, some molecules move backward temporarily.
c. After a compression wave, some molecules move upward temporarily.
d. After a compression wave, some molecules move downward temporarily.

FUN IN PHYSICS

The Physics of Surfing

Many people enjoy surfing in the ocean. For some surfers, the bigger the wave, the better. In one area off the coast of central California, waves can reach heights of up to 50 feet in certain times of the year (Figure 13.6).

Figure 13.6 A surfer negotiates a steep take-off on a winter day in California while his friend watches. (Ljsurf, Wikimedia Commons)

How do waves reach such extreme heights? Other than unusual causes, such as when earthquakes produce tsunami waves, most huge waves are caused simply by interactions between the wind and the surface of the water. The wind pushes up against the surface of the water and transfers energy to the water in the process. The stronger the wind, the more energy transferred. As waves start to form, a larger surface area becomes in contact with the wind, and even more energy is transferred from the wind to the water, thus creating higher waves. Intense storms create the fastest winds, kicking up massive waves that travel out from the origin of the storm. Longer-lasting storms and those storms that affect a larger area of the ocean create the biggest waves since they transfer more energy. The cycle of the tides from the Moon's gravitational pull also plays a small role in creating waves.

Actual ocean waves are more complicated than the idealized model of the simple transverse wave with a perfect sinusoidal shape. Ocean waves are examples of *orbital progressive waves*, where water particles at the surface follow a circular path from the crest to the trough of the passing wave, then cycle back again to their original position. This cycle repeats with each passing wave.

As waves reach shore, the water depth decreases and the energy of the wave is compressed into a smaller volume. This creates higher waves—an effect known as *shoaling*.

Since the water particles along the surface move from the crest to the trough, surfers hitch a ride on the cascading water, gliding along the surface. If ocean waves work exactly like the idealized transverse waves, surfing would be much less exciting as it would simply involve standing on a board that bobs up and down in place, just like the seagull in the previous figure.

Additional information and illustrations about the scientific principles behind surfing can be found in the "Using Science to Surf Better!" (http://www.openstax.org/l/28Surf) video.

GRASP CHECK

If we lived in a parallel universe where ocean waves were longitudinal, what would a surfer's motion look like?

a. The surfer would move side-to-side/back-and-forth vertically with no horizontal motion.
b. The surfer would forward and backward horizontally with no vertical motion.

Check Your Understanding

1. What is a wave?
   a. A wave is a force that propagates from the place where it was created.
   b. A wave is a disturbance that propagates from the place where it was created.
   c. A wave is matter that provides volume to an object.
   d. A wave is matter that provides mass to an object.

2. Do all waves require a medium to travel? Explain.
   a. No, electromagnetic waves do not require any medium to propagate.
   b. No, mechanical waves do not require any medium to propagate.
   c. Yes, both mechanical and electromagnetic waves require a medium to propagate.
   d. Yes, all transverse waves require a medium to travel.

3. What is a pulse wave?
   a. A pulse wave is a sudden disturbance with only one wave generated.
   b. A pulse wave is a sudden disturbance with only one or a few waves generated.

c. A pulse wave is a gradual disturbance with only one or a few waves generated.
d. A pulse wave is a gradual disturbance with only one wave generated.

4. Is the following statement true or false? A pebble dropped in water is an example of a pulse wave.
a. False
b. True

5. What are the categories of mechanical waves based on the type of motion?
a. Both transverse and longitudinal waves
b. Only longitudinal waves
c. Only transverse waves
d. Only surface waves

6. In which direction do the particles of the medium oscillate in a transverse wave?
a. Perpendicular to the direction of propagation of the transverse wave
b. Parallel to the direction of propagation of the transverse wave

13.2 Wave Properties: Speed, Amplitude, Frequency, and Period

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Define amplitude, frequency, period, wavelength, and velocity of a wave
- Relate wave frequency, period, wavelength, and velocity
- Solve problems involving wave properties

Section Key Terms

wavelength	wave velocity

Wave Variables

In the chapter on motion in two dimensions, we defined the following variables to describe harmonic motion:

- Amplitude—maximum displacement from the equilibrium position of an object oscillating around such equilibrium position
- Frequency—number of events per unit of time
- Period—time it takes to complete one oscillation

For waves, these variables have the same basic meaning. However, it is helpful to word the definitions in a more specific way that applies directly to waves:

- Amplitude—distance between the resting position and the maximum displacement of the wave
- Frequency—number of waves passing by a specific point per second
- Period—time it takes for one wave cycle to complete

In addition to amplitude, frequency, and period, their wavelength and wave velocity also characterize waves. The **wavelength** λ is the distance between adjacent identical parts of a wave, parallel to the direction of propagation. The **wave velocity** v_w is the speed at which the disturbance moves.

TIPS FOR SUCCESS

Wave velocity is sometimes also called the *propagation velocity* or *propagation speed* because the disturbance propagates from one location to another.

Consider the periodic water wave in Figure 13.7. Its wavelength is the distance from crest to crest or from trough to trough. The wavelength can also be thought of as the distance a wave has traveled after one complete cycle—or one period. The time for one complete up-and-down motion is the simple water wave's period T. In the figure, the wave itself moves to the right with a wave velocity v_w. Its amplitude X is the distance between the resting position and the maximum displacement—either the crest or the trough—of the wave. It is important to note that this movement of the wave is actually the *disturbance* moving to the right, not the water itself; otherwise, the bird would move to the right. Instead, the seagull bobs up and down in place as waves pass underneath, traveling a total distance of $2X$ in one cycle. However, as mentioned in the text feature on surfing, actual ocean waves are more complex than this simplified example.

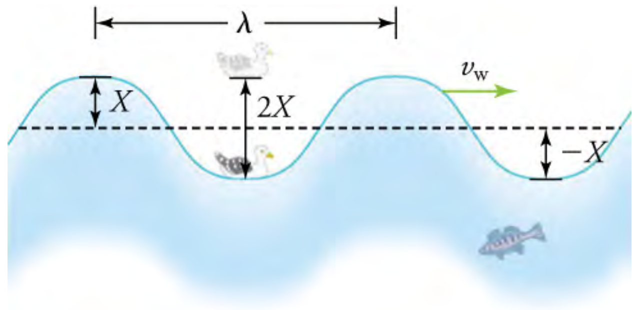


Figure 13.7 The wave has a wavelength λ, which is the distance between adjacent identical parts of the wave. The up-and-down disturbance of the surface propagates parallel to the surface at a speed v_w.

WATCH PHYSICS

Amplitude, Period, Frequency, and Wavelength of Periodic Waves

This video is a continuation of the video “Introduction to Waves” from the "Types of Waves" section. It discusses the properties of a periodic wave: amplitude, period, frequency, wavelength, and wave velocity.

[Click to view content (https://www.openstax.org/l/28wavepro)](https://www.openstax.org/l/28wavepro)

TIPS FOR SUCCESS

The crest of a wave is sometimes also called the *peak*.

GRASP CHECK

If you are on a boat in the trough of a wave on the ocean, and the wave amplitude is 1 m, what is the wave height from your position?

a. 1 m
b. 2 m
c. 4 m
d. 8 m

The Relationship between Wave Frequency, Period, Wavelength, and Velocity

Since wave frequency is the number of waves per second, and the period is essentially the number of seconds per wave, the relationship between frequency and period is

$$f = \frac{1}{T} \tag{13.1}$$

or

$$T = \frac{1}{f}, \tag{13.2}$$

just as in the case of harmonic motion of an object. We can see from this relationship that a higher frequency means a shorter period. Recall that the unit for frequency is hertz (Hz), and that 1 Hz is one cycle—or one wave—per second.

The speed of propagation v_w is the distance the wave travels in a given time, which is one wavelength in a time of one period. In equation form, it is written as

$$v_w = \frac{\lambda}{T} \tag{13.3}$$

or

$$v_w = f\lambda. \tag{13.4}$$

From this relationship, we see that in a medium where v_w is constant, the higher the frequency, the smaller the wavelength. See Figure 13.8.

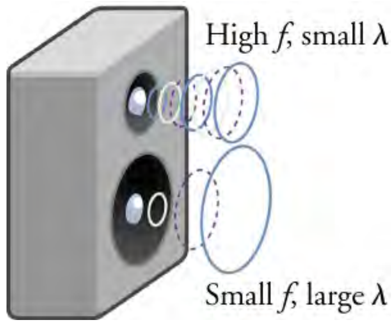


Figure 13.8 Because they travel at the same speed in a given medium, low-frequency sounds must have a greater wavelength than high-frequency sounds. Here, the lower-frequency sounds are emitted by the large speaker, called a woofer, while the higher-frequency sounds are emitted by the small speaker, called a tweeter.

These fundamental relationships hold true for all types of waves. As an example, for water waves, v_w is the speed of a surface wave; for sound, v_w is the speed of sound; and for visible light, v_w is the speed of light. The amplitude X is completely independent of the speed of propagation v_w and depends only on the amount of energy in the wave.

Snap Lab

Waves in a Bowl

In this lab, you will take measurements to determine how the amplitude and the period of waves are affected by the transfer of energy from a cork dropped into the water. The cork initially has some potential energy when it is held above the water—the greater the height, the higher the potential energy. When it is dropped, such potential energy is converted to kinetic energy as the cork falls. When the cork hits the water, that energy travels through the water in waves.

- Large bowl or basin
- Water
- Cork (or ping pong ball)
- Stopwatch
- Measuring tape

Instructions

Procedure

1. Fill a large bowl or basin with water and wait for the water to settle so there are no ripples.
2. Gently drop a cork into the middle of the bowl.
3. Estimate the wavelength and the period of oscillation of the water wave that propagates away from the cork. You can estimate the period by counting the number of ripples from the center to the edge of the bowl while your partner times it. This information, combined with the bowl measurement, will give you the wavelength when the correct formula is used.
4. Remove the cork from the bowl and wait for the water to settle again.
5. Gently drop the cork at a height that is different from the first drop.
6. Repeat Steps 3 to 5 to collect a second and third set of data, dropping the cork from different heights and recording the resulting wavelengths and periods.

7. Interpret your results.

GRASP CHECK

A cork is dropped into a pool of water creating waves. Does the wavelength depend upon the height above the water from which the cork is dropped?

a. No, only the amplitude is affected.
b. Yes, the wavelength is affected.

LINKS TO PHYSICS

Geology: Physics of Seismic Waves

Figure 13.9 The destructive effect of an earthquake is a palpable evidence of the energy carried in the earthquake waves. The Richter scale rating of earthquakes is related to both their amplitude and the energy they carry. (Petty Officer 2nd Class Candice Villarreal, U.S. Navy)

Geologists rely heavily on physics to study earthquakes since earthquakes involve several types of wave disturbances, including disturbance of Earth's surface and pressure disturbances under the surface. Surface earthquake waves are similar to surface waves on water. The waves under Earth's surface have both longitudinal and transverse components. The longitudinal waves in an earthquake are called pressure waves (P-waves) and the transverse waves are called shear waves (S-waves). These two types of waves propagate at different speeds, and the speed at which they travel depends on the rigidity of the medium through which they are traveling. During earthquakes, the speed of P-waves in granite is significantly higher than the speed of S-waves. Both components of earthquakes travel more slowly in less rigid materials, such as sediments. P-waves have speeds of 4 to 7 km/s, and S-waves have speeds of 2 to 5 km/s, but both are faster in more rigid materials. The P-wave gets progressively farther ahead of the S-wave as they travel through Earth's crust. For that reason, the time difference between the P- and S-waves is used to determine the distance to their source, the epicenter of the earthquake.

We know from seismic waves produced by earthquakes that parts of the interior of Earth are liquid. Shear or transverse waves cannot travel through a liquid and are not transmitted through Earth's core. In contrast, compression or longitudinal waves can pass through a liquid and they do go through the core.

All waves carry energy, and the energy of earthquake waves is easy to observe based on the amount of damage left behind after the ground has stopped moving. Earthquakes can shake whole cities to the ground, performing the work of thousands of wrecking balls. The amount of energy in a wave is related to its amplitude. Large-amplitude earthquakes produce large ground displacements and greater damage. As earthquake waves spread out, their amplitude decreases, so there is less damage the farther they get from the source.

GRASP CHECK

What is the relationship between the propagation speed, frequency, and wavelength of the S-waves in an earthquake?

a. The relationship between the propagation speed, frequency, and wavelength is $v_{\mathrm{w}} = f\lambda$.
b. The relationship between the propagation speed, frequency, and wavelength is $v_{\mathrm{w}} = \frac{f}{\lambda}$.
c. The relationship between the propagation speed, frequency, and wavelength is $v_{\mathrm{w}} = \frac{\lambda}{f}$.

d. The relationship between the propagation speed, frequency, and wavelength is $v_{\mathrm{w}} = \sqrt{f\lambda}$.

Virtual Physics

Wave on a String

Click to view content (http://www.openstax.org/l/28wavestring)

In this animation, watch how a string vibrates in slow motion by choosing the Slow Motion setting. Select the No End and Manual options, and wiggle the end of the string to make waves yourself. Then switch to the Oscillate setting to generate waves automatically. Adjust the frequency and the amplitude of the oscillations to see what happens. Then experiment with adjusting the damping and the tension.

GRASP CHECK

Which of the settings—amplitude, frequency, damping, or tension—changes the amplitude of the wave as it propagates? What does it do to the amplitude?

a. Frequency; it decreases the amplitude of the wave as it propagates.
b. Frequency; it increases the amplitude of the wave as it propagates.
c. Damping; it decreases the amplitude of the wave as it propagates.
d. Damping; it increases the amplitude of the wave as it propagates.

Solving Wave Problems

WORKED EXAMPLE

Calculate the Velocity of Wave Propagation: Gull in the Ocean

Calculate the wave velocity of the ocean wave in the previous figure if the distance between wave crests is 10.0 m and the time for a seagull to bob up and down is 5.00 s.

STRATEGY

The values for the wavelength $(\lambda = 10.0 \ \ m)$ and the period $(T = 5.00 \ \mathrm{s})$ are given and we are asked to find v_w Therefore, we can use $v_w = \frac{\lambda}{T}$ to find the wave velocity.

Solution

Enter the known values into $v_w = \frac{\lambda}{T}$

$$v_w = \frac{10.0 \ \mathrm{m}}{5.00 \ \mathrm{s}} = 2.\ 00 \ \mathrm{m/s}. \tag{13.5}$$

Discussion

This slow speed seems reasonable for an ocean wave. Note that in the figure, the wave moves to the right at this speed, which is different from the varying speed at which the seagull bobs up and down.

WORKED EXAMPLE

Calculate the Period and the Wave Velocity of a Toy Spring

The woman in creates two waves every second by shaking the toy spring up and down. (a)What is the period of each wave? (b) If each wave travels 0.9 meters after one complete wave cycle, what is the velocity of wave propagation?

STRATEGY FOR (A)

To find the period, we solve for $T = \frac{1}{f}$, given the value of the frequency $(f = 2\mathrm{s}^{-1})$.

Solution for (a)

Enter the known value into $T = \frac{1}{f}$

$$T = \frac{1}{2\ \text{s}^{-1}} = 0.\ 5\ \text{s}. \tag{13.6}$$

STRATEGY FOR (B)

Since one definition of wavelength is the distance a wave has traveled after one complete cycle—or one period—the values for the wavelength ($\lambda = 0.9\ \ m$) as well as the frequency are given. Therefore, we can use $v_\text{w} = f\lambda$ to find the wave velocity.

Solution for (b)

Enter the known values into $v_\text{w} = f\lambda$

$v_\text{w} = f\lambda = (2\ \text{s}^{-1})(0.9\ \text{m}) = 1.8\ \text{m/s}.$

Discussion

We could have also used the equation $v_\text{w} = \frac{\lambda}{T}$ to solve for the wave velocity since we already know the value of the period ($T = 0.5\ \text{s}$) from our calculation in part (a), and we would come up with the same answer.

Practice Problems

7. The frequency of a wave is 10 Hz. What is its period?
   a. The period of the wave is 100 s.
   b. The period of the wave is 10 s.
   c. The period of the wave is 0.01 s.
   d. The period of the wave is 0.1 s.

8. What is the velocity of a wave whose wavelength is 2 m and whose frequency is 5 Hz?
   a. 20 m/s
   b. 2.5 m/s
   c. 0.4 m/s
   d. 10 m/s

Check Your Understanding

9. What is the amplitude of a wave?
   a. A quarter of the total height of the wave
   b. Half of the total height of the wave
   c. Two times the total height of the wave
   d. Four times the total height of the wave

10. What is meant by the wavelength of a wave?
    a. The wavelength is the distance between adjacent identical parts of a wave, parallel to the direction of propagation.
    b. The wavelength is the distance between adjacent identical parts of a wave, perpendicular to the direction of propagation.
    c. The wavelength is the distance between a crest and the adjacent trough of a wave, parallel to the direction of propagation.
    d. The wavelength is the distance between a crest and the adjacent trough of a wave, perpendicular to the direction of propagation.

11. How can you mathematically express wave frequency in terms of wave period?
    a. $f = \frac{1}{T}$
    b. $f = \left(\frac{1}{T}\right)^2$
    c. $\text{f} = \text{T}$
    d. $f = (T)^2$

12. When is the wavelength directly proportional to the period of a wave?

a. When the velocity of the wave is halved
b. When the velocity of the wave is constant
c. When the velocity of the wave is doubled
d. When the velocity of the wave is tripled

13.3 Wave Interaction: Superposition and Interference

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe superposition of waves
- Describe interference of waves and distinguish between constructive and destructive interference of waves
- Describe the characteristics of standing waves
- Distinguish reflection from refraction of waves

Section Key Terms

antinode	constructive interference	destructive interference	inversion	nodes
reflection	refraction	standing wave	superposition	

Superposition of Waves

Most waves do not look very simple. They look more like the waves in Figure 13.10, rather than the simple water wave considered in the previous sections, which has a perfect sinusoidal shape.

Figure 13.10 These waves result from the superposition of several waves from different sources, producing a complex pattern. (Waterborough, Wikimedia Commons)

Most waves appear complex because they result from two or more simple waves that combine as they come together at the same place at the same time—a phenomenon called **superposition**.

Waves superimpose by adding their disturbances; each disturbance corresponds to a force, and all the forces add. If the disturbances are along the same line, then the resulting wave is a simple addition of the disturbances of the individual waves, that is, their amplitudes add.

Wave Interference

The two special cases of superposition that produce the simplest results are pure constructive interference and pure destructive interference.

Pure **constructive interference** occurs when two identical waves arrive at the same point exactly in phase. When waves are exactly in phase, the crests of the two waves are precisely aligned, as are the troughs. Refer to Figure 13.11. Because the disturbances add, the pure constructive interference of two waves with the same amplitude produces a wave that has twice the amplitude of the two individual waves, but has the same wavelength.

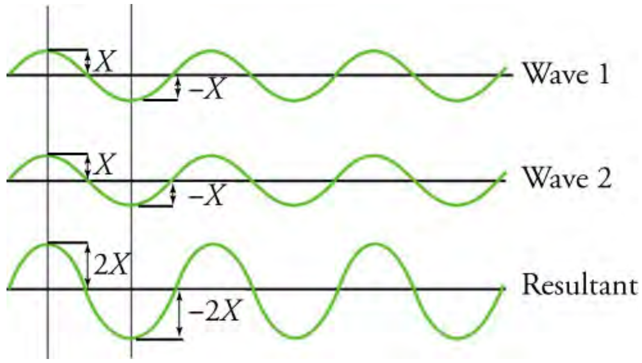


Figure 13.11 The pure constructive interference of two identical waves produces a wave with twice the amplitude but the same wavelength.

Figure 13.12 shows two identical waves that arrive exactly *out* of phase—that is, precisely aligned crest to trough—producing pure **destructive interference**. Because the disturbances are in opposite directions for this superposition, the resulting amplitude is zero for pure destructive interference; that is, the waves completely cancel out each other.

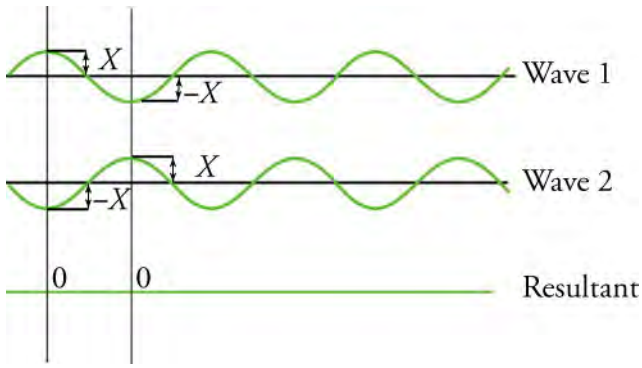


Figure 13.12 The pure destructive interference of two identical waves produces zero amplitude, or complete cancellation.

While pure constructive interference and pure destructive interference can occur, they are not very common because they require precisely aligned identical waves. The superposition of most waves that we see in nature produces a combination of constructive and destructive interferences.

Waves that are not results of pure constructive or destructive interference can vary from place to place and time to time. The sound from a stereo, for example, can be loud in one spot and soft in another. The varying loudness means that the sound waves add partially constructively and partially destructively at different locations. A stereo has at least two speakers that create sound waves, and waves can reflect from walls. All these waves superimpose.

An example of sounds that vary over time from constructive to destructive is found in the combined whine of jet engines heard by a stationary passenger. The volume of the combined sound can fluctuate up and down as the sound from the two engines varies in time from constructive to destructive.

The two previous examples considered waves that are similar—both stereo speakers generate sound waves with the same amplitude and wavelength, as do the jet engines. But what happens when two waves that are not similar, that is, having different amplitudes and wavelengths, are superimposed? An example of the superposition of two dissimilar waves is shown in Figure 13.13. Here again, the disturbances add and subtract, but they produce an even more complicated-looking wave. The resultant wave from the combined disturbances of two dissimilar waves looks much different than the idealized sinusoidal shape of a periodic wave.

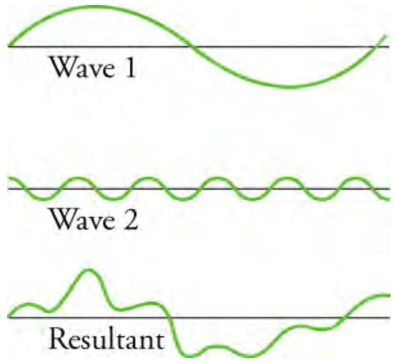


Figure 13.13 The superposition of nonidentical waves exhibits both constructive and destructive interferences.

Virtual Physics

Wave Interference

[Click to view content (http://www.openstax.org/l/28interference)](http://www.openstax.org/l/28interference)

In this simulation, make waves with a dripping faucet, an audio speaker, or a laser by switching between the water, sound, and light tabs. Contrast and compare how the different types of waves behave. Try rotating the view from top to side to make observations. Then experiment with adding a second source or a pair of slits to create an interference pattern.

GRASP CHECK

In the water tab, compare the waves generated by one drip versus two drips. What happens to the amplitude of the waves when there are two drips? Is this constructive or destructive interference? Why would this be the case?

a. The amplitude of the water waves remains same because of the destructive interference as the drips of water hit the surface at the same time.
b. The amplitude of the water waves is canceled because of the destructive interference as the drips of water hit the surface at the same time.
c. The amplitude of water waves remains same because of the constructive interference as the drips of water hit the surface at the same time.
d. The amplitude of water waves doubles because of the constructive interference as the drips of water hit the surface at the same time.

Standing Waves

Sometimes waves do not seem to move and they appear to just stand in place, vibrating. Such waves are called **standing waves** and are formed by the superposition of two or more waves moving in opposite directions. The waves move through each other with their disturbances adding as they go by. If the two waves have the same amplitude and wavelength, then they alternate between constructive and destructive interference. Standing waves created by the superposition of two identical waves moving in opposite directions are illustrated in Figure 13.14.

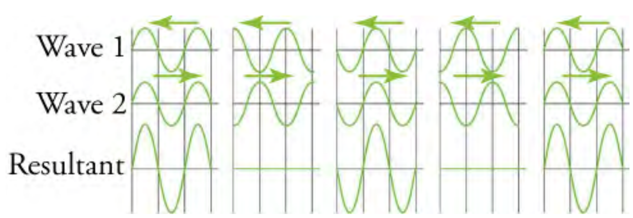


Figure 13.14 A standing wave is created by the superposition of two identical waves moving in opposite directions. The oscillations are at fixed locations in space and result from alternating constructive and destructive interferences.

As an example, standing waves can be seen on the surface of a glass of milk in a refrigerator. The vibrations from the refrigerator motor create waves on the milk that oscillate up and down but do not seem to move across the surface. The two waves that produce standing waves may be due to the reflections from the side of the glass.

Earthquakes can create standing waves and cause constructive and destructive interferences. As the earthquake waves travel along the surface of Earth and reflect off denser rocks, constructive interference occurs at certain points. As a result, areas closer to the epicenter are not damaged while areas farther from the epicenter are damaged.

Standing waves are also found on the strings of musical instruments and are due to reflections of waves from the ends of the string. Figure 13.15 and Figure 13.16 show three standing waves that can be created on a string that is fixed at both ends. When the wave reaches the fixed end, it has nowhere else to go but back where it came from, causing the reflection. The **nodes** are the points where the string does not move; more generally, the nodes are the points where the wave disturbance is zero in a standing wave. The fixed ends of strings must be nodes, too, because the string cannot move there.

The **antinode** is the location of maximum amplitude in standing waves. The standing waves on a string have a frequency that is related to the propagation speed v_w of the disturbance on the string. The wavelength λ is determined by the distance between the points where the string is fixed in place.

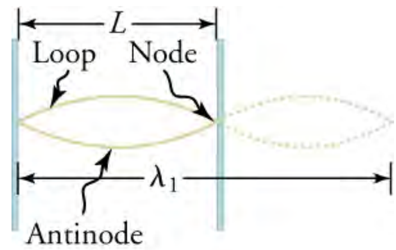


Figure 13.15 The figure shows a string oscillating with its maximum disturbance as the antinode.

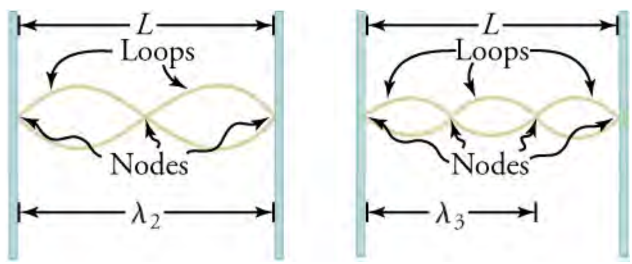


Figure 13.16 The figure shows a string oscillating with multiple nodes.

Reflection and Refraction of Waves

As we saw in the case of standing waves on the strings of a musical instrument, **reflection** is the change in direction of a wave when it bounces off a barrier, such as a fixed end. When the wave hits the fixed end, it changes direction, returning to its source. As it is reflected, the wave experiences an **inversion**, which means that it flips vertically. If a wave hits the fixed end with a crest, it will return as a trough, and vice versa (Henderson 2015). Refer to Figure 13.17.

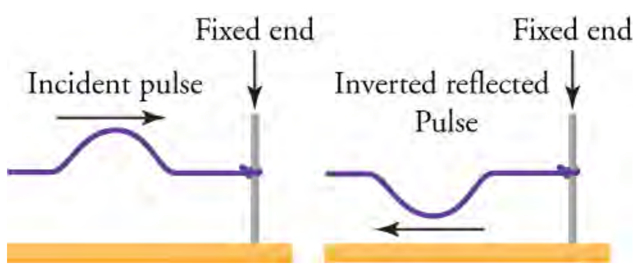


Figure 13.17 A wave is inverted after reflection from a fixed end.

TIPS FOR SUCCESS

If the end is not fixed, it is said to be a *free end*, and no inversion occurs. When the end is loosely attached, it reflects without inversion, and when the end is not attached to anything, it does not reflect at all. You may have noticed this while changing the settings from Fixed End to Loose End to No End in the Waves on a String PhET simulation.

Rather than encountering a fixed end or barrier, waves sometimes pass from one medium into another, for instance, from air into water. Different types of media have different properties, such as density or depth, that affect how a wave travels through them. At the boundary between media, waves experience **refraction**—they change their path of propagation. As the wave bends, it also changes its speed and wavelength upon entering the new medium. Refer to Figure 13.18.

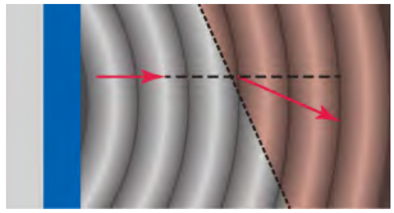


Figure 13.18 A wave refracts as it enters a different medium.

For example, water waves traveling from the deep end to the shallow end of a swimming pool experience refraction. They bend in a path closer to perpendicular to the surface of the water, propagate slower, and decrease in wavelength as they enter shallower water.

Check Your Understanding

13. What is the superposition of waves?
  a. When a single wave splits into two different waves at a point
  b. When two waves combine at the same place at the same time

14. How do waves superimpose on one another?
  a. By adding their frequencies
  b. By adding their wavelengths
  c. By adding their disturbances
  d. By adding their speeds

15. What is interference of waves?
  a. Interference is a superposition of two waves to form a resultant wave with higher or lower frequency.
  b. Interference is a superposition of two waves to form a wave of larger or smaller amplitude.
  c. Interference is a superposition of two waves to form a resultant wave with higher or lower velocity.
  d. Interference is a superposition of two waves to form a resultant wave with longer or shorter wavelength.

16. Is the following statement true or false? The two types of interference are constructive and destructive interferences.
  a. True
  b. False

17. What are standing waves?
  a. Waves that appear to remain in one place and do not seem to move
  b. Waves that seem to move along a trajectory

18. How are standing waves formed?
  a. Standing waves are formed by the superposition of two or more waves moving in opposite directions.
  b. Standing waves are formed by the superposition of two or more waves moving in the same direction.
  c. Standing waves are formed by the superposition of two or more waves moving in perpendicular directions.
  d. Standing waves are formed by the superposition of two or more waves moving in any arbitrary directions.

19. What is the reflection of a wave?
  a. The reflection of a wave is the change in amplitude of a wave when it bounces off a barrier.
  b. The reflection of a wave is the change in frequency of a wave when it bounces off a barrier.
  c. The reflection of a wave is the change in velocity of a wave when it bounces off a barrier.
  d. The reflection of a wave is the change in direction of a wave when it bounces off a barrier.

20. What is inversion of a wave?

a. Inversion occurs when a wave reflects off a fixed end, and the wave amplitude changes sign.
b. Inversion occurs when a wave reflects off a loose end, and the wave amplitude changes sign.
c. Inversion occurs when a wave reflects off a fixed end without the wave amplitude changing sign.
d. Inversion occurs when a wave reflects off a loose end without the wave amplitude changing sign.

KEY TERMS

antinode location of maximum amplitude in standing waves

constructive interference when two waves arrive at the same point exactly in phase; that is, the crests of the two waves are precisely aligned, as are the troughs

destructive interference when two identical waves arrive at the same point exactly out of phase that is precisely aligned crest to trough

inversion vertical flipping of a wave after reflection from a fixed end

longitudinal wave wave in which the disturbance is parallel to the direction of propagation

mechanical wave wave that requires a medium through which it can travel

medium solid, liquid, or gas material through which a wave propagates

nodes points where the string does not move; more generally, points where the wave disturbance is zero in a standing wave

periodic wave wave that repeats the same oscillation for several cycles and is associated with simple harmonic motion

pulse wave sudden disturbance with only one wave or a few waves generated

reflection change in direction of a wave at a boundary or fixed end

refraction bending of a wave as it passes from one medium to another medium with a different density

standing wave wave made by the superposition of two waves of the same amplitude and wavelength moving in opposite directions and which appears to vibrate in place

superposition phenomenon that occurs when two or more waves arrive at the same point

transverse wave wave in which the disturbance is perpendicular to the direction of propagation

wave disturbance that moves from its source and carries energy

wave velocity speed at which the disturbance moves; also called the propagation velocity or propagation speed

wavelength distance between adjacent identical parts of a wave

SECTION SUMMARY

13.1 Types of Waves

- A wave is a disturbance that moves from the point of creation and carries energy but not mass.
- Mechanical waves must travel through a medium.
- Sound waves, water waves, and earthquake waves are all examples of mechanical waves.
- Light is not a mechanical wave since it can travel through a vacuum.
- A periodic wave is a wave that repeats for several cycles, whereas a pulse wave has only one crest or a few crests and is associated with a sudden disturbance.
- Periodic waves are associated with simple harmonic motion.
- A transverse wave has a disturbance perpendicular to its direction of propagation, whereas a longitudinal wave has a disturbance parallel to its direction of propagation.

13.2 Wave Properties: Speed, Amplitude, Frequency, and Period

- A wave is a disturbance that moves from the point of creation at a wave velocity v_w.
- A wave has a wavelength λ, which is the distance between adjacent identical parts of the wave.
- The wave velocity and the wavelength are related to the wave's frequency and period by $v_\mathrm{w} = \frac{\lambda}{T}$ or $v_\mathrm{w} = f\lambda$.
- The time for one complete wave cycle is the period T.
- The number of waves per unit time is the frequency f.
- The wave frequency and the period are inversely related to one another.

13.3 Wave Interaction: Superposition and Interference

- Superposition is the combination of two waves at the same location.
- Constructive interference occurs when two identical waves are superimposed exactly in phase.
- Destructive interference occurs when two identical waves are superimposed exactly out of phase.
- A standing wave is a wave produced by the superposition of two waves. It varies in amplitude but does not propagate.
- The nodes are the points where there is no motion in standing waves.
- An antinode is the location of maximum amplitude of a standing wave.
- Reflection causes a wave to change direction.
- Inversion occurs when a wave reflects from a fixed end.
- Refraction causes a wave's path to bend and occurs when a wave passes from one medium into another medium with a different density.

KEY EQUATIONS

13.2 Wave Properties: Speed, Amplitude, Frequency, and Period

wave velocity	$v_\mathrm{w} = \frac{\lambda}{T}$ or $v_\mathrm{w} = f\lambda$
period	$T = \frac{1}{f}$
frequency	$f = \frac{1}{T}$

CHAPTER REVIEW

Concept Items

13.1 Types of Waves

1. Do water waves push water from one place to another? Explain.
   a. No, water waves transfer only energy from one place to another.
   b. Yes, water waves transfer water from one place to another.

2. With reference to waves, what is a trough?
   a. the lowermost position of a wave
   b. the uppermost position of a wave
   c. the final position of a wave
   d. the initial position of the wave

3. Give an example of longitudinal waves.
   a. light waves
   b. water waves in a lake
   c. sound waves in air
   d. seismic waves in Earth's surface

4. What does the speed of a mechanical wave depend on?
   a. the properties of the material through which it travels
   b. the shape of the material through which it travels
   c. the size of the material through which it travels
   d. the color of the material through which it travels

13.2 Wave Properties: Speed, Amplitude, Frequency, and Period

5. Which characteristic of a transverse wave is measured along the direction of propagation?
   a. The amplitude of a transverse wave is measured along the direction of propagation.
   b. The amplitude and the wavelength of a transverse wave are measured along the direction of propagation.
   c. The wavelength of a transverse wave is measured along the direction of propagation.
   d. The displacement of the particles of the medium in a transverse wave is measured along the direction of propagation.

6. Which kind of seismic waves cannot travel through liquid?
   a. compressional waves
   b. P-waves
   c. longitudinal waves
   d. S-waves

7. What is the period of a wave?
   a. the time that a wave takes to complete a half cycle
   b. the time that a wave takes to complete one cycle
   c. the time that a wave takes to complete two cycles
   d. the time that a wave takes to complete four cycles

8. When the period of a wave increases, what happens to its frequency?
   a. Its frequency decreases.
   b. Its frequency increases.
   c. Its frequency remains the same.

13.3 Wave Interaction: Superposition and Interference

9. Is this statement true or false? The amplitudes of waves add up only if they are propagating in the same line.
   a. True
   b. False

10. Why is sound from a stereo louder in one part of the room and softer in another?
    a. Sound is louder in parts of the room where the density is greatest. Sound is softer in parts of the room where density is smallest.
    b. Sound is louder in parts of the room where the density is smallest. Sound is softer in parts of the room where density is greatest.
    c. Sound is louder in parts of the room where constructive interference occurs and softer in parts where destructive interference occurs.
    d. Sound is louder in parts of the room where destructive interference occurs and softer in parts where constructive interference occurs.

11. In standing waves on a string, what does the frequency depend on?
    a. The frequency depends on the propagation speed and the density of the string.

b. The frequency depends on the propagation speed and the length of the string.
c. The frequency depends on the density and the length of the string.
d. The frequency depends on the propagation speed, the density, and the length of the string.

12. Is the following statement true or false? Refraction is useful in fiber optic cables for transmitting signals.
a. False
b. True

13. What is refraction?
a. Refraction is the phenomenon in which waves change their path of propagation at the interface of two media with different densities.
b. Refraction is the phenomenon in which waves change their path of propagation at the interface of two media with the same density.
c. Refraction is the phenomenon in which waves become non-periodic at the boundary of two media with different densities.
d. Refraction is the phenomenon in which waves become non-periodic at the boundary of two media with the same density.

Critical Thinking Items

13.1 Types of Waves

14. Give an example of a wave that propagates only through a solid.
a. Light wave
b. Sound wave
c. Seismic wave
d. Surface wave

15. Can mechanical waves be periodic waves?
a. No, mechanical waves cannot be periodic waves.
b. Yes, mechanical waves can be periodic.

16. In a sound wave, which parameter of the medium varies with every cycle?
a. The density of the medium varies with every cycle.
b. The mass of the medium varies with every cycle.
c. The resistivity of the medium varies with every cycle.
d. The volume of the medium varies with every cycle.

17. What is a transverse wave in an earthquake called?
a. L-wave
b. P-wave
c. S-wave
d. R-wave

13.2 Wave Properties: Speed, Amplitude, Frequency, and Period

18. If the horizontal distance, that is, the distance in the direction of propagation, between a crest and the adjacent trough of a sine wave is 1 m, what is the wavelength of the wave?
a. 0.5 m
b. 1 m
c. 2 m
d. 4 m

19. How is the distance to the epicenter of an earthquake determined?
a. The wavelength difference between P-waves and S-waves is used to measure the distance to the epicenter.
b. The time difference between P-waves and S-waves is used to measure the distance to the epicenter.
c. The frequency difference between P-waves and S-waves is used to measure the distance to the epicenter.
d. The phase difference between P-waves and S-waves is used to measure the distance to the epicenter.

20. Two identical waves superimpose in pure constructive interference. What is the height of the resultant wave if the amplitude of each of the waves is 1 m?
a. 1 m
b. 2 m
c. 3 m
d. 4 m

13.3 Wave Interaction: Superposition and Interference

21. Two identical waves with an amplitude X superimpose in a way that pure constructive interference occurs. What is the amplitude of the resultant wave?
a. $\frac{X}{2}$
b. X
c. $2X$
d. X^2

22. In which kind of wave is the amplitude at each point constant?
a. Seismic waves
b. Pulse wave
c. Standing waves
d. Electromagnetic waves

23. Which property of a medium causes refraction?
a. Conductivity
b. Opacity
c. Ductility

d. Density

24. What is added together when two waves superimpose?
a. Amplitudes
b. Wavelengths
c. Velocities

Problems

13.2 Wave Properties: Speed, Amplitude, Frequency, and Period

25. If a seagull sitting in water bobs up and down once every 2 seconds and the distance between two crests of the water wave is 3 m, what is the velocity of the wave?
a. 1.5 m/s
b. 3 m/s
c. 6 m/s
d. 12 m/s

26. A boat in the trough of a wave takes 3 seconds to reach the highest point of the wave. The velocity of the wave is 5 m/s. What is its wavelength?
a. 0.83 m
b. 15 m
c. 30 m
d. 180 m

Performance Task

13.3 Wave Interaction: Superposition and Interference

27. Ocean waves repeatedly crash against beaches and coasts. Their energy can lead to erosion and collapse of land. Scientists and engineers need to study how waves interact with beaches in order to assess threats to coastal communities and construct breakwater systems. In this task, you will construct a wave tank and fill it with water. Simulate a beach by placing sand at one end. Create waves by moving a piece of wood or plastic up and down in the water. Measure or estimate the wavelength, period, frequency, and amplitude of the wave, and observe the effect of the wave on the sand. Produce waves of different amplitudes and frequencies, and record your observations each time. Use mathematical representations to demonstrate the relationships between different wave properties. Change the position of the sand to create a steeper beach and record your observations. Give a qualitative analysis of the effects of the waves on the beach. What kind of wave causes the most damage? At what height, wavelength, and frequency do waves *break*? How does the steepness of the beach affect the waves?

TEST PREP

Multiple Choice

13.1 Types of Waves

28. What kind of waves are sound waves?
a. Mechanical waves
b. Electromagnetic waves

29. What kind of a wave does a tuning fork create?
a. Pulse wave
b. Periodic wave
c. Electromagnetic wave

30. What kind of waves are electromagnetic waves?
a. Longitudinal waves
b. Transverse waves
c. Mechanical waves
d. P-waves

31. With reference to waves, what is a *disturbance?*
a. It refers to the resistance produced by some particles of a material.
b. It refers to an oscillation produced by some energy that creates a wave.
c. It refers to the wavelength of the wave.
d. It refers to the speed of the wave.

13.2 Wave Properties: Speed, Amplitude, Frequency, and Period

32. Which of these is not a characteristic of a wave?
a. amplitude
b. period
c. mass
d. velocity

33. If you are in a boat at a resting position, how much will your height change when you are hit by the peak of a wave with a height of 2 m?
a. 0 m
b. 1 m
c. 2 m
d. 4 m

34. What is the period of a wave with a frequency of 0.5 Hz?

a. 0.5 s
b. 1 s
c. 2 s
d. 3 s

35. What is the relation between the amplitude of a wave and its speed?
a. The amplitude of a wave is independent of its speed.
b. The amplitude of a wave is directly proportional to its speed.
c. The amplitude of a wave is directly proportional to the square of the inverse of its speed.
d. The amplitude of a wave is directly proportional to the inverse of its speed.

36. What does the speed of seismic waves depend on?
a. The speed of seismic waves depends on the size of the medium.
b. The speed of seismic waves depends on the shape of the medium.
c. The speed of seismic waves depends on the rigidity of the medium.

13.3 Wave Interaction: Superposition and Interference

37. What is added together when two waves superimpose?
a. amplitudes
b. wavelengths
c. velocities

38. Pure constructive interference occurs between two waves when they have the same _____.
a. frequency and are in phase
b. frequency and are out of phase
c. amplitude and are in phase
d. amplitude and are out of phase

39. What kind(s) of interference can occur between two identical waves moving in opposite directions?
a. Constructive interference only
b. Destructive interference only
c. Both constructive and destructive interference
d. Neither constructive nor destructive interference

40. What term refers to the bending of light at the junction of two media?
a. interference
b. diffraction
c. scattering
d. refraction

41. Which parameter of a wave gets affected after superposition?
a. wavelength
b. direction
c. amplitude
d. frequency

42. When do the amplitudes of two waves get added?
a. When their amplitudes are the same
b. When their amplitudes are different
c. When they propagate in perpendicular directions
d. When they are propagating along the same line in opposite directions

Short Answer

13.1 Types of Waves

43. Give an example of a non-mechanical wave.
a. A radio wave is an example of a nonmechanical wave.
b. A sound wave is an example of a nonmechanical wave.
c. A surface wave is an example of a nonmechanical wave.
d. A seismic wave is an example of a nonmechanical wave.

44. How is sound produced by an electronic speaker?
a. The cone of a speaker vibrates to create small changes in the temperature of the air.
b. The cone of a speaker vibrates to create small changes in the pressure of the air.
c. The cone of a speaker vibrates to create small changes in the volume of the air.
d. The cone of a speaker vibrates to create small changes in the resistance of the air.

45. What kind of wave is thunder?
a. Transverse wave
b. Pulse wave
c. R-wave
d. P-wave

46. Are all ocean waves perfectly sinusoidal?
a. No, all ocean waves are not perfectly sinusoidal.
b. Yes, all ocean waves are perfectly sinusoidal.

47. What are orbital progressive waves?
a. Waves that force the particles of the medium to follow a linear path from the crest to the trough
b. Waves that force the particles of the medium to follow a circular path from the crest to the trough
c. Waves that force the particles of the medium to follow a zigzag path from the crest to the trough
d. Waves that force the particles of the medium to

follow a random path from the crest to the trough

48. Give an example of orbital progressive waves.
   a. light waves
   b. ocean waves
   c. sound waves
   d. seismic waves

13.2 Wave Properties: Speed, Amplitude, Frequency, and Period

49. What is the relation between the amplitude and height of a transverse wave?
   a. The height of a wave is half of its amplitude.
   b. The height of a wave is equal to its amplitude.
   c. The height of a wave is twice its amplitude.
   d. The height of a wave is four times its amplitude.

50. If the amplitude of a water wave is 0.2 m and its frequency is 2 Hz, how much distance would a bird sitting on the water's surface move with every wave? How many times will it do this every second?
   a. The bird will go up and down a distance of 0.4 m. It will do this twice per second.
   b. The bird will go up and down a distance of 0.2 m. It will do this twice per second.
   c. The bird will go up and down a distance of 0.4 m. It will do this once per second.
   d. The bird will go up and down a distance of 0.2 m. It will do this once per second.

51. What is the relation between the amplitude and the frequency of a wave?
   a. The amplitude and the frequency of a wave are independent of each other.
   b. The amplitude and the frequency of a wave are equal.
   c. The amplitude decreases with an increase in the frequency of a wave.
   d. The amplitude increases with an increase in the frequency of a wave.

52. What is the relation between a wave's energy and its amplitude?
   a. There is no relation between the energy and the amplitude of a wave.
   b. The magnitude of the energy is equal to the magnitude of the amplitude of a wave.
   c. The energy of a wave increases with an increase in the amplitude of the wave.
   d. The energy of a wave decreases with an increase in the amplitude of a wave.

53. A wave travels 2 m every 2 cycles. What is its wavelength?
   a. 4 m
   b. 2 m
   c. 0.5 m
   d. 1 m

54. A water wave propagates in a river at 6 m/s. If the river moves in the opposite direction at 3 m/s, what is the effective velocity of the wave?
   a. 3 m/s
   b. 6 m/s
   c. 9 m/s
   d. 18 m/s

13.3 Wave Interaction: Superposition and Interference

55. Is this statement true or false? Spherical waves can superimpose.
   a. True
   b. False

56. Is this statement true or false? Waves can superimpose if their frequencies are different.
   a. True
   b. False

57. When does pure destructive interference occur?
   a. When two waves with equal frequencies that are perfectly in phase and propagating along the same line superimpose.
   b. When two waves with unequal frequencies that are perfectly in phase and propagating along the same line superimpose.
   c. When two waves with unequal frequencies that are perfectly out of phase and propagating along the same line superimpose.
   d. When two waves with equal frequencies that are perfectly out of phase and propagating along the same line superimpose.

58. Is this statement true or false? The amplitude of one wave is affected by the amplitude of another wave only when they are precisely aligned.
   a. True
   b. False

59. Why does a standing wave form on a guitar string?
   a. due to superposition with the reflected waves from the ends of the string
   b. due to superposition with the reflected waves from the walls of the room
   c. due to superposition with waves generated from the body of the guitar

60. Is the following statement true or false? A standing wave is a superposition of two identical waves that are in phase and propagating in the same direction.

a. True
b. False

61. Why do water waves traveling from the deep end to the shallow end of a swimming pool experience refraction?
a. Because the pressure of water at the two ends of the pool is same
b. Because the pressures of water at the two ends of the pool are different
c. Because the density of water at the two ends of the pool is same
d. Because the density of water at the two ends of the pool is different

62. Is the statement true or false? Waves propagate faster in a less dense medium if the stiffness is the same.
a. True
b. False

Extended Response

13.1 Types of Waves

63. Why can light travel through outer space while sound cannot?
a. Sound waves are mechanical waves and require a medium to propagate. Light waves can travel through a vacuum.
b. Sound waves are electromagnetic waves and require a medium to propagate. Light waves can travel through a vacuum.
c. Light waves are mechanical waves and do not require a medium to propagate; sound waves require a medium to propagate.
d. Light waves are longitudinal waves and do not require a medium to propagate; sound waves require a medium to propagate.

64. Do periodic waves require a medium to travel through?
a. No, the requirement of a medium for propagation does not depend on whether the waves are pulse waves or periodic waves.
b. Yes, the requirement of a medium for propagation depends on whether the waves are pulse waves or periodic waves.

65. How is the propagation of sound in solids different from that in air?
a. Sound waves in solids are transverse, whereas in air, they are longitudinal.
b. Sound waves in solids are longitudinal, whereas in air, they are transverse.
c. Sound waves in solids can be both longitudinal and transverse, whereas in air, they are longitudinal.
d. Sound waves in solids are longitudinal, whereas in air, they can be both longitudinal and transverse.

13.2 Wave Properties: Speed, Amplitude, Frequency, and Period

66. A seagull is sitting in the water surface and a simple water wave passes under it. What sort of motion does the gull experience? Why?
a. The gull experiences mostly side-to-side motion and moves with the wave in its direction.
b. The gull experiences mostly side-to-side motion but does not move with the wave in its direction.
c. The gull experiences mostly up-and-down motion and moves with the wave in its direction.
d. The gull experiences mostly up-and-down motion but does not move in the direction of the wave.

67. Why does a good-quality speaker have a woofer and a tweeter?
a. In a good-quality speaker, sounds with high frequencies or short wavelengths are reproduced accurately by woofers, while sounds with low frequencies or long wavelengths are reproduced accurately by tweeters.
b. Sounds with high frequencies or short wavelengths are reproduced more accurately by tweeters, while sounds with low frequencies or long wavelengths are reproduced more accurately by woofers.

68. The time difference between a 2 km/s S-wave and a 6 km/s P-wave recorded at a certain point is 10 seconds. How far is the epicenter of the earthquake from that point?
a. 15 m
b. 30 m
c. 15 km
d. 30 km

13.3 Wave Interaction: Superposition and Interference

69. Why do water waves sometimes appear like a complex criss-cross pattern?
a. The crests and the troughs of waves traveling in the same direction combine to form a criss-cross pattern.
b. The crests and the troughs of waves traveling in different directions combine to form a criss-cross pattern.

70. What happens when two dissimilar waves interfere?
a. pure constructive interference
b. pure destructive interference
c. a combination of constructive and destructive

interference

71. Occasionally, during earthquakes, areas near the epicenter are not damaged while those farther away are damaged. Why could this occur?

a. Destructive interference results in waves with greater amplitudes being formed in places farther away from the epicenter.
b. Constructive interference results in waves with greater amplitudes being formed in places farther away from the epicenter.
c. The standing waves of great amplitudes are formed in places farther away from the epicenter.
d. The pulse waves of great amplitude are formed in places farther away from the epicenter.

72. Why does an object appear to be distorted when you view it through a glass of water?

a. The glass and the water reflect the light in different directions. Hence, the object appears to be distorted.
b. The glass and the water absorb the light by different amounts. Hence, the object appears to be distorted.
c. Water, air, and glass are media with different densities. Light rays refract and bend when they pass from one medium to another. Hence, the object appears to be distorted.
d. The glass and the water disperse the light into its components. Hence, the object appears to be distorted.

CHAPTER 14
Sound

Figure 14.1 This tree fell some time ago. When it fell, particles in the air were disturbed by the energy of the tree hitting the ground. This disturbance of matter, which our ears have evolved to detect, is called sound. (B.A. Bowen Photography)

Chapter Outline

INTRODUCTION If a tree falls in a forest (see Figure 14.1) and no one is there to hear it, does it make a sound? The answer to this old philosophical question depends on how you define sound. If sound only exists when someone is around to perceive it, then the falling tree produced no sound. However, in physics, we know that colliding objects can disturb the air, water or other matter surrounding them. As a result of the collision, the surrounding particles of matter began vibrating in a wave-like fashion. This is a sound wave. Consequently, if a tree collided with another object in space, no one would hear it, because no sound would be produced. This is because, in space, there is no air, water or other matter to be disturbed and produce sound waves. In this chapter, we'll learn more about the wave properties of sound, and explore hearing, as well as some special uses for sound.

14.1 Speed of Sound, Frequency, and Wavelength

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Relate the characteristics of waves to properties of sound waves
- Describe the speed of sound and how it changes in various media
- Relate the speed of sound to frequency and wavelength of a sound wave

Section Key Terms

rarefaction sound

Properties of Sound Waves

Sound is a wave. More specifically, sound is defined to be a disturbance of matter that is transmitted from its source outward. A disturbance is anything that is moved from its state of equilibrium. Some sound waves can be characterized as periodic waves, which means that the atoms that make up the matter experience simple harmonic motion.

A vibrating string produces a sound wave as illustrated in Figure 14.2, Figure 14.3, and Figure 14.4. As the string oscillates back and forth, part of the string's energy goes into compressing and expanding the surrounding air. This creates slightly higher and lower pressures. The higher pressure… regions are compressions, and the low pressure regions are **rarefactions**. The pressure disturbance moves through the air as longitudinal waves with the same frequency as the string. Some of the energy is lost in the form of thermal energy transferred to the air. You may recall from the chapter on waves that areas of compression and rarefaction in longitudinal waves (such as sound) are analogous to crests and troughs in transverse waves.

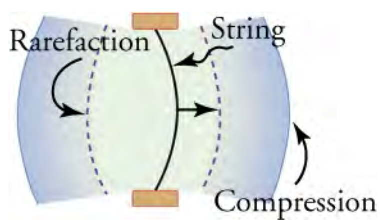


Figure 14.2 A vibrating string moving to the right compresses the air in front of it and expands the air behind it.

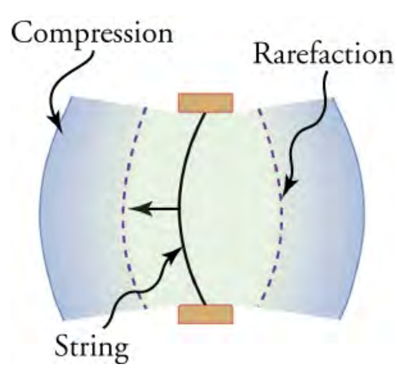


Figure 14.3 As the string moves to the left, it creates another compression and rarefaction as the particles on the right move away from the string.

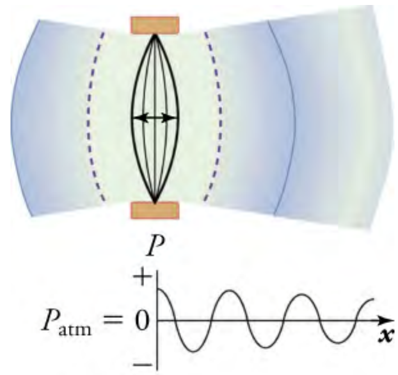


Figure 14.4 After many vibrations, there is a series of compressions and rarefactions that have been transmitted from the string as a sound wave. The graph shows gauge pressure (P_{gauge}) versus distance x from the source. Gauge pressure is the pressure relative to atmospheric pressure; it is positive for pressures above atmospheric pressure, and negative for pressures below it. For ordinary, everyday sounds, pressures vary only slightly from average atmospheric pressure.

The amplitude of a sound wave decreases with distance from its source, because the energy of the wave is spread over a larger and larger area. But some of the energy is also absorbed by objects, such as the eardrum in Figure 14.5, and some of the energy is converted to thermal energy in the air. Figure 14.4 shows a graph of gauge pressure versus distance from the vibrating string. From this figure, you can see that the compression of a longitudinal wave is analogous to the peak of a transverse wave, and the rarefaction of a longitudinal wave is analogous to the trough of a transverse wave. Just as a transverse wave alternates between peaks and troughs, a longitudinal wave alternates between compression and rarefaction.

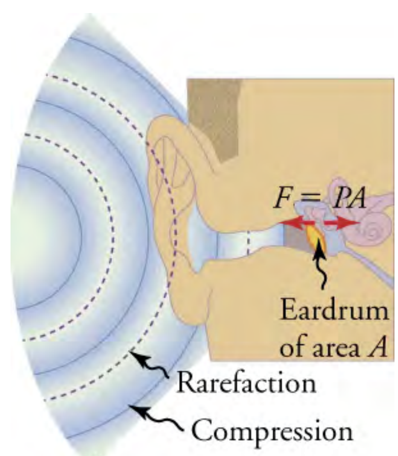


Figure 14.5 Sound wave compressions and rarefactions travel up the ear canal and force the eardrum to vibrate. There is a net force on the eardrum, since the sound wave pressures differ from the atmospheric pressure found behind the eardrum. A complicated mechanism converts the vibrations to nerve impulses, which are then interpreted by the brain.

The Speed of Sound

The speed of sound varies greatly depending upon the medium it is traveling through. The speed of sound in a medium is determined by a combination of the medium's rigidity (or compressibility in gases) and its density. The more rigid (or less compressible) the medium, the faster the speed of sound. The greater the density of a medium, the slower the speed of sound. The speed of sound in air is low, because air is compressible. Because liquids and solids are relatively rigid and very difficult to compress, the speed of sound in such media is generally greater than in gases. Table 14.1 shows the speed of sound in various media. Since temperature affects density, the speed of sound varies with the temperature of the medium through which it's traveling to some extent, especially for gases.

Medium	v_w (m/s)
Gases at 0 °C	
Air	331
Carbon dioxide	259
Oxygen	316
Helium	965
Hydrogen	1290
Liquids at 20 °C	
Ethanol	1160
Mercury	1450
Water, fresh	1480
Sea water	1540
Human tissue	1540
Solids (longitudinal or bulk)	
Vulcanized rubber	54
Polyethylene	920
Marble	3810
Glass, Pyrex	5640
Lead	1960
Aluminum	5120
Steel	5960

Table 14.1 Speed of Sound in Various Media

The Relationship Between the Speed of Sound and the Frequency and Wavelength of a Sound Wave

Figure 14.6 When fireworks explode in the sky, the light energy is perceived before the sound energy. Sound travels more slowly than light does. (Dominic Alves, Flickr)

Sound, like all waves, travels at certain speeds through different media and has the properties of frequency and wavelength. Sound travels much slower than light—you can observe this while watching a fireworks display (see Figure 14.6), since the flash of an explosion is seen before its sound is heard.

The relationship between the speed of sound, its frequency, and wavelength is the same as for all waves:

$$v = f\lambda, \tag{14.1}$$

where v is the speed of sound (in units of m/s), f is its frequency (in units of hertz), and λ is its wavelength (in units of meters). Recall that wavelength is defined as the distance between adjacent identical parts of a wave. The wavelength of a sound, therefore, is the distance between adjacent identical parts of a sound wave. Just as the distance between adjacent crests in a transverse wave is one wavelength, the distance between adjacent compressions in a sound wave is also one wavelength, as shown in Figure 14.7. The frequency of a sound wave is the same as that of the source. For example, a tuning fork vibrating at a given frequency would produce sound waves that oscillate at that same frequency. The frequency of a sound is the number of waves that pass a point per unit time.

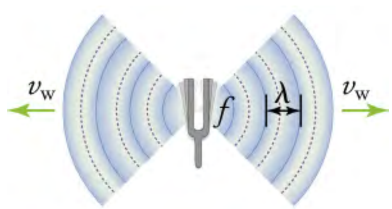


Figure 14.7 A sound wave emanates from a source vibrating at a frequency f, propagates at v, and has a wavelength λ .

One of the more important properties of sound is that its speed is nearly independent of frequency. If this were not the case, and high-frequency sounds traveled faster, for example, then the farther you were from a band in a football stadium, the more the sound from the low-pitch instruments would lag behind the high-pitch ones. But the music from all instruments arrives in cadence independent of distance, and so all frequencies must travel at nearly the same speed.

Recall that $v = f\lambda$, and in a given medium under fixed temperature and humidity, v is constant. Therefore, the relationship between f and λ is inverse: The higher the frequency, the shorter the wavelength of a sound wave.

The speed of sound can change when sound travels from one medium to another. However, the frequency usually remains the same because it is like a driven oscillation and maintains the frequency of the original source. If v changes and f remains the same, then the wavelength λ must change. Since $v = f\lambda$, the higher the speed of a sound, the greater its wavelength for a given frequency.

Virtual Physics

Sound

Click to view content (https://www.openstax.org/l/28sound)

This simulation lets you see sound waves. Adjust the frequency or amplitude (volume) and you can see and hear how the wave changes. Move the listener around and hear what she hears. Switch to the Two Source Interference tab or the Interference by Reflection tab to experiment with interference and reflection.

TIPS FOR SUCCESS

Make sure to have audio enabled and set to Listener rather than Speaker, or else the sound will not vary as you move the listener around.

GRASP CHECK

In the first tab, Listen to a Single Source, move the listener as far away from the speaker as possible, and then change the frequency of the sound wave. You may have noticed that there is a delay between the time when you change the setting and the time when you hear the sound get lower or higher in pitch. Why is this?

a. Because, intensity of the sound wave changes with the frequency.
b. Because, the speed of the sound wave changes when the frequency is changed.
c. Because, loudness of the sound wave takes time to adjust after a change in frequency.
d. Because it takes time for sound to reach the listener, so the listener perceives the new frequency of sound wave after a delay.

Is there a difference in the amount of delay depending on whether you make the frequency higher or lower? Why?

a. Yes, the speed of propagation depends only on the frequency of the wave.
b. Yes, the speed of propagation depends upon the wavelength of the wave, and wavelength changes as the frequency changes.
c. No, the speed of propagation depends only on the wavelength of the wave.
d. No, the speed of propagation is constant in a given medium; only the wavelength changes as the frequency changes.

Snap Lab

Voice as a Sound Wave

In this lab you will observe the effects of blowing and speaking into a piece of paper in order to compare and contrast different sound waves.

- sheet of paper
- tape
- table

Instructions

Procedure

1. Suspend a sheet of paper so that the top edge of the paper is fixed and the bottom edge is free to move. You could tape the top edge of the paper to the edge of a table, for example.
2. Gently blow air near the edge of the bottom of the sheet and note how the sheet moves.
3. Speak softly and then louder such that the sounds hit the edge of the bottom of the paper, and note how the sheet moves.
4. Interpret the results.

GRASP CHECK

Which sound wave property increases when you are speaking more loudly than softly?

a. amplitude of the wave
b. frequency of the wave
c. speed of the wave

d. wavelength of the wave

WORKED EXAMPLE

What Are the Wavelengths of Audible Sounds?

Calculate the wavelengths of sounds at the extremes of the audible range, 20 and 20,000 Hz, in conditions where sound travels at 348.7 m/s.

STRATEGY

To find wavelength from frequency, we can use $v = f\lambda$.

Solution

(1) Identify the knowns. The values for v and f are given.

(2) Solve the relationship between speed, frequency and wavelength for λ .

$$\lambda = \frac{v}{f}. \tag{14.2}$$

(3) Enter the speed and the minimum frequency to give the maximum wavelength.

$$\lambda_{\text{max}} = \frac{348.7 \text{ m/s}}{20 \text{ Hz}} = 17 \text{ m} \approx 20 \text{ m (1 sig. figure)} \tag{14.3}$$

(4) Enter the speed and the maximum frequency to give the minimum wavelength.

$$\lambda_{\text{min}} = \frac{348.7 \text{ m/s}}{20,000 \text{ Hz}} = 0.017 \text{ m} \approx 2 \text{ cm (1 sig. figure)} \tag{14.4}$$

Discussion

Because the product of f multiplied by λ equals a constant velocity in unchanging conditions, the smaller f is, the larger λ must be, and vice versa. Note that you can also easily rearrange the same formula to find frequency or velocity.

Practice Problems

1. What is the speed of a sound wave with frequency 2000 Hz and wavelength 0.4 m?
   a. 5×10^3 m/s
   b. 3.2×10^2 m/s
   c. 2×10^{-4} m/s
   d. 8×10^2 m/s

2. Dogs can hear frequencies of up to 45 kHz. What is the wavelength of a sound wave with this frequency traveling in air at 0°C?
   a. 2.0×10^7 m
   b. 1.5×10^7 m
   c. 1.4×10^2 m
   d. 7.4×10^{-3} m

LINKS TO PHYSICS

Echolocation

Figure 14.8 A bat uses sound echoes to find its way about and to catch prey. The time for the echo to return is directly proportional to the distance.

Echolocation is the use of reflected sound waves to locate and identify objects. It is used by animals such as bats, dolphins and whales, and is also imitated by humans in SONAR—Sound Navigation and Ranging—and echolocation technology.

Bats, dolphins and whales use echolocation to navigate and find food in their environment. They locate an object (or obstacle) by emitting a sound and then sensing the reflected sound waves. Since the speed of sound in air is constant, the time it takes for the sound to travel to the object and back gives the animal a sense of the distance between itself and the object. This is called *ranging*. Figure 14.8 shows a bat using echolocation to sense distances.

Echolocating animals identify an object by comparing the relative intensity of the sound waves returning to each ear to figure out the angle at which the sound waves were reflected. This gives information about the direction, size and shape of the object. Since there is a slight distance in position between the two ears of an animal, the sound may return to one of the ears with a bit of a delay, which also provides information about the position of the object. For example, if a bear is directly to the right of a bat, the echo will return to the bat's left ear later than to its right ear. If, however, the bear is directly ahead of the bat, the echo would return to both ears at the same time. For an animal without a sense of sight such as a bat, it is important to know *where* other animals are as well as *what* they are; their survival depends on it.

Principles of echolocation have been used to develop a variety of useful sensing technologies. SONAR, is used by submarines to detect objects underwater and measure water depth. Unlike animal echolocation, which relies on only one transmitter (a mouth) and two receivers (ears), manmade SONAR uses many transmitters and beams to get a more accurate reading of the environment. Radar technologies use the echo of radio waves to locate clouds and storm systems in weather forecasting, and to locate aircraft for air traffic control. Some new cars use echolocation technology to sense obstacles around the car, and warn the driver who may be about to hit something (or even to automatically parallel park). Echolocation technologies and training systems are being developed to help visually impaired people navigate their everyday environments.

GRASP CHECK

If a predator is directly to the left of a bat, how will the bat know?

a. The echo would return to the left ear first.
b. The echo would return to the right ear first.

Check Your Understanding

3. What is a rarefaction?
   a. Rarefaction is the high-pressure region created in a medium when a longitudinal wave passes through it.
   b. Rarefaction is the low-pressure region created in a medium when a longitudinal wave passes through it.
   c. Rarefaction is the highest point of amplitude of a sound wave.
   d. Rarefaction is the lowest point of amplitude of a sound wave.

4. What sort of motion do the particles of a medium experience when a sound wave passes through it?
   a. Simple harmonic motion

b. Circular motion
c. Random motion
d. Translational motion

5. What does the speed of sound depend on?
   a. The wavelength of the wave
   b. The size of the medium
   c. The frequency of the wave
   d. The properties of the medium

6. What property of a gas would affect the speed of sound traveling through it?
   a. The volume of the gas
   b. The flammability of the gas
   c. The mass of the gas
   d. The compressibility of the gas

14.2 Sound Intensity and Sound Level

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Relate amplitude of a wave to loudness and energy of a sound wave
- Describe the decibel scale for measuring sound intensity
- Solve problems involving the intensity of a sound wave
- Describe how humans produce and hear sounds

Section Key Terms

amplitude	decibel	hearing	loudness
pitch	sound intensity	sound intensity level	

Amplitude, Loudness and Energy of a Sound Wave

Figure 14.9 Noise on crowded roadways like this one in Delhi makes it hard to hear others unless they shout. (Lingaraj G J, Flickr)

In a quiet forest, you can sometimes hear a single leaf fall to the ground. But in a traffic jam filled with honking cars, you may have to shout just so the person next to you can hear Figure 14.9.The loudness of a sound is related to how energetically its source is vibrating. In cartoons showing a screaming person, the cartoonist often shows an open mouth with a vibrating uvula (the hanging tissue at the back of the mouth) to represent a loud sound coming from the throat. Figure 14.10 shows such a cartoon depiction of a bird loudly expressing its opinion.

A useful quantity for describing the loudness of sounds is called **sound intensity**. In general, the intensity of a wave is the power per unit area carried by the wave. Power is the rate at which energy is transferred by the wave. In equation form, intensity I is

$$I = \frac{P}{A}, \tag{14.5}$$

where P is the power through an area A. The SI unit for I is W/m^2. The intensity of a sound depends upon its pressure amplitude.

The relationship between the intensity of a sound wave and its pressure amplitude (or pressure variation Δp) is

$$I = \frac{(\Delta p)^2}{2\rho v_w}, \tag{14.6}$$

where ρ is the density of the material in which the sound wave travels, in units of kg/m^3, and v is the speed of sound in the medium, in units of m/s. Pressure amplitude has units of pascals (Pa) or N/m^2. Note that Δp is half the difference between the maximum and minimum pressure in the sound wave.

We can see from the equation that the intensity of a sound is proportional to its amplitude squared. The pressure variation is proportional to the amplitude of the oscillation, and so I varies as $(\Delta p)^2$. This relationship is consistent with the fact that the sound wave is produced by some vibration; the greater its pressure amplitude, the more the air is compressed during the vibration. Because the power of a sound wave is the rate at which energy is transferred, the energy of a sound wave is also proportional to its amplitude squared.

TIPS FOR SUCCESS

Pressure is usually denoted by capital P, but we are using a lowercase p for pressure in this case to distinguish it from power P above.

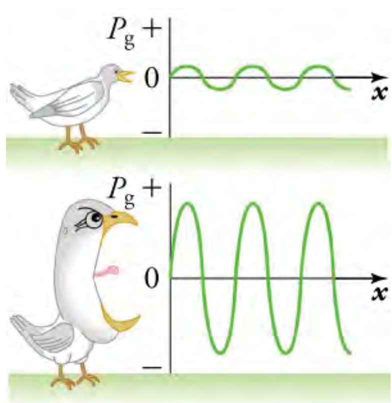


Figure 14.10 Graphs of the pressures in two sound waves of different intensities. The more intense sound is produced by a source that has larger-amplitude oscillations and has greater pressure maxima and minima. Because pressures are higher in the greater-intensity sound, it can exert larger forces on the objects it encounters.

The Decibel Scale

You may have noticed that when people talk about the loudness of a sound, they describe it in units of **decibels** rather than watts per meter squared. While sound intensity (in W/m^2) is the SI unit, the **sound intensity level** in decibels (dB) is more relevant for how humans perceive sounds. The way our ears perceive sound can be more accurately described by the logarithm of the intensity of a sound rather than the intensity of a sound directly. The sound intensity level β is defined to be

$$\beta\ (\text{dB}) = 10 \log_{10}\left(\frac{I}{I_0}\right), \tag{14.7}$$

where I is sound intensity in watts per meter squared, and $I_0 = 10^{-12}\ W/m^2$ is a reference intensity. I_0 is chosen as the reference point because it is the lowest intensity of sound a person with normal hearing can perceive. The decibel level of a sound having an intensity of $10^{-12}\ W/m^2$ is $\beta = 0$ dB, because $\log_{10} 1 = 0$. That is, the threshold of human hearing is 0 decibels.

Each factor of 10 in intensity corresponds to 10 dB. For example, a 90 dB sound compared with a 60 dB sound is 30 dB greater, or three factors of 10 (that is, 10^3 times) as intense. Another example is that if one sound is 10^7 as intense as another, it is 70 dB higher.

Since β is defined in terms of a ratio, it is unit-less. The unit called *deci*bel (dB) is used to indicate that this ratio is multiplied by 10. The sound intensity level is not the same as sound intensity—it tells you the *level* of the sound relative to a reference intensity rather than the actual intensity.

Snap Lab

Feeling Sound

In this lab, you will play music with a heavy beat to literally feel the vibrations and explore what happens when the volume changes.

- CD player or portable electronic device connected to speakers
- rock or rap music CD or mp3
- a lightweight table

Procedure

1. Place the speakers on a light table, and start playing the CD or mp3.
2. Place your hand gently on the table next to the speakers.
3. Increase the volume and note the level when the table just begins to vibrate as the music plays.
4. Increase the reading on the volume control until it doubles. What has happened to the vibrations?

GRASP CHECK

Do you think that when you double the volume of a sound wave you are doubling the sound intensity level (in dB) or the sound intensity (in W/m^2)? Why?

a. The sound intensity in W/m^2, because it is a closer measure of how humans perceive sound.
b. The sound intensity level in dB because it is a closer measure of how humans perceive sound.
c. The sound intensity in W/m^2 because it is the only unit to express the intensity of sound.
d. The sound intensity level in dB because it is the only unit to express the intensity of sound.

Solving Sound Wave Intensity Problems

WORKED EXAMPLE

Calculating Sound Intensity Levels: Sound Waves

Calculate the sound intensity level in decibels for a sound wave traveling in air at 0 °C and having a pressure amplitude of 0.656 Pa.

STRATEGY

We are given Δp, so we can calculate I using the equation $I = \frac{(\Delta p)^2}{2\rho v}$. Using I, we can calculate β straight from its definition in $\beta\,(d\text{B}) = 10\ \log_{10}\left(\frac{I}{I_0}\right)\beta\,(\text{dB}) = 10\ \log_{10}\left(\frac{I}{I_0}\right)$.

Solution

(1) Identify knowns:

Sound travels at 331 m/s in air at 0 °C.

Air has a density of 1.29 kg/m³ at atmospheric pressure and 0°C.

(2) Enter these values and the pressure amplitude into $I = \frac{(\Delta p)^2}{2\rho v_w}$.

$$I = \frac{(\Delta p)^2}{2\rho v_w} = \frac{(0.656\ \ \text{Pa})^2}{2\left(1.29\ \text{kg/m}^3\right)(331\ \ \text{m/s})} = 5.04 \times 10^{-4}\ \ \text{W/m}^2.$$

(3) Enter the value for I and the known value for I_0 into $\beta\ (\text{dB}) = 10\ \log_{10}\left(\frac{I}{I_0}\right)$. Calculate to find the sound intensity level in decibels.

$$10\ \log_{10}\left(5.04 \times 10^8\right) = 10\,(8.70)\ \ \text{dB} = 87.0\ \ \text{dB}.$$

Discussion

This 87.0 dB sound has an intensity five times as great as an 80 dB sound. So a factor of five in intensity corresponds to a difference of 7 dB in sound intensity level. This value is true for any intensities differing by a factor of five.

WORKED EXAMPLE

Change Intensity Levels of a Sound: What Happens to the Decibel Level?

Show that if one sound is twice as intense as another, it has a sound level about 3 dB higher.

STRATEGY

You are given that the ratio of two intensities is 2 to 1, and are then asked to find the difference in their sound levels in decibels. You can solve this problem using of the properties of logarithms.

Solution

(1) Identify knowns:

The ratio of the two intensities is 2 to 1, or: $\frac{I_2}{I_1} = 2.00$.

We want to show that the difference in sound levels is about 3 dB. That is, we want to show

$$\beta_2 - \beta_1 = 3 \text{ dB}. \tag{14.8}$$

Note that

$$\log_{10} b - \log_{10} a = \log_{10}\left(\frac{b}{a}\right). \tag{14.9}$$

(2) Use the definition of β to get

$$\beta_2 - \beta_1 = 10 \log_{10}\left(\frac{I_2}{I_1}\right) = 10 \log_{10} 2.00 = 10\,(0.301) \text{ dB}. \tag{14.10}$$

Therefore, $\beta_2 - \beta_1 = 3.01 \text{ dB}$.

Discussion

This means that the two sound intensity levels differ by 3.01 dB, or about 3 dB, as advertised. Note that because only the ratio I_2/I_1 is given (and not the actual intensities), this result is true for any intensities that differ by a factor of two. For example, a 56.0 dB sound is twice as intense as a 53.0 dB sound, a 97.0 dB sound is half as intense as a 100 dB sound, and so on.

Practice Problems

7. Calculate the intensity of a wave if the power transferred is 10 W and the area through which the wave is transferred is 5 square meters.
   a. 200 W / m^2
   b. 50 W / m^2
   c. 0.5 W / m^2
   d. 2 W / m^2

8. Calculate the sound intensity for a sound wave traveling in air at 0°C and having a pressure amplitude of 0.90 Pa.
   a. $1.8 \times 10^{-3} \text{W/m}^2$
   b. $4.2 \times 10^{-3} \text{ W/m}^2$
   c. $1.1 \times 10^{3} \text{ W/m}^2$
   d. $9.5 \times 10^{-4} \text{ W/m}^2$

Hearing and Voice

People create sounds by pushing air up through their lungs and through elastic folds in the throat called vocal cords. These folds open and close rhythmically, creating a pressure buildup. As air travels up and past the vocal cords, it causes them to vibrate. This vibration escapes the mouth along with puffs of air as sound. A voice changes in pitch when the muscles of the larynx relax or tighten, changing the tension on the vocal chords. A voice becomes louder when air flow from the lungs increases, making the amplitude of the sound pressure wave greater.

Hearing is the perception of sound. It can give us plenty of information—such as pitch, loudness, and direction. Humans can normally hear frequencies ranging from approximately 20 to 20,000 Hz. Other animals have hearing ranges different from that of humans. Dogs can hear sounds as high as 45,000 Hz, whereas bats and dolphins can hear up to 110,000 Hz sounds. You may have noticed that dogs respond to the sound of a dog whistle which produces sound out of the range of human hearing.

Sounds below 20 Hz are called infrasound, whereas those above 20,000 Hz are ultrasound. The perception of frequency is called **pitch**, and the perception of intensity is called **loudness**.

The way we hear involves some interesting physics. The sound wave that hits our ear is a pressure wave. The ear converts sound waves into electrical nerve impulses, similar to a microphone.

Figure 14.11 shows the anatomy of the ear with its division into three parts: the outer ear or ear canal; the middle ear, which runs from the eardrum to the cochlea; and the inner ear, which is the cochlea itself. The body part normally referred to as the ear is technically called the pinna.

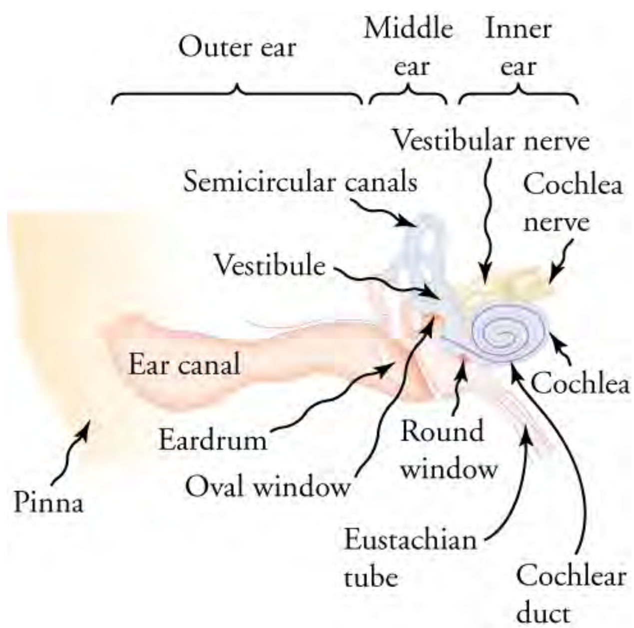


Figure 14.11 The illustration shows the anatomy of the human ear.

The outer ear, or ear canal, carries sound to the eardrum protected inside of the ear. The middle ear converts sound into mechanical vibrations and applies these vibrations to the cochlea. The lever system of the middle ear takes the force exerted on the eardrum by sound pressure variations, amplifies it and transmits it to the inner ear via the oval window. Two muscles in the middle ear protect the inner ear from very intense sounds. They react to intense sound in a few milliseconds and reduce the force transmitted to the cochlea. This protective reaction can also be triggered by your own voice, so that humming during a fireworks display, for example, can reduce noise damage.

Figure 14.12 shows the middle and inner ear in greater detail. As the middle ear bones vibrate, they vibrate the cochlea, which contains fluid. This creates pressure waves in the fluid that cause the tectorial membrane to vibrate. The motion of the tectorial membrane stimulates tiny cilia on specialized cells called hair cells. These hair cells, and their attached neurons, transform the motion of the tectorial membrane into electrical signals that are sent to the brain.

The tectorial membrane vibrates at different positions based on the frequency of the incoming sound. This allows us to detect the pitch of sound. Additional processing in the brain also allows us to determine which direction the sound is coming from (based on comparison of the sound's arrival time and intensity between our two ears).

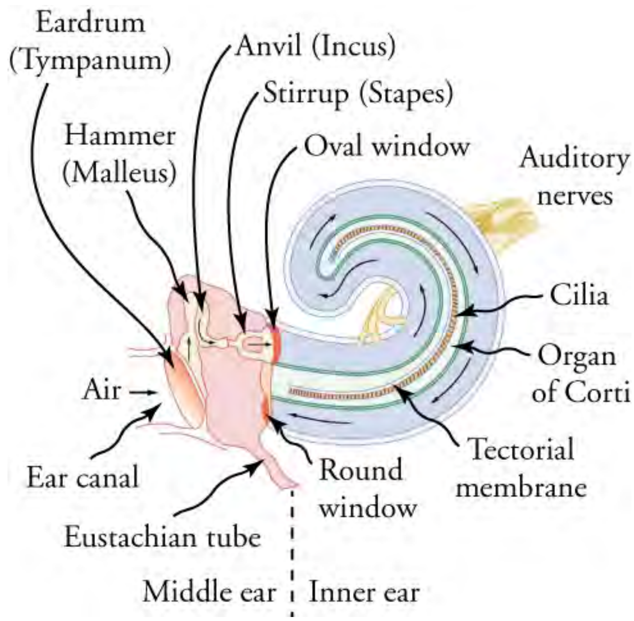


Figure 14.12 The inner ear, or cochlea, is a coiled tube about 3 mm in diameter and 3 cm in length when uncoiled. As the stapes vibrates against the oval window, it creates pressure waves that travel through fluid in the cochlea. These waves vibrate the tectorial membrane, which bends the cilia and stimulates nerves in the organ of Corti. These nerves then send information about the sound to the brain.

FUN IN PHYSICS

Musical Instruments

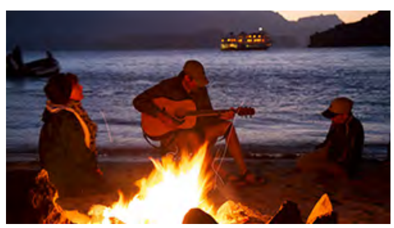


Figure 14.13 Playing music, also known as "rocking out", involves creating vibrations using musical instruments. (John Norton)

Yet another way that people make sounds is through playing musical instruments (see the previous figure). Recall that the perception of frequency is called pitch. You may have noticed that the pitch range produced by an instrument tends to depend upon its size. Small instruments, such as a piccolo, typically make high-pitch sounds, while larger instruments, such as a tuba, typically make low-pitch sounds. High-pitch means small wavelength, and the size of a musical instrument is directly related to the wavelengths of sound it produces. So a small instrument creates short-wavelength sounds, just as a large instrument creates long-wavelength sounds.

Most of us have excellent relative pitch, which means that we can tell whether one sound has a different frequency from another. We can usually distinguish one sound from another if the frequencies of the two sounds differ by as little as 1 Hz. For example, 500.0 and 501.5 Hz are noticeably different.

Musical notes are particular sounds that can be produced by most instruments, and are the building blocks of a song. In Western music, musical notes have particular names, such as A-sharp, C, or E-flat. Some people can identify musical notes just by listening to them. This rare ability is called *perfect*, or *absolute, pitch*.

When a violin plays middle C, there is no mistaking it for a piano playing the same note. The reason is that each instrument produces a distinctive set of frequencies and intensities. We call our perception of these combinations of frequencies and intensities the *timbre* of the sound. It is more difficult to quantify timbre than loudness or pitch. Timbre is more subjective. Evocative adjectives such as dull, brilliant, warm, cold, pure, and rich are used to describe the timbre of a sound rather than quantities with units, which makes for a difficult topic to dissect with physics. So the consideration of timbre takes us into the realm of perceptual psychology, where higher-level processes in the brain are dominant. This is also true for other perceptions of sound, such as music and noise. But as a teenager, you are likely already aware that one person's music may be another person's noise.

GRASP CHECK

If you turn up the volume of your stereo, will the pitch change? Why or why not?

a. No, because pitch does not depend on intensity.
b. Yes, because pitch is directly related to intensity.

Check Your Understanding

9. What is sound intensity?

a. Intensity is the energy per unit area carried by a wave.
b. Intensity is the energy per unit volume carried by a wave.
c. Intensity is the power per unit area carried by a wave.
d. Intensity is the power per unit volume carried by a wave.

10. How is power defined with reference to a sound wave?

a. Power is the rate at which energy is transferred by a sound wave.
b. Power is the rate at which mass is transferred by a sound wave.
c. Power is the rate at which amplitude of a sound wave changes.
d. Power is the rate at which wavelength of a sound wave changes.

11. What word or phrase is used to describe the loudness of sound?

a. frequency or oscillation
b. intensity level or decibel
c. timbre
d. pitch

12. What is the mathematical expression for sound intensity level β?

a. $\beta\,(\mathrm{dB}) = 10\log_{10}\left(\frac{I_0}{I}\right)$
b. $\beta\,(\mathrm{dB}) = 20\log_{10}\left(\frac{I}{I_0}\right)$
c. $\beta(dB) = 20\log_{10}\left(\frac{I_0}{I}\right)$
d. $\beta\,(\mathrm{dB}) = 10\log_{10}\left(\frac{I}{I_0}\right)$

13. What is the range frequencies that humans are capable of hearing?

a. $20\ \mathrm{Hz}$ to$200,000\ \mathrm{Hz}$
b. $2\ \mathrm{Hz}$ to $50,000\ \mathrm{Hz}$
c. $2\ \mathrm{Hz}$ to $2,000\ \mathrm{Hz}$
d. $20\ \mathrm{Hz}$ to $20,000\ \mathrm{Hz}$

14. How do humans change the pitch of their voice?

a. Relaxing or tightening their glottis
b. Relaxing or tightening their uvula
c. Relaxing or tightening their tongue
d. Relaxing or tightening their larynx

References

Nave, R. Vocal sound production—HyperPhysics. Retrieved from http://hyperphysics.phy-astr.gsu.edu/hbase/music/voice.html

14.3 Doppler Effect and Sonic Booms

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the Doppler effect of sound waves
- Explain a sonic boom
- Calculate the frequency shift of sound from a moving object by the Doppler shift formula, and calculate the speed of an object by the Doppler shift formula

Section Key Terms

Doppler effect	sonic boom

The Doppler Effect of Sound Waves

The **Doppler effect** is a change in the observed pitch of a sound, due to relative motion between the source and the observer. An example of the Doppler effect due to the motion of a source occurs when you are standing still, and the sound of a siren coming from an ambulance shifts from high-pitch to low-pitch as it passes by. The closer the ambulance is to you, the more sudden the shift. The faster the ambulance moves, the greater the shift. We also hear this shift in frequency for passing race cars, airplanes, and trains. An example of the Doppler effect with a stationary source and moving observer is if you ride a train past a stationary warning bell, you will hear the bell's frequency shift from high to low as you pass by.

What causes the Doppler effect? Let's compare three different scenarios: Sound waves emitted by a stationary source (Figure 14.14), sound waves emitted by a moving source (Figure 14.15), and sound waves emitted by a stationary source but heard by moving observers (Figure 14.16). In each case, the sound spreads out from the point where it was emitted.

If the source and observers are stationary, then observers on either side see the same wavelength and frequency as emitted by the source. But if the source is moving and continues to emit sound as it travels, then the air compressions (crests) become closer together in the direction in which it's traveling and farther apart in the direction it's traveling away from. Therefore, the wavelength is shorter in the direction the source is moving (on the right in Figure 14.15), and longer in the opposite direction (on the left in Figure 14.15).

Finally, if the observers move, as in Figure 14.16, the frequency at which they receive the compressions changes. The observer moving toward the source receives them at a higher frequency (and therefore shorter wavelength), and the person moving away from the source receives them at a lower frequency (and therefore longer wavelength).

Figure 14.14 Sounds emitted by a source spread out in spherical waves. Because the source, observers, and air are stationary, the wavelength and frequency are the same in all directions and to all observers.

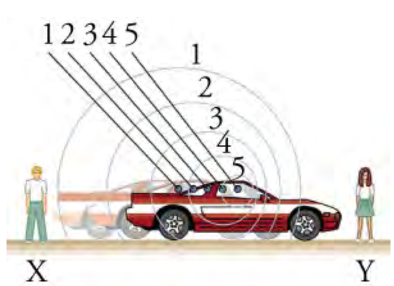


Figure 14.15 Sounds emitted by a source moving to the right spread out from the points at which they were emitted. The wavelength is

reduced and, consequently, the frequency is increased in the direction of motion, so that the observer on the right hears a higher-pitch sound. The opposite is true for the observer on the left, where the wavelength is increased and the frequency is reduced.

Figure 14.16 The same effect is produced when the observers move relative to the source. Motion toward the source increases frequency as the observer on the right passes through more wave crests than she would if stationary. Motion away from the source decreases frequency as the observer on the left passes through fewer wave crests than he would if stationary.

We know that wavelength and frequency are related by $v = f\lambda$, where v is the fixed speed of sound. The sound moves in a medium and has the same speed v in that medium whether the source is moving or not. Therefore, f multiplied by λ is a constant. Because the observer on the right in Figure 14.15 receives a shorter wavelength, the frequency she perceives must be higher. Similarly, the observer on the left receives a longer wavelength and therefore perceives a lower frequency.

The same thing happens in Figure 14.16. A higher frequency is perceived by the observer moving toward the source, and a lower frequency is perceived by an observer moving away from the source. In general, then, relative motion of source and observer toward one another increases the perceived frequency. Relative motion apart decreases the perceived frequency. The greater the relative speed is, the greater the effect.

WATCH PHYSICS

Introduction to the Doppler Effect

This video explains the Doppler effect visually.

Click to view content (https://www.openstax.org/l/28doppler)

GRASP CHECK

If you are standing on the sidewalk facing the street and an ambulance drives by with its siren blaring, at what point will the frequency that you observe most closely match the actual frequency of the siren?

a. when it is coming toward you
b. when it is going away from you
c. when it is in front of you

For a stationary observer and a moving source of sound, the frequency (f_{obs}) of sound perceived by the observer is

$$f_{obs} = f_s \left(\frac{v_w}{v_w \pm v_s} \right), \tag{14.11}$$

where f_s is the frequency of sound from a source, v_s is the speed of the source along a line joining the source and observer, and v_w is the speed of sound. The minus sign is used for motion toward the observer and the plus sign for motion away from the observer.

TIPS FOR SUCCESS

Rather than just memorizing rules, which are easy to forget, it is better to think about the rules of an equation intuitively. Using a minus sign in $f_{obs} = f_s \left(\frac{v_w}{v_w \pm v_s} \right)$ will decrease the denominator and increase the observed frequency, which is consistent with the expected outcome of the Doppler effect when the source is moving toward the observer. Using a plus sign will increase the denominator and decrease the observed frequency, consistent with what you would expect for the source moving away from the observer. This may be more helpful to keep in mind rather than memorizing the fact that "the minus sign is used for motion toward the observer and the plus sign for motion away from the observer."

Note that the greater the speed of the source, the greater the Doppler effect. Similarly, for a stationary source and moving observer, the frequency perceived by the observer f_{obs} is given by

$$f_{obs} = f_s \left(\frac{v_w \pm v_{obs}}{v_w} \right), \tag{14.12}$$

where v_{obs} is the speed of the observer along a line joining the source and observer. Here the plus sign is for motion toward the source, and the minus sign is for motion away from the source.

Sonic Booms

What happens to the sound produced by a moving source, such as a jet airplane, that approaches or even exceeds the speed of sound? Suppose a jet airplane is coming nearly straight at you, emitting a sound of frequency f_{s}. The greater the plane's speed, v_{s}, the greater the Doppler shift and the greater the value of f_{obs}. Now, as v_{s} approaches the speed of sound, v_{w}, f_{obs} approaches infinity, because the denominator in $f_{obs} = f_s \left(\frac{v_w}{v_w - v_s} \right)$ approaches zero.

This result means that at the speed of sound, in front of the source, each wave is superimposed on the previous one because the source moves forward at the speed of sound. The observer gets them all at the same instant, and so the frequency is theoretically infinite. If the source exceeds the speed of sound, no sound is received by the observer until the source has passed, so that the sounds from the source when it was approaching are stacked up with those from it when receding, creating a **sonic boom**. A sonic boom is a constructive interference of sound created by an object moving faster than sound.

An aircraft creates two sonic booms, one from its nose and one from its tail (see Figure 14.17). During television coverage of space shuttle landings, two distinct booms could often be heard. These were separated by exactly the time it would take the shuttle to pass by a point. Observers on the ground often do not observe the aircraft creating the sonic boom, because it has passed by before the shock wave reaches them. If the aircraft flies close by at low altitude, pressures in the sonic boom can be destructive enough to break windows. Because of this, supersonic flights are banned over populated areas of the United States.

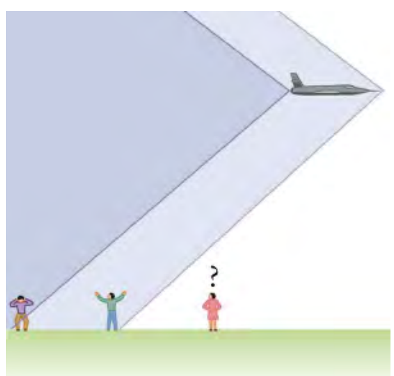


Figure 14.17 Two sonic booms, created by the nose and tail of an aircraft, are observed on the ground after the plane has passed by.

Solving Problems Using the Doppler Shift Formula

WATCH PHYSICS

Doppler Effect Formula for Observed Frequency

This video explains the Doppler effect formula for cases when the source is moving toward the observer.

Click to view content (https://www.openstax.org/l/28dopplerform)

GRASP CHECK

Let's say that you have a rare phobia where you are afraid of the Doppler effect. If you see an ambulance coming your way, what would be the best strategy to minimize the Doppler effect and soothe your Doppleraphobia?

a. Stop moving and become stationary till it passes by.
b. Run toward the ambulance.
c. Run alongside the ambulance.

WATCH PHYSICS

Doppler Effect Formula When Source is Moving Away

This video explains the Doppler effect formula for cases when the source is moving away from the observer.

[Click to view content (https://www.openstax.org/l/28doppleraway)](https://www.openstax.org/l/28doppleraway)

GRASP CHECK

Sal uses two different formulas for the Doppler effect-one for when the source is moving toward the observer and another for when the source is moving away. However, in this textbook we use only one formula. Explain.

a. The combined formula that can be used is, Use (+) when the source is moving toward the observer and (–) when the source is moving away from the observer.
b. The combined formula that can be used is, $f_{obs} = f_s \left(\frac{v_w \pm v_s}{v_w}\right)$. Use (+) when the source is moving away from the observer and (–) when the source is moving toward the observer.
c. The combined formula that can be used is, $f_{obs} = f_s \left(\frac{v_w}{v_w \pm v_s}\right)$. Use (+) when the source is moving toward the observer and (–) when the source is moving away from the observer.
d. The combined formula that can be used is, $f_{obs} = f_s \left(\frac{v_w}{v_w \pm v_s}\right)$. Use (+) when the source is moving away from the observer and (–) when the source is moving toward the observer.

WORKED EXAMPLE

Calculate Doppler Shift: A Train Horn

Suppose a train that has a 150 Hz horn is moving at 35 m/s in still air on a day when the speed of sound is 340 m/s. What frequencies are observed by a stationary person at the side of the tracks as the train approaches and after it passes?

Strategy

To find the observed frequency, $f_{obs} = f_s \left(\frac{v_w}{v_w \pm v_s}\right)$ must be used because the source is moving. The minus sign is used for the approaching train, and the plus sign for the receding train.

Solution

(1) Enter known values into $f_{obs} = f_s \left(\frac{v_w}{v_w - v_s}\right)$ to calculate the frequency observed by a stationary person as the train approaches:

$$f_{obs} = f_s \left(\frac{v_w}{v_w - v_s}\right) = (150 \text{ Hz}) \left(\frac{340 \text{ m/s}}{340 \text{ m/s} - 35 \text{ m/s}}\right) = 167 \text{ Hz} \approx 170 \text{ Hz (2 sig. figs.)}$$

(2) Use the same equation but with the plus sign to find the frequency heard by a stationary person as the train recedes.

$$f_{obs} = f_s \left(\frac{v_w}{v_w + v_s}\right) = (150 \text{ Hz}) \left(\frac{340 \text{ m/s}}{340 \text{ m/s} + 35 \text{ m/s}}\right) = 136 \text{ Hz} \approx 140 \text{ Hz (2 sig. figs.)}$$

Discussion

The numbers calculated are valid when the train is far enough away that the motion is nearly along the line joining the train and the observer. In both cases, the shift is significant and easily noticed. Note that the shift is approximately 20 Hz for motion toward and approximately 10 Hz for motion away. The shifts are not symmetric.

Practice Problems

15. What is the observed frequency when the source having frequency $3.0\ \text{kHz}$ is moving towards the observer at a speed of $1.0 \times 10^2\ \text{m/s}$ and the speed of sound is $331\ \text{m/s}$?
   a. $3.0\ \text{kHz}$
   b. $3.5\ \text{kHz}$
   c. $2.3\ \text{kHz}$
   d. $4.3\ \text{kHz}$

16. A train is moving away from you at a speed of $50.0\ \text{m/s}$. If you are standing still and hear the whistle at a frequency of $305\ \text{Hz}$, what is the actual frequency of the produced whistle? (Assume speed of sound to be $331\ \text{m/s}$.)
   a. $259\ \text{Hz}$
   b. $205\ \text{Hz}$
   c. $405\ \text{Hz}$
   d. $351\ \text{Hz}$

Check Your Understanding

17. What is the Doppler effect?
   a. The Doppler effect is a change in the observed speed of a sound due to the relative motion between the source and the observer.
   b. The Doppler effect is a change in the observed frequency of a sound due to the relative motion between the source and the observer.
   c. The Doppler effect is a change in the observed intensity of a sound due to the relative motion between the source and the observer.
   d. The Doppler effect is a change in the observed timbre of a sound, due to the relative motion between the source and the observer.

18. Give an example of the Doppler effect caused by motion of the source.
   a. The sound of a vehicle horn shifts from low-pitch to high-pitch as we move towards it.
   b. The sound of a vehicle horn shifts from low-pitch to high-pitch as we move away from it.
   c. The sound of a vehicle horn shifts from low-pitch to high-pitch as it passes by.
   d. The sound of a vehicle horn shifts from high-pitch to low-pitch as it passes by.

19. What is a sonic boom?
   a. It is a destructive interference of sound created by an object moving faster than sound.
   b. It is a constructive interference of sound created by an object moving faster than sound.
   c. It is a destructive interference of sound created by an object moving slower than sound.
   d. It is a constructive interference of sound created by an object moving slower than sound.

20. What is the relation between speed of source and value of observed frequency when the source is moving towards the observer?
   a. They are independent of each other.
   b. The greater the speed, the greater the value of observed frequency.
   c. The greater the speed, the smaller the value of observed frequency.
   d. The speed of the sound is directly proportional to the square of the frequency observed.

14.4 Sound Interference and Resonance

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe resonance and beats
- Define fundamental frequency and harmonic series
- Contrast an open-pipe and closed-pipe resonator
- Solve problems involving harmonic series and beat frequency

Section Key Terms

beat	beat frequency	damping	fundamental	harmonics
natural frequency	overtones	resonance	resonate	

Resonance and Beats

Sit in front of a piano sometime and sing a loud brief note at it while pushing down on the sustain pedal. It will sing the same note back at you—the strings that have the same frequencies as your voice, are resonating in response to the forces from the sound waves that you sent to them. This is a good example of the fact that objects—in this case, piano strings—can be forced to oscillate but oscillate best at their natural frequency.

A driving force (such as your voice in the example) puts energy into a system at a certain frequency, which is not necessarily the same as the natural frequency of the system. Over time the energy dissipates, and the amplitude gradually reduces to zero- this is called **damping**. The **natural frequency** is the frequency at which a system would oscillate if there were no driving and no damping force. The phenomenon of driving a system with a frequency equal to its natural frequency is called **resonance**, and a system being driven at its natural frequency is said to **resonate**.

Most of us have played with toys where an object bobs up and down on an elastic band, something like the paddle ball suspended from a finger in Figure 14.18. At first you hold your finger steady, and the ball bounces up and down with a small amount of damping. If you move your finger up and down slowly, the ball will follow along without bouncing much on its own. As you increase the frequency at which you move your finger up and down, the ball will respond by oscillating with increasing amplitude. When you drive the ball at its natural frequency, the ball's oscillations increase in amplitude with each oscillation for as long as you drive it. As the driving frequency gets progressively higher than the resonant or natural frequency, the amplitude of the oscillations becomes smaller, until the oscillations nearly disappear and your finger simply moves up and down with little effect on the ball.

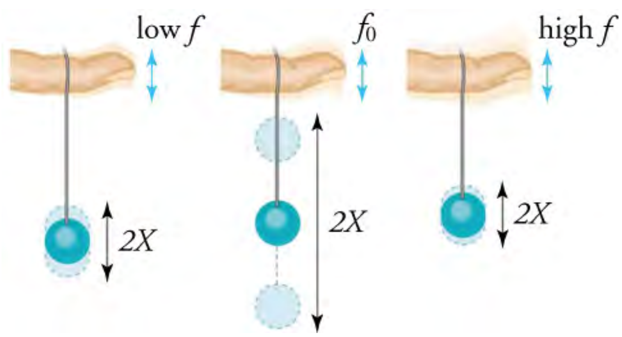


Figure 14.18 The paddle ball on its rubber band moves in response to the finger supporting it. If the finger moves with the natural frequency of the ball on the rubber band, then a resonance is achieved, and the amplitude of the ball's oscillations increases dramatically. At higher and lower driving frequencies, energy is transferred to the ball less efficiently, and it responds with lower-amplitude oscillations.

Another example is that when you tune a radio, you adjust its resonant frequency so that it oscillates only at the desired station's broadcast (driving) frequency. Also, a child on a swing is driven (pushed) by a parent at the swing's natural frequency to reach the maximum amplitude (height). In all of these cases, the efficiency of energy transfer from the driving force into the oscillator is best at resonance.

Figure 14.19 Some types of headphones use the phenomena of constructive and destructive interference to cancel out outside noises.

All sound resonances are due to constructive and destructive interference. Only the resonant frequencies interfere constructively to form standing waves, while others interfere destructively and are absent. From the toot made by blowing over a bottle to the recognizability of a great singer's voice, resonance and standing waves play a vital role in sound.

Interference happens to all types of waves, including sound waves. In fact, one way to support that something *is a wave* is to observe interference effects. Figure 14.19 shows a set of headphones that employs a clever use of sound interference to cancel noise. To get destructive interference, a fast electronic analysis is performed, and a second sound is introduced with its maxima and minima exactly reversed from the incoming noise.

In addition to resonance, superposition of waves can also create beats. **Beats** are produced by the superposition of two waves with slightly different frequencies but the same amplitude. The waves alternate in time between constructive interference and destructive interference, giving the resultant wave an amplitude that varies over time. (See the resultant wave in Figure 14.20).

This wave fluctuates in amplitude, or beats, with a frequency called the **beat frequency**. The equation for beat frequency is

$$f_B = |f_1 - f_2|, \tag{14.13}$$

where f_1 and f_2 are the frequencies of the two original waves. If the two frequencies of sound waves are similar, then what we hear is an average frequency that gets louder and softer at the beat frequency.

TIPS FOR SUCCESS

Don't confuse the beat frequency with the regular frequency of a wave resulting from superposition. While the beat frequency is given by the formula above, and describes the frequency of the beats, the actual frequency of the wave resulting from superposition is the average of the frequencies of the two original waves.

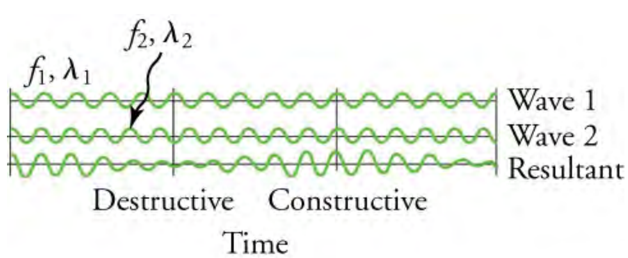


Figure 14.20 Beats are produced by the superposition of two waves of slightly different frequencies but identical amplitudes. The waves alternate in time between constructive interference and destructive interference, giving the resulting wave a time-varying amplitude.

Virtual Physics

Wave Interference

Click to view content (https://www.openstax.org/l/28interference)

For this activity, switch to the Sound tab. Turn on the Sound option, and experiment with changing the frequency and amplitude, and adding in a second speaker and a barrier.

GRASP CHECK

According to the graph, what happens to the amplitude of pressure over time. What is this phenomenon called, and what causes it ?

a. The amplitude decreases over time. This phenomenon is called damping. It is caused by the dissipation of energy.
b. The amplitude increases over time. This phenomenon is called feedback. It is caused by the gathering of energy.
c. The amplitude oscillates over time. This phenomenon is called echoing. It is caused by fluctuations in energy.

Fundamental Frequency and Harmonics

Suppose we hold a tuning fork near the end of a tube that is closed at the other end, as shown in Figure 14.21, Figure 14.22, and Figure 14.23. If the tuning fork has just the right frequency, the air column in the tube resonates loudly, but at most frequencies it vibrates very little. This means that the air column has only certain natural frequencies. The figures show how a resonance at the lowest of these natural frequencies is formed. A disturbance travels down the tube at the speed of sound and bounces off the closed end. If the tube is just the right length, the reflected sound arrives back at the tuning fork exactly half a cycle later, and it interferes constructively with the continuing sound produced by the tuning fork. The incoming and reflected sounds form a standing wave in the tube as shown.

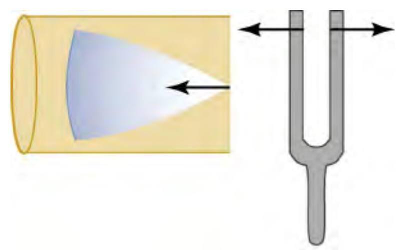


Figure 14.21 Resonance of air in a tube closed at one end, caused by a tuning fork. A disturbance moves down the tube.

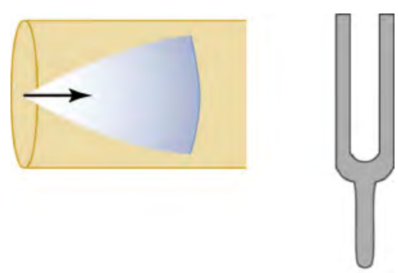


Figure 14.22 Resonance of air in a tube closed at one end, caused by a tuning fork. The disturbance reflects from the closed end of the tube.

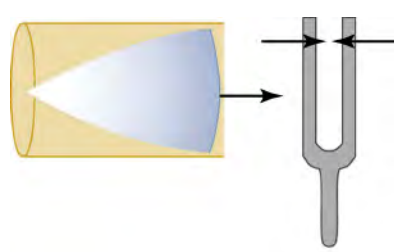


Figure 14.23 Resonance of air in a tube closed at one end, caused by a tuning fork. If the length of the tube L is just right, the disturbance gets back to the tuning fork half a cycle later and interferes constructively with the continuing sound from the tuning fork. This interference forms a standing wave, and the air column resonates.

The standing wave formed in the tube has its maximum air displacement (an antinode) at the open end, and no displacement (a node) at the closed end. Recall from the last chapter on waves that motion is unconstrained at the antinode, and halted at the node. The distance from a node to an antinode is one-fourth of a wavelength, and this equals the length of the tube; therefore, $\lambda = 4L$. This same resonance can be produced by a vibration introduced at or near the closed end of the tube, as shown in Figure 14.24.

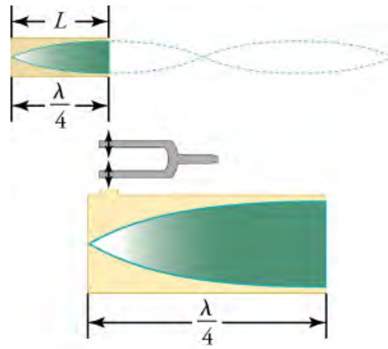


Figure 14.24 The same standing wave is created in the tube by a vibration introduced near its closed end.

Since maximum air displacements are possible at the open end and none at the closed end, there are other, shorter wavelengths that can resonate in the tube see Figure 14.25). Here the standing wave has three-fourths of its wavelength in the tube, or $L = (3/4)\lambda'$, so that $\lambda' = 4L/3$. There is a whole series of shorter-wavelength and higher-frequency sounds that resonate in the tube.

We use specific terms for the resonances in any system. The lowest resonant frequency is called the **fundamental**, while all higher resonant frequencies are called **overtones**. All resonant frequencies are multiples of the fundamental, and are called **harmonics**. The fundamental is the first harmonic, the first overtone is the second harmonic, and so on. Figure 14.26 shows the fundamental and the first three overtones (the first four harmonics) in a tube closed at one end.

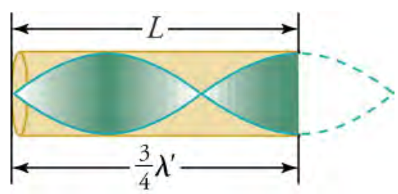


Figure 14.25 Another resonance for a tube closed at one end. This has maximum air displacements at the open end, and none at the closed end. The wavelength is shorter, with three-fourths λ' equaling the length of the tube, so that $\lambda' = 4L/3$. This higher-frequency vibration is the first overtone.

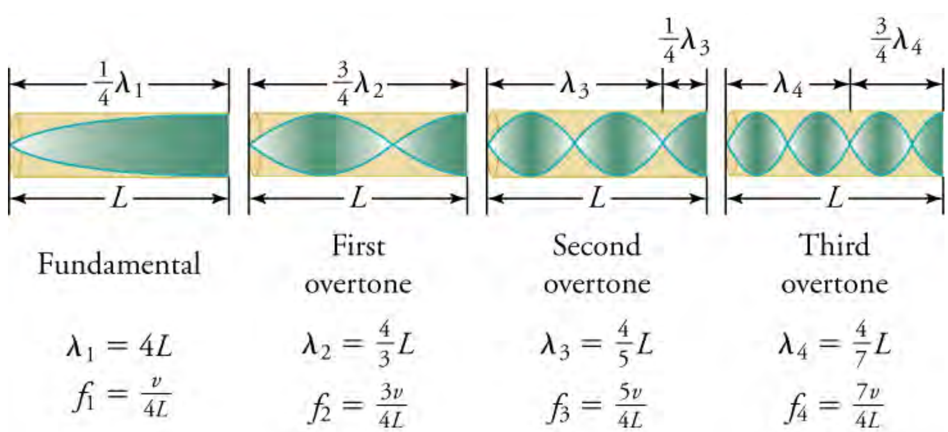


Figure 14.26 The fundamental and three lowest overtones for a tube closed at one end. All have maximum air displacements at the open end and none at the closed end.

The fundamental and overtones can be present at the same time in a variety of combinations. For example, the note middle C on a trumpet sounds very different from middle C on a clarinet, even though both instruments are basically modified versions of a tube closed at one end. The fundamental frequency is the same (and usually the most intense), but the overtones and their mix of intensities are different. This mix is what gives musical instruments (and human voices) their distinctive characteristics, whether they have air columns, strings, or drumheads. In fact, much of our speech is determined by shaping the cavity formed by the throat and mouth and positioning the tongue to adjust the fundamental and combination of overtones.

Open-Pipe and Closed-Pipe Resonators

The resonant frequencies of a tube closed at one end (known as a closed-pipe resonator) are $f_n = n\frac{v}{4L},\ n = 1, 3, 5...,$

where f_1 is the fundamental, f_3 is the first overtone, and so on. Note that the resonant frequencies depend on the speed of sound v and on the length of the tube L.

Another type of tube is one that is *open* at both ends (known as an open-pipe resonator). Examples are some organ pipes, flutes, and oboes. The air columns in tubes open at both ends have maximum air displacements at both ends. (See Figure 14.27). Standing waves form as shown.

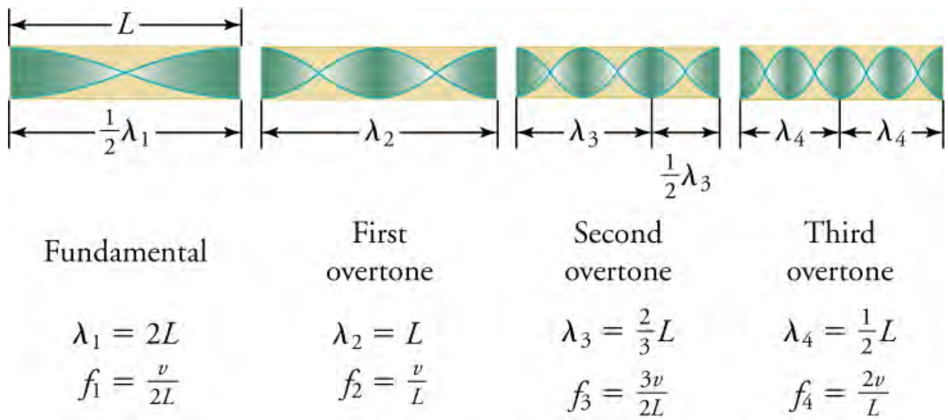


Figure 14.27 The resonant frequencies of a tube open at both ends are shown, including the fundamental and the first three overtones. In all cases the maximum air displacements occur at both ends of the tube, giving it different natural frequencies than a tube closed at one end.

The resonant frequencies of an open-pipe resonator are

$$f_n = n\frac{v}{2L},\ n = 1, 2, 3...,$$

where f_1 is the fundamental, f_2 is the first overtone, f_3 is the second overtone, and so on. Note that a tube open at both ends has a fundamental frequency twice what it would have if closed at one end. It also has a different spectrum of overtones than a tube closed at one end. So if you had two tubes with the same fundamental frequency but one was open at both ends and the other was closed at one end, they would sound different when played because they have different overtones.

Middle C, for example, would sound richer played on an open tube since it has more overtones. An open-pipe resonator has more overtones than a closed-pipe resonator because it has even multiples of the fundamental as well as odd, whereas a closed tube has only odd multiples.

In this section we have covered resonance and standing waves for wind instruments, but vibrating strings on stringed instruments also resonate and have fundamentals and overtones similar to those for wind instruments.

Solving Problems Involving Harmonic Series and Beat Frequency

WORKED EXAMPLE

Finding the Length of a Tube for a Closed-Pipe Resonator

If sound travels through the air at a speed of 344 m/s, what should be the length of a tube closed at one end to have a fundamental frequency of 128 Hz?

Strategy

The length L can be found by rearranging the equation $f_n = n\frac{v}{4L}$.

Solution

(1) Identify knowns.

- The fundamental frequency is 128 Hz.
- The speed of sound is 344 m/s.

(2) Use $f_n = n\frac{v_w}{4L}$ to find the fundamental frequency ($n = 1$).

$$f_1 = \frac{v}{4L} \tag{14.14}$$

(3) Solve this equation for length.

$$L = \frac{v}{4f_1} \tag{14.15}$$

(4) Enter the values of the speed of sound and frequency into the expression for L.

$$L = \frac{v}{4f_1} = \frac{344\ \text{ m/s}}{4\,(128\ \text{ Hz})} = 0.672\ \text{ m} \tag{14.16}$$

Discussion

Many wind instruments are modified tubes that have finger holes, valves, and other devices for changing the length of the resonating air column and therefore, the frequency of the note played. Horns producing very low frequencies, such as tubas, require tubes so long that they are coiled into loops.

WORKED EXAMPLE

Finding the Third Overtone in an Open-Pipe Resonator

If a tube that's open at both ends has a fundamental frequency of 120 Hz, what is the frequency of its third overtone?

Strategy

Since we already know the value of the fundamental frequency (n = 1), we can solve for the third overtone (n = 4) using the equation $f_n = n\frac{v}{2L}$.

Solution

Since fundamental frequency (n = 1) is

$$f_1 = \frac{v}{2L}, \tag{14.17}$$

and

$$f_4 = 4\frac{v}{2L}, f_4 = 4f_1 = 4(120 \text{ Hz}) = 480 \text{ Hz}. \tag{14.18}$$

Discussion

To solve this problem, it wasn't necessary to know the length of the tube or the speed of the air because of the relationship between the fundamental and the third overtone. This example was of an open-pipe resonator; note that for a closed-pipe resonator, the third overtone has a value of n = 7 (not n = 4).

WORKED EXAMPLE

Using Beat Frequency to Tune a Piano

Piano tuners use beats routinely in their work. When comparing a note with a tuning fork, they listen for beats and adjust the string until the beats go away (to zero frequency). If a piano tuner hears two beats per second, and the tuning fork has a frequency of 256 Hz, what are the possible frequencies of the piano?

Strategy

Since we already know that the beat frequency f_B is 2, and one of the frequencies (let's say f_2) is 256 Hz, we can use the equation $f_B = |f_1 - f_2|$ to solve for the frequency of the piano f_1.

Solution

Since $f_B = |f_1 - f_2|$,

we know that either $f_B = f_1 - f_2$ or $-f_B = f_1 - f_2$.

Solving for f_1,

$$f_1 = f_B + f_2 \text{ or } f_1 = -f_B + f_2 . \tag{14.19}$$

Substituting in values,

$$f_1 = 2 + 256\ \text{ Hz or } f_1 = -2 + 256\ \text{ Hz} \tag{14.20}$$

So,

$$f_1 = 258\ \text{ Hz or } 254 \text{ Hz.} \tag{14.21}$$

Discussion

The piano tuner might not initially be able to tell simply by listening whether the frequency of the piano is too high or too low and must tune it by trial and error, making an adjustment and then testing it again. If there are even more beats after the adjustment, then the tuner knows that he went in the wrong direction.

Practice Problems

21. Two sound waves have frequencies 250 Hz and 280 Hz. What is the beat frequency produced by their superposition?

a. 290 Hz
b. 265 Hz
c. 60 Hz
d. 30 Hz

22. What is the length of a pipe closed at one end with fundamental frequency 350 Hz? (Assume the speed of sound in air is 331 m/s.)

a. 26 cm
b. 26 m
c. 24 m
d. 24 cm

Check Your Understanding

23. What is damping?

a. Over time the energy increases and the amplitude gradually reduces to zero. This is called damping.
b. Over time the energy dissipates and the amplitude gradually increases. This is called damping.
c. Over time the energy increases and the amplitude gradually increases. This is called damping.
d. Over time the energy dissipates and the amplitude gradually reduces to zero. This is called damping.

24. What is resonance? When can you say that the system is resonating?

a. The phenomenon of driving a system with a frequency equal to its natural frequency is called resonance, and a system being driven at its natural frequency is said to resonate.
b. The phenomenon of driving a system with a frequency higher than its natural frequency is called resonance, and a system being driven at its natural frequency does not resonate.
c. The phenomenon of driving a system with a frequency equal to its natural frequency is called resonance, and a system being driven at its natural frequency does not resonate.
d. The phenomenon of driving a system with a frequency higher than its natural frequency is called resonance, and a system being driven at its natural frequency is said to resonate.

25. In the tuning fork and tube experiment, in case a standing wave is formed, at what point on the tube is the maximum disturbance from the tuning fork observed? Recall that the tube has one open end and one closed end.
   a. At the midpoint of the tube
   b. Both ends of the tube
   c. At the closed end of the tube
   d. At the open end of the tube

26. In the tuning fork and tube experiment, when will the air column produce the loudest sound?
   a. If the tuning fork vibrates at a frequency twice that of the natural frequency of the air column.
   b. If the tuning fork vibrates at a frequency lower than the natural frequency of the air column.
   c. If the tuning fork vibrates at a frequency higher than the natural frequency of the air column.
   d. If the tuning fork vibrates at a frequency equal to the natural frequency of the air column.

27. What is a closed-pipe resonator?
   a. A pipe or cylindrical air column closed at both ends
   b. A pipe with an antinode at the closed end
   c. A pipe with a node at the open end
   d. A pipe or cylindrical air column closed at one end

28. Give two examples of open-pipe resonators.
   a. piano, violin
   b. drum, tabla
   c. rlectric guitar, acoustic guitar
   d. flute, oboe

KEY TERMS

amplitude the amount that matter is disrupted during a sound wave, as measured by the difference in height between the crests and troughs of the sound wave.

beat a phenomenon produced by the superposition of two waves with slightly different frequencies but the same amplitude

beat frequency the frequency of the amplitude fluctuations of a wave

damping the reduction in amplitude over time as the energy of an oscillation dissipates

decibel a unit used to describe sound intensity levels

Doppler effect an alteration in the observed frequency of a sound due to relative motion between the source and the observer

fundamental the lowest-frequency resonance

harmonics the term used to refer to the fundamental and its overtones

hearing the perception of sound

loudness the perception of sound intensity

natural frequency the frequency at which a system would oscillate if there were no driving and no damping forces

overtones all resonant frequencies higher than the fundamental

pitch the perception of the frequency of a sound

rarefaction a low-pressure region in a sound wave

resonance the phenomenon of driving a system with a frequency equal to the system's natural frequency

resonate to drive a system at its natural frequency

sonic boom a constructive interference of sound created by an object moving faster than sound

sound a disturbance of matter that is transmitted from its source outward by longitudinal waves

sound intensity the power per unit area carried by a sound wave

sound intensity level the level of sound relative to a fixed standard related to human hearing

SECTION SUMMARY

14.1 Speed of Sound, Frequency, and Wavelength

- Sound is one type of wave.
- Sound is a disturbance of matter that is transmitted from its source outward in the form of longitudinal waves.
- The relationship of the speed of sound v, its frequency f, and its wavelength λ is given by $v = f\lambda$, which is the same relationship given for all waves.
- The speed of sound depends upon the medium through which the sound wave is travelling.
- In a given medium at a specific temperature (or density), the speed of sound v is the same for all frequencies and wavelengths.

14.2 Sound Intensity and Sound Level

- The intensity of a sound is proportional to its amplitude squared.
- The energy of a sound wave is also proportional to its amplitude squared.
- Sound intensity level in decibels (dB) is more relevant for how humans perceive sounds than sound intensity (in W/m^2), even though sound intensity is the SI unit.
- Sound intensity level is not the same as sound intensity—it tells you the *level* of the sound relative to a reference intensity rather than the actual intensity.
- Hearing is the perception of sound and involves that transformation of sound waves into vibrations of parts within the ear. These vibrations are then transformed into neural signals that are interpreted by the brain.
- People create sounds by pushing air up through their lungs and through elastic folds in the throat called vocal cords.

14.3 Doppler Effect and Sonic Booms

- The Doppler effect is a shift in the observed frequency of a sound due to motion of either the source or the observer.
- The observed frequency is greater than the actual source's frequency when the source and the observer are moving closer together, either by the source moving toward the observer or the observer moving toward the source.
- A sonic boom is constructive interference of sound created by an object moving faster than sound.

14.4 Sound Interference and Resonance

- A system's natural frequency is the frequency at which the system will oscillate if not affected by driving or damping forces.
- A periodic force driving a harmonic oscillator at its natural frequency produces resonance. The system is said to resonate.
- Beats occur when waves of slightly different frequencies are superimposed.
- In air columns, the lowest-frequency resonance is called the fundamental, whereas all higher resonant frequencies are called overtones. Collectively, they are

called harmonics.

- The resonant frequencies of a tube closed at one end are $f_n = n\frac{v}{4L},\ n = 1, 3, 5...$, where f_1 is the fundamental and L is the length of the tube.
- The resonant frequencies of a tube open at both ends are $f_n = n\frac{v}{2L},\ n = 1, 2, 3...$

KEY EQUATIONS

14.1 Speed of Sound, Frequency, and Wavelength

speed of sound	$v = f\lambda$

14.2 Sound Intensity and Sound Level

intensity	$I = \frac{P}{A}$
sound intensity	$I = \frac{(\Delta p)^2}{2\rho v_w}$
sound intensity level	$\beta\ (\text{dB}) = 10\ \log_{10}\left(\frac{I}{I_0}\right)$

14.3 Doppler Effect and Sonic Booms

Doppler effect observed frequency (moving source)	$f_{obs} = f_s\left(\frac{v_w}{v_w \pm v_s}\right)$
Doppler effect observed frequency (moving observer)	$f_{obs} = f_s\left(\frac{v_w \pm v_{obs}}{v_w}\right)$

14.4 Sound Interference and Resonance

beat frequency	$f_B = \lvert f_1 - f_2 \rvert$
resonant frequencies of a closed-pipe resonator	$f_n = n\frac{v}{4L},\ n = 1, 3, 5...$
resonant frequencies of an open-pipe resonator	$f_n = n\frac{v}{2L},\ n = 1, 2, 3...$

CHAPTER REVIEW

Concept Items

14.1 Speed of Sound, Frequency, and Wavelength

1. What is the amplitude of a sound wave perceived by the human ear?
   a. loudness
   b. pitch
   c. intensity
   d. timbre

2. The compressibility of air and hydrogen is almost the same. Which factor is the reason that sound travels faster in hydrogen than in air?
   a. Hydrogen is more dense than air.
   b. Hydrogen is less dense than air.
   c. Hydrogen atoms are heavier than air molecules.
   d. Hydrogen atoms are lighter than air molecules.

14.2 Sound Intensity and Sound Level

3. What is the mathematical relationship between intensity, power, and area?
   a. $I = \frac{P}{A^2}$
   b. $I = PA$
   c. $I = \frac{A}{P}$
   d. $I = \frac{P}{A}$

4. How does the "decibel" get its name?
   a. The meaning of deci is “hundred” and the number of decibels is one-hundredth of the logarithm to base 10 of the ratio of two sound intensities.
   b. The meaning of deci is "ten" and the number of decibels is one-tenth of the logarithm to base 10 of the ratio of two sound intensities.
   c. The meaning of deci is “one-hundredth” and the number of decibels is hundred times the logarithm to base 10 of the ratio of two sound intensities.
   d. The meaning of deci is “one-tenth” and the number of decibels is ten times the logarithm to base 10 of the ratio of two sound intensities.

5. What is “timbre” of sound?
   a. Timbre is the quality of the sound that distinguishes it from other sound

b. Timbre is the loudness of the sound that distinguishes it from other sound.
c. Timbre is the pitch of the sound that distinguishes it from other sound.
d. Timbre is the wavelength of the sound that distinguishes it from other sound.

14.3 Doppler Effect and Sonic Booms

6. Two sources of sound producing the same frequency are moving towards you at different speeds. Which one would sound more high-pitched?
   a. the one moving slower
   b. the one moving faster

7. When the speed of the source matches the speed of sound, what happens to the amplitude of the sound wave? Why?
   a. It approaches zero. This is because all wave crests are superimposed on one another through constructive interference.
   b. It approaches infinity. This is because all wave crests are superimposed on one another through constructive interference.
   c. It approaches zero, because all wave crests are superimposed on one another through destructive interference.
   d. It approaches infinity, because all wave crests are superimposed on one another through destructive interference.

8. What is the mathematical expression for the frequency perceived by the observer in the case of a stationary observer and a moving source?
   a. $f_{obs} = f_s \left(\frac{v_w}{v_s \pm v_w} \right)$
   b. $f_{obs} = f_s \left(\frac{v_w \pm v_s}{v_w} \right)$
   c. $f_{obs} = f_s \left(\frac{v_s \pm v_w}{v_w} \right)$
   d. $f_{obs} = f_s \left(\frac{v_w}{v_w \pm v_s} \right)$

14.4 Sound Interference and Resonance

9. When does a yo-yo travel the farthest from the finger?
   a. when the amplitude of the finger moving up and down is greater than the amplitude of the yo-yo
   b. when the amplitude of the finger moving up and down is less than the amplitude of the yo-yo
   c. when the frequency of the finger moving up and down is equal to the resonant frequency of the yo-yo
   d. when the frequency of the finger moving up and down is different from the resonant frequency of the yo-yo

10. What is the difference between harmonics and overtones?
    a. Harmonics are all multiples of the fundamental frequency. The first overtone is actually the first harmonic.
    b. Harmonics are all multiples of the fundamental frequency. The first overtone is actually the second harmonic.
    c. Harmonics are all multiples of the fundamental frequency. The second overtone is actually the first harmonic.
    d. Harmonics are all multiples of the fundamental frequency. The third overtone is actually the second harmonic.

11. What kind of waves form in pipe resonators?
    a. damped waves
    b. propagating waves
    c. high-frequency waves
    d. standing waves

12. What is the natural frequency of a system?
    a. The natural frequency is the frequency at which a system oscillates when it undergoes forced vibration.
    b. The natural frequency is the frequency at which a system oscillates when it undergoes damped oscillation.
    c. The natural frequency is the frequency at which a system oscillates when it undergoes free vibration without a driving force or damping.
    d. The natural frequency is the frequency at which a system oscillates when it undergoes forced vibration with damping.

Critical Thinking Items

14.1 Speed of Sound, Frequency, and Wavelength

13. What can be said about the frequency of a monotonous sound?
    a. It decreases with time.
    b. It decreases with distance.
    c. It increases with distance.
    d. It remains constant.

14. A scientist notices that a sound travels faster through a solid material than through the air. Which of the following can explain this?
    a. Solid materials are denser than air.
    b. Solid materials are less dense than air.
    c. A solid is more rigid than air.
    d. A solid is easier to compress than air.

14.2 Sound Intensity and Sound Level

15. Which property of the wave is related to its intensity? How?
    a. The frequency of the wave is related to the intensity of the sound. The larger-frequency oscillations indicate greater pressure maxima and minima, and the pressure is higher in greater-intensity sound.
    b. The wavelength of the wave is related to the intensity of the sound. The longer-wavelength oscillations indicate greater pressure maxima and minima, and the pressure is higher in greater-intensity sound.
    c. The amplitude of the wave is related to the intensity of the sound. The larger-amplitude oscillations indicate greater pressure maxima and minima, and the pressure is higher in greater-intensity sound.
    d. The speed of the wave is related to the intensity of the sound. The higher-speed oscillations indicate greater pressure maxima and minima, and the pressure is higher in greater-intensity sound.

16. Why is decibel (dB) used to describe loudness of sound?
    a. Because, human ears have an inverse response to the amplitude of sound.
    b. Because, human ears have an inverse response to the intensity of sound.
    c. Because, the way our ears perceive sound can be more accurately described by the amplitude of a sound rather than the intensity of a sound directly.
    d. Because, the way our ears perceive sound can be more accurately described by the logarithm of the intensity of a sound rather than the intensity of a sound directly.

17. How can humming while shooting a gun reduce ear damage?
    a. Humming can trigger those two muscles in the outer ear that react to intense sound produced while shooting and reduce the force transmitted to the cochlea.
    b. Humming can trigger those three muscles in the outer ear that react to intense sound produced while shooting and reduce the force transmitted to the cochlea.
    c. Humming can trigger those two muscles in the middle ear that react to intense sound produced while shooting and reduce the force transmitted to the cochlea.
    d. Humming can trigger those three muscles in the middle ear that react to intense sound produced while shooting and reduce the force transmitted to the cochlea.

18. A particular sound, S1, has an intensity 3 times that of another sound, S2. What is the difference in sound intensity levels measured in decibels?
    a. 9.54 dB
    b. 6.02 dB
    c. 3.01 dB
    d. 4.77 dB

14.3 Doppler Effect and Sonic Booms

19. When the source of sound is moving through the air, does the speed of sound change with respect to a stationary person standing nearby?
    a. Yes
    b. No

20. Why is no sound heard by the observer when an object approaches him at a speed faster than that of sound?
    a. If the source exceeds the speed of sound, then destructive interference occurs and no sound is heard by the observer when an object approaches him.
    b. If the source exceeds the speed of sound, the frequency of sound produced is beyond the audible range of sound.
    c. If the source exceeds the speed of sound, all the sound waves produced approach minimum intensity and no sound is heard by the observer when an object approaches him.
    d. If the source exceeds the speed of sound, all the sound waves produced are behind the source. Hence, the observer hears the sound only after the source has passed.

21. Does the Doppler effect occur when the source and observer are both moving towards each other? If so, how would this affect the perceived frequency?
    a. Yes, the perceived frequency will be even lower in this case than if only one of the two were moving.
    b. No, the Doppler effect occurs only when an observer is moving towards a source.
    c. No, the Doppler effect occurs only when a source is moving towards an observer.
    d. Yes, the perceived frequency will be even higher in this case than if only one of the two were moving.

14.4 Sound Interference and Resonance

22. When does the amplitude of an oscillating system become maximum?
    a. When two sound waves interfere destructively.
    b. When the driving force produces a transverse wave in the system.
    c. When the driving force of the oscillator to the oscillating system is at a maximum amplitude.
    d. When the frequency of the oscillator equals the

natural frequency of the oscillating system.

23. How can a standing wave be formed with the help of a tuning fork and a closed-end tube of appropriate length?
  a. If the tube is just the right length, the reflected sound arrives back at the tuning fork exactly half a cycle later, and it interferes constructively with the continuing sound produced by the tuning fork.
  b. If the tube is just the right length, the reflected sound arrives back at the tuning fork exactly half a cycle later, and it interferes destructively with the continuing sound produced by the tuning fork.
  c. If the tube is just the right length, the reflected sound arrives back at the tuning fork exactly one full cycle later, and it interferes constructively with the continuing sound produced by the tuning fork.
  d. If the tube is just the right length, the reflected sound arrives back at the tuning fork exactly one full cycle later, and it interferes destructively with the continuing sound produced by the tuning fork.

24. A tube open at both ends has a fundamental frequency of 500 Hz. What will the frequency be if one end is closed?
  a. 1000 Hz
  b. 500 Hz
  c. 125 Hz
  d. 250 Hz

Problems

14.1 Speed of Sound, Frequency, and Wavelength

25. A bat produces a sound at $17,250 \text{ Hz}$ and wavelength 0.019 m. What is the speed of the sound?
  a. $1.7 \times 10^6 \text{ m/s}$
  b. $8.6 \times 10^5 \text{ m/s}$
  c. $1.15 \times 10^{-6} \text{ m/s}$
  d. $3.28 \times 10^2 \text{ m/s}$

26. A sound wave with frequency of 80 Hz is traveling through air at 0°C. By how much will its wavelength change when it enters aluminum?
  a. 68 m
  b. 64 m
  c. 4 m
  d. 60 m

14.2 Sound Intensity and Sound Level

27. Calculate the sound intensity for a sound wave traveling through air at 15° C and having a pressure amplitude of 0.80 Pa. (Hint—Speed of sound in air at 15° C is 340 m/s .)
  a. 9.6×10^{-3} W / m²
  b. 7.7×10^{-3} W / m²
  c. 9.6×10^{-4} W / m²
  d. 7.7×10^{-4} W / m²

28. The sound level in dB of a sound traveling through air at 0°C is 97 dB. Calculate its pressure amplitude.
  a. 4.3 Pa
  b. 0.20 Pa
  c. 0.04 Pa
  d. 2.1 Pa

14.3 Doppler Effect and Sonic Booms

29. An ambulance is moving away from you. You are standing still and you hear its siren at a frequency of 101 Hz. You know that the actual frequency of the siren is 105 Hz. What is the speed of the ambulance? (Assume the speed of sound to be 331 m/s.)
  a. 17.07 m/s
  b. 16.55 m/s
  c. 14.59 m/s
  d. 13.1 m/s

30. An ambulance passes you at a speed of 15.0 m/s. If its siren has a frequency of 995 Hz, what is difference in the frequencies you perceive before and after it passes you? (Assume the speed of sound in air is 331 m/s.)
  a. 47.0 Hz
  b. 43.0 Hz
  c. 94.9 Hz
  d. 90.0 Hz

14.4 Sound Interference and Resonance

31. What is the length of an open-pipe resonator with a fundamental frequency of 400.0 Hz? (Assume the speed of sound is 331 m/s.)
  a. 165.1 cm
  b. 82.22 cm
  c. 20.25 cm
  d. 41.38 cm

32. An open-pipe resonator has a fundamental frequency of 250 Hz. By how much would its length have to be changed to get a fundamental frequency of 300.0 Hz? (Assume the speed of sound is 331 m/s.)
  a. 77.32 cm
  b. 44.09 cm
  c. 32.16 cm
  d. 11.03 cm

Performance Task

14.4 Sound Interference and Resonance

33. Design and make an open air resonator capable of playing at least three different pitches (frequencies) of sound using a selection of bamboo of varying widths and lengths, which can be obtained at a local hardware store. Choose a piece of bamboo for creating a musical pipe. Calculate the length required for a certain frequency to resonate and then mark the locations where holes should be placed in the pipe to achieve their desired pitches. Use a simple hand drill or ask your wood shop department for help drilling holes. Use tuning forks to test and calibrate your instrument. Demonstrate your pipe for the class.

TEST PREP

Multiple Choice

14.1 Speed of Sound, Frequency, and Wavelength

34. What properties does a loud, shrill whistle have?
- a. high amplitude, high frequency
- b. high amplitude, low frequency
- c. low amplitude, high frequency
- d. low amplitude, low frequency

35. What is the speed of sound in fresh water at 20 degrees Celsius?
- a. 5960 m/s
- b. 1540 m/s
- c. 331 m/s
- d. 1480 m/s

36. A tuning fork oscillates at a frequency of 512 Hz, creating sound waves. How many waves will reach the eardrum of a person near that fork in 2 seconds?
- a. 512
- b. 128
- c. 256
- d. 1024

37. Why does the amplitude of a sound wave decrease with distance from its source?
- a. The amplitude of a sound wave decreases with distance from its source, because the frequency of the sound wave decreases.
- b. The amplitude of a sound wave decreases with distance from its source, because the speed of the sound wave decreases.
- c. The amplitude of a sound wave decreases with distance from its source, because the wavelength of the sound wave increases.
- d. The amplitude of a sound wave decreases with distance from its source, because the energy of the wave is spread over a larger and larger area.

38. Does the elasticity of the medium affect the speed of sound? How?
- a. No, there is no relationship that exists between the speed of sound and elasticity of the medium.
- b. Yes. When particles are more easily compressed in a medium, sound does not travel as quickly through the medium.
- c. Yes. When the particles in a medium do not compress much, sound does not travel as quickly through the medium.
- d. No, the elasticity of a medium affects frequency and wavelength, not wave speed.

14.2 Sound Intensity and Sound Level

39. Which of the following terms is a useful quantity to describe the loudness of a sound?
- a. intensity
- b. frequency
- c. pitch
- d. wavelength

40. What is the unit of sound intensity level?
- a. decibels
- b. hertz
- c. watts

41. If a particular sound S1 is 5 times more intense than another sound S2, then what is the difference in sound intensity levels in dB for these two sounds?
- a. 5 dB
- b. 6 dB
- c. 7 dB

42. By what minimum amount should frequencies vary for humans to be able to distinguish two separate sounds?
- a. 100 Hz
- b. 10 Hz
- c. 5 Hz
- d. 1 Hz

43. Why is I^{o} chosen as the reference for sound intensity?
- a. Because, it is the highest intensity of sound a person with normal hearing can perceive at a frequency of 100 Hz.
- b. Because, it is the lowest intensity of sound a person with normal hearing can perceive at a frequency of 100 Hz.

c. Because, it is the highest intensity of sound a person with normal hearing can perceive at a frequency of 1000 Hz.
d. Because, it is the lowest intensity of sound a person with normal hearing can perceive at a frequency of 1000 Hz.

14.3 Doppler Effect and Sonic Booms

44. In which of the following situations is the Doppler effect absent?
a. The source and the observer are moving towards each other.
b. The observer is moving toward the source.
c. The source is moving away from the observer.
d. Neither the source nor the observer is moving relative to one another.

45. What does the occurrence of the sonic boom depend on?
a. speed of the source
b. frequency of source
c. amplitude of source
d. distance of observer from the source

46. What is the observed frequency when the observer is moving away from the source at 125 m/s? The source frequency is 237 Hz and the speed of sound is 325 m/s .
a. 303 Hz
b. 259 Hz
c. 201 Hz
d. 146 Hz

47. How will your perceived frequency change if the source is moving towards you?
a. The frequency will become lower.
b. The frequency will become higher.

14.4 Sound Interference and Resonance

48. Observation of which phenomenon can be considered proof that something is a wave?
a. interference
b. noise
c. reflection
d. conduction

49. Which of the resonant frequencies has the greatest amplitude?
a. The first harmonic
b. The second harmonic
c. The first overtone
d. The second overtone

50. What is the fundamental frequency of an open-pipe resonator?
a. 3 v/2 L
b. 2 v/L
c. v/L
d. v/2 L

51. What is the beat frequency produced by the superposition of two waves with frequencies 300 Hz and 340 Hz?
a. 640 Hz
b. 320 Hz
c. 20 Hz
d. 40 Hz

Short Answer

14.1 Speed of Sound, Frequency, and Wavelength

52. What component of a longitudinal sound wave is analogous to a trough of a transverse wave?
a. compression
b. rarefaction
c. node
d. antinode

53. What is the frequency of a sound wave as perceived by the human ear?
a. timbre
b. loudness
c. intensity
d. pitch

54. What properties of a solid determine the speed of sound traveling through it?
a. mass and density
b. rigidity and density
c. volume and density
d. shape and rigidity

55. Does the density of a medium affect the speed of sound?
a. No
b. Yes

56. Does a bat make use of the properties of sound waves to locate its prey?
a. No
b. Yes

57. Do the properties of a sound wave change when it travels from one medium to another?
a. No
b. Yes

14.2 Sound Intensity and Sound Level

58. When a passing driver has his stereo turned up, you cannot even hear what the person next to you is saying. Why is this so?

a. The sound from the passing car's stereo has a higher amplitude and hence higher intensity compared to the intensity of the sound coming from the person next to you. The higher intensity corresponds to greater loudness, so the first sound dominates the second.

b. The sound from the passing car's stereo has a higher amplitude and hence lower intensity compared to the intensity of the sound coming from the person next to you. The lower intensity corresponds to greater loudness, so the first sound dominates the second.

c. The sound from the passing car's stereo has a higher frequency and hence higher intensity compared to the intensity of the sound coming from the person next to you. The higher frequency corresponds to greater loudness so the first sound dominates the second.

d. The sound from the passing car's stereo has a lower frequency and hence higher intensity compared to the intensity of the sound coming from the person next to you. The lower frequency corresponds to greater loudness, so the first sound dominates the second.

59. For a constant area, what is the relationship between intensity of a sound wave and power?

a. The intensity is inversely proportional to the power transmitted by the wave, for a constant area.

b. The intensity is inversely proportional to the square of the power transmitted by the wave, for a constant area. $I = \frac{1}{P^2}$

c. The intensity is directly proportional to the square of the power transmitted by the wave, for a constant area. $I = \mathrm{P}^2$

d. The intensity is directly proportional to the power transmitted by the wave, for a constant area. $I = \frac{P}{A}$

60. What does I stand for in the equation $\beta(\mathrm{dB}) = 10\ \log_{10}\left(\frac{I}{I_0}\right)$? What is its unit?

a. Yes, I is the sound intensity in watts per meter squared in the equation, $\beta(\mathrm{dB}) = 10 \log_{10}\left(\frac{I}{I_0}\right)$.

b. I is the sound illuminance and its SI unit is lumen per meter squared.

c. I is the sound intensity and its SI unit is watts per meter cubed.

d. I is the sound intensity and its SI unit is watts per meter squared.

61. Why is the reference intensity $I_0 = 10^{-12}\ \mathrm{W/m^2}$?

a. The upper limit of human hearing is 100 decibels, i.e. $\beta = 100\,\mathrm{dB}$. For $\beta = 100\,\mathrm{dB}$, $I_0 = 10^{-12}\ \mathrm{W/m^2}$.

b. The lower threshold of human hearing is 10 decibels, i.e. $\beta = 10\,\mathrm{dB}$. For $\beta = 10\,\mathrm{dB}$, $I_0 = 10^{-12}\ \mathrm{W/m^2}$

c. The upper limit of human hearing is 10 decibels, i.e. $\beta = 10\,\mathrm{dB}$. For $\beta = 10\,\mathrm{dB}$, $I_0 = 10^{-12}\ \mathrm{W/m^2}$

d. The lower threshold of human hearing is 0 decibels, i.e., $\beta = 0\,\mathrm{dB}$. For $\beta = 0\,\mathrm{dB}$, $I_0 = 10^{-12}\ \mathrm{W/m^2}$

62. Given that the sound intensity level of a particular wave is 82 dB, what will be the sound intensity for that wave?

a. $I = 1.6 \times 10^{-6}\ \mathrm{W/m^2}$

b. $I = 82 \times 10^{-12}\ \mathrm{W/m^2}$

c. $I = 8.2 \times 10^{-12}\ \mathrm{W/m^2}$

d. $I = 1.6 \times 10^{-4}\ \mathrm{W/m^2}$

63. For a sound wave with intensity $1.58 \times 10^{-4}\ \mathrm{W/m^2}$, calculate the pressure amplitude given that the sound travels through air at $0^\circ\mathrm{C}$.

a. 0.734 Pa

b. 3.67 Pa

c. 0.135 Pa

d. 0.367 Pa

64. Which nerve carries auditory information to the brain?

a. buccal nerve

b. peroneal nerve

c. cochlear nerve

d. mandibular nerve

65. Why do some smaller instruments, such as piccolos, produce higher-pitched sounds than larger instruments, such as tubas?

a. Smaller instruments produce sounds with shorter wavelengths, and thus higher frequencies.

b. Smaller instruments produce longer wavelength, and thus higher amplitude, sounds.

c. Smaller instruments produce lower amplitude, and thus longer wavelength sounds.

d. Smaller instruments produce higher amplitude, and thus lower frequency, sounds.

14.3 Doppler Effect and Sonic Booms

66. How will your perceived frequency change if you move away from a stationary source of sound?

a. The frequency will become lower.

b. The frequency will be doubled.

c. The frequency will be tripled.

d. The frequency will become higher.

67. True or false—The Doppler effect also occurs with waves other than sound waves.
a. False
b. True

68. A source of sound is moving towards you. How will what you hear change if the speed of the source increases?
a. The sound will become more high-pitched.
b. The sound will become more low-pitched.
c. The pitch of the sound will not change.

69. Do sonic booms continue to be created when an object is traveling at supersonic speeds?
a. No, a sonic boom is created only when the source exceeds the speed of sound.
b. Yes, sonic booms continue to be created when an object is traveling at supersonic speeds.

70. Suppose you are driving at a speed of $20.0\ \text{m/s}$ and you hear the sound of a bell at a frequency of $400.0\ \text{Hz}$. What is the actual frequency of the bell if the speed of sound is $335\ \text{m/s}$?
a. $f_s = 401\ \text{Hz}$ or $f_s = 315\ \text{Hz}$
b. $f_s = 385\ \text{Hz}$ or $f_s = 419\ \text{Hz}$
c. $f_s = 415\ \text{Hz}$ or $f_s = 366\ \text{Hz}$
d. $f_s = 425\ \text{Hz}$ or $f_s = 377\ \text{Hz}$

71. What is the frequency of a stationary sound source if you hear it at 1200.0 Hz while moving towards it at a speed of 50.0 m/s? (Assume speed of sound to be 331 m/s.)
a. 1410 Hz
b. 1380 Hz
c. 1020 Hz
d. 1042 Hz

14.4 Sound Interference and Resonance

72. What is the actual frequency of the wave produced as a result of superposition of two waves?
a. It is the average of the frequencies of the two original waves that were superimposed.
b. It is the difference between the frequencies of the two original waves that were superimposed.
c. It is the product of the frequencies of the two original waves that were superimposed.
d. It is the sum of the frequencies of the two original waves that were superimposed.

73. Can beats be produced through a phenomenon different from resonance? How?
a. No, beats can be produced only by resonance.
b. Yes, beats can be produced by superimposition of any two waves having slightly different frequencies.

74. How is human speech produced?
a. Human speech is produced by shaping the cavity formed by the throat and mouth, the vibration of vocal cords, and using the tongue to adjust the fundamental frequency and combination of overtones.
b. Human speech is produced by shaping the cavity formed by the throat and mouth into a closed pipe and using tongue to adjust the fundamental frequency and combination of overtones.
c. Human speech is produced only by the vibrations of the tongue.
d. Human speech is produced by elongating the vocal cords.

75. What is the possible number of nodes and antinodes along one full wavelength of a standing wave?
a. 2 nodes and 3 antinodes or 2 antinodes and 3 nodes.
b. 2 nodes and 2 antinodes or 3 antinodes and 3 nodes.
c. 3 nodes and 3 antinodes or 2 antinodes and 2 nodes.
d. 6 nodes and 4 antinodes or 6 antinodes and 4 nodes.

76. In a pipe resonator, which frequency will be the least intense of those given below?
a. second overtone frequency
b. first overtone frequency
c. fundamental frequency
d. third overtone frequency

77. A flute is an open-pipe resonator. If a flute is $60\ \text{cm}$ long, what is the longest wavelength it can produce?
a. $240\ \text{cm}$
b. $180\ \text{cm}$
c. $60\ \text{cm}$
d. $120\ \text{cm}$

78. What is the frequency of the second overtone of a closed-pipe resonator with a length of $22.0\ \text{cm}$? (Assume the speed of sound is $331\ \text{m/s}$.)
a. $7520\ \text{Hz}$
b. $1510\ \text{Hz}$
c. $376\ \text{Hz}$
d. $1880\ \text{Hz}$

79. An open-pipe resonator has a fundamental frequency of $220\ \text{Hz}$ when the speed of sound is $331\ \text{m/s}$. What will its fundamental frequency be when the speed of sound is $350\ \text{m/s}$?
a. $690\ \text{Hz}$
b. $470\ \text{Hz}$
c. $110\ \text{Hz}$
d. $230\ \text{Hz}$

Extended Response

14.1 Speed of Sound, Frequency, and Wavelength

80. How is a human able to hear sounds?

a. Sound waves cause the eardrum to vibrate. A complicated mechanism converts the vibrations to nerve impulses, which are perceived by the person as sound.
b. Sound waves cause the ear canal to vibrate. A complicated mechanism converts the vibrations to nerve impulses, which are perceived by the person as sound.
c. Sound waves transfer electrical impulses to the eardrum. A complicated mechanism converts the electrical impulses to sound.
d. Sound waves transfer mechanical vibrations to the ear canal, and the eardrum converts them to electrical impulses.

81. Why does sound travel faster in iron than in air even though iron is denser than air?

a. The density of iron is greater than that of air. However, the rigidity of iron is much greater than that of air. Hence, sound travels faster in it.
b. The density of iron is greater than that of air. However, the rigidity of iron is much less than that of air. Hence, sound travels faster in it.
c. The density of iron is greater than that of air. However, the rigidity of iron is equal to that of air. Hence, sound travels faster in it.
d. The mass of iron is much less than that of air and the rigidity of iron is much greater than that of air. Hence, sound travels faster in it.

82. Is the speed of sound dependent on its frequency?

a. No
b. Yes

14.2 Sound Intensity and Sound Level

83. Why is the sound from a tire burst louder than that from a finger snap?

a. The sound from the tire burst has higher pressure amplitudes, hence it can exert smaller force on the eardrum.
b. The sound from the tire burst has lower pressure amplitudes, hence it can exert smaller force on the eardrum.
c. The sound from the tire burst has lower pressure amplitudes, hence it can exert larger force on the ear drum.
d. The sound from the tire burst has higher pressure amplitudes, hence it can exert larger force on the eardrum.

84. Sound A is 1000 times more intense than Sound B. What will be the difference in decibels in their sound intensity levels?

a. $5\,\text{dB}$
b. $10\,\text{dB}$
c. $3\,\text{dB}$
d. $30\,\text{dB}$

85. The ratio of the pressure amplitudes of two sound waves traveling through water at 0°C is 4.0. What will be the difference in their sound intensity levels in dB?

a. $1.2\,\text{dB}$
b. $6.0\,\text{dB}$
c. $0.60\,\text{dB}$
d. $12\,\text{dB}$

86. Which of the following most closely models how sound is produced by the vocal cords?

a. A person plucks a string.
b. A person blows over the mouth of a half-filled glass bottle.
c. A person strikes a hammer against a hard surface.
d. A person blows through a small slit in a wide, stretched rubber band.

14.3 Doppler Effect and Sonic Booms

87. True or false—The Doppler effect occurs only when the sound source is moving.

a. False
b. True

88. True or false—The observed frequency becomes infinite when the source is moving at the speed of sound.

a. False
b. True

89. You are driving alongside a train. You hear its horn at a pitch that is lower than the actual frequency. What should you do to match the speed of the train? Why?

a. In order to match the speed of the train, one would need to increase or decrease the speed of his/her car because a lower pitch means that either the train (the source) is moving away or that you (the observer) are moving away.
b. In order to match the speed of the train, one would need to drive at a constant speed because a lower pitch means that the train and the car are at the same speed.

14.4 Sound Interference and Resonance

90. How are the beat frequency and the regular frequency of a wave resulting from superposition of two waves different?

a. Beat frequency is the sum of two frequencies and regular frequency is the difference between frequencies of two original waves.
b. Beat frequency is the difference between the constituent frequencies, but the regular frequency is the average of the frequencies of the two original waves.
c. Beat frequency is the sum of two frequencies and regular frequency is the average of frequencies of two original waves.
d. Beat frequency is the average of frequencies of two original waves and regular frequency is the sum of two original frequencies.

91. In the tuning fork and tube experiment, if resonance is formed for $L = \lambda/4$, where L is the length of the tube and λ is the wavelength of the sound wave, can resonance also be formed for a wavelength $\lambda = 4L/9$? Why?

a. The frequency formed is a harmonic and first overtone so resonance will occur.
b. The frequency formed is a harmonic and second overtone so resonance will occur.
c. The frequency formed is a harmonic and third overtone so resonance will occur.
d. The frequency formed is a harmonic and fourth overtone so resonance will occur.

92. True or false—An open-pipe resonator has more overtones than a closed-pipe resonator.

a. False
b. True

93. A flute has finger holes for changing the length of the resonating air column, and therefore, the frequency of the note played. How far apart are two holes that, when closed, play two frequencies that are $300.0\ \text{Hz}$ apart, if the first hole is $20.0\ \text{cm}$ away from the mouthpiece of the flute?

a. $0.31\ \text{m}$
b. $0.24\ \text{m}$
c. $0.04\ \text{m}$
d. $0.11\ \text{m}$

CHAPTER 15
Light

Figure 15.1 Human eyes detect these orange *sea goldie* fish swimming over a coral reef in the blue waters of the Gulf of Eilat, in the Red Sea, using visible light. (credit: David Darom, Wikimedia Commons)

Chapter Outline

INTRODUCTION The beauty of a coral reef, the warm radiance of sunshine, the sting of sunburn, the X-ray revealing a broken bone, even microwave popcorn—all are brought to us by electromagnetic waves. The list of the various types of electromagnetic waves, ranging from radio transmission waves to nuclear gamma-ray (γ-ray) emissions, is interesting in itself.

Even more intriguing is that all of these different phenomena are manifestations of the same thing—electromagnetic waves (see Figure 15.1). What are electromagnetic waves? How are they created, and how do they travel? How can we understand their widely varying properties? What is the relationship between electric and magnetic effects? These and other questions will be explored.

15.1 The Electromagnetic Spectrum

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Define the electromagnetic spectrum, and describe it in terms of frequencies and wavelengths
- Describe and explain the differences and similarities of each section of the electromagnetic spectrum and the applications of radiation from those sections

Section Key Terms

electric field	electromagnetic radiation (EMR)	magnetic field	Maxwell's equations

The Electromagnetic Spectrum

We generally take light for granted, but it is a truly amazing and mysterious form of energy. Think about it: Light travels to Earth across millions of kilometers of empty space. When it reaches us, it interacts with matter in various ways to generate almost all the energy needed to support life, provide heat, and cause weather patterns. Light is a form of **electromagnetic radiation (EMR)**. The term *light* usually refers to visible light, but this is not the only form of EMR. As we will see, visible light occupies a narrow band in a broad range of types of electromagnetic radiation.

Electromagnetic radiation is generated by a moving electric charge, that is, by an electric current. As you will see when you study electricity, an electric current generates both an **electric field**, **E**, and a **magnetic field**, **B**. These fields are perpendicular to each other. When the moving charge oscillates, as in an alternating current, an EM wave is propagated. Figure 15.2 shows how an electromagnetic wave moves away from the source—indicated by the ~ symbol.

WATCH PHYSICS

Electromagnetic Waves and the Electromagnetic Spectrum

This video, link below, is closely related to the following figure. If you have questions about EM wave properties, the EM spectrum, how waves propagate, or definitions of any of the related terms, the answers can be found in this video (http://www.openstax.org/l/28EMWaves) .

Click to view content (https://www.openstax.org/l/28EMWaves)

GRASP CHECK

In an electromagnetic wave, how are the magnetic field, the electric field, and the direction of propagation oriented to each other?

a. All three are parallel to each other and are along the *x*-axis.
b. All three are mutually perpendicular to each other.
c. The electric field and magnetic fields are parallel to each other and perpendicular to the direction of propagation.
d. The magnetic field and direction of propagation are parallel to each other along the *y*-axis and perpendicular to the electric field.

Virtual Physics

Radio Waves and Electromagnetic Fields

Click to view content (https://www.openstax.org/l/28Radiowaves)

This simulation demonstrates wave propagation. The EM wave is propagated from the broadcast tower on the left, just as in Figure 15.2. You can make the wave yourself or allow the animation to send it. When the wave reaches the antenna on the right, it causes an oscillating current. This is how radio and television signals are transmitted and received.

GRASP CHECK

Where do radio waves fall on the electromagnetic spectrum?

a. Radio waves have the same wavelengths as visible light.
b. Radio waves fall on the high-frequency side of visible light.
c. Radio waves fall on the short-wavelength side of visible light.
d. Radio waves fall on the low-frequency side of visible light.

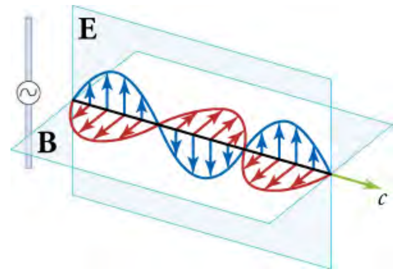


Figure 15.2 A part of the electromagnetic wave sent out from an oscillating charge at one instant in time. The electric and magnetic fields (**E** and **B**) are in phase, and they are perpendicular to each other and to the direction of propagation. For clarity, the waves are shown only along one direction, but they propagate out in other directions too.

From your study of sound waves, recall these features that apply to all types of waves:

- Wavelength—The distance between two wave crests or two wave troughs, expressed in various metric measures of distance
- Frequency—The number of wave crests that pass a point per second, expressed in hertz (Hz or s^{-1})
- Amplitude: The height of the crest above the null point

As mentioned, electromagnetic radiation takes several forms. These forms are characterized by a range of frequencies. Because frequency is inversely proportional to wavelength, any form of EMR can also be represented by its range of wavelengths. Figure 15.3 shows the frequency and wavelength ranges of various types of EMR. With how many of these types are you familiar?

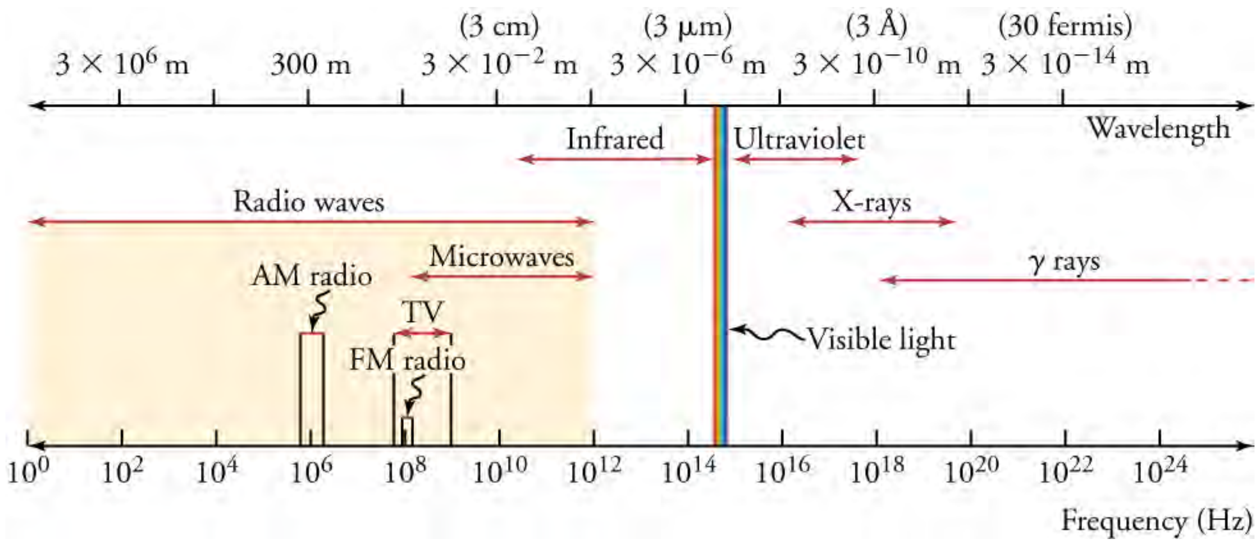


Figure 15.3 The electromagnetic spectrum, showing the major categories of electromagnetic waves. The range of frequencies and wavelengths is remarkable. The dividing line between some categories is distinct, whereas other categories overlap.

Take a few minutes to study the positions of the various types of radiation on the EM spectrum, above. Sometimes all radiation with frequencies lower than those of visible light are referred to as infrared (IR) radiation. This includes radio waves, which overlap with the frequencies used for media broadcasts of TV and radio signals. The microwave radiation that you see on the diagram is the same radiation that is used in a microwave oven. What we feel as radiant heat is also a form of low-frequency EMR.

All the high-frequency radiation to the right of visible light is sometimes referred to as ultraviolet (UV) radiation. This includes X-rays and gamma (γ) rays. The narrow band that is visible light extends from lower-frequency red light to higher-frequency violet light, thus the terms are *infrared* (below red) and *ultraviolet* (beyond violet).

BOUNDLESS PHYSICS

Maxwell's Equations

The Scottish physicist James Clerk Maxwell (1831–1879) is regarded widely to have been the greatest theoretical physicist of the nineteenth century. Although he died young, Maxwell not only formulated a complete electromagnetic theory, represented by **Maxwell's equations**, he also developed the kinetic theory of gases, and made significant contributions to the understanding of color vision and the nature of Saturn's rings.

Maxwell brought together all the work that had been done by brilliant physicists, such as Ørsted, Coulomb, Ampere, Gauss, and Faraday, and added his own insights to develop the overarching theory of electromagnetism. Maxwell's equations are paraphrased here in words because their mathematical content is beyond the level of this text. However, the equations illustrate how apparently simple mathematical statements can elegantly unite and express a multitude of concepts—why mathematics is the language of science.

Maxwell's Equations

1. Electric field lines originate on positive charges and terminate on negative charges. The electric field is defined as the force per unit charge on a test charge, and the strength of the force is related to the electric constant, ε_0.
2. Magnetic field lines are continuous, having no beginning or end. No magnetic monopoles are known to exist. The strength of the magnetic force is related to the magnetic constant, μ_0.
3. A changing magnetic field induces an electromotive force (emf) and, hence, an electric field. The direction of the emf opposes the change, changing direction of the magnetic field.
4. Magnetic fields are generated by moving charges or by changing electric fields.

Maxwell's complete theory shows that electric and magnetic forces are not separate, but different manifestations of the same thing—the electromagnetic force. This classical unification of forces is one motivation for current attempts to unify the four basic forces in nature—the gravitational, electromagnetic, strong nuclear, and weak nuclear forces. The weak nuclear and electromagnetic forces have been unified, and further unification with the strong nuclear force is expected; but, the unification of the gravitational force with the other three has proven to be a real head-scratcher.

One final accomplishment of Maxwell was his development in 1855 of a process that could produce color photographic images. In 1861, he and photographer Thomas Sutton worked together on this process. The color image was achieved by projecting red, blue, and green light through black-and-white photographs of a tartan ribbon, each photo itself exposed in different-colored light. The final image was projected onto a screen (see Figure 15.4).

Figure 15.4 Maxwell and Sutton's photograph of a colored ribbon. This was the first durable color photograph. The plaid tartan of the Scots made a *colorful* photographic subject.

GRASP CHECK

Describe electromagnetic force as explained by Maxwell's equations.

a. According to Maxwell's equations, electromagnetic force gives rise to electric force and magnetic force.
b. According to Maxwell's equations, electric force and magnetic force are different manifestations of electromagnetic force.
c. According to Maxwell's equations, electric force is the cause of electromagnetic force.
d. According to Maxwell's equations, magnetic force is the cause of electromagnetic force.

Characteristics of Electromagnetic Radiation

All the EM waves mentioned above are basically the same form of radiation. They can all travel across empty space, and they all travel at the speed of light in a vacuum. The basic difference between types of radiation is their differing frequencies. Each frequency has an associated wavelength. As frequency increases across the spectrum, wavelength decreases. Energy also increases with frequency. Because of this, higher frequencies penetrate matter more readily. Some of the properties and uses of the various EM spectrum bands are listed in Table 15.1.

Types of EM Waves	Production	Applications	Life Sciences Aspect	Issues
Radio and TV	Accelerating charges	Communications, remote controls	MRI	Requires controls for band use
Microwaves	Accelerating charges & thermal agitation	Communications, microwave ovens, radar	Deep heating	Cell phone use
Infrared	Thermal agitation & electronic transitions	Thermal imaging, heating	Absorption by atmosphere	Greenhouse effect
Visible Light	Thermal agitation & electronic transitions	All pervasive	Photosynthesis, human vision	
Ultraviolet	Thermal agitation & electronic transitions	Sterilization, slowing abnormal growth of cells	Vitamin D production	Ozone depletion, causes cell damage
X-rays	Inner electronic transitions & fast collisions	Medical, security	Medical diagnosis, cancer therapy	Causes cell damage
Gamma Rays	Nuclear decay	Nuclear medicine, security	Medical diagnosis, cancer therapy	Causes cell damage, radiation damage

Table 15.1 Electromagnetic Waves This table shows how each type of EM radiation is produced, how it is applied, as well as environmental and health issues associated with it.

The narrow band of visible light is a combination of the colors of the rainbow. Figure 15.5 shows the section of the EM spectrum that includes visible light. The frequencies corresponding to these wavelengths are $4.0 \times 10^{14}\ \mathrm{s}^{-1}$ at the red end to $7.9 \times 10^{14}\ \mathrm{s}^{-1}$ at the violet end. This is a very narrow range, considering that the EM spectrum spans about 20 orders of magnitude.

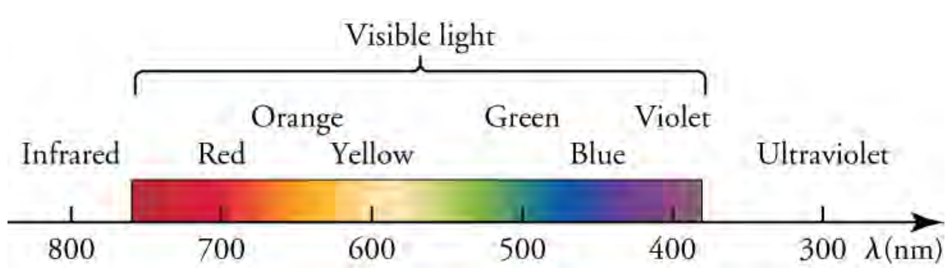


Figure 15.5 A small part of the electromagnetic spectrum that includes its visible components. The divisions between infrared, visible, and ultraviolet are not perfectly distinct, nor are the divisions between the seven rainbow colors

TIPS FOR SUCCESS

Wavelengths of visible light are often given in nanometers, nm. One nm equals 10^{-9} m. For example, yellow light has a wavelength of about 600 nm, or 6×10^{-7} m.

As a child, you probably learned the color wheel, shown on the left in Figure 15.6. It helps if you know what color results when you mix different colors of paint together. Mixing two of the primary *pigment* colors—magenta, yellow, or cyan—together results in a secondary color. For example, mixing cyan and yellow makes green. This is called *subtractive* color mixing. Mixing different colors of *light* together is quite different. The diagram on the right shows *additive* color mixing. In this case, the primary colors are red, green, and blue, and the secondary colors are cyan, magenta, and yellow. Mixing pigments and mixing light are different because materials absorb light by a different set of rules than does the perception of light by the eye. Notice that, when all colors are subtracted, the result is no color, or black. When all colors are added, the result is white light. We see the reverse of this when white sunlight is separated into the visible spectrum by a prism or by raindrops when a rainbow appears in the sky.

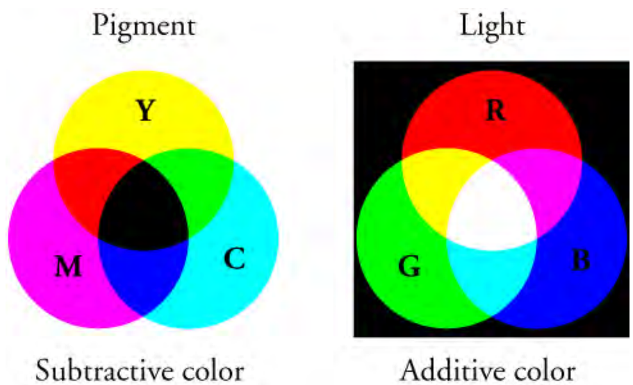


Figure 15.6 Mixing colored pigments follows the subtractive color wheel, and mixing colored light follows the additive color wheel.

Virtual Physics

Color Vision

Click to view content (https://www.openstax.org/l/28Colorvision)

This video demonstrates additive color and color filters. Try all the settings except *Photons*.

GRASP CHECK

Explain why only light from a blue bulb passes through the blue filter.

a. A blue filter absorbs blue light.
b. A blue filter reflects blue light.
c. A blue filter absorbs all visible light other than blue light.
d. A blue filter reflects all of the other colors of light and absorbs blue light.

LINKS TO PHYSICS

Animal Color Perception

The physics of color perception has interesting links to zoology. Other animals have very different views of the world than humans, especially with respect to which colors can be seen. Color is detected by cells in the eye called *cones*. Humans have three cones that are sensitive to three different ranges of electromagnetic wavelengths. They are called red, blue, and green cones, although these colors do not correspond exactly to the centers of the three ranges. The ranges of wavelengths that each cone detects are red, 500 to 700 nm; green, 450 to 630 nm; and blue, 400 to 500 nm.

Most primates also have three kinds of cones and see the world much as we do. Most mammals other than primates only have two cones and have a less colorful view of things. Dogs, for example see blue and yellow, but are color blind to red and green. You might think that *simpler* species, such as fish and insects, would have less sophisticated vision, but this is not the case. Many birds, reptiles, amphibians, and insects have four or five different cones in their eyes. These species don't have a wider range of perceived colors, but they see more hues, or combinations of colors. Also, some animals, such as bees or rattlesnakes, see a range of colors that is as broad as ours, but shifted into the ultraviolet or infrared.

These differences in color perception are generally adaptations that help the animals survive. Colorful tropical birds and fish display some colors that are too subtle for us to see. These colors are believed to play a role in the species mating rituals. Figure 15.7 shows the colors visible and the color range of vision in humans, bees, and dogs.

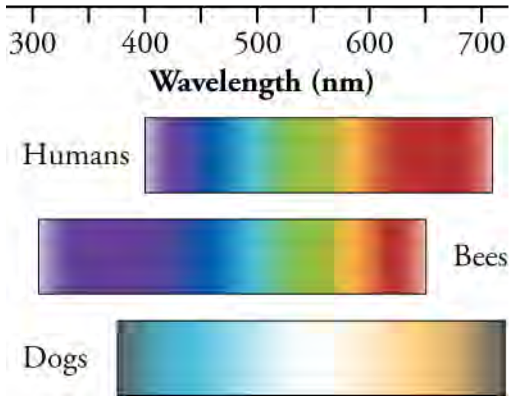


Figure 15.7 Humans, bees, and dogs see colors differently. Dogs see fewer colors than humans, and bees see a different range of colors.

GRASP CHECK

The belief that bulls are enraged by seeing the color red is a misconception. What did you read in this Links to Physics that shows why this belief is incorrect?

a. Bulls are color-blind to every color in the spectrum of colors.
b. Bulls are color-blind to the blue colors in the spectrum of colors.
c. Bulls are color-blind to the red colors in the spectrum of colors.
d. Bulls are color-blind to the green colors in the spectrum of colors.

Humans have found uses for every part of the electromagnetic spectrum. We will take a look at the uses of each range of frequencies, beginning with visible light. Most of our uses of visible light are obvious; without it our interaction with our surroundings would be much different. We might forget that nearly all of our food depends on the photosynthesis process in plants, and that the energy for this process comes from the visible part of the spectrum. Without photosynthesis, we would also have almost no oxygen in the atmosphere.

The low-frequency, infrared region of the spectrum has many applications in media broadcasting. Television, radio, cell phone, and remote-control devices all broadcast and/or receive signals with these wavelengths. AM and FM radio signals are both low-frequency radiation. They are in different regions of the spectrum, but that is not their basic difference. AM and FM are abbreviations for *amplitude modulation* and *frequency modulation*. Information in AM signals has the form of changes in *amplitude* of the radio waves; information in FM signals has the form of changes in wave *frequency*.

Another application of long-wavelength radiation is found in microwave ovens. These appliances cook or warm food by irradiating it with EM radiation in the microwave frequency range. Most kitchen microwaves use a frequency of 2.45×10^9 Hz. These waves have the right amount of energy to cause polar molecules, such as water, to rotate faster. Polar molecules are those that have a partial charge separation. The rotational energy of these molecules is given up to surrounding matter as heat. The first microwave ovens were called *Radaranges* because they were based on radar technology developed during World War II.

Radar uses radiation with wavelengths similar to those of microwaves to detect the location and speed of distant objects, such as airplanes, weather formations, and motor vehicles. Radar information is obtained by receiving and analyzing the echoes of microwaves reflected by an object. The speed of the object can be measured using the Doppler shift of the returning waves. This is the same effect you learned about when you studied sound waves. Like sound waves, EM waves are shifted to higher frequencies by an object moving toward an observer, and to lower frequencies by an object moving away from the observer. Astronomers use this same Doppler effect to measure the speed at which distant galaxies are moving away from us. In this case, the shift in frequency is called the *red shift*, because visible frequencies are shifted toward the lower-frequency, red end of the spectrum.

Exposure to any radiation with frequencies greater than those of visible light carries some health hazards. All types of radiation in this range are known to cause cell damage. The danger is related to the high energy and penetrating ability of these EM waves. The likelihood of being harmed by any of this radiation depends largely on the amount of exposure. Most people try to reduce exposure to UV radiation from sunlight by using sunscreen and protective clothing. Physicians still use X-rays to diagnose medical problems, but the intensity of the radiation used is extremely low. Figure 15.8 shows an X-ray image of a patient's chest cavity.

One medical-imaging technique that involves no danger of exposure is magnetic resonance imaging (MRI). MRI is an important imaging and research tool in medicine, producing highly detailed two- and three-dimensional images. Radio waves are broadcast, absorbed, and reemitted in a resonance process that is sensitive to the density of nuclei, usually hydrogen nuclei—protons.

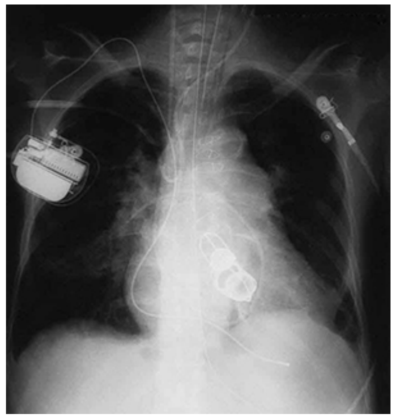


Figure 15.8 This shadow X-ray image shows many interesting features, such as artificial heart valves, a pacemaker, and wires used to close the sternum. (credit: P.P. Urone)

Check Your Understanding

1. Identify the fields produced by a moving charged particle.
   a. Both an electric field and a magnetic field will be produced.
   b. Neither a magnetic field nor an electric field will be produced.
   c. A magnetic field, but no electric field will be produced.
   d. Only the electric field, but no magnetic field will be produced.

2. X-rays carry more energy than visible light. Compare the frequencies and wavelengths of these two types of EM radiation.
   a. Visible light has higher frequencies and shorter wavelengths than X-rays.
   b. Visible light has lower frequencies and shorter wavelengths than X-rays.
   c. Visible light has higher frequencies and longer wavelengths than X-rays.
   d. Visible light has lower frequencies and longer wavelengths than X-rays.

3. How does wavelength change as frequency increases across the EM spectrum?
   a. The wavelength increases.
   b. The wavelength first increases and then decreases.
   c. The wavelength first decreases and then increases.
   d. The wavelength decreases.

4. Why are X-rays used in imaging of broken bones, rather than radio waves?
   a. X-rays have higher penetrating energy than radio waves.
   b. X-rays have lower penetrating energy than radio waves.
   c. X-rays have a lower frequency range than radio waves.
   d. X-rays have longer wavelengths than radio waves.

5. Identify the fields that make up an electromagnetic wave.
   a. both an electric field and a magnetic field
   b. neither a magnetic field nor an electric field

c. only a magnetic field, but no electric field
d. only an electric field, but no magnetic field

15.2 The Behavior of Electromagnetic Radiation

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the behavior of electromagnetic radiation
- Solve quantitative problems involving the behavior of electromagnetic radiation

Section Key Terms

illuminance	interference	lumens	luminous flux	lux	polarized light

Types of Electromagnetic Wave Behavior

In a vacuum, all electromagnetic radiation travels at the same incredible speed of 3.00×10^8 m/s, which is equal to 671 million miles per hour. This is one of the fundamental physical constants. It is referred to as the speed of light and is given the symbol *c*. The space between celestial bodies is a near vacuum, so the light we see from the Sun, stars, and other planets has traveled here at the speed of light. Keep in mind that all EM radiation travels at this speed. All the different wavelengths of radiation that leave the Sun make the trip to Earth in the same amount of time. That trip takes 8.3 minutes. Light from the nearest star, besides the Sun, takes 4.2 years to reach Earth, and light from the nearest galaxy—a dwarf galaxy that orbits the Milky Way—travels 25,000 years on its way to Earth. You can see why we call very long distances *astronomical*.

When light travels through a physical medium, its speed is always less than the speed of light. For example, light travels in water at three-fourths the value of *c*. In air, light has a speed that is just slightly slower than in empty space: 99.97 percent of *c*. Diamond slows light down to just 41 percent of *c*. When light changes speeds at a boundary between media, it also changes direction. The greater the difference in speeds, the more the path of light bends. In other chapters, we look at this bending, called *refraction*, in greater detail. We introduce refraction here to help explain a phenomenon called thin-film interference.

Have you ever wondered about the rainbow colors you often see on soap bubbles, oil slicks, and compact discs? This occurs when light is both refracted by and reflected from a very thin film. The diagram shows the path of light through such a thin film. The symbols n_1, n_2, and n_3 indicate that light travels at different speeds in each of the three materials. Learn more about this topic in the chapter on diffraction and interference.

Figure 15.9 shows the result of thin film interference on the surface of soap bubbles. Because ray 2 travels a greater distance, the two rays become *out of phase*. That is, the crests of the two emerging waves are no longer moving together. This causes **interference**, which reinforces the intensity of the wavelengths of light that create the bands of color. The color bands are separated because each color has a different wavelength. Also, the thickness of the film is not uniform, and different thicknesses cause colors of different wavelengths to interfere in different places. Note that the film must be very, very thin—somewhere in the vicinity of the wavelengths of visible light.

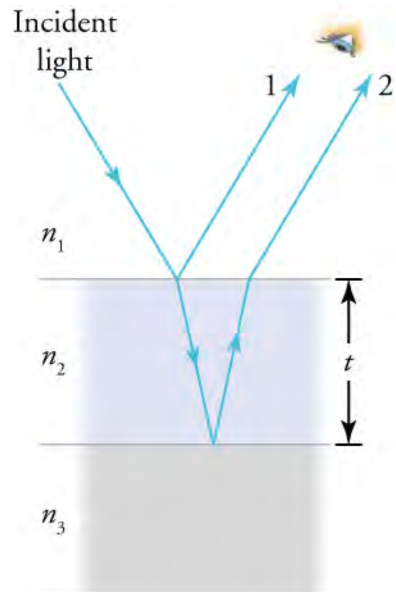


Figure 15.9 Light striking a thin film is partially reflected (ray 1) and partially refracted at the top surface. The refracted ray is partially reflected at the bottom surface and emerges as ray 2. These rays will interfere in a way that depends on the thickness of the film and the indices of refraction of the various media.

You have probably experienced how polarized sunglasses reduce glare from the surface of water or snow. The effect is caused by the wave nature of light. Looking back at , we see that the electric field moves in only one direction perpendicular to the direction of propagation. Light from most sources vibrates in all directions perpendicular to propagation. Light with an electric field that vibrates in only one direction is called *polarized*. A diagram of **polarized light** would look like .

Polarized glasses are an example of a polarizing filter. These glasses absorb most of the horizontal light waves and transmit the vertical waves. This cuts down glare, which is caused by horizontal waves. Figure 15.10 shows how waves traveling along a rope can be used as a model of how a polarizing filter works. The oscillations in one rope are in a vertical plane and are said to be vertically polarized. Those in the other rope are in a horizontal plane and are horizontally polarized. If a vertical slit is placed on the first rope, the waves pass through. However, a vertical slit blocks the horizontally polarized waves. For EM waves, the direction of the electric field oscillation is analogous to the disturbances on the ropes.

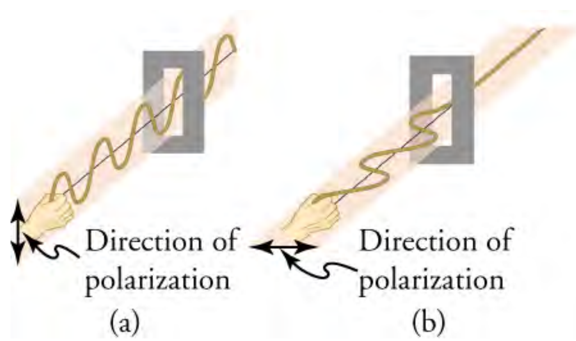


Figure 15.10 The transverse oscillations in one rope are in a vertical plane, and those in the other rope are in a horizontal plane. The first is said to be vertically polarized, and the other is said to be horizontally polarized. Vertical slits pass vertically polarized waves and block horizontally polarized waves.

Light can also be polarized by reflection. Most of the light reflected from water, glass, or any highly reflective surface is polarized horizontally. Figure 15.11 shows the effect of a polarizing lens on light reflected from the surface of water.

Figure 15.11 These two photographs of a river show the effect of a polarizing filter in reducing glare in light reflected from the surface of water. Part (b) of this figure was taken with a polarizing filter and part (a) was taken without. As a result, the reflection of clouds and sky observed in part (a) is not observed in part (b). Polarizing sunglasses are particularly useful on snow and water.

WATCH PHYSICS

Polarization of Light, Linear and Circular

This video explains the polarization of light in great detail. Before viewing the video, look back at the drawing of an electromagnetic wave from the previous section. Try to visualize the two-dimensional drawing in three dimensions.

[Click to view content (https://www.openstax.org/l/28Polarization)](https://www.openstax.org/l/28Polarization)

GRASP CHECK

How do polarized glasses reduce glare reflected from the ocean?

a. They block horizontally polarized and vertically polarized light.
b. They are transparent to horizontally polarized and vertically polarized light.
c. They block horizontally polarized rays and are transparent to vertically polarized rays.
d. They are transparent to horizontally polarized light and block vertically polarized light.

Snap Lab

Polarized Glasses

- EYE SAFETY—Looking at the Sun directly can cause permanent eye damage. Avoid looking directly at the Sun.

- two pairs of polarized sunglasses
  OR
- two lenses from one pair of polarized sunglasses

Procedure

1. Look through both or either polarized lens and record your observations.
2. Hold the lenses, one in front of the other. Hold one lens stationary while you slowly rotate the other lens. Record your observations, including the relative angles of the lenses when you make each observation.
3. Find a reflective surface on which the Sun is shining. It could be water, glass, a mirror, or any other similar smooth surface. The results will be more dramatic if the sunlight strikes the surface at a sharp angle.
4. Observe the appearance of the surface with your naked eye and through one of the polarized lenses.
5. Observe any changes as you slowly rotate the lens, and note the angles at which you see changes.

GRASP CHECK

If you buy sunglasses in a store, how can you be sure that they are polarized?

a. When one pair of sunglasses is placed in front of another and rotated in the plane of the body, the light passing through the sunglasses will be blocked at two positions due to refraction of light.
b. When one pair of sunglasses is placed in front of another and rotated in the plane of the body, the light passing through the sunglasses will be blocked at two positions due to reflection of light.
c. When one pair of sunglasses is placed in front of another and rotated in the plane of the body, the light passing through the sunglasses will be blocked at two positions due to the polarization of light.
d. When one pair of sunglasses is placed in front of another and rotated in the plane of the body, the light passing through the sunglasses will be blocked at two positions due to the bending of light waves.

Quantitative Treatment of Electromagnetic Waves

We can use the speed of light, c, to carry out several simple but interesting calculations. If we know the distance to a celestial object, we can calculate how long it takes its light to reach us. Of course, we can also make the reverse calculation if we know the time it takes for the light to travel to us. For an object at a very great distance from Earth, it takes many years for its light to reach us. This means that we are looking at the object as it existed in the distant past. The object may, in fact, no longer exist. Very large distances in the universe are measured in light years. One light year is the distance that light travels in one year, which is 9.46×10^{12} kilometers or 5.88×10^{12} miles (…and 10^{12} is a trillion!).

A useful equation involving c is

$$c = f\lambda \tag{15.1}$$

where f is frequency in Hz, and λ is wavelength in meters.

WORKED EXAMPLE

Frequency and Wavelength Calculation

For example, you can calculate the frequency of yellow light with a wavelength of 6.00×10^{-7} m.

STRATEGY

Rearrange the equation to solve for frequency.

$$f = \frac{c}{\lambda} \tag{15.2}$$

Solution

Substitute the values for the speed of light and wavelength into the equation.

$$f = \frac{3.00 \times 10^{8}\ \text{m/s}}{6.00 \times 10^{-7}\ \text{m}} = 5.00 \times 10^{14}\ \text{s}^{-1} = 5.00 \times 10^{14}\ \text{Hz} \tag{15.3}$$

Discussion

Manipulating exponents of 10 in a fraction can be tricky. Be sure you keep track of the + and – exponents correctly. Checking back to the diagram of the electromagnetic spectrum in the previous section shows that 10^{14} is a reasonable order of magnitude for the frequency of yellow light.

The frequency of a wave is proportional to the energy the wave carries. The actual proportionality constant will be discussed in a later chapter. Since frequency is inversely proportional to wavelength, we also know that wavelength is inversely proportional to energy. Keep these relationships in mind as general rules.

The rate at which light is radiated from a source is called **luminous flux**, P, and it is measured in **lumens** (lm). Energy-saving light bulbs, which provide more luminous flux for a given use of electricity, are now available. One of these bulbs is called a *compact fluorescent lamp*; another is an *LED* (light-emitting diode) bulb. If you wanted to replace an old incandescent bulb with an energy saving bulb, you would want the new bulb to have the same brightness as the old one. To compare bulbs accurately, you would need to compare the lumens each one puts out. Comparing wattage—that is, the electric power used—would be misleading. Both wattage and lumens are stated on the packaging.

The luminous flux of a bulb might be 2,000 lm. That accounts for all the light radiated in all directions. However, what we really need to know is how much light falls on an object, such as a book, at a specific distance. The number of lumens per square meter is called **illuminance**, and is given in units of **lux** (lx). Picture a light bulb in the middle of a sphere with a 1-m radius. The total surface of the sphere equals $4\pi r^2$ m^2. The illuminance then is given by

$$\text{illuminance} = \frac{P}{4\pi r^2}. \tag{15.4}$$

What happens if the radius of the sphere is increased 2 m? The illuminance is now only one-fourth as great, because the r^2 term in the denominator is 4 instead of 1. Figure 15.12 shows how illuminance decreases with the inverse square of the distance.

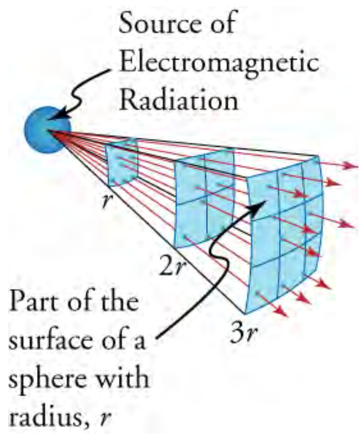


Figure 15.12 The diagram shows why the illuminance varies inversely with the square of the distance from a source of light.

WORKED EXAMPLE

Calculating Illuminance

A woman puts a new bulb in a floor lamp beside an easy chair. If the luminous flux of the bulb is rated at 2,000 lm, what is the illuminance on a book held 2.00 m from the bulb?

STRATEGY

Choose the equation and list the knowns.

Equation: $\text{illuminance} = \frac{P}{4\pi r^2}$

P = 2,000 lm

π = 3.14

r = 2.00 m

Solution

Substitute the known values into the equation.

$$\begin{aligned}\text{illuminance} &= \frac{P}{4\pi r^2} \\ &= \frac{2{,}000 \text{ lm}}{4(3.14)(2.00)^2 \text{ m}^2} \\ &= 39.8 \text{ lm/m}^2 \\ &= 39.8 \text{ lx}\end{aligned}$$

Discussion

Try some other distances to illustrate how greatly light fades with distance from its source. For example, at 3 m the illuminance is only 17.7 lux. Parents often scold children for reading in light that is too dim. Instead of shouting, “You’ll ruin your eyes!” it might be better to explain the inverse square law of illuminance to the child.

Practice Problems

6. Red light has a wavelength of 7.0 × 10–7 m and a frequency of 4.3 × 1014 Hz. Use these values to calculate the speed of light in a vacuum.

a. 3 × 1020 m/s
b. 3 × 1015 m/s
c. 3 × 1014 m/s
d. 3 × 108 m/s

7. A light bulb has a luminous flux of 942 lumens. What is the illuminance on a surface $3.00\ \text{m}$ from the bulb when it is lit?

a. $33.32\ \text{lx}$
b. $26.15\ \text{lx}$
c. $2.77\ \text{lx}$
d. $8.33\ \text{lx}$

Check Your Understanding

8. Give an example of a place where light travels at the speed of 3.00×10^8 m/s.

a. outer space
b. water
c. Earth's atmosphere
d. quartz glass

9. Explain in terms of distances and the speed of light why it is currently very unlikely that humans will visit planets that circle stars other than our Sun.

a. The spacecrafts used for travel are very heavy and thus very slow.
b. Spacecrafts do not have a constant source of energy to run them.
c. If a spacecraft could attain a maximum speed equal to that of light, it would still be too slow to cover astronomical distances.
d. Spacecrafts can attain a maximum speed equal to that of light, but it is difficult to locate planets around stars.

KEY TERMS

electric field a field that tells us the force per unit charge at all locations in space around a charge distribution

electromagnetic radiation (EMR) radiant energy that consists of oscillating electric and magnetic fields

illuminance number of lumens per square meter, given in units of lux (lx)

interference increased or decreased light intensity caused by the phase differences between waves

lumens unit of measure for luminous flux

luminous flux rate at which light is radiated from a source

lux unit of measure for illuminance

magnetic field the directional lines around a magnetic material that indicates the direction and magnitude of the magnetic force

Maxwell's equations equations that describe the interrelationship between electric and magnetic fields, and how these fields combine to form electromagnetic radiation

polarized light light whose electric field component vibrates in a specific plane

SECTION SUMMARY

15.1 The Electromagnetic Spectrum

- The electromagnetic spectrum is made up of a broad range of frequencies of electromagnetic radiation.
- All frequencies of EM radiation travel at the same speed in a vacuum and consist of an electric field and a magnetic field. The types of EM radiation have different frequencies and wavelengths, and different energies and penetrating ability.

15.2 The Behavior of Electromagnetic Radiation

- EM radiation travels at different speeds in different media, produces colors on thin films, and can be polarized to oscillate in only one direction.
- Calculations can be based on the relationship among the speed, frequency, and wavelength of light, and on the relationship among luminous flux, illuminance, and distance.

KEY EQUATIONS

15.2 The Behavior of Electromagnetic Radiation

frequency and wavelength	$c = f\lambda$
illuminance	$\text{illuminance} = \frac{P}{4\pi r^2}$

CHAPTER REVIEW

Concept Items

15.1 The Electromagnetic Spectrum

1. Use the concepts on which Maxwell's equations are based to explain why a compass needle is deflected when the compass is brought near a wire that is carrying an electric current.
    a. The charges in the compass needle and the charges in the electric current have interacting electric fields, causing the needle to deflect.
    b. The electric field from the moving charges in the current interacts with the magnetic field of the compass needle, causing the needle to deflect.
    c. The magnetic field from the moving charges in the current interacts with the electric field of the compass needle, causing the needle to deflect.
    d. The moving charges in the current produce a magnetic field that interacts with the compass needle's magnetic field, causing the needle to deflect.

2. Consider these colors of light: yellow, blue, and red. Part A. Put these light waves in order according to wavelength, from shortest wavelength to longest wavelength. Part B. Put these light waves in order according to frequency, from lowest frequency to highest frequency.
    a. wavelength: blue, yellow, red
       frequency: blue, yellow, red
    b. wavelength: red, yellow, blue
       frequency: red, yellow, blue
    c. wavelength: red, yellow, blue
       frequency: blue, yellow, red
    d. wavelength: blue, yellow, red
       frequency: red, yellow, blue

3. Describe the location of gamma rays on the electromagnetic spectrum.

a. At the high-frequency and long-wavelength end of the spectrum
b. At the high-frequency and short-wavelength end of the spectrum
c. At the low-frequency and long-wavelength end of the spectrum
d. At the low-frequency and short-wavelength end of the spectrum

4. In which region of the electromagnetic spectrum would you find radiation that is invisible to the human eye and has low energy?
   a. Long-wavelength and high-frequency region
   b. Long-wavelength and low-frequency region
   c. Short-wavelength and high-frequency region
   d. Short-wavelength and low-frequency region

15.2 The Behavior of Electromagnetic Radiation

5. Light travels at different speeds in different media. Put these media in order, from the slowest light speed to the fastest light speed: air, diamond, vacuum, water.
   a. diamond, water, air, vacuum
   b. vacuum, diamond, air, water
   c. diamond, air, water, vacuum
   d. air, diamond, water, vacuum

6. Visible light has wavelengths in the range of about 400 to 800 nm. What does this indicate about the approximate thickness of the wall of a soap bubble? Explain your answer.
   a. The thickness of the bubble wall is ten times that of the wavelength of light.
   b. The thickness of the bubble wall is similar to that of the wavelength of light.
   c. The thickness of the bubble wall is half the wavelength of light.
   d. The thickness of the bubble wall equals the cube of the wavelength of light.

7. Bright sunlight is reflected from an icy pond. You look at the glare of the reflected light through polarized glasses. When you take the glasses off, rotate them 90°, and look through one of the lenses again, the light you see becomes brighter. Explain why the light you see changes.
   a. The glass blocks horizontally polarized light, and the light reflected from the icy pond is, in part, polarized horizontally.
   b. The glass blocks vertically polarized light, and the light reflected from the icy pond is, in part, polarized vertically.
   c. The glass allows horizontally polarized light to pass, and the light reflected from the icy pond is, in part, polarized vertically.
   d. The glass allows horizontally polarized light to pass, and the light reflected from the icy pond is, in part, polarized horizontally.

Critical Thinking Items

15.1 The Electromagnetic Spectrum

8. Standing in front of a fire, we can sense both its heat and its light. How are the light and heat radiated by the fire the same, and how are they different?
   a. Both travel as waves, but only light waves are a form of electromagnetic radiation.
   b. Heat and light are both forms of electromagnetic radiation, but light waves have higher frequencies.
   c. Heat and light are both forms of electromagnetic radiation, but heat waves have higher frequencies.
   d. Heat and light are both forms of electromagnetic radiation, but light waves have higher wavelengths.

9. Light shines on a picture of the subtractive color wheel. The light is a mixture of red, blue, and green light.
   Part A—Which part of the color wheel will look blue? Explain in terms of absorbed and reflected light.
   Part B—Which part of the color wheel will look yellow? Explain in terms of absorbed and reflected light.
   a. A. The yellow section of the wheel will look blue because it will reflect blue light and absorb red and green.
      B. The blue section of the wheel will look yellow because it will reflect red and green light and absorb blue.
   b. A. The blue section of the wheel will look blue because it will absorb blue light and reflect red and green.
      B. The yellow section of the wheel will look yellow because it will absorb red and green light and reflect blue.
   c. A. The yellow section of the wheel will look blue because it will absorb blue light and reflect red and green.
      B. The blue section of the wheel will look yellow because it will absorb red and green light and reflect blue.
   d. A. The blue section of the wheel will look blue because it will reflect blue light and absorb red and green.
      B. The yellow section of the wheel will look yellow because it will reflect red and green light and absorb blue.

10. Part A. When you stand in front of an open fire, you can sense light waves with your eyes. You sense another type of electromagnetic radiation as heat. What is this other type of radiation?
Part B. How is this other type of radiation different front light waves?

a. A. X-rays
B. The X-rays have higher frequencies and shorter wavelengths than the light waves.

b. A. X-rays
B. The X-rays have lower frequencies and longer wavelengths than the light waves.

c. A. infrared rays
B. The infrared rays have higher frequencies and shorter wavelengths than the light waves.

d. A. infrared rays
B. The infrared rays have lower frequencies and longer wavelengths than the light waves.

11. Overexposure to this range of EM radiation is dangerous, and yet it is used by doctors to diagnose medical problems.
Part A—Identify the type of radiation.
Part B—Locate the position of this radiation on the EM spectrum by comparing its frequency and wavelength to visible light.
Part C—Explain why this radiation is both dangerous and therapeutic in terms of its energy, based on your answer to Part B.

a. A. X-rays
B. X-rays have shorter wavelengths ($1 \times 10^{-8} - 5 \times 10^{-12}$ m) and higher frequencies ($3 \times 10^{16} - 6 \times 10^{19}$ Hz) than visible light ($7.5 \times 10^{-7} - 4.0 \times 10^{-7}$ m; $4.0 \times 10^{14} - 7.5 \times 10^{14}$ Hz).
C. X-rays have low energies because of their high frequencies, and so can penetrate matter to greater depths.

b. A. X-rays
B. X-rays have shorter wavelengths ($1 \times 10^{-8} - 5 \times 10^{-12}$ m) and higher frequencies ($3 \times 10^{10} - 6 \times 10^{13}$ Hz) than visible light ($7.5 \times 10^{-7} - 4.0 \times 10^{-7}$ m; $4.0 \times 10^{14} - 7.5 \times 10^{14}$ Hz).
C. X-rays have low energies because of their low frequencies, and so can penetrate matter to greater depths.

c. A. X-rays B. X-rays have longer wavelengths ($1 \times 10^{-6} - 5 \times 10^{-7}$ m) and higher frequencies ($3 \times 10^{15} - 6 \times 10^{15}$ Hz) than visible light ($7.5 \times 10^{-7} - 4.0 \times 10^{-7}$ m; $4.0 \times 10^{14} - 7.5 \times 10^{14}$ Hz).
C. X-rays have high energies because of their high frequencies, and therefore can penetrate matter to greater depths.

d. A. X-rays
B. X-rays have shorter wavelengths ($1 \times 10^{-8} - 5 \times 10^{-12}$ m) and higher frequencies ($3 \times 10^{16} - 6 \times 10^{19}$ Hz) than visible light ($7.5 \times 10^{-7} - 4.0 \times 10^{-7}$ m; $4.0 \times 10^{14} - 7.5 \times 10^{14}$ Hz).
C. X-rays have high energies because of their high frequencies, and so can penetrate matter to greater depths.

15.2 The Behavior of Electromagnetic Radiation

12. Explain how thin-film interference occurs. Discuss in terms of the meaning of interference and the pathways of light waves.

a. For a particular thickness of film, light of a given wavelength that reflects from the outer and inner film surfaces is completely in phase, and so undergoes constructive interference.
b. For a particular thickness of film, light of a given wavelength that reflects from the outer and inner surfaces is completely in phase, and so undergoes destructive interference.
c. For a particular thickness of film, light of a given wavelength that reflects from the outer and inner film surfaces is completely out of phase, and so undergoes constructive interference.
d. For a particular thickness of film, light of a given wavelength that reflects from the outer and inner film surfaces is completely out of phase, and so undergoes no interference.

13. When you move a rope up and down, waves are created. If the waves pass through a slot, they will be affected differently, depending on the orientation of the slot. Using the rope waves and the slot as a model, explain how polarizing glasses affect light waves.

a. If the wave—electric field—is vertical and slit—polarizing molecules in the glass—is horizontal, the wave will pass.
b. If the wave—electric field— is vertical and slit—polarizing molecules in the glass—is vertical, the wave will not pass.
c. If the wave—electric field—is horizontal and slit—polarizing molecules in the glass—is horizontal, the wave will pass.
d. If the wave—electric field—is horizontal and slit—polarizing molecules in the glass—is horizontal, the wave will not pass.

Problems

15.2 The Behavior of Electromagnetic Radiation

14. Visible light has a range of wavelengths from about 400 nm to 800 nm . What is the range of frequencies for visible light?
   a. 3.75×10^6 Hz to 7.50×10^6 Hz
   b. 3.75 Hz to 7.50 Hz
   c. 3.75×10^{-7} Hz to 7.50×10^{-7} Hz
   d. 3.75×10^{14} Hz to 7.50×10^{14} Hz

15. Light travels through the wall of a soap bubble that is 600 nm thick and is reflected from the inner surface back into the air. Assume the bubble wall is mostly water and that light travels in water at 75 percent of the speed of light in vacuum. How many seconds behind will the light reflected from the inner surface arrive compared to the light that was reflected from the outer surface?
   a. 4.0×10^{-8} s
   b. 5.3×10^{-6} s
   c. 2.65×10^{-15} s
   d. 5.3×10^{-15} s

Performance Task

15.2 The Behavior of Electromagnetic Radiation

16. Design an experiment to observe the phenomenon of thin-film interference. Observe colors of visible light, and relate each color to its corresponding wavelength. Comparison with the magnitudes of visible light wavelength will give an appreciation of just how very thin a thin film is. Thin-film interference has a number of practical applications, such as anti-reflection coatings and optical filters. Thin films used in filters can be designed to reflect or transmit specific wavelengths of light. This is done by depositing a film one molecular layer at a time from a vapor, thus allowing the thickness of the film to be exactly controlled.

- EYE SAFETY—Chemicals in this lab are poisonous if ingested. If chemicals are ingested, inform your teacher immediately.
- FUMES—Certain chemicals or chemical reactions in this lab create a vapor that is harmful if inhaled. Follow your teacher's instructions for the use of fume hoods and other safety apparatus designed to prevent fume inhalation. Never smell or otherwise breath in any chemicals or vapors in the lab.
- FLAMMABLE—Chemicals in this lab are highly flammable and can ignite, especially if exposed to a spark or open flame. Follow your teacher's instructions carefully on how to handle flammable chemicals. Do not expose any chemical to a flame or other heat source unless specifically instructed by your teacher.
- HAND WASHING—Some materials may be hazardous if in extended contact with the skin. Be sure to wash your hands with soap after handling and disposing of these materials during the lab.
- WASTE—Some things in this lab are hazardous and need to be disposed of properly. Follow your teacher's instructions for disposal of all items.

- A large flat tray with raised sides, such as a baking tray
- Small volumes of motor oil, lighter fluid or a penetrating oil of the type used to loosen rusty bolts, and cooking oil
- Water
- A camera

a. Thin-film interference causes colors to appear on the surface of a thin transparent layer. Do you expect to see a pattern to the colors?
b. How could you make a permanent record of your observations?
c. What data would you need to look up to help explain any patterns that you see?
d. What could explain colors failing to appear under some conditions?

TEST PREP

Multiple Choice

15.1 The Electromagnetic Spectrum

17. Which type of EM radiation has the shortest wavelengths?
   a. gamma rays
   b. infrared waves
   c. blue light
   d. microwaves

18. Which form of EM radiation has the most penetrating ability?
   a. red light
   b. microwaves
   c. gamma rays
   d. infrared radiation

19. Why are high-frequency gamma rays more dangerous to humans than visible light?
   a. Gamma rays have a lower frequency range than

visible light.
b. Gamma rays have a longer wavelength range than visible light.
c. Gamma rays have greater energy than visible light for penetrating matter.
d. Gamma rays have less energy than visible light for penetrating matter.

20. A dog would have a hard time stalking and catching a red bird hiding in a field of green grass. Explain this in terms of cone cells and color perception.
a. Dogs are red-green color-blind because they can see only blue and yellow through two kinds of cone cells present in their eyes.
b. Dogs are only red color-blind because they can see only blue and yellow through two kinds of cones cells present in their eyes.
c. Dogs are only green color-blind because they can see only blue and yellow through two kinds of cones cells present in their eyes.
d. Dogs are color-blind because they have only rods and no cone cells present in their eyes.

15.2 The Behavior of Electromagnetic Radiation

21. To compare the brightness of light bulbs for sale in a store, you should look on the labels to see how they are rated in terms of ____.
a. frequency
b. watts
c. amps
d. lumens

22. What is the wavelength of red light with a frequency of 4.00×10^{14} Hz?
a. 2.50×10^{14} m
b. 4.00×10^{15} m
c. 2.50×10^{6} m
d. 4.00×10^{-7} m

23. What is the distance of one light year in kilometers?
a. 2.59×10^{10} km
b. 1.58×10^{11} km
c. 2.63×10^{9} km
d. 9.46×10^{12} km

24. How does the illuminance of light change when the distance from the light source is tripled? Cite the relevant equation and explain how it supports your answer.
a. $\text{Illuminance} = \frac{P}{4\pi r^2}$; if distance is tripled, then the illuminance increases by 19 times.
b. $\text{Illuminance} = \frac{P}{4\pi r}$; if distance is tripled, then the illuminance decreases by 13 times.
c. $\text{Illuminance} = P\cdot 4\pi r^2$; if distance is tripled, then the illuminance decreases by 9 times.
d. $\text{Illuminance} = P\cdot 4\pi r$; if distance tripled, then the illuminance increases by 3 times.

25. A light bulb has an illuminance of 19.9 lx at a distance of 2 m . What is the luminous flux of the bulb?
a. 500 lm
b. 320 lm
c. 250 lm
d. 1,000 lm

Short Answer

15.1 The Electromagnetic Spectrum

26. Describe one way in which heat waves—infrared radiation—are different from sound waves.
a. Sound waves are transverse waves, whereas heat waves—infrared radiation—are longitudinal waves.
b. Sound waves have shorter wavelengths than heat waves.
c. Sound waves require a medium, whereas heat waves—infrared radiation—do not.
d. Sound waves have higher frequencies than heat waves.

27. Describe the electric and magnetic fields that make up an electromagnetic wave in terms of their orientation relative to each other and their phases.
a. They are perpendicular to and out of phase with each other.
b. They are perpendicular to and in phase with each other.
c. They are parallel to and out of phase with each other.
d. They are parallel to and in phase with each other.

28. Explain how X-radiation can be harmful and how it can be a useful diagnostic tool.
a. Overexposure to X-rays can cause HIV, though normal levels of X-rays can be used for sterilizing needles.
b. Overexposure to X-rays can cause cancer, though in limited doses X-rays can be used for imaging internal body parts.
c. Overexposure to X-rays causes diabetes, though normal levels of X-rays can be used for imaging internal body parts.
d. Overexposure to X-rays causes cancer, though normal levels of X-rays can be used for reducing

cholesterol in the blood.

29. Explain how sunlight is the original source of the energy in the food we eat.
   a. Sunlight is converted into chemical energy by plants; this energy is released when we digest food.
   b. Sunlight is converted into chemical energy by animals; this energy is released when we digest food.
   c. Sunlight is converted into chemical energy by fish; this energy is released when we digest food.
   d. Sunlight is converted into chemical energy by humans; this energy is released when we digest food.

15.2 The Behavior of Electromagnetic Radiation

30. Describe what happens to the path of light when the light slows down as it passes from one medium to another?
   a. The path of the light remains the same.
   b. The path of the light becomes circular.
   c. The path of the light becomes curved.
   d. The path of the light changes.

31. What is it about the nature of light reflected from snow that causes skiers to wear polarized sunglasses?
   a. The reflected light is polarized in the vertical direction.
   b. The reflected light is polarized in the horizontal direction.
   c. The reflected light has less intensity than the incident light.
   d. The reflected light has triple the intensity of the incident light.

32. How many lumens are radiated from a candle which has an illuminance of 3.98 lx at a distance of 2.00 m?
   a. 400 lm
   b. 100 lm
   c. 50 lm
   d. 200 lm

33. Saturn is 1.43×1012 m from the Sun. How many minutes does it take the Sun's light to reach Saturn?
   a. 7.94×10^9 minutes
   b. 3.4×10^4 minutes
   c. 3.4×10^{-6} minutes
   d. 79.4 minutes

Extended Response

15.1 The Electromagnetic Spectrum

34. A frequency of red light has a wavelength of 700 nm.
Part A—Compare the wavelength and frequency of violet light to red light.
Part B—Identify a type of radiation that has lower frequencies than red light.
Part C—Identify a type of radiation that has shorter wavelengths than violet light.
   a. A. Violet light has a lower frequency and longer wavelength than red light.
      B. ultraviolet radiation
      C. infrared radiation
   b. A. Violet light has a lower frequency and longer wavelength than red light.
      B. infrared radiation
      C. ultraviolet radiation
   c. A. Violet light has a higher frequency and shorter wavelength than red light.
      B. ultraviolet radiation
      C. infrared radiation
   d. A. Violet light has a higher frequency and shorter wavelength than red light.
      B. infrared radiation
      C. ultraviolet radiation

35. A mixture of red and green light is shone on each of the subtractive colors.
Part A—Which of these colors of light are reflected from magenta?
Part B—Which of these colors of light are reflected from yellow?
Part C—Which these colors of light are reflected from cyan?
   a. Part A. red and green
      Part B. green
      Part C. red
   b. Part A. red and green
      Part B. red
      Part C. green
   c. Part A. green
      Part B. red and green
      Part C. red
   d. Part A. red
      Part B. red and green
      Part C. green

15.2 The Behavior of Electromagnetic Radiation

36. Explain why we see the colorful effects of thin-film interference on the surface of soap bubbles and oil slicks, but not on the surface of a window pane or clear plastic bag.

a. The thickness of a window pane or plastic bag is more than the wavelength of light, and interference occurs for thicknesses smaller than the wavelength of light.
b. The thickness of a window pane or plastic bag is less than the wavelength of light, and interference occurs for thicknesses similar to the wavelength of light.
c. The thickness of a window pane or plastic bag is more than the wavelength of light, and interference occurs for thicknesses similar to the wavelength of light.
d. The thickness of a window pane or plastic bag is less than the wavelength of light, and interference occurs for thicknesses larger than the wavelength of light.

37. The Occupational Safety and Health Administration (OSHA) recommends an illuminance of $500\ \mathrm{lx}$ for desktop lighting. An office space has lighting hung $2.50\ \mathrm{m}$ above desktop level that provides only $300\ \mathrm{lx}$. To what height would the lighting fixtures have to be lowered to provide $500\ \mathrm{lx}$ on desktops?

a. $1.22\ \mathrm{m}$
b. $1.09\ \mathrm{m}$
c. $0.96\ \mathrm{m}$
d. $1.94\ \mathrm{m}$

CHAPTER 16
Mirrors and Lenses

Figure 16.1 Flat, smooth surfaces reflect light to form mirror images. (credit: NASA Goddard Photo and Video, via Flickr)

Chapter Outline

INTRODUCTION “In another moment Alice was through the glass, and had jumped lightly down into the Looking-glass room.”

—Through the Looking Glass *by Lewis Carol*

Through the Looking Glass tells of the adventures of Alice after she steps from the real world, through a mirror, and into the virtual world. In this chapter we examine the optical meanings of real and virtual, as well as other concepts that make up the field of optics.

The light from this page or screen is formed into an image by the lens of your eyes, much as the lens of the camera that made the photograph at the beginning of this chapter. Mirrors, like lenses, can also form images, which in turn are captured by your eyes.

Optics is the branch of physics that deals with the behavior of visible light and other electromagnetic waves. For now, we concentrate on the propagation of light and its interaction with matter.

It is convenient to divide optics into two major parts based on the size of objects that light encounters. When light interacts with an object that is several times as large as the light’s wavelength, its observable behavior is similar to a ray; it does not display its wave characteristics prominently. We call this part of optics *geometric optics*. This chapter focuses on situations for which geometric optics is suited.

16.1 Reflection

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain reflection from mirrors, describe image formation as a consequence of reflection from mirrors, apply ray diagrams to predict and interpret image and object locations, and describe applications of mirrors
- Perform calculations based on the law of reflection and the equations for curved mirrors

Section Key Terms

angle of incidence	angle of reflection	central axis	concave mirror	convex mirror
diffused	focal length	focal point	geometric optics	law of reflection
law of refraction	ray	real image	specular	virtual image

Characteristics of Mirrors

There are three ways, as shown in Figure 16.2, in which light can travel from a source to another location. It can come directly from the source through empty space, such as from the Sun to Earth. Light can travel to an object through various media, such as air and glass. Light can also arrive at an object after being reflected, such as by a mirror. In all these cases, light is modeled as traveling in a straight line, called a ***ray***. Light may change direction when it encounters the surface of a different material (such as a mirror) or when it passes from one material to another (such as when passing from air into glass). It then continues in a straight line—that is, as a ray. The word *ray* comes from mathematics. Here it means a straight line that originates from some point. It is acceptable to visualize light rays as laser rays (or even science fiction depictions of *ray guns*).

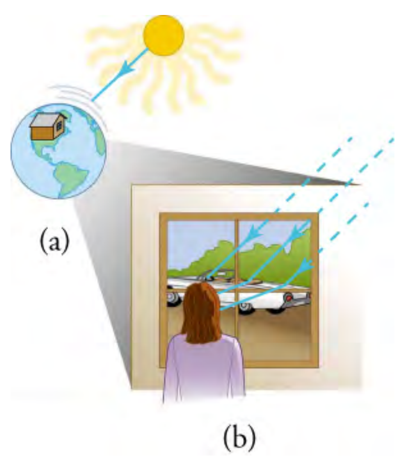


Figure 16.2 Three methods for light to travel from a source to another location are shown. (a) Light reaches the upper atmosphere of Earth by traveling through empty space directly from the source (the Sun). (b) This light can reach a person in one of two ways. It can travel through a medium, such as air or glass, and typically travels from one medium to another. It can also reflect from an object, such as a mirror.

Because light moves in straight lines, that is, as rays, and changes directions when it interacts with matter, it can be described through geometry and trigonometry. This part of optics, described by straight lines and angles, is therefore called ***geometric optics***. There are two laws that govern how light changes direction when it interacts with matter: the **law of reflection**, for situations in which light bounces off matter; and the **law of refraction**, for situations in which light passes through matter. In this section, we consider the geometric optics of **reflection**.

Whenever we look into a mirror or squint at sunlight glinting from a lake, we are seeing a reflection. How does the reflected light travel from the object to your eyes? The law of reflection states: The **angle of reflection**, θ_r, equals the **angle of incidence**, θ_i

. This law governs the behavior of all waves when they interact with a smooth surface, and therefore describe the behavior of light waves as well. The reflection of light is simplified when light is treated as a ray. This concept is illustrated in Figure 16.3, which also shows how the angles are measured relative to the line perpendicular to the surface at the point where the light ray strikes it. This perpendicular line is also called the *normal line*, or just the *normal*. Light reflected in this way is referred to as **specular** (from the Latin word for *mirror*: *speculum*).

We expect to see reflections from smooth surfaces, but Figure 16.4, illustrates how a rough surface reflects light. Because the light is reflected from different parts of the surface at different angles, the rays go in many different directions, so the reflected light is **diffused**. Diffused light allows you to read a printed page from almost any angle because some of the rays go in different directions. Many objects, such as people, clothing, leaves, and walls, have rough surfaces and can be seen from many angles. A mirror, on the other hand, has a smooth surface and reflects light at specific angles.

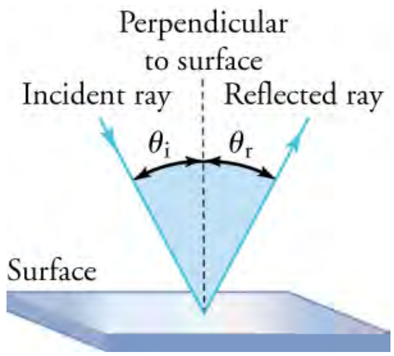


Figure 16.3 The law of reflection states that the angle of reflection, $\boldsymbol{\theta_r}$, equals the angle of incidence, $\boldsymbol{\theta_i}$. The angles are measured relative to the line perpendicular to the surface at the point where the ray strikes the surface. The incident and reflected rays, along with the normal, lie in the same plane.

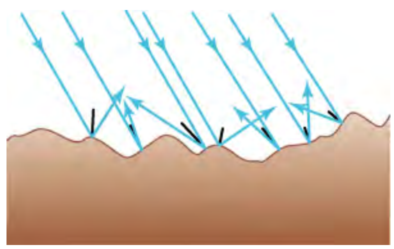


Figure 16.4 Light is diffused when it reflects from a rough surface. Here, many parallel rays are incident, but they are reflected at many different angles because the surface is rough.

When we see ourselves in a mirror, it appears that our image is actually behind the mirror. We see the light coming from a direction determined by the law of reflection. The angles are such that our image is exactly the same distance behind the mirror, d_i, as the distance we stand away from the mirror, d_o. Although these mirror images make objects appear to be where they cannot be (such as behind a solid wall), the images are not figments of our imagination. Mirror images can be photographed and videotaped by instruments and look just as they do to our eyes, which are themselves optical instruments. An image in a mirror is said to be a **virtual image**, as opposed to a **real image**. A virtual image is formed when light rays appear to diverge from a point without actually doing so.

Figure 16.5 helps illustrate how a flat mirror forms an image. Two rays are shown emerging from the same point, striking the mirror, and reflecting into the observer's eye. The rays can diverge slightly, and both still enter the eye. If the rays are extrapolated backward, they seem to originate from a common point behind the mirror, allowing us to locate the image. The paths of the reflected rays into the eye are the same as if they had come directly from that point behind the mirror. Using the law of reflection—the angle of reflection equals the angle of incidence—we can see that the image and object are the same distance from the mirror. This is a virtual image, as defined earlier.

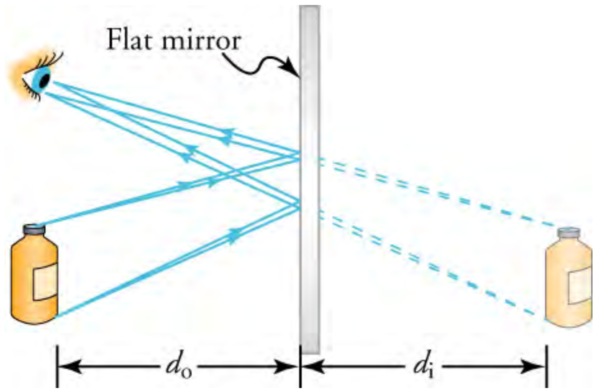


Figure 16.5 When two sets of rays from common points on an object are reflected by a flat mirror into the eye of an observer, the reflected rays seem to originate from behind the mirror, which determines the position of the virtual image.

FUN IN PHYSICS

Mirror Mazes

Figure 16.6 is a chase scene from an old silent film called *The Circus,* starring Charlie Chaplin. The chase scene takes place in a mirror maze. You may have seen such a maze at an amusement park or carnival. Finding your way through the maze can be very difficult. Keep in mind that only one image in the picture is real—the others are virtual.

Figure 16.6 Charlie Chaplin is in a mirror maze. Which image is real?

One of the earliest uses of mirrors for creating the illusion of space is seen in the Palace of Versailles, the former home of French royalty. Construction of the Hall of Mirrors (Figure 16.7) began in 1678. It is still one of the most popular tourist attractions at Versailles.

Figure 16.7 Tourists love to wander in the Hall of Mirrors at the Palace of Versailles. (credit: Michal Osmenda, Flickr)

GRASP CHECK

Only one Charlie in this image (Figure 16.8) is real. The others are all virtual images of him. Can you tell which is real? Hint—His hat is tilted to one side.

Figure 16.8

a. The virtual images have their hats tilted to the right.
b. The virtual images have their hats tilted to the left.
c. The real images have their hats tilted to the right.
d. The real images have their hats tilted to the left.

WATCH PHYSICS

Virtual Image

This video explains the creation of virtual images in a mirror. It shows the location and orientation of the images using ray diagrams, and relates the perception to the human eye.

Click to view content (https://openstax.org/l/28Virtualimage)

Compare the distance of an object from a mirror to the apparent distance of its virtual image behind the mirror.

a. The distances of the image and the object from the mirror are the same.
b. The distances of the image and the object from the mirror are always different.
c. The image is formed at infinity if the object is placed near the mirror.
d. The image is formed near the mirror if the object is placed at infinity.

Some mirrors are curved instead of flat. A mirror that curves inward is called a ***concave mirror***, whereas one that curves outward is called a ***convex mirror***. Pick up a well-polished metal spoon and you can see an example of each type of curvature. The side of the spoon that holds the food is a concave mirror; the back of the spoon is a convex mirror. Observe your image on both sides of the spoon.

TIPS FOR SUCCESS

You can remember the difference between concave and convex by thinking, *Concave means caved in*.

Ray diagrams can be used to find the point where reflected rays converge or appear to converge, or the point from which rays appear to diverge. This is called the ***focal point***, F. The distance from F to the mirror along the **central axis** (the line perpendicular to the center of the mirror's surface) is called the ***focal length***, *f*. Figure 16.9 shows the focal points of concave and convex mirrors.

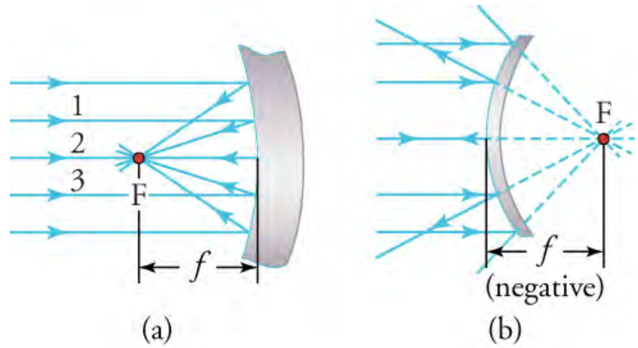


Figure 16.9 (a, b) The focal length for the concave mirror in (a), formed by converging rays, is in front of the mirror, and has a positive value. The focal length for the convex mirror in (b), formed by diverging rays, appears to be behind the mirror, and has a negative value.

Images formed by a concave mirror vary, depending on which side of the focal point the object is placed. For any object placed on the far side of the focal point with respect to the mirror, the rays converge in front of the mirror to form a real image, which can be projected onto a surface, such as a screen or sheet of paper However, for an object located inside the focal point with respect to the concave mirror, the image is virtual. For a convex mirror the image is always virtual—that is, it appears to be behind the mirror. The ray diagrams in Figure 16.10 show how to determine the nature of the image formed by concave and convex mirrors.

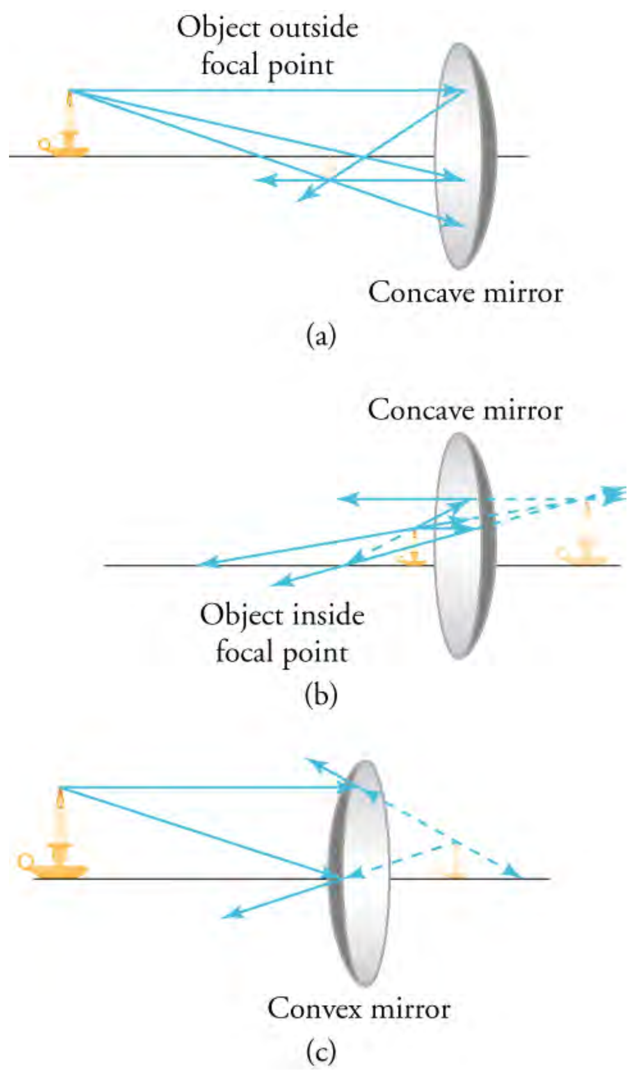


Figure 16.10 (a) The image of an object placed outside the focal point of a concave mirror is inverted and real. (b) The image of an object

placed inside the focal point of a concave mirror is erect and virtual. (c) The image of an object formed by a convex mirror is erect and virtual.

The information in Figure 16.10 is summarized in Table 16.1.

Type of Mirror	Object to Mirror Distance, d_o	Image Characteristics
Concave	$d_o > f$	Real and inverted
Concave	$d_o < f$	Virtual and erect
Convex	$d_o <$ or $> f$	Virtual and erect

Table 16.1 Curved Mirror Images This table details the type and orientation of images formed by concave and convex mirrors.

Snap Lab

Concave and Convex Mirrors

- Silver spoon and silver polish, or a new spoon made of any shiny metal

Instructions

Procedure

1. Choose any small object with a top and a bottom, such as a short nail or tack, or a coin, such as a quarter. Observe the object's reflection on the back of the spoon.
2. Observe the reflection of the object on the front (bowl side) of the spoon when held away from the spoon at a distance of several inches.
3. Observe the image while slowly moving the small object toward the bowl of the spoon. Continue until the object is all the way inside the bowl of the spoon.
4. You should see one point where the object disappears and then reappears. This is the focal point.

WATCH PHYSICS

Parabolic Mirrors and Real Images

This video uses ray diagrams to show the special feature of parabolic mirrors that makes them ideal for either projecting light energy in parallel rays, with the source being at the focal point of the parabola, or for collecting at the focal point light energy from a distant source.

Click to view content (https://www.openstax.org/l/28Parabolic)

Explain why using a parabolic mirror for a car headlight throws much more light on the highway than a flat mirror.

a. The rays do not polarize after reflection.
b. The rays are dispersed after reflection.
c. The rays are polarized after reflection.
d. The rays become parallel after reflection.

You should be able to notice everyday applications of curved mirrors. One common example is the use of security mirrors in stores, as shown in Figure 16.11.

Figure 16.11 Security mirrors are convex, producing a smaller, upright image. Because the image is smaller, a larger area is imaged compared with what would be observed for a flat mirror; hence, security is improved. (credit: Laura D'Alessandro, Flickr)

Some telescopes also use curved mirrors and no lenses (except in the eyepieces) both to magnify images and to change the path of light. Figure 16.12 shows a Schmidt-Cassegrain telescope. This design uses a spherical primary concave mirror and a convex secondary mirror. The image is projected onto the focal plane by light passing through the perforated primary mirror. The effective focal length of such a telescope is the focal length of the primary mirror multiplied by the magnification of the secondary mirror. The result is a telescope with a focal length much greater than the length of the telescope itself.

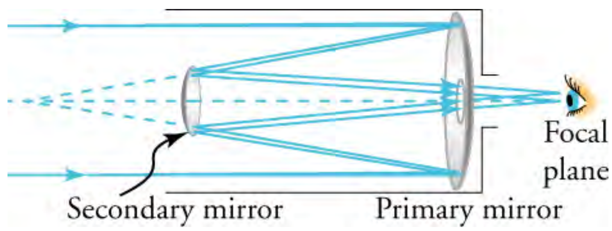


Figure 16.12 This diagram shows the design of a Schmidt–Cassegrain telescope.

A parabolic concave mirror has the very useful property that all light from a distant source, on reflection by the mirror surface, is directed to the focal point. Likewise, a light source placed at the focal point directs all the light it emits in parallel lines away from the mirror. This case is illustrated by the ray diagram in Figure 16.13. The light source in a car headlight, for example, is located at the focal point of a parabolic mirror.

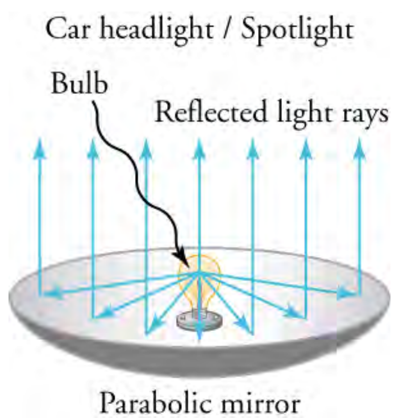


Figure 16.13 The bulb in this ray diagram of a car headlight is located at the focal point of a parabolic mirror.

Parabolic mirrors are also used to collect sunlight and direct it to a focal point, where it is transformed into heat, which in turn can be used to generate electricity. This application is shown in Figure 16.14.

Figure 16.14 Parabolic trough collectors are used to generate electricity in southern California. (credit: kjkolb, Wikimedia Commons)

Using a concave mirror, you look at the reflection of a faraway object. The image size changes if you move the object closer to the mirror. Why does the image disappear entirely when the object is at the mirror's focal point?

a. The height of the image became infinite.
b. The height of the object became zero.
c. The intensity of intersecting light rays became zero.
d. The intensity of intersecting light rays increased.

The Application of the Curved Mirror Equations

Curved mirrors and the images they create involve a fairly small number of variables: the mirror's radius of curvature, R; the focal length, f; the distances of the object and image from the mirror, d_o and d_i, respectively; and the heights of the object and image, h_o and h_i, respectively. The signs of these values indicate whether the image is inverted, erect (upright), real, or virtual. We now look at the equations that relate these variables and apply them to everyday problems.

Figure 16.15 shows the meanings of most of the variables we will use for calculations involving curved mirrors.

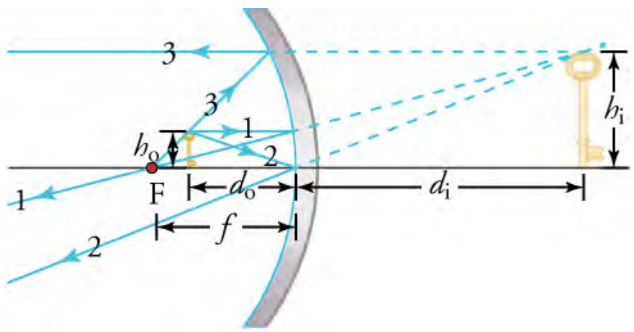


Figure 16.15 Look for the variables, d_o, d_i, h_o, h_i, and f in this figure.

The basic equation that describes both lenses and mirrors is the lens/mirror equation

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}.$$

This equation can be rearranged several ways. For example, it may be written to solve for focal length.

$$f = \frac{d_i d_o}{d_o + d_i}$$

Magnification, m, is the ratio of the size of the image, h_i, to the size of the object, h_o. The value of m can be calculated in two ways.

$$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$$

This relationship can be written to solve for any of the variables involved. For example, the height of the image is given by

$$h_i = -h_o\left(\frac{d_i}{d_o}\right).$$

We saved the simplest equation for last. The radius of curvature of a curved mirror, R, is simply twice the focal length.

$$R = 2f$$

We can learn important information from the algebraic sign of the result of a calculation using the previous equations:

- A negative d_i indicates a virtual image; a positive value indicates a real image
- A negative h_i indicates an inverted image; a positive value indicates an erect image
- For concave mirrors, f is positive; for convex mirrors, f is negative

Now let's apply these equations to solve some problems.

WORKED EXAMPLE

Calculating Focal Length

A person standing 6.0 m from a convex security mirror forms a virtual image that appears to be 1.0 m behind the mirror. What is the focal length of the mirror?

STRATEGY

The person is the object, so $d_o = 6.0$ m. We know that, for this situation, d_o is positive. The image is virtual, so the value for the image distance is negative, so $d_i = -1.0$ m.

Now, use the appropriate version of the lens/mirror equation to solve for focal length by substituting the known values.

Solution

$$f = \frac{d_i d_o}{d_o + d_i} = \frac{(-1.0)(6.0)}{6.0 + (-1.0)} = \frac{-6.0}{5.0} = -1.2 \text{ m}$$

Discussion

The negative result is expected for a convex mirror. This indicates the focal point is behind the mirror.

WORKED EXAMPLE

Calculating Object Distance

Electric room heaters use a concave mirror to reflect infrared (IR) radiation from hot coils. Note that IR radiation follows the same law of reflection as visible light. Given that the mirror has a radius of curvature of 50.0 cm and produces an image of the coils 3.00 m in front of the mirror, where are the coils with respect to the mirror?

STRATEGY

We are told that the concave mirror projects a real image of the coils at an image distance $d_i = 3.00$ m. The coils are the object, and we are asked to find their location—that is, to find the object distance d_o. We are also given the radius of curvature of the mirror, so that its focal length is $f = R/2 = 25.0$ cm (a positive value, because the mirror is concave, or converging). We can use the lens/mirror equation to solve this problem.

Solution

Because d_i and f are known, the lens/mirror equation can be used to find d_o.

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o} \tag{16.1}$$

Rearranging to solve for d_o, we have

$$d_o = \frac{d_i f}{d_i - f}. \tag{16.2}$$

Entering the known quantities gives us

$$d_o = \frac{(3.00 \times 10^2)(25.0)}{(3.00 \times 10^2) - 25.0} = 27.3 \text{ cm}. \tag{16.3}$$

Discussion

Note that the object (the coil filament) is farther from the mirror than the mirror's focal length. This is a *case 1* image ($d_o > f$ and f positive), consistent with the fact that a real image is formed. You get the most concentrated thermal energy directly in front of the mirror and 3.00 m away from it. In general, this is not desirable because it could cause burns. Usually, you want the rays to emerge parallel, and this is accomplished by having the filament at the focal point of the mirror.

Note that the filament here is not much farther from the mirror than the focal length, and that the image produced is considerably farther away.

Practice Problems

1. A concave mirror has a radius of curvature of 0.8 m. What is the focal length of the mirror?
   a. -0.8 m
   b. -0.4 m
   c. 0.4 m
   d. 0.8 m

2. What is the focal length of a makeup mirror that produces a magnification of 1.50 when a person's face is 12.0 cm away? Construct a ray diagram using paper, a pencil and a ruler to confirm your calculation.
   a. −36.0 cm
   b. −7.20 cm
   c. 7.20 cm
   d. 36.0 cm

Check Your Understanding

3. How does the object distance, d_o, compare with the focal length, f, for a concave mirror that produces an image that is real and inverted?
   a. $d_o > f$, where d_o and f are object distance and focal length, respectively.
   b. $d_o < f$, where d_o and f are object distance and focal length, respectively.
   c. $d_o = f$, where do and f are object distance and focal length, respectively.
   d. $d_o = 0$, where do is the object distance.

4. Use the law of reflection to explain why it is not a good idea to polish a mirror with sandpaper.
   a. The surface becomes smooth, and a smooth surface produces a sharp image.
   b. The surface becomes irregular, and an irregular surface produces a sharp image.
   c. The surface becomes smooth, and a smooth surface transmits light, but does not reflect it.
   d. The surface becomes irregular, and an irregular surface produces a blurred image.

5. An object is placed in front of a concave mirror at a distance that is greater than the focal length of the mirror. Will the image produced by the mirror be real or virtual? Will it be erect or inverted?
   a. It is real and erect.
   b. It is real and inverted.
   c. It is virtual and inverted.
   d. It is virtual and erect.

16.2 Refraction

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain refraction at media boundaries, predict the path of light after passing through a boundary (Snell's law), describe the index of refraction of materials, explain total internal reflection, and describe applications of refraction and total internal reflection
- Perform calculations based on the law of refraction, Snell's law, and the conditions for total internal reflection

Section Key Terms

angle of refraction	corner reflector	critical angle	dispersion	incident ray
index of refraction	refracted ray	Snell's law	total internal reflection	

The Law of Refraction

You may have noticed some odd optical phenomena when looking into a fish tank. For example, you may see the same fish appear to be in two different places (Figure 16.16). This is because light coming to you from the fish changes direction when it leaves the tank and, in this case, light rays traveling along two different paths both reach our eyes. The changing of a light ray's direction (loosely called *bending*) when it passes a boundary between materials of different composition, or between layers in single material where there are changes in temperature and density, is called *refraction*. Refraction is responsible for a tremendous range of optical phenomena, from the action of lenses to voice transmission through optical fibers.

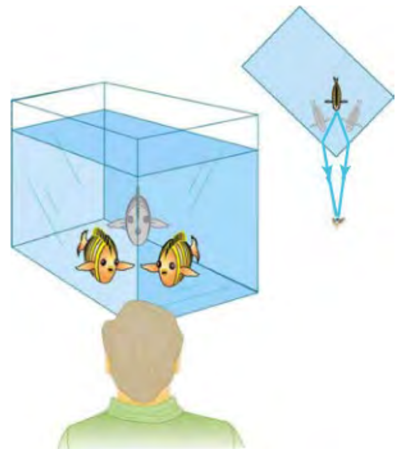


Figure 16.16 Looking at the fish tank as shown, we can see the same fish in two different locations, because light changes directions when it passes from water to air. In this case, light rays traveling on two different paths change direction as they travel from water to air, and so reach the observer. Consequently, the fish appears to be in two different places. This bending of light is called *refraction* and is responsible for many optical phenomena.

Why does light change direction when passing from one material (medium) to another? It is because light changes speed when going from one material to another. This behavior is typical of all waves and is especially easy to apply to light because light waves have very small wavelengths, and so they can be treated as rays. Before we study the law of refraction, it is useful to discuss the speed of light and how it varies between different media.

The speed of light is now known to great precision. In fact, the speed of light in a vacuum, c, is so important, and is so precisely known, that it is accepted as one of the basic physical quantities, and has the fixed value

$$c = 2.9972458 \times 10^8\,\text{m/s} \approx 3.00 \times 10^8\,\text{m/s} \tag{16.4}$$

where the approximate value of 3.00×10^8 m/s is used whenever three-digit precision is sufficient. The speed of light through matter is less than it is in a vacuum, because light interacts with atoms in a material. The speed of light depends strongly on the type of material, given that its interaction with different atoms, crystal lattices, and other substructures varies. We define the **index of refraction**, n, of a material to be

$$n = \frac{c}{v},$$

where v is the observed speed of light in the material. Because the speed of light is always less than c in matter and equals c only in a vacuum, the index of refraction (plural: indices of refraction) is always greater than or equal to one.

Table 16.2 lists the indices of refraction in various common materials.

Medium	*n*
Gases at 0 °C and 1 atm	

Table 16.2 Indices of Refraction The table lists the indices of refraction for various materials that are transparent to light. Note, that light travels the slowest in the materials with the greatest indices of refraction.

Medium	n
Air	1.000293
Carbon dioxide	1.00045
Hydrogen	1.000139
Oxygen	1.000271
Liquids at 20 °C	
Benzene	1.501
Carbon disulfide	1.628
Carbon tetrachloride	1.461
Ethanol	1.361
Glycerin	1.473
Water, fresh	1.333
Solids at 20 °C	
Diamond	2.419
Fluorite	1.434
Glass, crown	1.52
Glass, flint	1.66
Ice at 0 °C	1.309
Plexiglas	1.51
Polystyrene	1.49
Quartz, crystalline	1.544
Quartz, fused	1.458
Sodium chloride	1.544

Table 16.2 Indices of Refraction The table lists the indices of refraction for various materials that are transparent to light. Note, that light travels the slowest in the materials with the greatest indices of refraction.

Medium	n
Zircon	1.923

Table 16.2 Indices of Refraction The table lists the indices of refraction for various materials that are transparent to light. Note, that light travels the slowest in the materials with the greatest indices of refraction.

Figure 16.17 provides an analogy for and a description of how a ray of light changes direction when it passes from one medium to another. As in the previous section, the angles are measured relative to a perpendicular to the surface at the point where the light ray crosses it. The change in direction of the light ray depends on how the speed of light changes. The change in the speed of light is related to the indices of refraction of the media involved. In the situations shown in Figure 16.17, medium 2 has a greater index of refraction than medium 1. This difference in index of refraction means that the speed of light is less in medium 2 than in medium 1. Note that, in Figure 16.17(a), the path of the ray moves closer to the perpendicular when the ray slows down. Conversely, in Figure 16.17(b), the path of the ray moves away from the perpendicular when the ray speeds up. The path is exactly reversible. In both cases, you can imagine what happens by thinking about pushing a lawn mower from a footpath onto grass, and vice versa. Going from the footpath to grass, the right front wheel is slowed and pulled to the side as shown. This is the same change in direction for light when it goes from a fast medium to a slow one. When going from the grass to the footpath, the left front wheel moves faster than the others, and the mower changes direction as shown. This, too, is the same change in direction as light going from slow to fast.

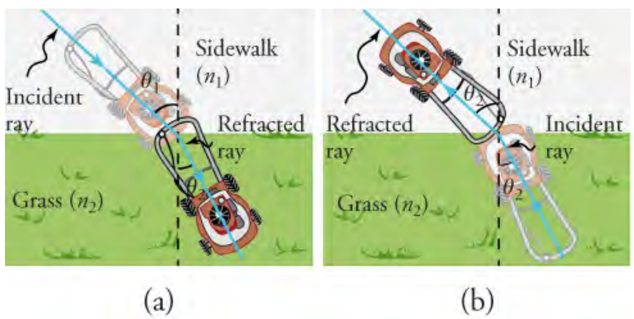


Figure 16.17 The change in direction of a light ray depends on how the speed of light changes when it crosses from one medium to another. For the situations shown here, the speed of light is greater in medium 1 than in medium 2. (a) A ray of light moves closer to the perpendicular when it slows down. This is analogous to what happens when a lawnmower goes from a footpath (medium 1) to grass (medium 2). (b) A ray of light moves away from the perpendicular when it speeds up. This is analogous to what happens when a lawnmower goes from grass (medium 2) to the footpath (medium 1). The paths are exactly reversible.

Snap Lab

Bent Pencil

A classic observation of refraction occurs when a pencil is placed in a glass filled halfway with water. Do this and observe the shape of the pencil when you look at it sideways through air, glass, and water.

- A full-length pencil
- A glass half full of water

Instructions

Procedure

1. Place the pencil in the glass of water.
2. Observe the pencil from the side.

3. Explain your observations.

Virtual Physics

Bending Light

Click to view content (https://www.openstax.org/l/28Bendinglight)

The Bending Light simulation in allows you to show light refracting as it crosses the boundaries between various media (download animation first to view). It also shows the reflected ray. You can move the protractor to the point where the light meets the boundary and measure the angle of incidence, the **angle of refraction**, and the angle of reflection. You can also insert a prism into the beam to view the spreading, or **dispersion**, of white light into colors, as discussed later in this section. Use the ray option at the upper left.

A light ray moving upward strikes a horizontal boundary at an acute angle relative to the perpendicular and enters the medium above the boundary. What must be true for the light to bend away from the perpendicular?

a. The medium below the boundary must have a greater index of refraction than the medium above.
b. The medium below the boundary must have a lower index of refraction than the medium above.
c. The medium below the boundary must have an index of refraction of zero.
d. The medium above the boundary must have an infinite index of refraction.

The amount that a light ray changes direction depends both on the incident angle and the amount that the speed changes. For a ray at a given incident angle, a large change in speed causes a large change in direction, and thus a large change in the angle of refraction. The exact mathematical relationship is the law of refraction, or ***Snell's law***, which is stated in equation form as

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \text{ or } \frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1}.$$

In terms of speeds, Snell's law becomes

$$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}. \tag{16.5}$$

Here, n_1 and n_2 are the indices of refraction for media 1 and 2, respectively, and θ_1 and θ_2 are the angles between the rays and the perpendicular in the respective media 1 and 2, as shown in Figure 16.17. The incoming ray is called the ***incident ray*** and the outgoing ray is called the ***refracted ray***. The associated angles are called the *angle of incidence* and the *angle of refraction*. Later, we apply Snell's law to some practical situations.

Dispersion is defined as the spreading of white light into the wavelengths of which it is composed. This happens because the index of refraction varies slightly with wavelength. Figure 16.18 shows how a prism disperses white light into the colors of the rainbow.

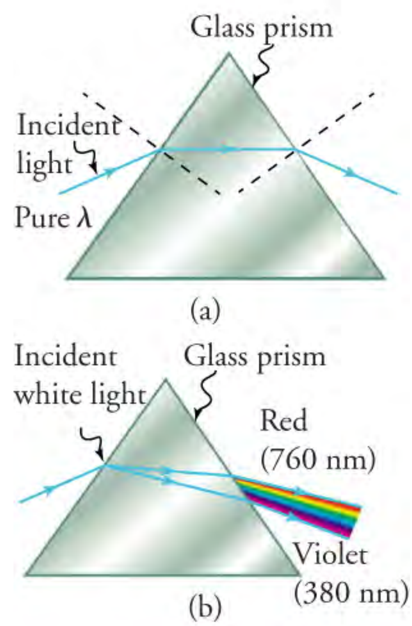


Figure 16.18 (a) A pure wavelength of light (λ) falls onto a prism and is refracted at both surfaces. (b) White light is dispersed by the prism (spread of light exaggerated). Because the index of refraction varies with wavelength, the angles of refraction vary with wavelength. A sequence of red to violet is produced, because the index of refraction increases steadily with decreasing wavelength.

Rainbows are produced by a combination of refraction and reflection. You may have noticed that you see a rainbow only when you turn your back to the Sun. Light enters a drop of water and is reflected from the back of the drop, as shown in Figure 16.19. The light is refracted both as it enters and as it leaves the drop. Because the index of refraction of water varies with wavelength, the light is dispersed and a rainbow is observed.

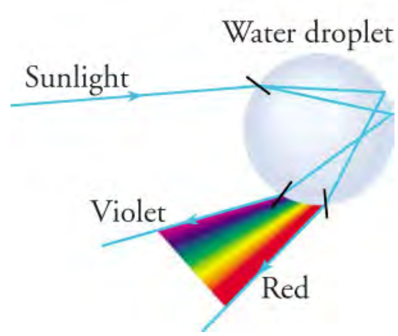


Figure 16.19 Part of the light falling on this water drop enters and is reflected from the back of the drop. This light is refracted and dispersed both as it enters and as it leaves the drop.

WATCH PHYSICS

Dispersion

This video explains how refraction disperses white light into its composite colors.

Click to view content (https://www.openstax.org/l/28Raindrop)

Which colors of the rainbow bend most when refracted?

a. Colors with a longer wavelength and higher frequency bend most when refracted.
b. Colors with a shorter wavelength and higher frequency bend most when refracted.
c. Colors with a shorter wavelength and lower frequency bend most when refracted.
d. Colors with a longer wavelength and a lower frequency bend most when refracted.

A good-quality mirror reflects more than 90 percent of the light that falls on it; the mirror absorbs the rest. But, it would be useful to have a mirror that reflects all the light that falls on it. Interestingly, we can produce total reflection using an aspect of refraction. Consider what happens when a ray of light strikes the surface between two materials, such as is shown in Figure 16.20(a). Part of the light crosses the boundary and is refracted; the rest is reflected. If, as shown in the figure, the index of refraction for the second medium is less than the first, the ray bends away from the perpendicular. Because $n_1 > n_2$, the angle of refraction is greater than the angle of incidence—that is, $\theta_2 > \theta_1$. Now, imagine what happens as the incident angle is increased. This causes θ_2 to increase as well. The largest the angle of refraction, θ_2, can be is 90°, as shown in Figure 16.20(b). The **critical angle**, θ_c, for a combination of two materials is defined to be the incident angle, θ_1, which produces an angle of refraction of 90°. That is, θ_c is the incident angle for which $\theta_2 = 90°$. If the incident angle, θ_1, is greater than the critical angle, as shown in Figure 16.20(c), then all the light is reflected back into medium 1, a condition called ***total internal reflection***.

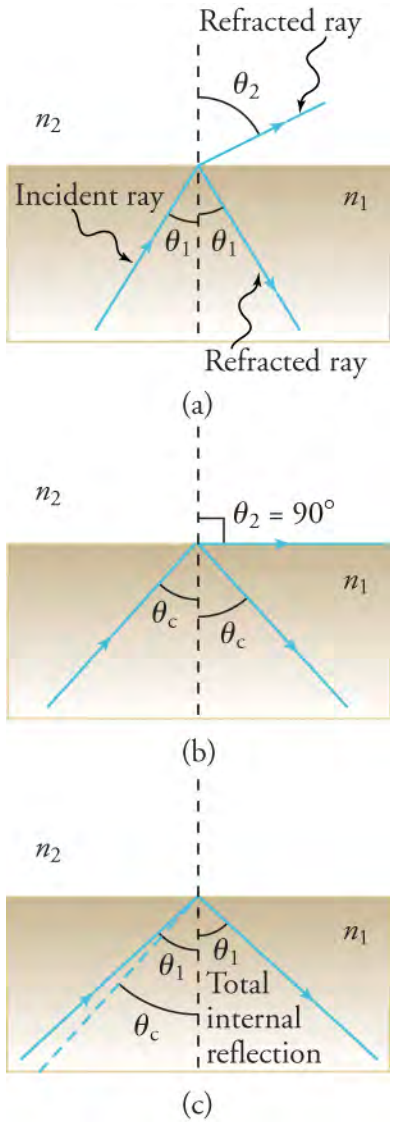


Figure 16.20 (a) A ray of light crosses a boundary where the speed of light increases and the index of refraction decreases—that is, $n_2 < n_1$. The refracted ray bends away from the perpendicular. (b) The critical angle, θ_c, is the one for which the angle of refraction is 90°. (c) Total internal reflection occurs when the incident angle is greater than the critical angle.

Recall that Snell's law states the relationship between angles and indices of refraction. It is given by

$$n_1 \sin\theta_1 = n_2 \sin\theta_2. \tag{16.6}$$

When the incident angle equals the critical angle ($\theta_1 = \theta_c$), the angle of refraction is 90° ($\theta_2 = 90°$). Noting that sin 90° = 1, Snell's law in this case becomes

$$n_1 \sin \theta_1 = n_2.$$

The critical angle, θ_c, for a given combination of materials is thus

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right),$$

for $n_1 > n_2$.

Total internal reflection occurs for any incident angle greater than the critical angle, θ_c, and it can only occur when the second medium has an index of refraction less than the first. Note that the previous equation is written for a light ray that travels in medium 1 and reflects from medium 2, as shown in Figure 16.20.

There are several important applications of total internal reflection. Total internal reflection, coupled with a large index of refraction, explains why diamonds sparkle more than other materials. The critical angle for a diamond-to-air surface is only 24.4°; so, when light enters a diamond, it has trouble getting back out (Figure 16.21). Although light freely enters the diamond at different angles, it can exit only if it makes an angle less than 24.4° with the normal to a given surface. Facets on diamonds are specifically intended to make this unlikely, so that the light can exit only in certain places. Diamonds with very few impurities are very clear, so the light makes many internal reflections and is concentrated at the few places it can exit—hence the sparkle.

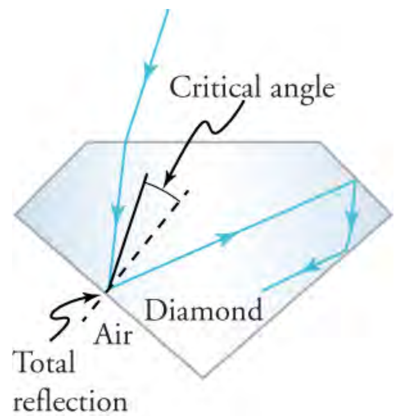


Figure 16.21 Light cannot escape a diamond easily because its critical angle with air is so small. Most reflections are total and the facets are placed so that light can exit only in particular ways, thus concentrating the light and making the diamond sparkle.

A light ray that strikes an object that consists of two mutually perpendicular reflecting surfaces is reflected back exactly parallel to the direction from which it came. This parallel reflection is true whenever the reflecting surfaces are perpendicular, and it is independent of the angle of incidence. Such an object is called a ***corner reflector*** because the light bounces from its inside corner. Many inexpensive reflector buttons on bicycles, cars, and warning signs have corner reflectors designed to return light in the direction from which it originates. Corner reflectors are perfectly efficient when the conditions for total internal reflection are satisfied. With common materials, it is easy to obtain a critical angle that is less than 45°. One use of these perfect *mirrors* is in binoculars, as shown in Figure 16.22. Another application is for periscopes used in submarines.

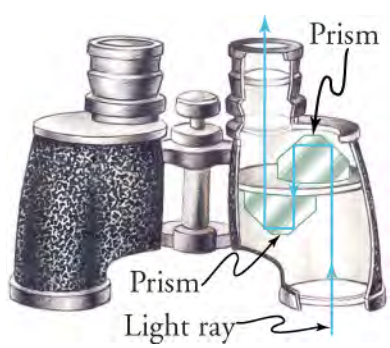


Figure 16.22 These binoculars use corner reflectors with total internal reflection to get light to the observer's eyes.

Fiber optics are one common application of total internal reflection. In communications, fiber optics are used to transmit telephone, internet, and cable TV signals, and they use the transmission of light down fibers of plastic or glass. Because the fibers are thin, light entering one is likely to strike the inside surface at an angle greater than the critical angle and, thus, be totally reflected (Figure 16.23). The index of refraction outside the fiber must be smaller than inside, a condition that is satisfied easily by coating the outside of the fiber with a material that has an appropriate refractive index. In fact, most fibers have a varying refractive index to allow more light to be guided along the fiber through total internal reflection. Rays are reflected around corners as shown in the figure, making the fibers into tiny *light pipes*.

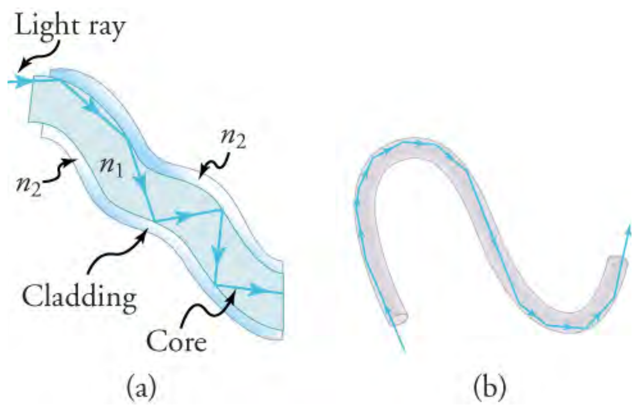


Figure 16.23 (a) Fibers in bundles are clad by a material that has a lower index of refraction than the core to ensure total internal reflection, even when fibers are in contact with one another. A single fiber with its cladding is shown. (b) Light entering a thin fiber may strike the inside surface at large, or grazing, angles, and is completely reflected if these angles exceed the critical angle. Such rays continue down the fiber, even following it around corners, because the angles of reflection and incidence remain large.

LINKS TO PHYSICS

Medicine: Endoscopes

A medical device called an *endoscope* is shown in Figure 16.24.

Figure 16.24 Endoscopes, such as the one drawn here, send light down a flexible fiber optic tube, which sends images back to a doctor in charge of performing a medical procedure.

The word *endoscope means looking inside*. Doctors use endoscopes to look inside hollow organs in the human body and inside body cavities. These devices are used to diagnose internal physical problems. Images may be transmitted to an eyepiece or sent to a video screen. Another channel is sometimes included to allow the use of small surgical instruments. Such surgical procedures include collecting biopsies for later testing, and removing polyps and other growths.

Identify the process that allows light and images to travel through a tube that is not straight.

a. The process is refraction of light.
b. The process is dispersion of light.
c. The process is total internal reflection of light.
d. The process is polarization of light.

Calculations with the Law of Refraction

The calculation problems that follow require application of the following equations:

$$n = \frac{c}{v}, \tag{16.8}$$

$$n_1 \sin\theta_1 = n_2 \sin\theta_2 \text{ or } \frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1},$$

and

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right), \text{ for } n_1 > n_2.$$

These are the equations for refractive index, the mathematical statement of the law of refraction (Snell's law), and the equation for the critical angle.

WATCH PHYSICS

Snell's Law Example 1

This video leads you through calculations based on the application of the equation that represents Snell's law.

Click to view content (https://www.openstax.org/l/28Snellslaw)

Which two types of variables are included in Snell's law?

a. The two types of variables are density of a material and the angle made by the light ray with the normal.
b. The two types of variables are density of a material and the thickness of a material.
c. The two types of variables are refractive index and thickness of each material.
d. The two types of variables are refractive index of a material and the angle made by a light ray with the normal.

WORKED EXAMPLE

Calculating Index of Refraction from Speed

Calculate the index of refraction for a solid medium in which the speed of light is 2.012×10^8 m/s, and identify the most likely substance, based on the previous table of indicies of refraction.

STRATEGY

We know the speed of light, *c*, is 3.00×10^8 m/s, and we are given *v*. We can simply plug these values into the equation for index of refraction, *n*.

Solution

$$n = \frac{c}{v} = \frac{3.00 \times 10^8 \text{m/s}}{2.012 \times 10^8 \text{m/s}} = 1.49 \tag{16.9}$$

This value matches that of polystyrene exactly, according to the table of indices of refraction (Table 16.2).

Discussion

The three-digit approximation for *c* is used, which in this case is all that is needed. Many values in the table are only given to three significant figures. Note that the units for speed cancel to yield a dimensionless answer, which is correct.

WORKED EXAMPLE

Calculating Index of Refraction from Angles

Suppose you have an unknown, clear solid substance immersed in water and you wish to identify it by finding its index of refraction. You arrange to have a beam of light enter it at an angle of 45.00°, and you observe the angle of refraction to be 40.30°. What are the index of refraction of the substance and its likely identity?

STRATEGY

We must use the mathematical expression for the law of refraction to solve this problem because we are given angle data, not speed data.

$$\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1} \tag{16.10}$$

The subscripts 1 and 2 refer to values for water and the unknown, respectively, where 1 represents the medium from which the light is coming and 2 is the new medium it is entering. We are given the angle values, and the table of indicies of refraction gives us n for water as 1.333. All we have to do before solving the problem is rearrange the equation

$$n_2 = \frac{n_1 \sin\theta_1}{\sin\theta_2}. \tag{16.11}$$

Solution

$$n_2 = \frac{(1.333)(0.7071)}{0.6468} = 1.457 \tag{16.12}$$

The best match from Table 16.2 is fused quartz, with $n = 1.458$.

Discussion

Note the relative sizes of the variables involved. For example, a larger angle has a larger sine value. This checks out for the two angles involved. Note that the smaller value of θ_2 compared with θ_1 indicates the ray has bent *toward* normal. This result is to be expected if the unknown substance has a greater n value than that of water. The result shows that this is the case.

WORKED EXAMPLE

Calculating Critical Angle

Verify that the critical angle for light going from water to air is 48.6°. (See Table 16.2, the table of indices of refraction.)

STRATEGY

First, choose the equation for critical angle

$$\theta_c = \sin^{-1}\left(\frac{n_2}{n_1}\right), \text{ for } n_1 > n_2. \tag{16.13}$$

Then, look up the n values for water, n_1, and air, n_2. Find the value of $\frac{n_2}{n_1}$. Last, find the angle that has a sine equal to this value and it compare with the given angle of 48.6°.

Solution

For water, $n_1 = 1.333$; for air, $n_2 = 1.0003$. So,

$$\begin{aligned} \tfrac{n_2}{n_1} &= \tfrac{1.0003}{1.333} = 0.7504 \\ \sin^{-1}(0.7504) &= 48.63°. \end{aligned} \tag{16.14}$$

Discussion

Remember, when we try to find a critical angle, we look for the angle at which light can no longer escape past a medium boundary by refraction. It is logical, then, to think of subscript 1 as referring to the medium the light is trying to leave, and subscript 2 as where it is trying (unsuccessfully) to go. So water is 1 and air is 2.

Practice Problems

6. The refractive index of ethanol is 1.36. What is the speed of light in ethanol?
   a. 2.25×108 m/s
   b. 2.21×107 m/s
   c. 2.25×109 m/s
   d. 2.21×108 m/s

7. The refractive index of air is 1.0003 and the refractive index of crystalline quartz is 1.544. What is the critical angle for a ray of light going from crystalline quartz into air?
   a. 49.61°
   b. 20.19°
   c. 0.6479 rad
   d. 0.7048 rad

Check Your Understanding

8. Which law is expressed by the equation $n_1 \sin \vartheta_1 = n_2 \sin \vartheta_2$?
  a. This is Ohm's law.
  b. This is Wien's displacement law.
  c. This is Snell's law.
  d. This is Newton's law.

9. Explain why the index of refraction is always greater than or equal to one.
  a. The formula for index of refraction, n, of a material is $n = \frac{\text{speed of light in a material}}{\text{speed of light in a vacuum}} = \frac{v}{c}$, where $\mathrm{v} > \mathrm{c}$, so n is always greater than one.
  b. The formula for index of refraction, n, of a material is $n = \frac{\text{speed of light in a vacuum}}{\text{speed of light in a material}} = \frac{c}{v}$, where $\mathrm{c} > \mathrm{v}$, so n is always greater than one.
  c. The formula for index of refraction, n, of a material is $n = \text{speed of light in a vacuum} \times \text{speed of light in a materaial} = c \times v$, where $\mathrm{c}, \mathrm{v} > 1$, so n is always greater than one.
  d. The formula for refractive index, n, of a material is $n = \frac{1}{\text{speed of light in a vacuum} \times \text{speed of light in a material}} = \frac{1}{c \times v}$, where $\mathrm{c} < \mathrm{v} < 1$, so n is always greater than one.

10. Write an equation that expresses the law of refraction.
  a. $\frac{n_1}{n_2} = \frac{\sin \theta_1}{\sin \theta_2}$
  b. $\frac{n_2}{n_1} = \left(\frac{\sin \theta_2}{\sin \theta_1}\right)^2$
  c. $\frac{n_1}{n_2} = \left(\frac{\sin \theta_2}{\sin \theta_1}\right)^2$
  d. $\frac{n_1}{n_2} = \frac{\sin \theta_2}{\sin \theta_1}$

16.3 Lenses

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe and predict image formation and magnification as a consequence of refraction through convex and concave lenses, use ray diagrams to confirm image formation, and discuss how these properties of lenses determine their applications
- Explain how the human eye works in terms of geometric optics
- Perform calculations, based on the thin-lens equation, to determine image and object distances, focal length, and image magnification, and use these calculations to confirm values determined from ray diagrams

Section Key Terms

aberration	chromatic aberration	concave lens	converging lens	convex lens
diverging lens	eyepiece	objective	ocular	parfocal

Characteristics of Lenses

Lenses are found in a huge array of optical instruments, ranging from a simple magnifying glass to the eye to a camera's zoom lens. In this section, we use the law of refraction to explore the properties of lenses and how they form images.

Some of what we learned in the earlier discussion of curved mirrors also applies to the study of lenses. Concave, convex, focal point F, and focal length *f* have the same meanings as before, except each measurement is made from the center of the lens instead of the surface of the mirror. The **convex lens** shown in Figure 16.25 has been shaped so that all light rays that enter it parallel to its central axis cross one another at a single point on the opposite side of the lens. The central axis, or axis, is defined to be a line normal to the lens at its center. Such a lens is called a **converging lens** because of the converging effect it has on light rays. An expanded view of the path of one ray through the lens is shown in Figure 16.25 to illustrate how the ray changes direction both as it enters and as it leaves the lens. Because the index of refraction of the lens is greater than that of air, the ray moves toward the perpendicular as it enters and away from the perpendicular as it leaves. (This is in accordance with the law of refraction.) As a result of the shape of the lens, light is thus bent toward the axis at both surfaces.

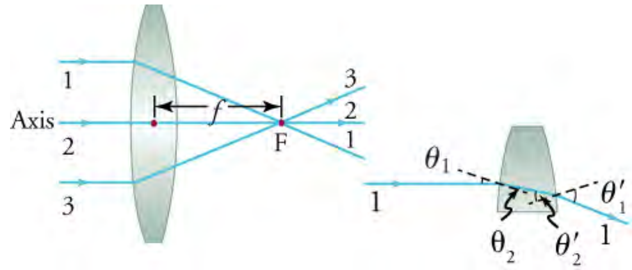


Figure 16.25 Rays of light entering a convex, or converging, lens parallel to its axis converge at its focal point, F. Ray 2 lies on the axis of the lens. The distance from the center of the lens to the focal point is the focal length, *f*, of the lens. An expanded view of the path taken by ray 1 shows the perpendiculars and the angles of incidence and refraction at both surfaces.

Note that rays from a light source placed at the focal point of a converging lens emerge parallel from the other side of the lens. You may have heard of the trick of using a converging lens to focus rays of sunlight to a point. Such a concentration of light energy can produce enough heat to ignite paper.

Figure 16.26 shows a **concave lens** and the effect it has on rays of light that enter it parallel to its axis (the path taken by ray 2 in the figure is the axis of the lens). The concave lens is a **diverging lens** because it causes the light rays to bend away (diverge) from its axis. In this case, the lens has been shaped so all light rays entering it parallel to its axis appear to originate from the same point, F, defined to be the focal point of a diverging lens. The distance from the center of the lens to the focal point is again called the *focal length*, or "*f*," of the lens. Note that the focal length of a diverging lens is defined to be negative. An expanded view of the path of one ray through the lens is shown in Figure 16.26 to illustrate how the shape of the lens, together with the law of refraction, causes the ray to follow its particular path and diverge.

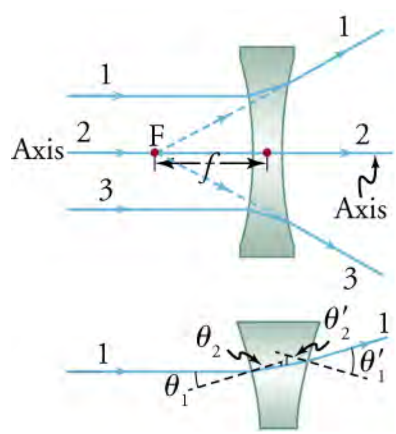


Figure 16.26 Rays of light enter a concave, or diverging, lens parallel to its axis diverge and thus appear to originate from its focal point, F. The dashed lines are not rays; they indicate the directions from which the rays appear to come. The focal length, *f*, of a diverging lens is negative. An expanded view of the path taken by ray 1 shows the perpendiculars and the angles of incidence and refraction at both surfaces.

The power, *P*, of a lens is very easy to calculate. It is simply the reciprocal of the focal length, expressed in meters

$$P = \frac{1}{f}. \tag{16.15}$$

The units of power are diopters, D, which are expressed in reciprocal meters. If the focal length is negative, as it is for the diverging lens in Figure 16.26, then the power is also negative.

In some circumstances, a lens forms an image at an obvious location, such as when a movie projector casts an image onto a screen. In other cases, the image location is less obvious. Where, for example, is the image formed by eyeglasses? We use ray tracing for thin lenses to illustrate how they form images, and we develop equations to describe the image-formation quantitatively. These are the rules for ray tracing:

1. A ray entering a converging lens parallel to its axis passes through the focal point, F, of the lens on the other side
2. A ray entering a diverging lens parallel to its axis seems to come from the focal point, F, on the side of the entering ray
3. A ray passing through the center of either a converging or a diverging lens does not change direction
4. A ray entering a converging lens through its focal point exits parallel to its axis
5. A ray that enters a diverging lens by heading toward the focal point on the opposite side exits parallel to the axis

Consider an object some distance away from a converging lens, as shown in Figure 16.27. To find the location and size of the image formed, we trace the paths of select light rays originating from one point on the object. In this example, the originating point is the top of a woman's head. Figure 16.27 shows three rays from the top of the object that can be traced using the ray-tracing rules just listed. Rays leave this point traveling in many directions, but we concentrate on only a few, which have paths that are easy to trace. The first ray is one that enters the lens parallel to its axis and passes through the focal point on the other side (rule 1). The second ray passes through the center of the lens without changing direction (rule 3). The third ray passes through the nearer focal point on its way into the lens and leaves the lens parallel to its axis (rule 4). All rays that come from the same point on the top of the person's head are refracted in such a way as to cross at the same point on the other side of the lens. The image of the top of the person's head is located at this point. Rays from another point on the object, such as the belt buckle, also cross at another common point, forming a complete image, as shown. Although three rays are traced in Figure 16.27, only two are necessary to locate the image. It is best to trace rays for which there are simple ray-tracing rules. Before applying ray tracing to other situations, let us consider the example shown in Figure 16.27 in more detail.

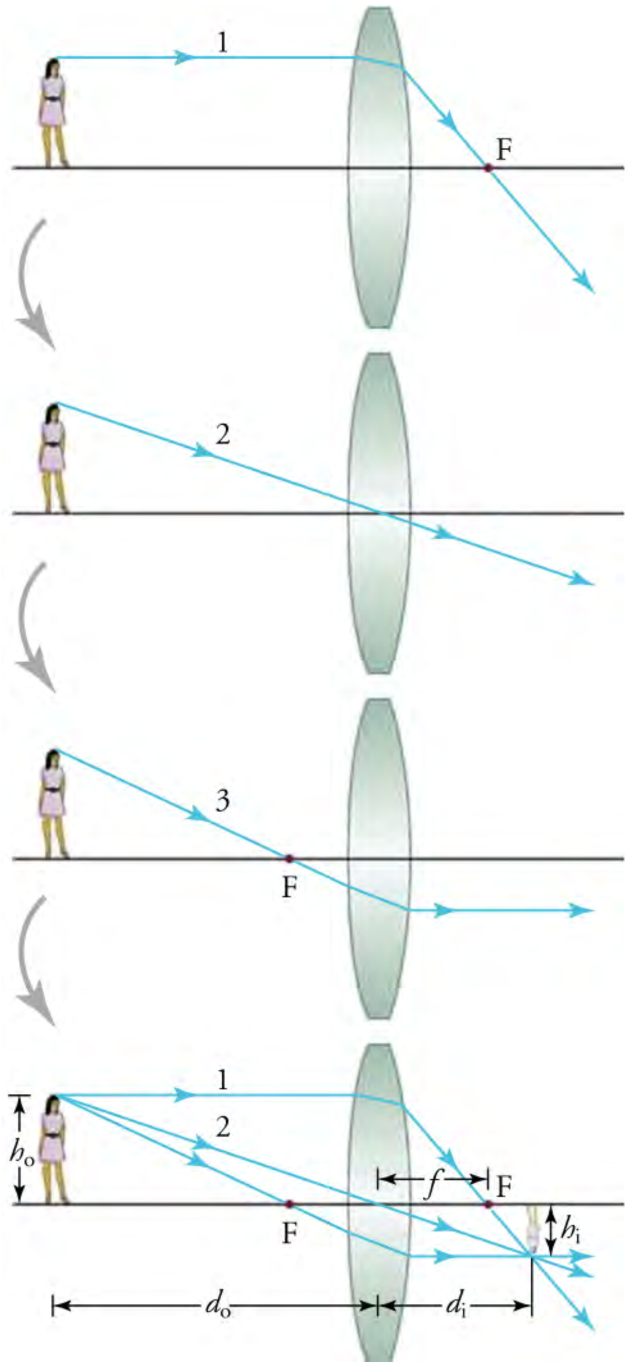


Figure 16.27 Ray tracing is used to locate the image formed by a lens. Rays originating from the same point on the object are traced. The three chosen rays each follow one of the rules for ray tracing, so their paths are easy to determine. The image is located at the point where the rays cross. In this case, a real image—one that can be projected on a screen—is formed.

The image formed in Figure 16.27 is a real image—meaning, it can be projected. That is, light rays from one point on the object actually cross at the location of the image and can be projected onto a screen, a piece of film, or the retina of an eye.

In Figure 16.27, the object distance, d_o, is greater than f. Now we consider a ray diagram for a convex lens where $d_o < f$, and another diagram for a concave lens.

Virtual Physics

Geometric Optics

[Click to view content (https://www.openstax.org/l/28Geometric)](https://www.openstax.org/l/28Geometric)

This animation shows you how the image formed by a convex lens changes as you change object distance, curvature radius, refractive index, and diameter of the lens. To begin, choose Principal Rays in the upper left menu and then try varying some of the parameters indicated at the upper center. Show Help supplies a few helpful labels.

How does the focal length, f, change with an increasing radius of curvature? How does f change with an increasing refractive index?

a. The focal length increases in both cases: when the radius of curvature and the refractive index increase.
b. The focal length decreases in both cases: when the radius of curvature and the refractive index increase.
c. The focal length increases when the radius of curvature increases; it decreases when the refractive index increases.
d. The focal length decreases when the radius of curvature increases; it increases in when the refractive index increases.

Type	Formed When	Image Type	d_i	M
Case 1	f positive, $d_o > f$	Real	Positive	Negative m >, <, or = −1
Case 2	f positive, $d_o < f$	Virtual	Negative	Positive $m > 1$
Case 3	f negative	Virtual	Negative	Positive $m < 1$

Table 16.3 Three Types of Images Formed by Lenses

The examples in Figure 16.27 and Figure 16.28 represent the three possible cases—case 1, case 2, and case 3—summarized in Table 16.3. In the table, m is magnification; the other symbols have the same meaning as they did for curved mirrors.

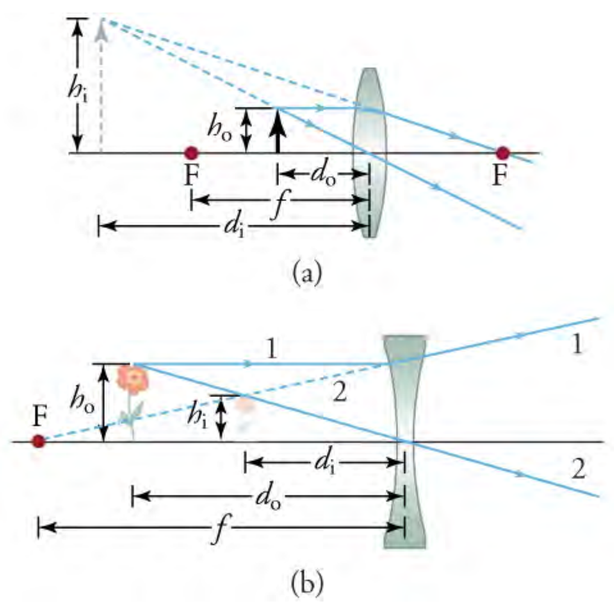


Figure 16.28 (a) The image is virtual and larger than the object. (b) The image is virtual and smaller than the object.

Snap Lab

Focal Length

- Temperature extremes—Very hot or very cold temperatures are encountered in this lab that can cause burns. Use protective mitts, eyewear, and clothing when handling very hot or very cold objects. Notify your teacher immediately of any burns.
- EYE SAFETY—Looking at the Sun directly can cause permanent eye damage. Do not look at the Sun through any lens.

- Several lenses
- A sheet of white paper
- A ruler or tape measure

Instructions

Procedure

1. Find several lenses and determine whether they are converging or diverging. In general, those that are thicker near the edges are diverging and those that are thicker near the center are converging.
2. On a bright, sunny day take the converging lenses outside and try focusing the sunlight onto a sheet of white paper.
3. Determine the focal lengths of the lenses. Have one partner slowly move the lens toward and away from the paper until you find the distance at which the light spot is at its brightest. Have the other partner measure the distance from the lens to the bright spot. Be careful, because the paper may start to burn, depending on the type of lens.

True or false—The bright spot that appears in focus on the paper is an image of the Sun.

a. True
b. False

Image formation by lenses can also be calculated from simple equations. We learn how these calculations are carried out near the end of this section.

Some common applications of lenses with which we are all familiar are magnifying glasses, eyeglasses, cameras, microscopes, and telescopes. We take a look at the latter two examples, which are the most complex. We have already seen the design of a telescope that uses only mirrors in . Figure 16.29 shows the design of a telescope that uses two lenses. Part (a) of the figure shows the design of the telescope used by Galileo. It produces an upright image, which is more convenient for many applications. Part (b) shows an arrangement of lenses used in many astronomical telescopes. This design produces an inverted image, which is less of a problem when viewing celestial objects.

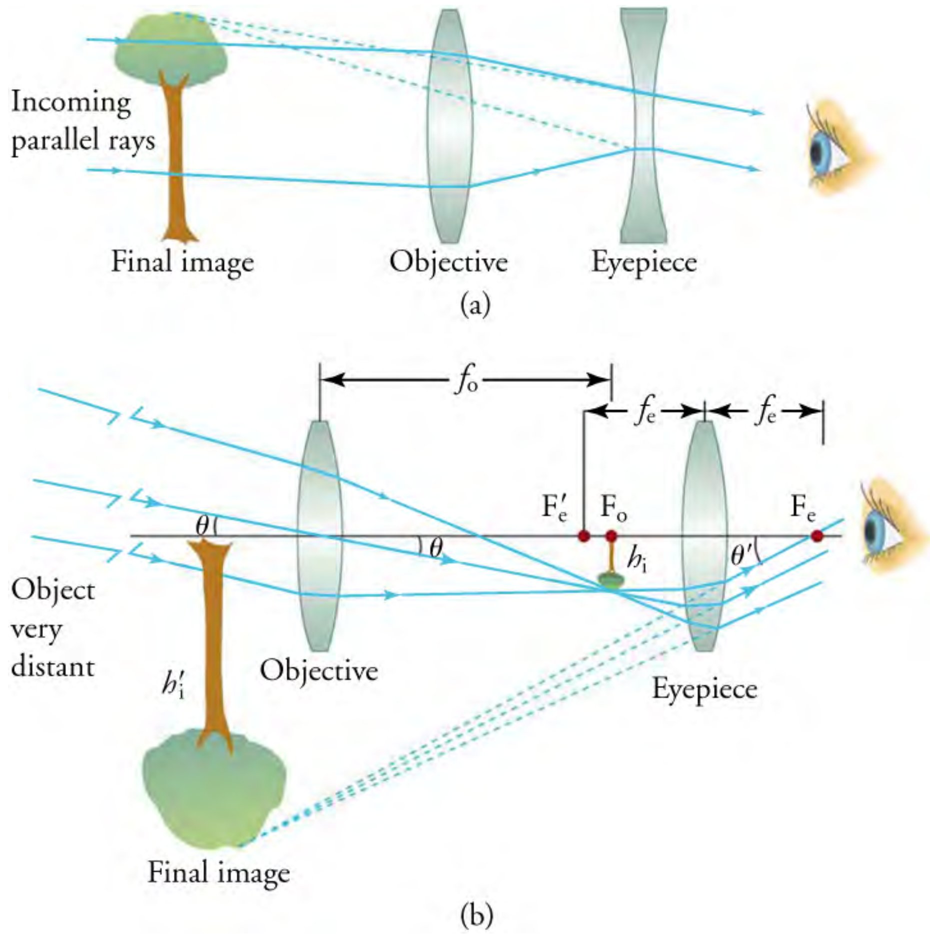


Figure 16.29 (a) Galileo made telescopes with a convex objective and a concave eyepiece. They produce an upright image and are used in spyglasses. (b) Most simple telescopes have two convex lenses. The objective forms a case 1 image, which is the object for the eyepiece. The eyepiece forms a case 2 final image that is magnified.

Figure 16.30 shows the path of light through a typical microscope. Microscopes were first developed during the early 1600s by eyeglass makers in the Netherlands and Denmark. The simplest compound microscope is constructed from two convex lenses, as shown schematically in Figure 16.30. The first lens is called the *objective lens*; it has typical magnification values from 5 × to 100 × . In standard microscopes, the **objectives** are mounted such that when you switch between them, the sample remains in focus. Objectives arranged in this way are described as ***parfocal***. The second lens, the **eyepiece**, also referred to as the ***ocular***, has several lenses that slide inside a cylindrical barrel. The focusing ability is provided by the movement of both the objective lens and the eyepiece. The purpose of a microscope is to magnify small objects, and both lenses contribute to the final magnification. In addition, the final enlarged image is produced in a location far enough from the observer to be viewed easily because the eye cannot focus on objects or images that are too close.

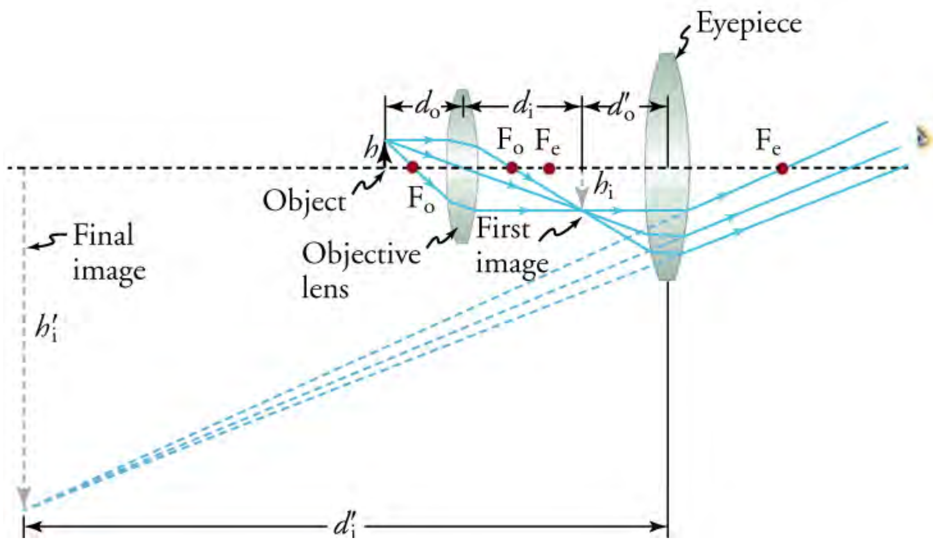


Figure 16.30 A compound microscope composed of two lenses, an objective and an eyepiece. The objective forms a case 1 image that is larger than the object. This first image is the object for the eyepiece. The eyepiece forms a case 2 final image that is magnified even further.

Real lenses behave somewhat differently from how they are modeled using rays diagrams or the thin-lens equations. Real lenses produce aberrations. An **aberration** is a distortion in an image. There are a variety of aberrations that result from lens size, material, thickness, and the position of the object. One common type of aberration is **chromatic aberration**, which is related to color. Because the index of refraction of lenses depends on color, or wavelength, images are produced at different places and with different magnifications for different colors. The law of reflection is independent of wavelength, so mirrors do not have this problem. This result is another advantage for the use of mirrors in optical systems such as telescopes.

Figure 16.31(a) shows chromatic aberration for a single convex lens, and its partial correction with a two-lens system. The index of refraction of the lens increases with decreasing wavelength, so violet rays are refracted more than red rays, and are thus focused closer to the lens. The diverging lens corrects this in part, although it is usually not possible to do so completely. Lenses made of different materials and with different dispersions may be used. For example, an achromatic doublet consisting of a converging lens made of crown glass in contact with a diverging lens made of flint glass can reduce chromatic aberration dramatically (Figure 16.31(b)).

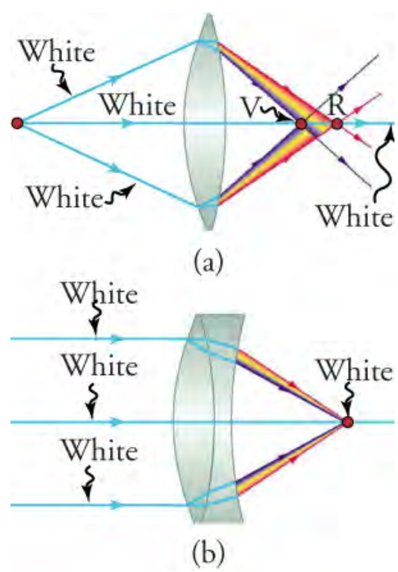


Figure 16.31 (a) Chromatic aberration is caused by the dependence of a lens's index of refraction on color (wavelength). The lens is more powerful for violet (V) than for red (R), producing images with different colors, locations, and magnifications. (b) Multiple-lens systems can correct chromatic aberrations in part, but they may require lenses of different materials and add to the expense of optical systems such as cameras.

Physics of the Eye

The eye is perhaps the most interesting of all optical instruments. It is remarkable in how it forms images and in the richness of detail and color they eye can detect. However, our eyes commonly need some correction to reach what is called *normal* vision, but should be called *ideal* vision instead. Image formation by our eyes and common vision correction are easy to analyze using geometric optics. Figure 16.32 shows the basic anatomy of the eye. The cornea and lens form a system that, to a good approximation, acts as a single thin lens. For clear vision, a real image must be projected onto the light-sensitive retina, which lies at a fixed distance from the lens. The lens of the eye adjusts its power to produce an image on the retina for objects at different distances. The center of the image falls on the fovea, which has the greatest density of light receptors and the greatest acuity (sharpness) in the visual field. There are no receptors at the place where the optic nerve meets the eye, which is called the *blind spot*. An image falling on this spot cannot be seen. The variable opening (or pupil) of the eye along with chemical adaptation allows the eye to detect light intensities from the lowest observable to 10^{10} times greater (without damage). Ten orders of magnitude is an incredible range of detection. Our eyes perform a vast number of functions, such as sense direction, movement, sophisticated colors, and distance. Processing of visual nerve impulses begins with interconnections in the retina and continues in the brain. The optic nerve conveys signals received by the eye to the brain.

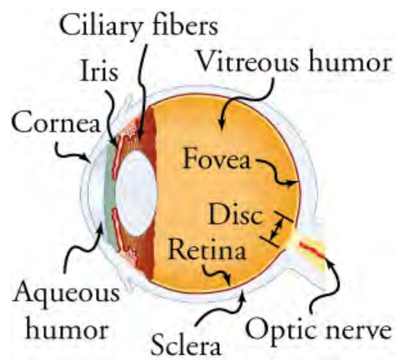


Figure 16.32 The cornea and lens of an eye act together to form a real image on the light-sensing retina, which has its densest concentration of receptors in the fovea, and a blind spot over the optic nerve. The power of the lens of an eye is adjustable to provide an image on the retina for varying object distances.

Refractive indices are crucial to image formation using lenses. Table 16.4 shows refractive indices relevant to the eye. The biggest change in the refractive index—and the one that causes the greatest bending of rays—occurs at the cornea rather than the lens. The ray diagram in Figure 16.33 shows image formation by the cornea and lens of the eye. The rays bend according to the refractive indices provided in Table 16.4. The cornea provides about two-thirds of the magnification of the eye because the speed of light changes considerably while traveling from air into the cornea. The lens provides the remaining magnification needed to produce an image on the retina. The cornea and lens can be treated as a single thin lens, although the light rays pass through several layers of material (such as the cornea, aqueous humor, several layers in the lens, and vitreous humor), changing direction at each interface. The image formed is much like the one produced by a single convex lens. This result is a case 1 image. Images formed in the eye are inverted, but the brain inverts them once more to make them seem upright.

Material	Index of Refraction
Water	1.33
Air	1.00
Cornea	1.38
Aqueous humor	1.34

*The index of refraction varies throughout the lens and is greatest at its center.

Table 16.4 Refractive Indices Relevant to the Eye

Material	Index of Refraction
Lens	1.41 average*
Vitreous humor	1.34

*The index of refraction varies throughout the lens and is greatest at its center.

Table 16.4 Refractive Indices Relevant to the Eye

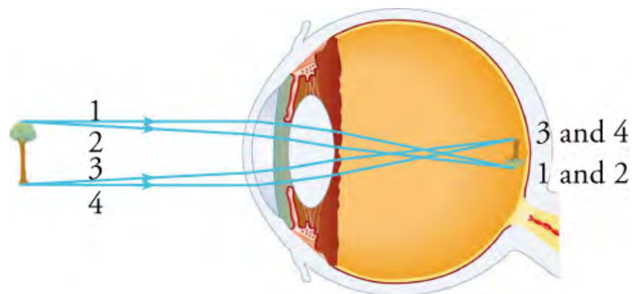


Figure 16.33 An image is formed on the retina, with light rays converging most at the cornea and on entering and exiting the lens. Rays from the top and bottom of the object are traced and produce an inverted real image on the retina. The distance to the object is drawn smaller than scale.

As noted, the image must fall precisely on the retina to produce clear vision—that is, the image distance, d_i, must equal the lens-to-retina distance. Because the lens-to-retina distance does not change, d_i must be the same for objects at all distances. The eye manages to vary the distance by varying the power (and focal length) of the lens to accommodate for objects at various distances. In Figure 16.33, you can see the small ciliary muscles above and below the lens that change the shape of the lens and, thus, the focal length.

The need for some type of vision correction is very common. Common vision defects are easy to understand, and some are simple to correct. Figure 16.34 illustrates two common vision defects. Nearsightedness, or myopia, is the inability to see distant objects clearly while close objects are in focus. The nearsighted eye *overconverges* the nearly parallel rays from a distant object, and the rays cross in front of the retina. More divergent rays from a close object are converged on the retina, producing a clear image. Farsightedness, or hyperopia, is the inability to see close objects clearly whereas distant objects may be in focus. A farsighted eye does not converge rays from a close object sufficiently to make the rays meet on the retina. Less divergent rays from a distant object can be converged for a clear image.

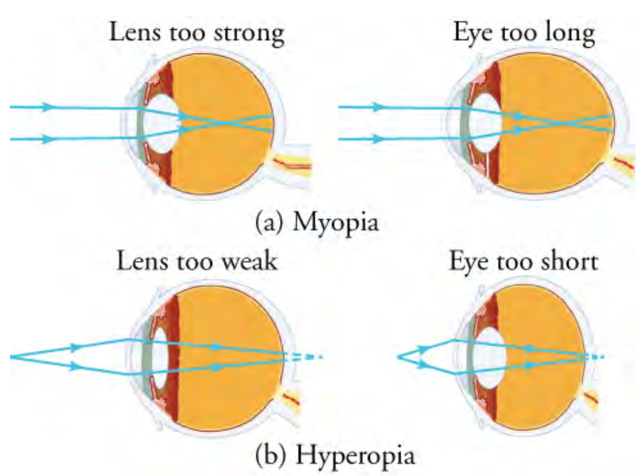


Figure 16.34 (a) The nearsighted (myopic) eye converges rays from a distant object in front of the retina; thus, they are diverging when they strike the retina, and produce a blurry image. This divergence can be caused by the lens of the eye being too powerful (in other words, too short a focal length) or the length of the eye being too great. (b) The farsighted (hyperopic) eye is unable to converge the rays from a close object by the time they strike the retina and produce ... blurry close vision. This poor convergence can be caused by insufficient power (in

other words, too long a focal length) in the lens or by the eye being too short.

Because the nearsighted eye overconverges light rays, the correction for nearsightedness involves placing a diverging spectacle lens in front of the eye. This lens reduces the power of an eye that has too short a focal length (Figure 16.35(a)). Because the farsighted eye *underconverges* light rays, the correction for farsightedness is to place a converging spectacle lens in front of the eye. This lens increases the power of an eye that has too long a focal length (Figure 16.35(b)).

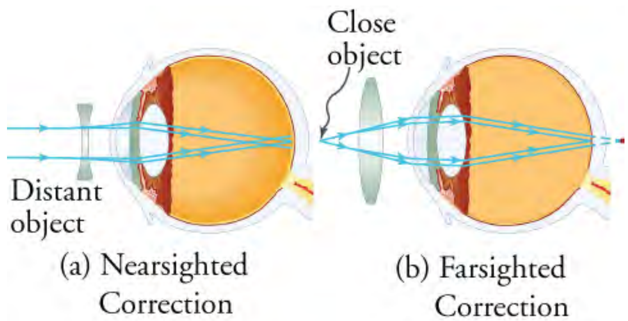


Figure 16.35 (a) Correction of nearsightedness requires a diverging lens that compensates for the overconvergence by the eye. The diverging lens produces an image closer to the eye than the object so that the nearsighted person can see it clearly. (b) Correction of farsightedness uses a converging lens that compensates for the underconvergence by the eye. The converging lens produces an image farther from the eye than the object so that the farsighted person can see it clearly. In both (a) and (b), the rays that meet at the retina represent corrected vision, and the other rays represent blurred vision without corrective lenses.

Calculations Using Lens Equations

As promised, there are no new equations to memorize. We can use equations already presented for solving problems involving curved mirrors. Careful analysis allows you to apply these equations to lenses. Here are the equations you need

$$P = \frac{1}{f},$$

where P is power, expressed in reciprocal meters (m^{-1}) rather than diopters (D), and f is focal length, expressed in meters (m). You also need

$$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o},$$

where, as before, d_o and d_i are object distance and image distance, respectively. Remember, this equation is usually more useful if rearranged to solve for one of the variables. For example,

$$d_i = \frac{f d_o}{d_o - f}.$$

The equations for magnification, m, are also the same as for mirrors

$$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o},$$

where h_i and h_o are the image height and object height, respectively. Remember, also, that a negative d_i value indicates a virtual image and a negative h_i value indicates an inverted image.

These are the steps to follow when solving a lens problem:

- Step 1. Examine the situation to determine that image formation by a lens is involved.
- Step 2. Determine whether ray tracing, the thin-lens equations, or both should be used. A sketch is very helpful even if ray tracing is not specifically required by the problem. Write useful symbols and values on the sketch.
- Step 3. Identify exactly what needs to be determined in the problem (identify the unknowns).
- Step 4. Make a list of what is given or can be inferred from the problem as stated (identify the knowns). It is helpful to determine whether the situation involves a case 1, 2, or 3 image. Although these are just names for types of images, they have certain characteristics (given in Table 16.3) that can be of great use in solving problems.

- Step 5. If ray tracing is required, use the ray-tracing rules listed earlier in this section.
- Step 6. Most quantitative problems require the use of the thin-lens equations. These equations are solved in the usual manner by substituting knowns and solving for unknowns. Several worked examples were included earlier and can serve as guides.
- Step 7. Check whether the answer is reasonable. Does it make sense? If you identified the type of image (case 1, 2, or 3) correctly, you should assess whether your answer is consistent with the type of image, magnification, and so on.

All problems will be solved by one or more of the equations just presented, with ray tracing used only for general analysis of the problem. The steps then simplify to the following:

1. Identify the unknown.
2. Identify the knowns.
3. Choose an equation, plug in the knowns, and solve for the unknown.

Here are some worked examples:

WORKED EXAMPLE

The Power of a Magnifying Glass

Strategy

The Sun is so far away that its rays are nearly parallel when they reach Earth. The magnifying glass is a convex (or converging) lens, focusing the nearly parallel rays of sunlight. Thus, the focal length of the lens is the distance from the lens to the spot, and its power, in diopters (D), is the inverse of this distance (in reciprocal meters).

Solution

The focal length of the lens is the distance from the center of the lens to the spot, which we know to be 8.00 cm. Thus,

$$f = 8.00 \text{ cm}. \tag{16.16}$$

To find the power of the lens, we must first convert the focal length to meters; then, we substitute this value into the equation for power.

$$P = \frac{1}{f} = \frac{1}{0.0800 \text{ m}} = 12.5 \text{ D} \tag{16.17}$$

Discussion

This result demonstrates a relatively powerful lens. Remember that the power of a lens in diopters should not be confused with the familiar concept of power in watts.

WORKED EXAMPLE

Image Formation by a Convex Lens

A clear glass light bulb is placed 0.75 m from a convex lens with a 0.50 m focal length, as shown in Figure 16.36. Use ray tracing to get an approximate location for the image. Then, use the mirror/lens equations to calculate (a) the location of the image and (b) its magnification. Verify that ray tracing and the thin-lens and magnification equations produce consistent results.

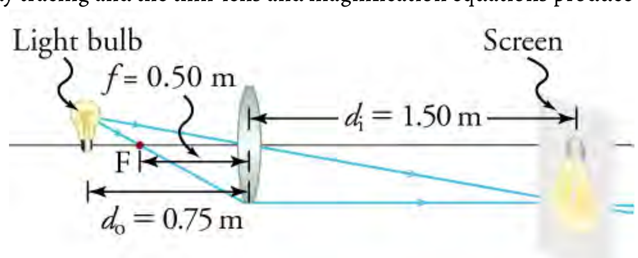


Figure 16.36 A light bulb placed 0.75 m from a lens with a 0.50 m focal length produces a real image on a poster board, as discussed in the previous example. Ray tracing predicts the image location and size.

Strategy

Because the object is placed farther away from a converging lens than the focal length of the lens, this situation is analogous to the one illustrated in the previous figure of a series of drawings showing a woman standing to the left of a lens. Ray tracing to scale should produce similar results for d_i. Numerical solutions for d_i and m can be obtained using the thin-lens and magnification equations, noting that $d_o = 0.75$ m and $f = 0.50$ m.

Solution

The ray tracing to scale in Figure 16.36 shows two rays from a point on the bulb's filament crossing about 1.50 m on the far side of the lens. Thus, the image distance, d_i, is about 1.50 m. Similarly, the image height based on ray tracing is greater than the object height by about a factor of two, and the image is inverted. Thus, m is about −2. The minus sign indicates the image is inverted. The lens equation can be rearranged to solve for d_i from the given information.

$$d_i = \frac{fd_o}{d_o - f} = \frac{(0.50)(0.75)}{0.75–0.50} = 1.5 \text{ m} \tag{16.18}$$

Now, we use $\frac{d_i}{d_o}$ to find m.

$$m = -\frac{d_i}{d_o} = -\frac{1.5}{0.75} = -2.0 \tag{16.19}$$

Discussion

Note that the minus sign causes the magnification to be negative when the image is inverted. Ray tracing and the use of the lens equation produce consistent results. The thin-lens equation gives the most precise results, and is limited only by the accuracy of the given information. Ray tracing is limited by the accuracy with which you draw, but it is highly useful both conceptually and visually.

WORKED EXAMPLE

Image Formation by a Concave Lens

Suppose an object, such as a book page, is held 6.50 cm from a concave lens with a focal length of −10.0 cm. Such a lens could be used in eyeglasses to correct pronounced nearsightedness. What magnification is produced?

Strategy

This example is identical to the preceding one, except that the focal length is negative for a concave or diverging lens. The method of solution is therefore the same, but the results are different in important ways.

Solution

$$d_i = \frac{fd_o}{d_o - f} = \frac{(-10.0)(6.50)}{6.50 - (-10.0)} = -3.94 \text{ cm} \tag{16.20}$$

Now the magnification equation can be used to find the magnification, m, because both d_i and d_o are known. Entering their values gives

$$m = -\frac{d_i}{d_o} = -\frac{-3.94}{6.50} = 0.606. \tag{16.21}$$

Discussion

A number of results in this example are true of all case 3 images. Magnification is positive (as calculated), meaning the image is upright. The magnification is also less than one, meaning the image is smaller than the object—in this case, a little more than half its size. The image distance is negative, meaning the image is on the same side of the lens as the object. The image is virtual. The image is closer to the lens than the object, because the image distance is smaller in magnitude than the object distance. The location of the image is not obvious when you look through a concave lens. In fact, because the image is smaller than the object, you may think it is farther away; however, the image is closer than the object—a fact that is useful in correcting nearsightedness.

WATCH PHYSICS

The Lens Equation and Problem Solving

The video shows calculations for both concave and convex lenses. It also explains real versus virtual images, erect versus inverted images, and the significance of negative and positive signs for the involved variables.

Click to view content (https://www.openstax.org/l/28Lenses)

If a lens has a magnification of $-\frac{1}{2}$, how does the image compare with the object in height and orientation?

a. The image is erect and is half as tall as the object.
b. The image is erect and twice as tall as the object.
c. The image is inverted and is half as tall as the object.
d. The image is inverted and is twice as tall as the object.

Practice Problems

11. A lens has a focal length of 12.5 cm. What is the power of the lens?
    a. The power of the lens is 0.0400 D.
    b. The power of the lens is 0.0800 D.
    c. The power of the lens is 4.00 D.
    d. The power of the lens is 8.00 D.

12. If a lens produces a 5.00 -cm tall image of an 8.00 -cm -high object when placed 10.0 cm from the lens, what is the apparent image distance? Construct a ray diagram using paper, a pencil, and a ruler to confirm your calculation.
    a. −3.12 cm
    b. −6.25 cm
    c. 3.12 cm
    d. 6.25 cm

Check Your Understanding

13. A lens has a magnification that is negative. What is the orientation of the image?
    a. Negative magnification means the image is erect and real.
    b. Negative magnification means the image is erect and virtual.
    c. Negative magnification means the image is inverted and virtual.
    d. Negative magnification means the image is inverted and real.

14. Which part of the eye controls the amount of light that enters?
    a. the pupil
    b. the iris
    c. the cornea
    d. the retina

15. An object is placed between the focal point and a convex lens. Describe the image that is formed in terms of its orientation, and whether the image is real or virtual.
    a. The image is real and erect.
    b. The image is real and inverted.
    c. The image is virtual and erect.
    d. The image is virtual and inverted.

16. A farsighted person buys a pair of glasses to correct her farsightedness. Describe the main symptom of farsightedness and the type of lens that corrects it.
    a. Farsighted people cannot focus on objects that are far away, but they can see nearby objects easily. A convex lens is used to correct this.
    b. Farsighted people cannot focus on objects that are close up, but they can see far-off objects easily. A concave lens is used to correct this.

c. Farsighted people cannot focus on objects that are close up, but they can see distant objects easily. A convex lens is used to correct this.
d. Farsighted people cannot focus on objects that are either close up or far away. A concave lens is used to correct this.

KEY TERMS

aberration a distortion in an image produced by a lens

angle of incidence the angle, with respect to the normal, at which a ray meets a boundary between media or a reflective surface

angle of reflection the angle, with respect to the normal, at which a ray leaves a reflective surface

angle of refraction the angle between the normal and the refracted ray

central axis a line perpendicular to the center of a lens or mirror extending in both directions

chromatic aberration an aberration related to color

concave lens a lens that causes light rays to diverge from the central axis

concave mirror a mirror with a reflective side that is curved inward

converging lens a convex lens

convex lens a lens that causes light rays to converge toward the central axis

convex mirror a mirror with a reflective side that is curved outward

critical angle an incident angle that produces an angle of refraction of 90°

dispersion separation of white light into its component wavelengths

diverging lens a concave lens

focal length the distance from the focal point to the mirror

focal point the point at which rays converge or appear to converge

incident ray the incoming ray toward a medium boundary or a reflective surface

index of refraction the speed of light in a vacuum divided by the speed of light in a given material

law of reflection the law that indicates the angle of reflection equals the angle of incidence

law of refraction the law that describes the relationship between refractive indices of materials on both sides of a boundary and the change in the path of light crossing the boundary, as given by the equation $n_1 \sin \theta_1 = n_2 \sin \theta_2$

ray light traveling in a straight line

real image an optical image formed when light rays converge and pass through the image, producing an image that can be projected onto a screen

refracted ray the light ray after it has been refracted

Snell's law the law of refraction expressed mathematically as $n_1 \sin \theta_1 = n_2 \sin \theta_2$

total internal reflection reflection of light traveling through a medium with a large refractive index at a boundary of a medium with a low refractive index under conditions such that refraction cannot occur

virtual image the point from which light rays appear to diverge without actually doing so

SECTION SUMMARY

16.1 Reflection

- The angle of reflection equals the angle of incidence.
- Plane mirrors and convex mirrors reflect virtual, erect images. Concave mirrors reflect light to form real, inverted images or virtual, erect images, depending on the location of the object.
- Image distance, height, and other characteristics can be calculated using the lens/mirror equation and the magnification equation.

16.2 Refraction

- The index of refraction for a material is given by the speed of light in a vacuum divided by the speed of light in that material.
- Snell's law states the relationship between indices of refraction, the incident angle, and the angle of refraction.
- The critical angle, θ_c, determines whether total internal refraction can take place, and can be calculated according to $\theta_c = sin^{-1}(\frac{n_2}{n_1})$.

16.3 Lenses

- The characteristics of images formed by concave and convex lenses can be predicted using ray tracing. Characteristics include real versus virtual, inverted versus upright, and size.
- The human eye and corrective lenses can be explained using geometric optics.
- Characteristics of images formed by lenses can be calculated using the mirror/lens equation.

KEY EQUATIONS

16.1 Reflection

lens/mirror equation (reciprocal version)	$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$
lens/mirror equation (solved version)	$f = \frac{d_i d_o}{d_o + d_i}$

magnification equation	$m = \frac{h_i}{h_o} = \frac{-d_i}{d_o}$
radius/focal length equation	$R = 2f$

critical angle	$\theta_c = \sin^{-1}(\frac{n_2}{n_1})$, for $n_1 > n_2$

16.2 Refraction

index of Refraction	$n = \frac{c}{v}$
Snell's law	$n_1 \sin\theta_1 = n_2 \sin\theta_2$ or $\frac{n_1}{n_2} = \frac{\sin\theta_2}{\sin\theta_1}$
>Snell's law in terms of speed	$\frac{\sin\theta_1}{\sin\theta_2} = \frac{v_1}{v_2}$

16.3 Lenses

power and focal length	$P = \frac{1}{f}$
mirror/lens (or thin-lens) equation	$\frac{1}{f} = \frac{1}{d_i} + \frac{1}{d_o}$
rearranged mirror/lens equation	$d_i = \frac{f d_o}{d_o - f}$
magnification equation	$m = \frac{h_i}{h_o} = -\frac{d_i}{d_o}$

CHAPTER REVIEW

Concept Items

16.1 Reflection

1. Part A. Can you see a virtual image? Part B. Can you photograph one? Explain your answers.
   a. A. yes; B. No, an image from a flat mirror cannot be photographed.
   b. A. no; B. Yes, an image from a flat mirror can be photographed.
   c. A. yes; B. Yes, an image from a flat mirror can be photographed.
   d. A. no; B. No, an image from a flat mirror cannot be photographed.

2. State the law of reflection.
   a. $\vartheta_r = \vartheta_i$, where ϑ_r is the angle of reflection and ϑ_i is the angle of incidence.
   b. $\vartheta_r > \vartheta_i$, where ϑ_r is the angle of reflection and ϑ_i is the angle of incidence.
   c. $\vartheta_r < \vartheta_i$, where ϑ_r is the angle of reflection and ϑ_i is the angle of incidence.
   d. $\vartheta_r = 0$, where ϑ_r is the angle of reflection.

16.2 Refraction

3. Does light change direction toward or away from the normal when it goes from air to water? Explain.
   a. The light bends away from the normal because the index of refraction of water is greater than that of air.
   b. The light bends away from the normal because the index of refraction of air is greater than that of water.
   c. The light bends toward the normal because the index of refraction of water is greater than that of air.
   d. The light bends toward the normal because the index of refraction of air is greater than that of water.

16.3 Lenses

4. An object is positioned in front of a lens with its base resting on the principal axis. Describe two rays that could be traced from the top of the object and through the lens that would locate the top of an image.
   a. A ray perpendicular to the axis and a ray through the center of the lens
   b. A ray parallel to the axis and a ray that does not pass through the center of the lens
   c. A ray parallel to the axis and a ray through the center of the lens
   d. A ray parallel to the axis and a ray that does not pass through the focal point

5. A person timing the moonrise looks at her watch and then at the rising moon. Describe what happened inside her eyes that allowed her to see her watch clearly one second and then see the moon clearly.
   a. The shape of the lens was changed by the sclera, and thus its focal length was also changed, so that each of the images focused on the retina.
   b. The shape of the lens was changed by the choroid, and thus its focal length was also changed, so that each of the images focused on the retina.
   c. The shape of the lens was changed by the iris, and thus its focal length was also changed, so that each of the images focused on the retina.

   d. The shape of the lens was changed by the muscles, and thus its focal length was also changed, so that each of the images focused on the retina.

6. For a concave lens, if the image distance, d_i, is negative, where does the image appear to be with respect to the object?
   a. The image always appears on the same side of the lens.
   b. The image appears on the opposite side of the lens.
   c. The image appears on the opposite side of the lens only if the object distance is greater than the focal length.
   d. The image appears on the same side of the lens only if the object distance is less than the focal length.

Critical Thinking Items

16.1 Reflection

7. Why are diverging mirrors often used for rear-view mirrors in vehicles? What is the main disadvantage of using such a mirror compared with a flat one?
   a. It gives a wide range of view. The image appears to be closer than the actual object.
   b. It gives a narrow range of view. The image appears to be farther than the actual object.
   c. It gives a narrow range of view. The image appears to be closer than the actual object.
   d. It gives a wide range of view. The image appears to be farther than the actual object.

16.2 Refraction

8. A high-quality diamond may be quite clear and colorless, transmitting all visible wavelengths with little absorption. Explain how it can sparkle with flashes of brilliant color when illuminated by white light.
   a. Diamond and air have a small difference in their refractive indices that results in a very small critical angle. The light that enters a diamond may exit at only a few points, and these points sparkle because many rays have been directed toward them.
   b. Diamond and air have a small difference in their refractive indices that results in a very large critical angle. The light that enters a diamond may exit at only a few points, and these points sparkle because many rays have been directed toward them.
   c. Diamond has a high index of refraction with respect to air, which results in a very small critical angle. The light that enters a diamond may exit at only a few points, and these points sparkle because many rays have been directed toward them.
   d. Diamond has a high index of refraction with respect to air, which results in a very large critical angle. The light that enters a diamond may exit at only a few points, and these points sparkle because many rays have been directed toward them.

9. The most common type of mirage is an illusion in which light from far-away objects is reflected by a pool of water that is not really there. Mirages are generally observed in deserts, where there is a hot layer of air near the ground. Given that the refractive index of air is less for air at higher temperatures, explain how mirages can be formed.
   a. The hot layer of air near the ground is lighter than the cooler air above it, but the difference in refractive index is small, which results in a large critical angle. The light rays coming from the horizon strike the hot air at large angles, so they are reflected as they would be from water.
   b. The hot layer of air near the ground is lighter than the cooler air above it, and the difference in refractive index is large, which results in a large critical angle. The light rays coming from the horizon strike the hot air at large angles, so they are reflected as they would be from water.
   c. The hot layer of air near the ground is lighter than the cooler air above it, but the difference in refractive index is small, which results in a small critical angle. The light rays coming from the horizon strike the hot air at large angles, so they are reflected as they would be from water.
   d. The hot layer of air near the ground is lighter than the cooler air above it, and the difference in the refractive index is large, which results in a small critical angle. The light rays coming from the horizontal strike the hot air at large angles, so they are reflected as they would be from water.

16.3 Lenses

10. When you focus a camera, you adjust the distance of the lens from the film. If the camera lens acts like a thin lens, why can it not be kept at a fixed distance from the film for both near and distant objects?
    a. To focus on a distant object, you need to increase the image distance.
    b. To focus on a distant object, you need to increase the focal length of the lens.
    c. To focus on a distant object, you need to decrease the focal length of the lens.
    d. To focus on a distant object, you may need to increase or decrease the focal length of the lens.

11. Part A—How do the refractive indices of the cornea,

aqueous humor, and the lens of the eye compare with the refractive index of air?
Part B—How do the comparisons in part A explain how images are focused on the retina?

a. (A) The cornea, aqueous humor, and lens of the eye have smaller refractive indices than air.
(B) Rays entering the eye are refracted away from the central axis, which causes them to meet at the focal point on the retina.
b. (A) The cornea, aqueous humor, and lens of the eye have greater refractive indices than air.
(B) Rays entering the eye are refracted away from the central axis, which causes them to meet at the focal point on the retina.
c. (A) The cornea, aqueous humor, and lens of the eye have smaller refractive indices than air.
(B) Rays entering the eye are refracted toward the central axis, which causes them to meet at the focal point on the retina.
d. (A) The cornea, aqueous humor, and lens of the eye have greater refractive indices than air.
(B) Rays entering the eye are refracted toward the central axis, which causes them to meet at the focal point on the retina.

Problems

16.1 Reflection

12. Some telephoto cameras use a mirror rather than a lens. What radius of curvature is needed for a concave mirror to replace a 0.800 -m focal-length telephoto lens?

a. 0.400 m
b. 1.60 m
c. 4.00 m
d. 16.0 m

13. What is the focal length of a makeup mirror that produces a magnification of 2.00 when a person's face is 8.00 cm away?

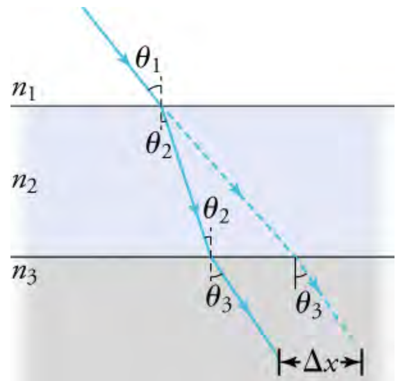
a. −16 cm
b. −5.3 cm
c. 5.3 cm
d. 16 cm

16.2 Refraction

14. An optical fiber uses flint glass ($n = 1.66$) clad with crown glass ($n = 1.52$) . What is the critical angle?

a. 33.2°
b. 23.7°
c. 0.92 rad
d. 1.16 rad

15. Suppose this figure represents a ray of light going from air ($n = 1.0003$) through crown glass ($n = 1.52$) into water, similar to a beam of light going into a fish tank.



Calculate the amount the ray is displaced by the glass (Δx), given that the incident angle is 40.0° and the glass is 1.00 cm thick.

a. 0.839 cm
b. 0.619 cm
c. 0.466 cm
d. 0.373 cm

16.3 Lenses

16. A camera's zoom lens has an adjustable focal length ranging from 80.0 to 200 mm . What is its range of powers?

a. The lowest power is 0.05 D and the highest power is 0.125 D.
b. The lowest power is 0.08 D and the highest power is 0.20 D.
c. The lowest power is 5.00 D and the highest power is 12.5 D.
d. The lowest power is 80 D and the highest power is 200 D.

17. Suppose a telephoto lens with a focal length of 200 mm is being used to photograph mountains 10.0 km away. (a) Where is the image? (b) What is the height of the image of a 1,000-m-high cliff on one of the mountains?

a. (a) The image is 0.200 m on the same side of the lens. (b) The height of the image is – 2.00 cm.
b. (a) The image is 0.200 m on the opposite side of the

lens. (b) The height of the image is – 2.00 cm.
c. (a) The image is 0.200 m on the opposite side of the lens. (b) The height of the image is +2.00 cm.
d. (a) The image is 0.100 m on the same side of the lens. (b) The height of the image is +2.00 cm.

Performance Task

16.3 Lenses

18. In this performance task, you will investigate the lens-like properties of a clear bottle.

- a water bottle or glass with a round cross-section and smooth, vertical sides
- enough water to fill the bottle
- a meter stick or tape measure
- a bright light source with a small bulb, such as a pen light
- a small bright object, such as a silver spoon.

Instructions

Procedure

1. Look through a clear glass or plastic bottle and describe what you see.
2. Next, fill the bottle with water and describe what you see.
3. Use the water bottle as a lens to produce the image of a bright object.
4. Estimate the focal length of the water bottle lens.
   a. How can you find the focal length of the lens using the light and a blank wall?
   b. How can you find the focal length of the lens using the bright object?
   c. Why did the water change the lens properties of the bottle?

TEST PREP

Multiple Choice

16.1 Reflection

19. In geometric optics, a straight line emerging from a point is called a (an) ________.
a. ray
b. focal point
c. image
d. object distance

20. An image of a 2.0 -cm object reflected from a mirror is 5.0 cm tall. What is the magnification of the mirror?
a. 0.4
b. 2.5
c. 3
d. 10

21. Can a virtual image be projected onto a screen with additional lenses or mirrors? Explain your answer.
a. Yes, the rays actually meet behind the lens or mirror.
b. No, the image is formed by rays that converge to a point in front of the mirror or lens.
c. Yes, any image that can be seen can be manipulated so that it can be projected onto a screen.
d. No, the image can only be perceived as being behind the lens or mirror.

16.2 Refraction

22. What does c represent in the equation $n = \frac{c}{v}$?
a. the critical angle
b. the refractive index
c. the speed of light in a vacuum
d. the speed of light in a transparent material

23. What is the term for the minimum angle at which a light ray is reflected back into a material and cannot pass into the surrounding medium?
a. critical angle
b. incident angle
c. angle of refraction
d. angle of reflection

24. Consider these indices of refraction: glass: 1.52, air: 1.0003, water: 1.333. Put these materials in order from the one in which the speed of light is fastest to the one in which it is slowest.
a. The speed of light in water > the speed of light in air > the speed of light in glass.
b. The speed of light in glass > the speed of light in water > the speed of light in air.
c. The speed of light in air > the speed of light in water > the speed of light in glass.
d. The speed of light in glass > the speed of light in air > the speed of light in water.

25. Explain why an object in water always appears to be at a depth that is more shallow than it actually is.
a. Because of the refraction of light, the light coming from the object bends toward the normal at the interface of water and air. This causes the object to appear at a location that is above the actual position of the object. Hence, the image appears to

be at a depth that is more shallow than the actual depth.

b. Because of the refraction of light, the light coming from the object bends away from the normal at the interface of water and air. This causes the object to appear at a location that is above the actual position of the object. Hence, the image appears to be at a depth that is more shallow than the actual depth.

c. Because of the refraction of light, the light coming from the object bends toward the normal at the interface of water and air. This causes the object to appear at a location that is below the actual position of the object. Hence, the image appears to be at a depth that is more shallow than the actual depth.

d. Because of the refraction of light, the light coming from the object bends away from the normal at the interface of water and air. This causes the object to appear at a location that is below the actual position of the object. Hence, the image appears to be at a depth that is more shallow than the actual depth.

16.3 Lenses

26. For a given lens, what is the height of the image divided by the height of the object ($\frac{h_i}{h_o}$) equal to?

a. power
b. focal length
c. magnification
d. radius of curvature

27. Which part of the eye has the greatest density of light receptors?

a. the lens
b. the fovea
c. the optic nerve
d. the vitreous humor

28. What is the power of a lens with a focal length of 10 cm?

a. 10 m^{-1}, or 10 D
b. 10 cm^{-1}, or 10 D
c. 10 m, or 10 D
d. 10 cm, or 10 D

29. Describe the cause of chromatic aberration.

a. Chromatic aberration results from the dependence of the frequency of light on the refractive index, which causes dispersion of different colors of light by a lens so that each color has a different focal point.
b. Chromatic aberration results from the dispersion of different wavelengths of light by a curved mirror so that each color has a different focal point.
c. Chromatic aberration results from the dependence of the reflection angle at a spherical mirror's surface on the distance of light rays from the principal axis so that different colors have different focal points.
d. Chromatic aberration results from the dependence of the wavelength of light on the refractive index, which causes dispersion of different colors of light by a lens so that each color has a different focal point.

Short Answer

16.1 Reflection

30. Distinguish between reflection and refraction in terms of how a light ray changes when it meets the interface between two media.

a. Reflected light penetrates the surface whereas refracted light is *bent* as it travels from one medium to the other.
b. Reflected light penetrates the surface whereas refracted light travels along a curved path.
c. Reflected light *bounces* from the surface whereas refracted light travels along a curved path.
d. Reflected light *bounces* from the surface whereas refracted light is *bent* as it travels from one medium to the other.

31. Sometimes light may be both reflected and refracted as it meets the surface of a different medium. Identify a material with a surface that when light travels through the air it is both reflected and refracted. Explain how this is possible.

a. Light passing through air is partially reflected and refracted when it meets a glass surface. It is reflected because glass has a smooth surface; it is refracted while passing into the transparent glass.
b. Light passing through air is partially reflected and refracted when it meets a glass surface. It is reflected because glass has a rough surface, and it is refracted while passing into the opaque glass.
c. Light passing through air is partially reflected and refracted when it meets a glass surface. It is reflected because glass has a smooth surface; it is refracted while passing into the opaque glass.
d. Light passing through air is partially reflected and refracted when it meets a glass surface. It is reflected because glass has a rough surface; it is refracted while passing into the transparent glass.

32. A concave mirror has a focal length of 5.00 cm. What is the image distance of an object placed 7.00 cm from the center of the mirror?

a. –17.5 cm
b. –2.92 cm
c. 2.92 cm
d. 17.5 cm

33. An 8.0 -cm tall object is placed 6.0 cm from a concave mirror with a magnification of –2.0. What are the image height and the image distance?

a. $h_i = -16$ cm, $d_i = -12$ cm
b. $h_i = -16$ cm, $d_i = 12$ cm
c. $h_i = 16$ cm, $d_i = -12$ cm
d. $h_i = 16$ cm, $d_i = 12$ cm

16.2 Refraction

34. At what minimum angle does total internal reflection of light occur if it travels from water ($n = 1.33$) toward ice ($n = 1.31$)?

a. 44.6°
b. 26.5°
c. 13.3°
d. 80.1°

35. Water floats on a liquid called *carbon tetrachloride*. The two liquids do not mix. A light ray passing from water into carbon tetrachloride has an incident angle of 45.0° and an angle of refraction of 40.1°. If the index of refraction of water is 1.33, what is the index of refraction of carbon tetrachloride?

a. 1.60
b. 1.49
c. 1.21
d. 1.46

36. Describe what happens to a light ray when it is refracted. Include in your explanation comparison of angles, comparison of refractive indices, and the term *normal*.

a. When a ray of light goes from one medium to another medium with a different refractive index, the ray changes its path as a result of interference. The angle between the ray and the normal (the line perpendicular to the surfaces of the two media) is greater in the medium with the greater refractive index.
b. When a ray of light goes from one medium to another medium with a different refractive index, the ray changes its path as a result of refraction. The angle between the ray and the normal (the line perpendicular to the surfaces of the two media) is less in the medium with the greater refractive index.
c. When a ray of light goes from one medium to another medium with a different refractive index, the ray does not change its path. The angle between the ray and the normal (the line parallel to the surfaces of the two media) is the same in both media.
d. When a ray of light goes from one medium to another medium with a different refractive index, the ray changes its path as a result of refraction. The angle between the ray and the normal (the line perpendicular to the surfaces of the two media) is less in the medium with the lower refractive index.

16.3 Lenses

37. What are two equivalent terms for a lens that always causes light rays to bend away from the principal axis?

a. a diverging lens or a convex lens
b. a diverging lens or a concave lens
c. a converging lens or a concave lens
d. a converging lens or a convex lens

38. Define the term *virtual image*.

a. A virtual image is an image that cannot be projected onto a screen.
b. A virtual image is an image that can be projected onto a screen.
c. A virtual image is an image that is formed on the opposite side of the lens from where the object is placed.
d. A virtual image is an image that is always bigger than the object.

39. Compare nearsightedness (myopia) and farsightedness (hyperopia) in terms of focal point.

a. The eyes of a nearsighted person have focal points beyond the retina. A farsighted person has eyes with focal points between the lens and the retina.
b. A nearsighted person has eyes with focal points between the lens and the retina. A farsighted person has eyes with focal points beyond the retina.
c. A nearsighted person has eyes with focal points between the lens and the choroid. A farsighted person has eyes with focal points beyond the choroid.
d. A nearsighted person has eyes with focal points between the lens and the retina. A farsighted person has eyes with focal points on the retina.

40. Explain how a converging lens corrects farsightedness.

a. A converging lens disperses the rays so they focus on the retina.
b. A converging lens bends the rays closer together so they do not focus on the retina.
c. A converging lens bends the rays closer together so they focus on the retina.

d. A converging lens disperses the rays so they do not focus on the retina.

41. Solve the equation $\frac{1}{d_o} + \frac{1}{d_i} = \frac{1}{f}$ for f in such a way that it is not expressed as a reciprocal.

a. $f = \frac{d_o+d_i}{d_o d_i}$
b. $f = \frac{d_o d_i}{d_i+d_o}$
c. $f = (d_i + d_o)$
d. $f = d_o d_i$

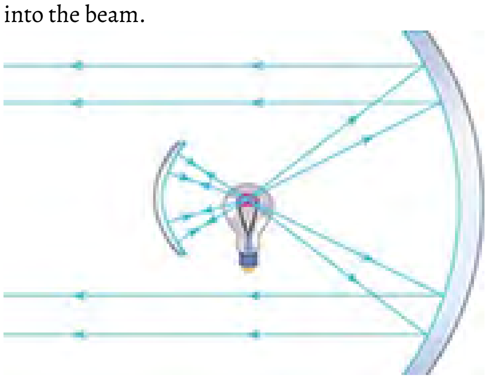
42. What is the magnification of a lens if it produces a 12-cm-high image of a 4 -cm -high object? The image is virtual and erect.

a. -3.00
b. $-\frac{1}{3.00}$
c. $\frac{1}{3.00}$
d. 3.00

Extended Response

16.1 Reflection

43. The diagram shows a lightbulb between two mirrors. One mirror produces a beam of light with parallel rays; the other keeps light from escaping without being put into the beam.



Where is the light source in relation to the focal point or radius of curvature of each mirror? Explain your answer.

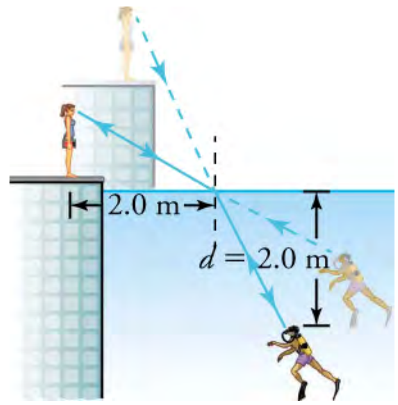
a. The bulb is at the center of curvature of the small mirror and at the focal point of the large mirror.
b. The bulb is at the focal point of the small mirror and at the focal point of the large mirror.
c. The bulb is at the center of curvature of the small mirror and at the center of curvature of the large mirror.
d. The bulb is at the focal point of the small mirror and at the center of curvature of the large mirror.

44. An object is placed $4.00\,\text{cm}$ in front of a mirror that has a magnification of 1.50. What is the radius of curvature of the mirror?

a. $-24.0\,\text{cm}$
b. $-4.80\,\text{cm}$
c. $4.80\,\text{cm}$
d. $24.0\,\text{cm}$

16.2 Refraction

45. A scuba diver training in a pool looks at his instructor, as shown in this figure. The angle between the ray in the water and the normal to the water is 25°.



What angle does the ray make from the instructor's face with the normal to the water ($n = 1.33$) at the point where the ray enters? Assume $n = 1.00$ for air.

a. 68°
b. 25°
c. 19°
d. 34°

46. Describe total internal reflection. Include a definition of the critical angle and how it is related to total internal reflection. Also, compare the indices of refraction of the interior material and the surrounding material.

a. When the interior material has a smaller index of refraction than the surrounding material, the incident ray may approach the boundary at an angle (called the *critical angle*) such that the refraction angle is 90° . The refracted ray cannot leave the interior, so it is reflected back inside and total internal reflection occurs.
b. When the interior material has a smaller index of refraction than the surrounding material, the incident ray may approach the boundary at an angle (called the *critical angle*) such that the refraction angle is less than 90° . The refracted ray cannot leave the interior, so it is reflected back inside and total internal reflection occurs.
c. When the interior material has the same index of refraction as the surrounding material, the incident ray approaches the boundary at an angle (called the *critical angle*) such that the refraction

angle is less than 90° . The refracted ray cannot leave the interior, so it is reflected back inside and total internal reflection occurs.

d. When the interior material has a greater index of refraction than the surrounding material, the incident ray may approach the boundary at an angle (called the *critical angle*) such that the refraction angle is 90° . The refracted ray cannot leave the interior, so it is reflected back inside and total internal reflection occurs.

16.3 Lenses

47. The muscles that change the shape of the lens in the eyes have become weak, causing vision problems for a person. In particular, the muscles cannot pull hard enough on the edges of the lens to make it less convex.
Part A—What condition does inability cause?
Part B—Where are images focused with respect to the retina?
Part C—Which type of lens corrects this person's problem? Explain.

a. Part A—This condition causes hyperopia.
Part B—Images are focused between the lens and the retina.
Part C—A converging lens gathers the rays slightly so they focus onto the retina.

b. Part A—This condition causes myopia.
Part B—Images are focused between the lens and the retina.
Part C—A converging lens gathers the rays slightly so they focus onto the retina.

c. Part A—This condition causes hyperopia.
Part B—Images are focused between the lens and the retina.
Part C—A diverging lens spreads the rays slightly so they focus onto the retina.

d. Part A—This condition causes myopia.
Part B—Images are focused between the lens and the retina.
Part C—A diverging lens spreads the rays slightly so they focus onto the retina.

48. If the lens-to-retina distance is $2.00\,\text{cm}$, what is the power of the eye when viewing an object $50.0\,\text{cm}$ away?

a. $-52.0\,\text{D}$
b. $0.52\,\text{D}$
c. $1.92\,\text{D}$
d. $52.0\,\text{D}$

CHAPTER 17
Diffraction and Interference

Figure 17.1 The colors reflected by this compact disc vary with angle and are not caused by pigments. Colors such as these are direct evidence of the wave character of light. (credit: Reggie Mathalone)

Chapter Outline

INTRODUCTION Examine a compact disc under white light, noting the colors observed and their locations on the disc. Using the CD, explore the spectra of a few light sources, such as a candle flame, an incandescent bulb, and fluorescent light. If you have ever looked at the reds, blues, and greens in a sunlit soap bubble and wondered how straw-colored soapy water could produce them, you have hit upon one of the many phenomena that can only be explained by the wave character of light. That and other interesting phenomena, such as the dispersion of white light into a rainbow of colors when passed through a narrow slit, cannot be explained fully by geometric optics. In such cases, light interacts with small objects and exhibits its wave characteristics. The topic of this chapter is the branch of optics that considers the behavior of light when it exhibits wave characteristics.

17.1 Understanding Diffraction and Interference

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain wave behavior of light, including diffraction and interference, including the role of constructive and destructive interference in Young's single-slit and double-slit experiments
- Perform calculations involving diffraction and interference, in particular the wavelength of light using data from a two-slit interference pattern

Section Key Terms

diffraction	Huygens's principle	monochromatic	wavefront

Diffraction and Interference

We know that visible light is the type of electromagnetic wave to which our eyes responds. As we have seen previously, light obeys the equation

$$c = f\lambda,$$

where $c = 3.00 \times 10^8$ m/s is the speed of light in vacuum, f is the frequency of the electromagnetic wave in Hz (or s^{-1}), and λ is its wavelength in m. The range of visible wavelengths is approximately 380 to 750 nm. As is true for all waves, light travels in straight lines and acts like a ray when it interacts with objects several times as large as its wavelength. However, when it interacts with smaller objects, it displays its wave characteristics prominently. Interference is the identifying behavior of a wave.

In Figure 17.2, both the ray and wave characteristics of light can be seen. The laser beam emitted by the observatory represents ray behavior, as it travels in a straight line. Passing a pure, one-wavelength beam through vertical slits with a width close to the wavelength of the beam reveals the wave character of light. Here we see the beam spreading out horizontally into a pattern of bright and dark regions that are caused by systematic constructive and destructive interference. As it is characteristic of wave behavior, interference is observed for water waves, sound waves, and light waves.

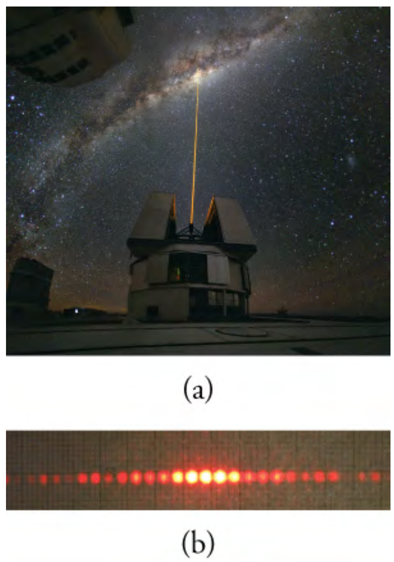


Figure 17.2 (a) The light beam emitted by a laser at the Paranal Observatory (part of the European Southern Observatory in Chile) acts like a ray, traveling in a straight line. (credit: Yuri Beletsky, European Southern Observatory) (b) A laser beam passing through a grid of vertical slits produces an interference pattern—characteristic of a wave. (credit: Shim'on and Slava Rybka, Wikimedia Commons)

That interference is a characteristic of energy propagation by waves is demonstrated more convincingly by water waves. Figure 17.3 shows water waves passing through gaps between some rocks. You can easily see that the gaps are similar in width to the wavelength of the waves and that this causes an interference pattern as the waves pass beyond the gaps. A cross-section across the waves in the foreground would show the crests and troughs characteristic of an interference pattern.

Figure 17.3 Incoming waves (at the top of the picture) pass through the gaps in the rocks and create an interference pattern (in the foreground).

Light has wave characteristics in various media as well as in a vacuum. When light goes from a vacuum to some medium, such as water, its speed and wavelength change, but its frequency, f, remains the same. The speed of light in a medium is $v = c/n$, where n is its index of refraction. If you divide both sides of the equation $c = f\lambda$ by n, you get $c/n = v = f\lambda/n$. Therefore, $v = f\lambda_n$, where λ_n is the wavelength in a medium, and

$$\lambda_n = \frac{\lambda}{n},$$

where λ is the wavelength in vacuum and n is the medium's index of refraction. It follows that the wavelength of light is smaller in any medium than it is in vacuum. In water, for example, which has $n = 1.333$, the range of visible wavelengths is (380 nm)/1.333 to (760 nm)/1.333, or $\lambda_n = 285–570$ nm. Although wavelengths change while traveling from one medium to another, colors do not, since colors are associated with frequency.

The Dutch scientist Christiaan Huygens (1629–1695) developed a useful technique for determining in detail how and where waves propagate. He used **wavefronts**, which are the points on a wave's surface that share the same, constant phase (such as all the points that make up the crest of a water wave). **Huygens's principle** states, "Every point on a wavefront is a source of wavelets that spread out in the forward direction at the same speed as the wave itself. The new wavefront is a line tangent to all of the wavelets."

Figure 17.4 shows how Huygens's principle is applied. A wavefront is the long edge that moves; for example, the crest or the trough. Each point on the wavefront emits a semicircular wave that moves at the propagation speed v. These are drawn later at a time, t, so that they have moved a distance $s = vt$. The new wavefront is a line tangent to the wavelets and is where the wave is located at time t. Huygens's principle works for all types of waves, including water waves, sound waves, and light waves. It will be useful not only in describing how light waves propagate, but also in how they interfere.

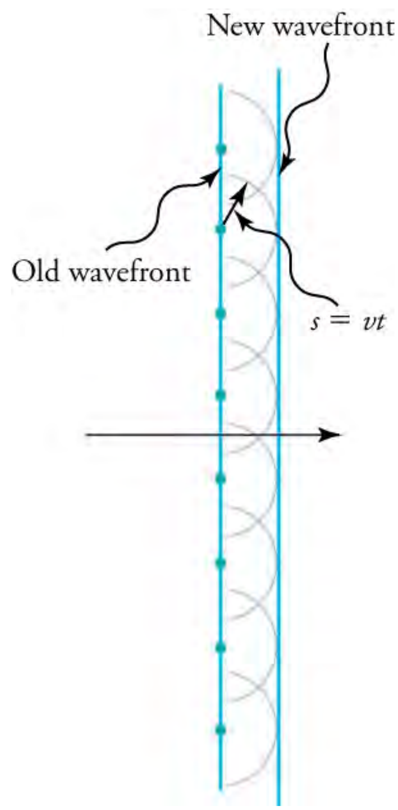


Figure 17.4 Huygens's principle applied to a straight wavefront. Each point on the wavefront emits a semicircular wavelet that moves a distance $s = vt$. The new wavefront is a line tangent to the wavelets.

What happens when a wave passes through an opening, such as light shining through an open door into a dark room? For light, you expect to see a sharp shadow of the doorway on the floor of the room, and you expect no light to bend around corners into other parts of the room. When sound passes through a door, you hear it everywhere in the room and, thus, you understand that sound spreads out when passing through such an opening. What is the difference between the behavior of sound waves and light waves in this case? The answer is that the wavelengths that make up the light are very short, so that the light acts like a ray. Sound has wavelengths on the order of the size of the door, and so it bends around corners.

If light passes through smaller openings, often called slits, you can use Huygens's principle to show that light bends as sound does (see Figure 17.5). The bending of a wave around the edges of an opening or an obstacle is called **diffraction**. Diffraction is a wave characteristic that occurs for all types of waves. If diffraction is observed for a phenomenon, it is evidence that the phenomenon is produced by waves. Thus, the horizontal diffraction of the laser beam after it passes through slits in Figure 17.2 is evidence that light has the properties of a wave.

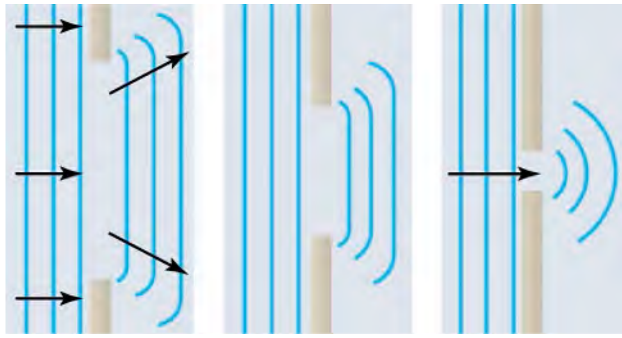


Figure 17.5 Huygens's principle applied to a straight wavefront striking an opening. The edges of the wavefront bend after passing through the opening, a process called diffraction. The amount of bending is more extreme for a small opening, consistent with the fact that wave characteristics are most noticeable for interactions with objects about the same size as the wavelength.

Once again, water waves present a familiar example of a wave phenomenon that is easy to observe and understand, as shown in Figure 17.6.

Figure 17.6 Ocean waves pass through an opening in a reef, resulting in a diffraction pattern. Diffraction occurs because the opening is similar in width to the wavelength of the waves.

WATCH PHYSICS

Single-Slit Interference

This video works through the math needed to predict diffraction patterns that are caused by single-slit interference.

Click to view content (https://www.openstax.org/l/28slit)

Which values of *m* denote the location of destructive interference in a single-slit diffraction pattern?

a. whole integers, excluding zero
b. whole integers
c. real numbers excluding zero
d. real numbers

The fact that Huygens's principle worked was not considered enough evidence to prove that light is a wave. People were also reluctant to accept light's wave nature because it contradicted the ideas of Isaac Newton, who was still held in high esteem. The acceptance of the wave character of light came after 1801, when the English physicist and physician Thomas Young (1773–1829) did his now-classic double-slit experiment (see Figure 17.7).

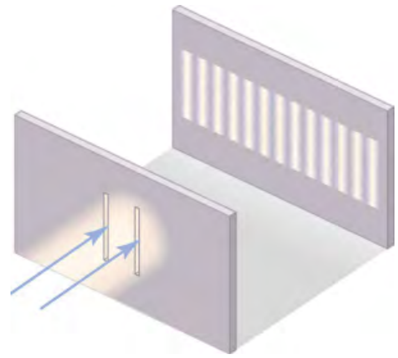


Figure 17.7 Young’s double-slit experiment. Here, light of a single wavelength passes through a pair of vertical slits and produces a diffraction pattern on the screen—numerous vertical light and dark lines that are spread out horizontally. Without diffraction and interference, the light would simply make two lines on the screen.

When light passes through narrow slits, it is diffracted into semicircular waves, as shown in Figure 17.8 (a). Pure constructive interference occurs where the waves line up crest to crest or trough to trough. Pure destructive interference occurs where they line up crest to trough. The light must fall on a screen and be scattered into our eyes for the pattern to be visible. An analogous pattern for water waves is shown in Figure 17.8 (b). Note that regions of constructive and destructive interference move out from the slits at well-defined angles to the original beam. Those angles depend on wavelength and the distance between the slits, as you will see below.

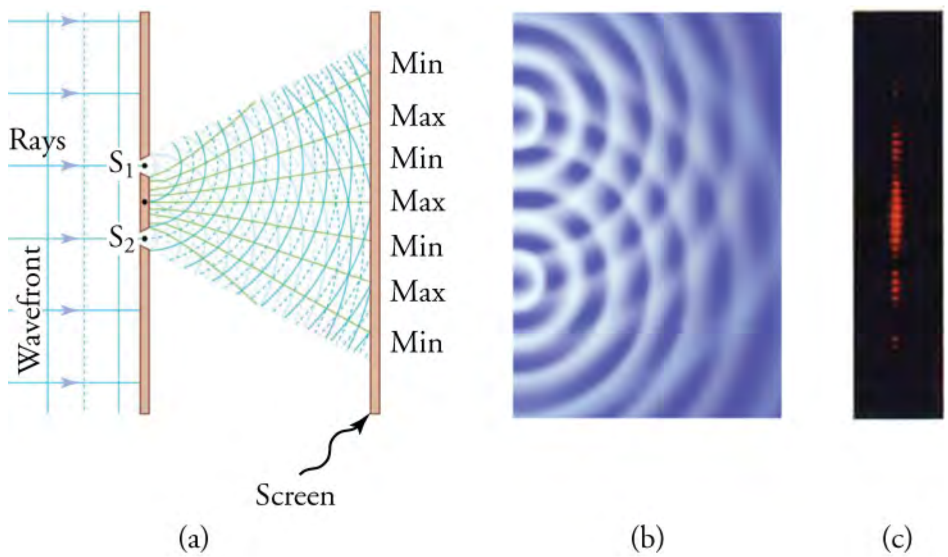


Figure 17.8 Double slits produce two sources of waves that interfere. (a) Light spreads out (diffracts) from each slit, because the slits are narrow. The waves overlap and interfere constructively (bright lines) and destructively (dark regions). You can only see the effect if the light falls onto a screen and is scattered into your eyes. (b) The double-slit interference pattern for water waves is nearly identical to that for light. Wave action is greatest in regions of constructive interference and least in regions of destructive interference. (c) When light that has passed through double slits falls on a screen, we see a pattern such as this.

Virtual Physics

Wave Interference

Click to view content (https://www.openstax.org/l/28interference)

This simulation demonstrates most of the wave phenomena discussed in this section. First, observe interference between two sources of electromagnetic radiation without adding slits. See how water waves, sound, and light all show interference patterns. Stay with light waves and use only one source. Create diffraction patterns with one slit and then with two. You may have to adjust slit width to see the pattern.

Visually compare the slit width to the wavelength. When do you get the best-defined diffraction pattern?

a. when the slit width is larger than the wavelength
b. when the slit width is smaller than the wavelength
c. when the slit width is comparable to the wavelength
d. when the slit width is infinite

Calculations Involving Diffraction and Interference

The fact that the wavelength of light of one color, or **monochromatic** light, can be calculated from its two-slit diffraction pattern in Young's experiments supports the conclusion that light has wave properties. To understand the basis of such calculations, consider how two waves travel from the slits to the screen. Each slit is a different distance from a given point on the screen. Thus different numbers of wavelengths fit into each path. Waves start out from the slits in phase (crest to crest), but they will end up out of phase (crest to trough) at the screen if the paths differ in length by half a wavelength, interfering destructively. If the paths differ by a whole wavelength, then the waves arrive in phase (crest to crest) at the screen, interfering constructively. More generally, if the paths taken by the two waves differ by any half-integral number of wavelengths $(\frac{1}{2}\lambda,\ \frac{3}{2}\lambda,\ \frac{5}{2}\lambda,\ \text{etc.})$, then destructive interference occurs. Similarly, if the paths taken by the two waves differ by any integral number of wavelengths $(\lambda,\ 2\lambda,\ 3\lambda,\ \text{etc.})$, then constructive interference occurs.

Figure 17.9 shows how to determine the path-length difference for waves traveling from two slits to a common point on a screen. If the screen is a large distance away compared with the distance between the slits, then the angle θ between the path and a line from the slits perpendicular to the screen (see the figure) is nearly the same for each path. That approximation and simple trigonometry show the length difference, ΔL, to be $d \sin\theta$, where d is the distance between the slits,

$$\Delta L = d\sin\theta.$$

To obtain constructive interference for a double slit, the path-length difference must be an integral multiple of the wavelength, or

$$d\sin\theta = m\lambda,\ \text{for } m = 0, 1, -1, 2, -2, \dots \text{(constructive)}.$$

Similarly, to obtain destructive interference for a double slit, the path-length difference must be a half-integral multiple of the wavelength, or

$$d\sin\theta = (m + \tfrac{1}{2})\lambda,\ \text{for } m = 0, 1, -1, 2, -2, \dots \text{(destructive)}.$$

The number m is the *order* of the interference. For example, $m = 4$ is fourth-order interference.

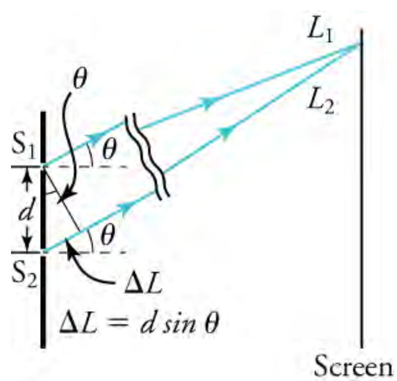


Figure 17.9 The paths from each slit to a common point on the screen differ by an amount $d \sin\theta$, assuming the distance to the screen is much greater than the distance between the slits (not to scale here).

Figure 17.10 shows how the intensity of the bands of constructive interference decreases with increasing angle.

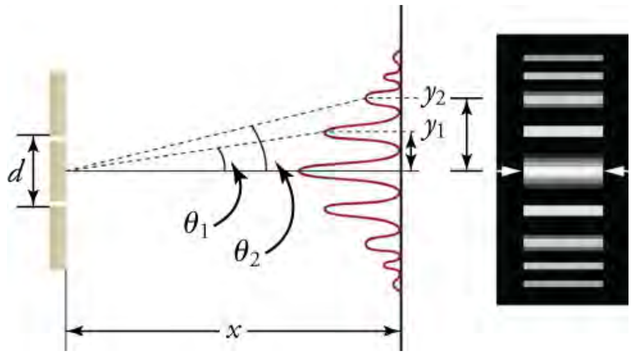


Figure 17.10 The interference pattern for a double slit has an intensity that falls off with angle. The photograph shows multiple bright and dark lines, or fringes, formed by light passing through a double slit.

Light passing through a single slit forms a diffraction pattern somewhat different from that formed by double slits. Figure 17.11 shows a single-slit diffraction pattern. Note that the central maximum is larger than those on either side, and that the intensity decreases rapidly on either side.

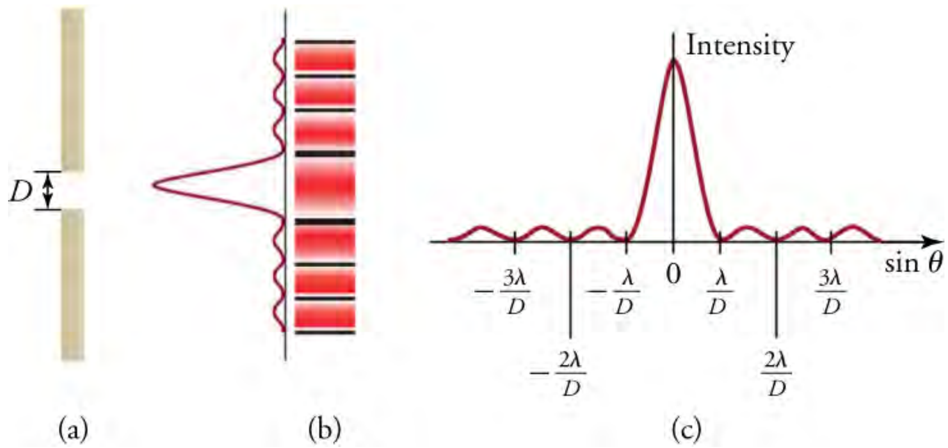


Figure 17.11 (a) Single-slit diffraction pattern. Monochromatic light passing through a single slit produces a central maximum and many smaller and dimmer maxima on either side. The central maximum is six times higher than shown. (b) The drawing shows the bright central maximum and dimmer and thinner maxima on either side. (c) The location of the minima are shown in terms of λ and D.

The analysis of single-slit diffraction is illustrated in Figure 17.12. Assuming the screen is very far away compared with the size of the slit, rays heading toward a common destination are nearly parallel. That approximation allows a series of trigonometric operations that result in the equations for the minima produced by destructive interference.

$$D \sin\theta = m\lambda$$

or

$$\frac{Dy}{L} = m\lambda$$

When rays travel straight ahead, they remain in phase and a central maximum is obtained. However, when rays travel at an angle θ relative to the original direction of the beam, each ray travels a different distance to the screen, and they can arrive in or out of phase. Thus, a ray from the center travels a distance $\lambda/2$ farther than the ray from the top edge of the slit, they arrive out of phase, and they interfere destructively. Similarly, for every ray between the top and the center of the slit, there is a ray between the center and the bottom of the slit that travels a distance $\lambda/2$ farther to the common point on the screen, and so interferes destructively. Symmetrically, there will be another minimum at the same angle below the direct ray.

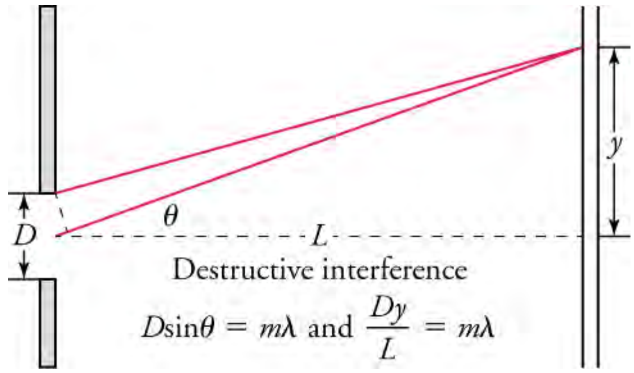


Figure 17.12 Equations for a single-slit diffraction pattern, where λ is the wavelength of light, D is the slit width, θ is the angle between a line from the slit to a minimum and a line perpendicular to the screen, L is the distance from the slit to the screen, y is the distance from the center of the pattern to the minimum, and m is a nonzero integer indicating the order of the minimum.

Below we summarize the equations needed for the calculations to follow.

The speed of light in a vacuum, c, the wavelength of the light, λ, and its frequency, f, are related as follows.

$$c = f\lambda$$

The wavelength of light in a medium, λ_n, compared to its wavelength in a vacuum, λ, is given by

$$\lambda_n = \frac{\lambda}{n}. \tag{17.1}$$

To calculate the positions of constructive interference for a double slit, the path-length difference must be an integral multiple, m, of the wavelength. λ

$$d\sin\theta = m\lambda,\ \text{for } m = 0, 1, -1, 2, -2, \ldots \text{(constructive)},$$

where d is the distance between the slits and θ is the angle between a line from the slits to the maximum and a line perpendicular to the barrier in which the slits are located. To calculate the positions of destructive interference for a double slit, the path-length difference must be a half-integral multiple of the wavelength:

$$d\sin\theta = (m + ½)\lambda,\ \text{for } m = 0, 1, -1, 2, -2, \ldots \text{(destructive)}.$$

For a single-slit diffraction pattern, the width of the slit, D, the distance of the first ($m = 1$) destructive interference minimum, y, the distance from the slit to the screen, L, and the wavelength, λ, are given by

$$\frac{Dy}{L} = \lambda.$$

Also, for single-slit diffraction,

$$D\sin\theta = m\lambda,$$

where θ is the angle between a line from the slit to the minimum and a line perpendicular to the screen, and m is the order of the minimum.

WORKED EXAMPLE

Two-Slit Interference

Suppose you pass light from a He-Ne laser through two slits separated by 0.0100 mm, and you find that the third bright line on a screen is formed at an angle of 10.95° relative to the incident beam. What is the wavelength of the light?

STRATEGY

The third bright line is due to third-order constructive interference, which means that $m = 3$. You are given $d = 0.0100$ mm and $\theta = 10.95°$. The wavelength can thus be found using the equation $d\sin\theta = m\lambda$ for constructive interference.

Solution

The equation is $d \sin \theta = m\lambda$. Solving for the wavelength, λ, gives

$$\lambda = \frac{d \sin \theta}{m}. \tag{17.2}$$

Substituting known values yields

$$\lambda = \frac{(0.0100 \ \text{mm})(\sin \ 10.95°)}{3} = 6.33 \ \times \ 10^{-4} \ \text{mm} = 633 \ \text{nm}. \tag{17.3}$$

Discussion

To three digits, 633 nm is the wavelength of light emitted by the common He-Ne laser. Not by coincidence, this red color is similar to that emitted by neon lights. More important, however, is the fact that interference patterns can be used to measure wavelength. Young did that for visible wavelengths. His analytical technique is still widely used to measure electromagnetic spectra. For a given order, the angle for constructive interference increases with λ, so spectra (measurements of intensity versus wavelength) can be obtained.

WORKED EXAMPLE

Single-Slit Diffraction

Visible light of wavelength 550 nm falls on a single slit and produces its second diffraction minimum at an angle of 45.0° relative to the incident direction of the light. What is the width of the slit?

STRATEGY

From the given information, and assuming the screen is far away from the slit, you can use the equation $D \sin \theta = m\lambda$ to find *D*.

Solution

Quantities given are $\lambda = 550$ nm, $m = 2$, and $\theta_2 = 45.0°$. Solving the equation $D \sin \theta = m\lambda$ for *D* and substituting known values gives

$$D = \frac{m\lambda}{\sin\theta} = \frac{2(550 \ \text{nm})}{\sin 45.0°} = 1.56 \ \times \ 10^{-6} \ \text{m}. \tag{17.4}$$

Discussion

You see that the slit is narrow (it is only a few times greater than the wavelength of light). That is consistent with the fact that light must interact with an object comparable in size to its wavelength in order to exhibit significant wave effects, such as this single-slit diffraction pattern.

Practice Problems

1. Monochromatic light from a laser passes through two slits separated by 0.00500 mm. The third bright line on a screen is formed at an angle of $18.0°$ relative to the incident beam. What is the wavelength of the light?
   a. 51.5 nm
   b. 77.3 nm
   c. 515 nm
   d. 773 nm

2. What is the width of a single slit through which 610-nm orange light passes to form a first diffraction minimum at an angle of 30.0°?
   a. 0.863 µm
   b. 0.704 µm
   c. 0.610 µm
   d. 1.22 µm

Check Your Understanding

3. Which aspect of a beam of monochromatic light changes when it passes from a vacuum into water, and how does it change?
   a. The wavelength first decreases and then increases.
   b. The wavelength first increases and then decreases.
   c. The wavelength increases.
   d. The wavelength decreases.

4. Go outside in the sunlight and observe your shadow. It has fuzzy edges, even if you do not. Is this a diffraction effect? Explain.
   a. This is a diffraction effect. Your whole body acts as the origin for a new wavefront.
   b. This is a diffraction effect. Every point on the edge of your shadow acts as the origin for a new wavefront.
   c. This is a refraction effect. Your whole body acts as the origin for a new wavefront.
   d. This is a refraction effect. Every point on the edge of your shadow acts as the origin for a new wavefront.

5. Which aspect of monochromatic green light changes when it passes from a vacuum into diamond, and how does it change?
   a. The wavelength first decreases and then increases.
   b. The wavelength first increases and then decreases.
   c. The wavelength increases.
   d. The wavelength decreases.

17.2 Applications of Diffraction, Interference, and Coherence

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain behaviors of waves, including reflection, refraction, diffraction, interference, and coherence, and describe applications based on these behaviors
- Perform calculations related to applications based on wave properties of light

Section Key Terms

differential interference contrast (DIC)	diffraction grating	iridescence	laser
monochromator	Rayleigh criterion	resolution	

Wave-Based Applications of Light

In 1917, Albert Einstein was thinking about photons and excited atoms. He considered an atom excited by a certain amount of energy and what would happen if that atom were hit by a photon with the same amount of energy. He suggested that the atom would emit a photon with that amount of energy, and it would be accompanied by the original photon. The exciting part is that you would have *two* photons with the same energy *and* they would be in phase. Those photons could go on to hit other excited atoms, and soon you would have a stream of in-phase photons. Such a light stream is said to be *coherent*. Some four decades later, Einstein's idea found application in a process called, *light amplification by stimulated emission of radiation*. Take the first letters of all the words (except *by* and "of") and write them in order. You get the word ***laser*** (see (a)), which is the name of the device that produces such a beam of light.

Laser beams are directional, very intense, and narrow (only about 0.5 mm in diameter). These properties lead to a number of applications in industry and medicine. The following are just a few examples:

- This chapter began with a picture of a compact disc (see). Those audio and data-storage devices began replacing cassette tapes during the 1990s. CDs are read by interpreting variations in reflections of a laser beam from the surface.
- Some barcode scanners use a laser beam.
- Lasers are used in industry to cut steel and other metals.
- Lasers are bounced off reflectors that astronauts left on the Moon. The time it takes for the light to make the round trip can be used to make precise calculations of the Earth-Moon distance.
- Laser beams are used to produce holograms. The name hologram means *entire picture* (from the Greek *holo-*, as in

holistic), because the image is three-dimensional. A viewer can move around the image and see it from different perspectives. Holograms take advantage of the wave properties of light, as opposed to traditional photography which is based on geometric optics. A holographic image is produced by constructive and destructive interference of a split laser beam.
- One of the advantages of using a laser as a surgical tool is that it is accompanied by very little bleeding.
- Laser eye surgery has improved the vision of many people, without the need for corrective lenses. A laser beam is used to change the shape of the lens of the eye, thus changing its focal length.

Virtual Physics

Lasers

Click to view content (https://www.openstax.org/l/28lasers)

This animation allows you to examine the workings of a laser. First view the picture of a real laser. Change the energy of the incoming photons, and see if you can match it to an excitation level that will produce pairs of coherent photons. Change the excitation level and try to match it to the incoming photon energy.

In the animation there is only one excited atom. Is that the case for a real laser? Explain.

a. No, a laser would have two excited atoms.
b. No, a laser would have several million excited atoms.
c. Yes, a laser would have only one excited atom.
d. No, a laser would have on the order of 10^{23} excited atoms.

An interesting thing happens if you pass light through a large number of evenly-spaced parallel slits. Such an arrangement of slits is called a **diffraction grating**. An interference pattern is created that is very similar to the one formed by double-slit diffraction (see and). A diffraction grating can be manufactured by scratching glass with a sharp tool to form a number of precisely positioned parallel lines, which act like slits. Diffraction gratings work both for transmission of light, as in Figure 17.13, and for reflection of light, as on the butterfly wings or the Australian opal shown in Figure 17.14, or the CD pictured in the opening illustration of this chapter. In addition to their use as novelty items, diffraction gratings are commonly used for spectroscopic dispersion and analysis of light. What makes them particularly useful is the fact that they form a sharper pattern than do double slits. That is, their bright regions are narrower and brighter, while their dark regions are darker. Figure 17.15 shows idealized graphs demonstrating the sharper pattern. Natural diffraction gratings occur in the feathers of certain birds. Tiny, fingerlike structures in regular patterns act as reflection gratings, producing constructive interference that gives the feathers colors not solely due to their pigmentation. The effect is called **iridescence**.

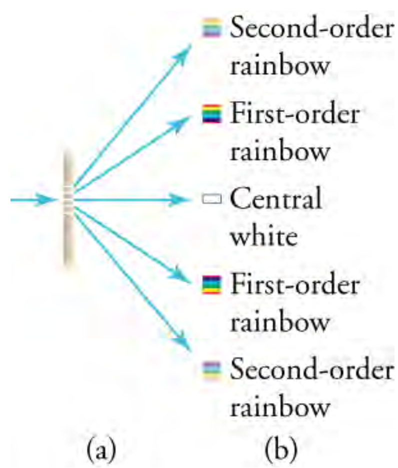


Figure 17.13 A diffraction grating consists of a large number of evenly-spaced parallel slits. (a) Light passing through the grating is diffracted in a pattern similar to a double slit, with bright regions at various angles. (b) The pattern obtained for white light incident on a grating. The central maximum is white, and the higher-order maxima disperse white light into a rainbow of colors.

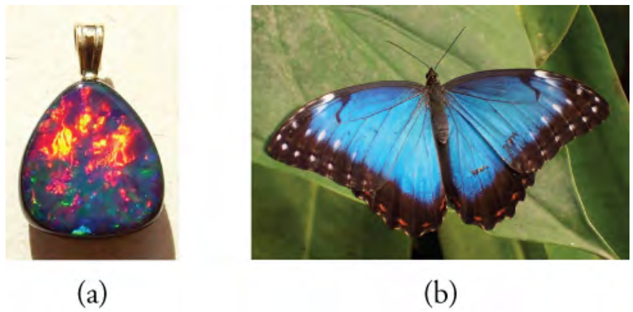


Figure 17.14 (a) This Australian opal and (b) the butterfly wings have rows of reflectors that act like reflection gratings, reflecting different colors at different angles. (credit: (a) Opals-On-Black.com, via Flickr (b) whologwhy, Flickr)

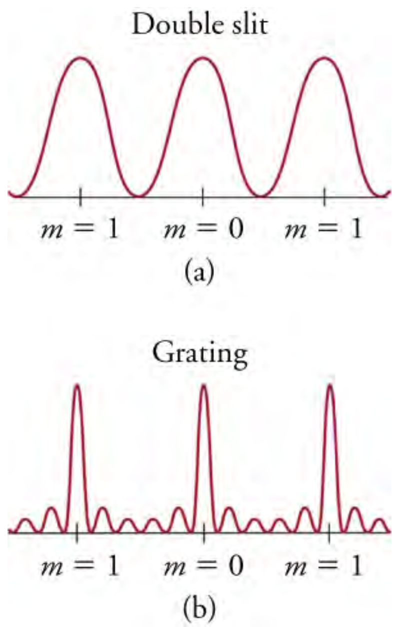


Figure 17.15 Idealized graphs of the intensity of light passing through a double slit (a) and a diffraction grating (b) for monochromatic light. Maxima can be produced at the same angles, but those for the diffraction grating are narrower, and hence sharper. The maxima become narrower and the regions between become darker as the number of slits is increased.

Snap Lab

Diffraction Grating

- A CD (compact disc) or DVD
- A measuring tape
- Sunlight near a white wall

Instructions

Procedure

1. Hold the CD in direct sunlight near the wall, and move it around until a circular rainbow pattern appears on the wall.
2. Measure the distance from the CD to the wall and the distance from the center of the circular pattern to a color in the rainbow. Use those two distances to calculate $\tan\theta$. Find $\sin\theta$.
3. Look up the wavelength of the color you chose. That is λ .
4. Solve $d\sin\theta = m\lambda$ for *d*.
5. Compare your answer to the usual spacing between CD tracks, which is 1,600 nm (1.6 µm).

How do you know what number to use for *m*?

a. Count the rainbow rings preceding the chosen color.

b. Calculate mfrom the frequency of the light of the chosen color.
c. Calculate m from the wavelength of the light of the chosen color.
d. The value of m is fixed for every color.

FUN IN PHYSICS

CD Players

Can you see the grooves on a CD or DVD (see Figure 17.16)? You may think you can because you know they are there, but they are extremely narrow—1,600 in a millimeter. Because the width of the grooves is similar to wavelengths of visible light, they form a diffraction grating. That is why you see rainbows on a CD. The colors are attractive, but they are incidental to the functions of storing and retrieving audio and other data.

Figure 17.16 For its size, this CD holds a surprising amount of information. Likewise, the CD player it is in houses a surprising number of electronic devices.

The grooves are actually one continuous groove that spirals outward from the center. Data are recorded in the grooves as binary code (zeroes and ones) in small pits. Information in the pits is detected by a laser that tracks along the groove. It gets even more complicated: The speed of rotation must be varied as the laser tracks toward the circumference so that the linear speed along the groove remains constant. There is also an error correction mechanism to prevent the laser beam from getting off track. A diffraction grating is used to create the first two maxima on either side of the track. If those maxima are not the same distance from the track, an error is indicated and then corrected.

The pits are reflective because they have been coated with a thin layer of aluminum. That allows the laser beam to be reflected back and directed toward a photodiode detector. The signal can then be processed and converted to the audio we hear.

The longest wavelength of visible light is about 780 nm . How does that compare to the distance between CD grooves?

a. The grooves are about 3 times the longest wavelength of visible light.
b. The grooves are about 2 times the longest wavelength of visible light.
c. The grooves are about 2 times the shortest wavelength of visible light.
d. The grooves are about 3 times the shortest wavelength of visible light.

LINKS TO PHYSICS

Biology: DIC Microscopy

If you were completely transparent, it would be hard to recognize you from your photograph. The same problem arises when using a traditional microscope to view or photograph small transparent objects such as cells and microbes. Microscopes using **differential interference contrast (DIC)** solve the problem by making it possible to view microscopic objects with enhanced contrast, as shown in Figure 17.17.

Figure 17.17 This aquatic organism was photographed with a DIC microscope. (credit: Public Library of Science)

A DIC microscope separates a polarized light source into two beams polarized at right angles to each other and coherent with each other, that is, in phase. After passing through the sample, the beams are recombined and realigned so they have the same plane of polarization. They then create an interference pattern caused by the differences in their optical path and the refractive indices of the parts of the sample they passed through. The result is an image with contrast and shadowing that could not be observed with traditional optics.

Where are diffraction gratings used? Diffraction gratings are key components of **monochromators**—devices that separate the various wavelengths of incoming light and allow a beam with only a specific wavelength to pass through. Monochromators are used, for example, in optical imaging of particular wavelengths from biological or medical samples. A diffraction grating can be chosen to specifically analyze a wavelength of light emitted by molecules in diseased cells in a biopsy sample, or to help excite strategic molecules in the sample with a selected frequency of light. Another important use is in optical fiber technologies where fibers are designed to provide optimum performance at specific wavelengths. A range of diffraction gratings is available for selecting specific wavelengths for such use.

Diffraction gratings are used in spectroscopes to separate a light source into its component wavelengths. When a material is heated to incandescence, it gives off wavelengths of light characteristic of the chemical makeup of the material. A pure substance will produce a spectrum that is unique, thus allowing identification of the substance. Spectroscopes are also used to measure wavelengths both shorter and longer than visible light. Such instruments have become especially useful to astronomers and chemists. Figure 17.18 shows a diagram of a spectroscope.

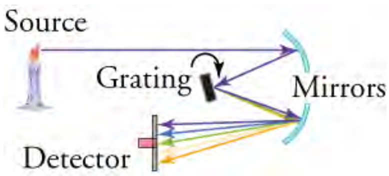


Figure 17.18 The diagram shows the function of a diffraction grating in a spectroscope.

Light diffracts as it moves through space, bending around obstacles and interfering constructively and destructively. While diffraction allows light to be used as a spectroscopic tool, it also limits the detail we can obtain in images.

Figure 17.19 (a) shows the effect of passing light through a small circular aperture. Instead of a bright spot with sharp edges, a spot with a fuzzy edge surrounded by circles of light is obtained. This pattern is caused by diffraction similar to that produced by a single slit. Light from different parts of the circular aperture interferes constructively and destructively. The effect is most noticeable when the aperture is small, but the effect is there for large apertures, too.

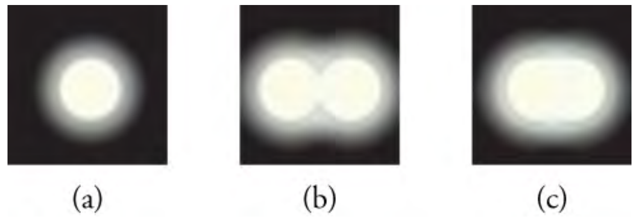


Figure 17.19 (a) Monochromatic light passed through a small circular aperture produces this diffraction pattern. (b) Two point light sources that are close to one another produce overlapping images because of diffraction. (c) If they are closer together, they cannot be resolved, that is, distinguished.

How does diffraction affect the detail that can be observed when light passes through an aperture? Figure 17.19 (b) shows the diffraction pattern produced by two point light sources that are close to one another. The pattern is similar to that for a single point source, and it is just barely possible to tell that there are two light sources rather than one. If they are closer together, as in Figure 17.19 (c), you cannot distinguish them, thus limiting the detail, or **resolution**, you can obtain. That limit is an inescapable consequence of the wave nature of light.

There are many situations in which diffraction limits the resolution. The acuity of vision is limited because light passes through the pupil, the circular aperture of the eye. Be aware that the diffraction-like spreading of light is due to the limited diameter of a light beam, not the interaction with an aperture. Thus light passing through a lens with a diameter of D shows the diffraction effect and spreads, blurring the image, just as light passing through an aperture of diameter D does. Diffraction limits the resolution of any system having a lens or mirror. Telescopes are also limited by diffraction, because of the finite diameter, D, of their primary mirror.

Why are diffraction gratings used in spectroscopes rather than just two slits?

a. The bands produced by diffraction gratings are dimmer but sharper than the bands produced by two slits.
b. The bands produced by diffraction gratings are brighter, though less sharp, than the bands produced by two slits.
c. The bands produced by diffraction gratings are brighter and sharper than the bands produced by two slits.
d. The bands produced by diffraction gratings are dimmer and less sharp, but more widely dispersed, than the bands produced by two slits.

Calculations Involving Diffraction Gratings and Resolution

Early in the chapter, it was mentioned that when light passes from one medium to another, its speed and wavelength change, but its frequency remains constant. The equation

$$\lambda_n = \frac{\lambda}{n}$$

shows how to the wavelength in a given medium, λ_n, is related to the wavelength in a vacuum, λ, and the refractive index, n, of the medium. The equation is useful for calculating the change in wavelength of a monochromatic laser beam in various media. The analysis of a diffraction grating is very similar to that for a double slit. As you know from the discussion of double slits in Young's double-slit experiment, light is diffracted by, and spreads out after passing through, each slit. Rays travel at an angle θ relative to the incident direction. Each ray travels a different distance to a common point on a screen far away. The rays start in phase, and they can be in or out of phase when they reach a screen, depending on the difference in the path lengths traveled. Each ray travels a distance that differs by $d \sin\theta$ from that of its neighbor, where d is the distance between slits. If $d \sin\theta$ equals an integral number of wavelengths, the rays all arrive in phase, and constructive interference (a maximum) is obtained. Thus, the condition necessary to obtain constructive interference for a diffraction grating is

$$d \sin\theta = m\lambda, \text{for } m = 0, 1, -1, 2, -2, \ldots,$$

where d is the distance between slits in the grating, λ is the wavelength of the light, and m is the order of the maximum. Note that this is exactly the same equation as for two slits separated by d. However, the slits are usually closer in diffraction gratings than in double slits, producing fewer maxima at larger angles.

WATCH PHYSICS

Diffraction Grating

This video (https://www.openstax.org/l/28diffraction) explains the geometry behind the diffraction pattern produced by a diffraction grating.

Click to view content (https://www.openstax.org/l/28diffraction)

The equation that gives the points of constructive interference produced by a diffraction grating is $d \sin\theta = m\lambda$. Why does that equation look familiar?

a. It is the same as the equation for destructive interference for a double-slit diffraction pattern.
b. It is the same as the equation for constructive interference for a double-slit diffraction pattern.
c. It is the same as the equation for constructive interference for a single-slit diffraction pattern.
d. It is the same as the equation for destructive interference for a single-slit diffraction pattern.

Just what is the resolution limit of an aperture or lens? To answer that question, consider the diffraction pattern for a circular aperture, which, similar to the diffraction pattern of light passing through a slit, has a central maximum that is wider and brighter than the maxima surrounding it (see Figure 17.19 (a)). It can be shown that, for a circular aperture of diameter D, the first minimum in the diffraction pattern occurs at $\theta = 1.22\lambda/D$, provided that the aperture is large compared with the wavelength of light, which is the case for most optical instruments. The accepted criterion for determining the diffraction limit to resolution based on diffraction was developed by Lord Rayleigh in the 19th century. The **Rayleigh criterion** for the diffraction limit to resolution states that two images are just resolvable when the center of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other. See Figure 17.20 (b). The first minimum is at an angle of $\theta = 1.22\lambda/D$, so that two point objects are just resolvable if they are separated by the angle

$$\theta = 1.22\frac{\lambda}{D},$$

where λ is the wavelength of the light (or other electromagnetic radiation) and D is the diameter of the aperture, lens, mirror, etc., with which the two objects are observed. In the expression above, θ has units of radians.

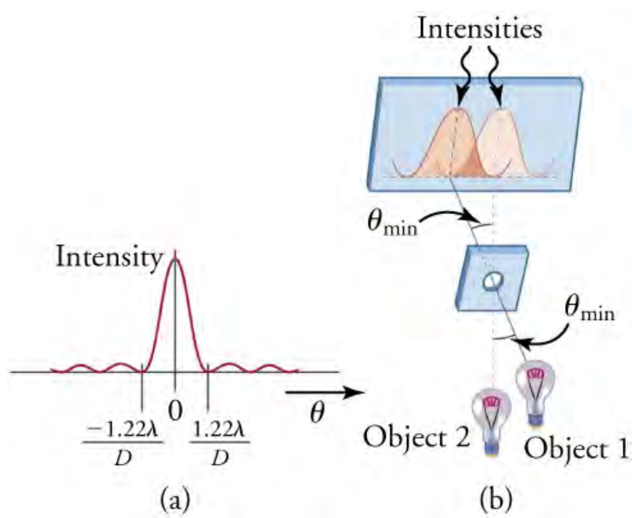


Figure 17.20 (a) Graph of intensity of the diffraction pattern for a circular aperture. Note that, similar to a single slit, the central maximum is wider and brighter than those to the sides. (b) Two point objects produce overlapping diffraction patterns. Shown here is the Rayleigh criterion for their being just resolvable. The central maximum of one pattern lies on the first minimum of the other.

Snap Lab

Resolution

- A sheet of white paper
- A black pen or pencil
- A measuring tape

Instructions

Procedure

1. Draw two lines several mm apart on a white sheet of paper.
2. Move away from the sheet as it is held upright, and measure the distance at which you can just distinguish (resolve) the lines as separate.
3. Use $\theta = 1.22\frac{\lambda}{D}$ to calculate D the diameter of your pupil. Use the distance between the lines and the maximum distance at which they were resolved to calculate θ . Use the average wavelength for visible light as the value for λ .
4. Compare your answer to the average pupil diameter of 3 mm.

Describe resolution in terms of minima and maxima of diffraction patterns.

a. The limit for resolution is when the minimum of the pattern for one of the lines is directly over the first minimum of the pattern for the other line.
b. The limit for resolution is when the maximum of the pattern for one of the lines is directly over the first minimum of the pattern for the other line.
c. The limit for resolution is when the maximum of the pattern for one of the lines is directly over the second minimum of the pattern for the other line.
d. The limit for resolution is when the minimum of the pattern for one of the lines is directly over the second maximum of the pattern for the other line.

WORKED EXAMPLE

Change of Wavelength

A monochromatic laser beam of green light with a wavelength of 550 nm passes from air to water. The refractive index of water is 1.33. What will be the wavelength of the light after it enters the water?

STRATEGY

You can assume that the refractive index of air is the same as that of light in a vacuum because they are so close. You then have all the information you need to solve for λ_n .

Solution

$$\lambda_n = \frac{\lambda}{n} = \frac{550 \text{ nm}}{1.33} = 414 \text{ nm} \tag{17.5}$$

Discussion

The refractive index of air is 1.0003, so the approximation holds for three significant figures. You would not see the light change color, however. Color is determined by frequency, not wavelength.

WORKED EXAMPLE

Diffraction Grating

A diffraction grating has 2000 lines per centimeter. At what angle will the first-order maximum form for green light with a wavelength of 520 nm?

STRATEGY

You are given enough information to calculate d, and you are given the values of λ and m. You will have to find the arcsin of a number to find θ .

Solution

First find *d*.

$$d = \frac{1 \text{ cm}}{2{,}000} = 5.00 \times 10^{-4} \text{ cm} = 5{,}000 \text{ nm} \tag{17.6}$$

Rearrange the equation for constructive interference conditions for a diffraction grating, and substitute the known values.

$$\begin{aligned} d \sin \theta &= m\lambda \\ \theta &= \sin^{-1} \frac{m\lambda}{d} \\ &= \sin^{-1} \left(\frac{(1)(520)}{5{,}000} \right) \\ &= 5.97 \end{aligned}$$

Discussion

This angle seems reasonable for the first maximum. Recall that the meaning of $\sin^{-1}$ (or arcsin) is *the angle with a sine that is (the unknown).* Remember that the value of $\sin \theta$ will not be greater than 1 for any value of θ .

WORKED EXAMPLE

Resolution

What is the minimum angular spread of a 633-nm-wavelength He-Ne laser beam that is originally 1.00 mm in diameter?

STRATEGY

The diameter of the beam is the same as if it were coming through an aperture of that size, so $D = 1.00$ mm. You are given λ, and you must solve for θ .

Solution

$$\theta = \frac{(1.22)\lambda}{D} = \frac{(1.22)(633 \text{ nm})}{1.00 \times 10^{6} \text{ nm}} = 7.72 \times 10^{-4} \text{ rad} = 0.0442° \tag{17.7}$$

Discussion

The conversion factor for radians to degrees is 1.000 radian = 57.3°. The spread is very small and would not be noticeable over short distances. The angle represents the angular separation of the central maximum and the first minimum.

Practice Problems

6. A beam of yellow light has a wavelength of 600 nm in a vacuum and a wavelength of 397 nm in Plexiglas. What is the refractive index of Plexiglas?
   a. 1.51
   b. 2.61
   c. 3.02
   d. 3.77
7. What is the angle between two just-resolved points of light for a 3.00 mm diameter pupil, assuming an average wavelength of 550 nm?
   a. 224 rad
   b. 183 rad
   c. 1.83×10^{-4} rad
   d. 2.24×10^{-4} rad

Check Your Understanding

8. How is an interference pattern formed by a diffraction grating different from the pattern formed by a double slit?

a. The pattern is colorful.
b. The pattern is faded.
c. The pattern is sharper.
d. The pattern is curved.

9. A beam of light always spreads out. Why can a beam not be produced with parallel rays to prevent spreading?
a. Light is always polarized.
b. Light is always reflected.
c. Light is always refracted.
d. Light is always diffracted.

10. Compare interference patterns formed by a double slit and by a diffraction grating in terms of brightness and narrowness of bands.
a. The pattern formed has broader and brighter bands.
b. The pattern formed has broader and duller bands.
c. The pattern formed has narrower and duller bands.
d. The pattern formed has narrower and brighter bands.

11. Describe the slits in a diffraction grating in terms of number and spacing, as compared to a two-slit diffraction setup.
a. The slits in a diffraction grating are broader, with space between them that is greater than the separation of the two slits in two-slit diffraction.
b. The slits in a diffraction grating are broader, with space between them that is the same as the separation of the two slits in two-slit diffraction.
c. The slits in a diffraction grating are narrower, with space between them that is the same as the separation of the two slits in two-slit diffraction.
d. The slits in a diffraction grating are narrower, with space between them that is greater than the separation of the two slits in two-slit diffraction.

KEY TERMS

differential interference contrast (DIC) separating a polarized light source into two beams polarized at right angles to each other and coherent with each other then, after passing through the sample, recombining and realigning the beams so they have the same plane of polarization, and then creating an interference pattern caused by the differences in their optical path and the refractive indices of the parts of the sample they passed through; the result is an image with contrast and shadowing that could not be observed with traditional optics

diffraction bending of a wave around the edges of an opening or an obstacle

diffraction grating many of evenly spaced slits having dimensions such that they produce an interference pattern

Huygens's principle Every point on a wavefront is a source of wavelets that spread out in the forward direction at the same speed as the wave itself; the new wavefront is a line tangent to all of the wavelets.

iridescence the effect that occurs when tiny, fingerlike structures in regular patterns act as reflection gratings, producing constructive interference that gives feathers colors not solely due to their pigmentation

laser acronym for a device that produces *light amplification by stimulated emission of radiation*

monochromatic one color

monochromator device that separates the various wavelengths of incoming light and allows a beam with only a specific wavelength to pass through

Rayleigh criterion two images are just resolvable when the center of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other

resolution degree to which two images can be distinguished from one another, which is limited by diffraction

wavefront points on a wave surface that all share an identical, constant phase

SECTION SUMMARY

17.1 Understanding Diffraction and Interference

- The wavelength of light varies with the refractive index of the medium.
- Slits produce a diffraction pattern if their width and separation are similar to the wavelength of light passing through them.
- Interference bands of a single-slit diffraction pattern can be predicted.
- Interference bands of a double-slit diffraction pattern can be predicted.

17.2 Applications of Diffraction, Interference, and Coherence

- The focused, coherent radiation emitted by lasers has many uses in medicine and industry.
- Characteristics of diffraction patterns produced with diffraction gratings can be determined.
- Diffraction gratings have been incorporated in many instruments, including microscopes and spectrometers.
- Resolution has a limit that can be predicted.

KEY EQUATIONS

17.1 Understanding Diffraction and Interference

speed of light, frequency, and wavelength	$c = f\lambda$
change of wavelength with index of refraction	$\lambda_n = \frac{\lambda}{n}$
two-slit constructive interference	$d \sin\theta = m\lambda$, for $m = 0, 1, -1, 2, -2, \ldots$
two-slit destructive interference	$d \sin\theta = (m + ½)\lambda$, for $m = 0, 1, -1, 2, -2, \ldots$
one-slit, first-order destructive interference; wavelength related to dimensions	$\frac{Dy}{L} = \lambda$
one-slit destructive interference	$D \sin\theta = m\lambda$

17.2 Applications of Diffraction, Interference, and Coherence

wavelength change with change in medium	$\lambda_n = \frac{\lambda}{n}$
diffraction grating constructive interference	$d \sin\theta = m\lambda$
resolution	$\theta = 1.22\frac{\lambda}{D}$

CHAPTER REVIEW

Concept Items

17.1 Understanding Diffraction and Interference

1. Which behavior of light is indicated by an interference pattern?
   a. ray behavior
   b. particle behavior
   c. corpuscular behavior
   d. wave behavior

2. Which behavior of light is indicated by diffraction?
   a. wave behavior
   b. particle behavior
   c. ray behavior
   d. corpuscular behavior

17.2 Applications of Diffraction, Interference, and Coherence

3. There is a principle related to resolution that is expressed by this equation.

$$\theta = \frac{\lambda}{D} \tag{17.8}$$

   What is that principle stated in full?

4. A principle related to resolution states, "Two images are just resolved when the center of the diffraction pattern of one is directly over the first minimum of the diffraction pattern of the other." Write the equation that expresses that principle.
   a. $\theta = 1.22\frac{D}{\lambda}$
   b. $\theta = \frac{D}{\lambda}$
   c. $\theta = \frac{\lambda}{D}$
   d. $\theta = 1.22\frac{\lambda}{D}$

5. Which statement completes this resolution? Two images are just resolved when —
   a. The center of the diffraction pattern of one image is directly over the central maximum of the diffraction pattern of the other.
   b. The center of the diffraction pattern of one image is directly over the central minimum of the diffraction pattern of the other
   c. The center of the diffraction pattern of one image is directly over the first minimum of the diffraction pattern of the other
   d. The center of the diffraction pattern of one is directly over the first maximum of the diffraction pattern of the other

Critical Thinking Items

17.1 Understanding Diffraction and Interference

6. Describe a situation in which bodies of water and a line of rocks could create a diffraction pattern similar to light passing through double slits. Include the arrangement of the rocks, the positions of the bodies of water, and the location of the diffraction pattern. Note the dimensions that are necessary for the production of the pattern.
   a. When waves from a small body of water pass through two widely separated openings and enter a larger body of water, a diffraction pattern is produced that is similar tothe diffraction pattern formed by light passing through two slits. The width of each opening is larger than the size of the wavelength of the waves.
   b. When waves from a large body of water pass through two narrow openings and enter a smaller body of water, a diffraction pattern is produced that is similar to the diffraction pattern formed by light passing through two slits. The widths and separation of the openings are similar to the size of the wavelength of the waves.
   c. When waves from a small body of water pass through two wide openings and enter a larger body of water, a diffraction pattern is produced that is similar tothe diffraction pattern formed by light passing through two slits. The separation between the openings is similar to the size of the wavelength of the waves.
   d. When waves from a large body of water pass through two wide openings and enter a smaller body of water, a diffraction pattern is produced that is similar to the diffraction pattern formed by light passing through two slits. The widths and

separation of the openings are larger than the size of the wavelength of the waves.

17.2 Applications of Diffraction, Interference, and Coherence

7. For what type of electromagnetic radiation would a grating with spacing greater than 800 nm be useful as a spectroscopic tool?
   a. It can be used to analyze spectra only in the infrared portion of the spectrum.
   b. It can be used to analyze spectra in the entire visible portion of the electromagnetic spectrum.
   c. It can only be used to analyze spectra in the short-wavelength visible.
   d. It can only be used to analyze spectra in the short-wavelength visible and ultraviolet.

8. A beam of green light has a wavelength of $500\ \text{nm}$ in a vacuum and a wavelength of $331\ \text{nm}$ in Plexiglas. What is the refractive index of Plexiglas?
   a. 1.12
   b. 1.25
   c. 1.51
   d. 4.53

Problems

17.1 Understanding Diffraction and Interference

9. What is the distance between two slits that produce a diffraction pattern with the first minimum at an angle of 45.0° when 410-nm violet light passes through the slits?
   a. 2,030 nm
   b. 1,450 nm
   c. 410 nm
   d. 290 nm

10. A breakwater at the entrance to a harbor consists of a rock barrier with a 50.0 – m -wide opening. Ocean waves with a 20.0-m wavelength approach the opening straight on. At what angle to the incident direction are the boats inside the harbor most protected against wave action?
   a. 11.5°
   b. 7.46°
   c. 5.74°
   d. 23.6°

17.2 Applications of Diffraction, Interference, and Coherence

11. A 500-nm beam of light passing through a diffraction grating creates its second band of constructive interference at an angle of 1.50°. How far apart are the slits in the grating?
   a. 38,200 nm
   b. 19,100 nm
   c. 667 nm
   d. 333 nm

12. The range of the visible-light spectrum is 380 nm to 780 nm. What is the maximum number of lines per centimeter a diffraction grating can have and produce a complete first-order spectrum for visible light?
   a. 26,300 lines/cm
   b. 13,200 lines/cm
   c. 6,410 lines/cm
   d. 12,820 lines/cm

Performance Task

17.2 Applications of Diffraction, Interference, and Coherence

13. In this performance task you will create one- and two-slit diffraction and observe the interference patterns that result.
   - A utility knife (a knife with a razor blade-like cutting edge)
   - Aluminum foil
   - A straight edge
   - A strong, small light source or a laser pointer
   - A tape measure
   - A white wall

Procedure

1. Cut a piece of aluminum foil about 15 cm × 15 cm.
2. Use the utility knife and the straight edge to cut a straight slit several cm long in the center of the foil square.
3. With the room darkened, one partner shines the light through the slit and toward the wall. The other partner observes the pattern on the wall. The partner with the light changes the distance from the foil to the wall and the distance from the light to the foil.
4. When the sharpest, brightest pattern possible is obtained, the partner who is not holding the foil and light makes measurements.
5. Measure the perpendicular (shortest) distance from

the slit to the wall, the distance from the center of the pattern to several of the dark bands, and the distance from the slit to the same dark bands.

6. Carefully make a second slit parallel to the first slit and 1 mm or less away.
7. Repeat steps 2 through 5, only this time measure the distances to bright bands.
NOTE—In your calculations, use 580 nm for λ if you used white light. If you used a colored laser pointer, look up the wavelength of the color. You may find it easier to calculate θ from its tangent rather than from its sine.

a. Which experiment gave the most distinct pattern—one or two slits?
b. What was the width of the single slit? Compare the calculated distance with the measured distance.
c. What was the distance between the two slits? Compare the calculated distance with the measured distance.

TEST PREP

Multiple Choice

17.1 Understanding Diffraction and Interference

14. Which remains unchanged when a monochromatic beam of light passes from air into water?

a. the speed of the light
b. the direction of the beam
c. the frequency of the light
d. the wavelength of the light

15. Two slits are separated by a distance of 3500 nm . If light with a wavelength of 500 nm passes through the slits and produces an interference pattern, the m = ________ order minimum appears at an angle of 30.0°.

a. 0
b. 1
c. 2
d. 3

16. In the sunlight, the shadow of a building has fuzzy edges even if the building does not. Is this a refraction effect? Explain.

a. Yes, this is a refraction effect, where every point on the building acts as the origin for a new wavefront.
b. Yes, this is a refraction effect, where the whole building acts as the origin for a new wavefront.
c. No, this is a diffraction effect, where every point on the edge of the building's shadow acts as the origin for a new wavefront.
d. No, this is a diffraction effect, where the whole building acts as the origin for a new wavefront.

17.2 Applications of Diffraction, Interference, and Coherence

17. Two images are just resolved when the center of the diffraction pattern of one is directly over ________ of the diffraction pattern of the other.

a. the center
b. the first minimum
c. the first maximum
d. the last maximum

18. Two point sources of $500\ \mathrm{nm}$ light are just resolvable as they pass through a small hole. The angle to the first minimum of one source is $0.100\ \mathrm{rad}$. What is the diameter of the hole?

a. $4.10\ \mu\mathrm{m}$
b. $5.73\ \mu\mathrm{m}$
c. $0.106\ \mu\mathrm{m}$
d. $6.10\ \mu\mathrm{m}$

19. Will a beam of light shining through a 1-mm hole behave any differently than a beam of light that is 1 mm wide as it leaves its source? Explain.?

a. Yes, the beam passing through the hole will spread out as it travels, because it is diffracted by the edges of the hole, whereas the 1 -mm beam, which encounters no diffracting obstacle, will not spread out.
b. Yes, the beam passing through the hole will be made *more parallel* by passing through the hole, and so will not spread out as it travels, whereas the unaltered wavefronts of the 1-mm beam will cause the beam to spread out as it travels.
c. No, both beams will remain the same width as they travel, and they will not spread out.
d. No, both beams will spread out as they travel.

20. A laser pointer emits a coherent beam of parallel light rays. Does the light from such a source spread out at all? Explain.

a. Yes, every point on a wavefront is not a source of wavelets, which prevent the spreading of light waves.
b. No, every point on a wavefront is not a source of wavelets, so that the beam behaves as a bundles of rays that travel in their initial direction.
c. No, every point on a wavefront is a source of

wavelets, which keep the beam from spreading.

d. Yes, every point on a wavefront is a source of wavelets, which cause the beam to spread out steadily as it moves forward.

Short Answer

17.1 Understanding Diffraction and Interference

21. Light passing through double slits creates a diffraction pattern. How would the spacing of the bands in the pattern change if the slits were closer together?

a. The bands would be closer together.
b. The bands would spread farther apart.
c. The bands would remain stationary.
d. The bands would fade and eventually disappear.

22. A beam of light passes through a single slit to create a diffraction pattern. How will the spacing of the bands in the pattern change if the width of the slit is increased?

a. The width of the spaces between the bands will remain the same.
b. The width of the spaces between the bands will increase.
c. The width of the spaces between the bands will decrease.
d. The width of the spaces between the bands will first decrease and then increase.

23. What is the wavelength of light falling on double slits separated by $2.00\ \mu\text{m}$ if the third-order maximum is at an angle of 60.0°?

a. $667\ \text{nm}$
b. $471\ \text{nm}$
c. $333\ \text{nm}$
d. $577\ \text{nm}$

24. What is the longest wavelength of light passing through a single slit of width 1.20 µm for which there is a first-order minimum?

a. 1.04 µm
b. 0.849 µm
c. 0.600 µm
d. 2.40 µm

17.2 Applications of Diffraction, Interference, and Coherence

25. Describe a diffraction grating and the interference pattern it produces.

a. A diffraction grating is a large collection of evenly spaced parallel lines that produces an interference pattern that is similar to but sharper and better dispersed than that of a double slit.
b. A diffraction grating is a large collection of randomly spaced parallel lines that produces an interference pattern that is similar to but less sharp or well-dispersed as that of a double slit.
c. A diffraction grating is a large collection of randomly spaced intersecting lines that produces an interference pattern that is similar to but sharper and better dispersed than that of a double slit.
d. A diffraction grating is a large collection of evenly spaced intersecting lines that produces an interference pattern that is similar to but less sharp or well-dispersed as that of a double slit.

26. Suppose pure-wavelength light falls on a diffraction grating. What happens to the interference pattern if the same light falls on a grating that has more lines per centimeter?

a. The bands will spread farther from the central maximum.
b. The bands will come closer to the central maximum.
c. The bands will not spread farther from the first maximum.
d. The bands will come closer to the first maximum.

27. How many lines per centimeter are there on a diffraction grating that gives a first-order maximum for 473 nm blue light at an angle of 25.0°?

a. 529,000 lines/cm
b. 50,000 lines/cm
c. 851 lines/cm
d. 8,934 lines/cm

28. What is the distance between lines on a diffraction grating that produces a second-order maximum for 760-nm red light at an angle of 60.0°?

a. 2.28×10^4 nm
b. 3.29×10^2 nm
c. 2.53×10^1 nm
d. 1.76×10^3 nm

Extended Response

17.1 Understanding Diffraction and Interference

29. Suppose you use a double slit to perform Young's double-slit experiment in air, and then repeat the experiment with the same double slit in water. Does the color of the light change? Do the angles to the same parts of the interference pattern get larger or smaller? Explain.

a. No, the color is determined by frequency. The magnitude of the angle decreases.
b. No, the color is determined by wavelength. The magnitude of the angle decreases.
c. Yes, the color is determined by frequency. The magnitude of the angle increases.
d. Yes, the color is determined by wavelength. The magnitude of the angle increases.

30. A double slit is located at a distance x from a screen, with the distance along the screen from the center given by y. When the distance d between the slits is relatively large, there will be numerous bright bands. For small angles (where $\sin\theta = \theta$, with θ in radians), what is the distance between fringes?

a. $\Delta y = \frac{d}{x\lambda}$
b. $\Delta y = \frac{xd}{\lambda}$
c. $\Delta y = \frac{\lambda}{xd}$
d. $\Delta y = \frac{x\lambda}{d}$

17.2 Applications of Diffraction, Interference, and Coherence

31. Compare the interference patterns of single-slit diffraction, double-slit diffraction, and a diffraction grating.

a. All three interference pattern produce identical bands.
b. A double slit produces the sharpest and most distinct bands.
c. A single slit produces the sharpest and most distinct bands.
d. The diffraction grating produces the sharpest and most distinct bands.

32. An electric current through hydrogen gas produces several distinct wavelengths of visible light. The light is projected onto a diffraction grating having $10{,}000$ lines per centimeter. What are the wavelengths of the hydrogen spectrum if the light forms first-order maxima at angles of $24.2°$, $25.7°$, $29.1°$, and $41.0°$?

a. $\lambda_1 = \left(10^3\ \text{nm}\right) \sin 24.2° = 410\ \text{nm}$
$\lambda_2 = \left(10^3\ \text{nm}\right) \sin 25.7° = 434\ \text{nm}$
$\lambda_3 = \left(10^3\ \text{nm}\right) \sin 29.1° = 486\ \text{nm}$
$\lambda_4 = \left(10^3\ \text{nm}\right) \sin 41.0° = 656\ \text{nm}$

b. $\lambda_1 = \left(10^3\ \text{nm}\right) \sin 41.0° = 410\ \text{nm}$
$\lambda_2 = \left(10^3\ \text{nm}\right) \sin 25.7° = 434\ \text{nm}$
$\lambda_3 = \left(10^3\ \text{nm}\right) \sin 29.1° = 486\ \text{nm}$
$\lambda_4 = \left(10^3\ \text{nm}\right) \sin 24.2° = 656\ \text{nm}$

c. $\lambda_1 = \left(10^3\ \text{nm}\right) \sin 24.2° = 410\ \text{nm}$
$\lambda_2 = \left(10^3\ \text{nm}\right) \sin 29.1° = 434\ \text{nm}$
$\lambda_3 = \left(10^3\ \text{nm}\right) \sin 25.7° = 486\ \text{nm}$
$\lambda_4 = \left(10^3\ \text{nm}\right) \sin 41.0° = 656\ \text{nm}$

d. $\lambda_1 = \left(10^3\ \text{nm}\right) \sin 41.0° = 410\ \text{nm}$
$\lambda_2 = \left(10^3\ \text{nm}\right) \sin 29.1° = 434\ \text{nm}$
$\lambda_3 = \left(10^3\ \text{nm}\right) \sin 25.7° = 486\ \text{nm}$
$\lambda_4 = \left(10^3\ \text{nm}\right) \sin 24.2° = 656\ \text{nm}$

CHAPTER 18
Static Electricity

Figure 18.1 This child's hair contains an imbalance of electrical charge (commonly called *static electricity*), which causes it to stand on end. The sliding motion stripped electrons away from the child's body, leaving him with an excess of positive charges, which repel each other along each strand of hair. (credit: Ken Bosma, Wikimedia Commons)

Chapter Outline

INTRODUCTION You may have been introduced to static electricity like the child sliding down the slide in the opening photograph (Figure 18.1). The *zap* that he is likely to receive if he touches a playmate or parent tends to bring home the lesson. But static electricity is more than just fun and games—it is put to use in many industries. The forces between electrically charged particles are used in technologies such as printers, pollution filters, and spray guns used for painting cars and trucks. Static *electricity* is the study of phenomena that involve an imbalance of electrical charge. Although creating this imbalance typically requires moving charge around, once the imbalance is created, it often remains static for a long time. The study of charge in motion is called *electromagnetism* and will be covered in a later chapter. What is electrical charge, how is it associated with objects, and what forces does it create? These are just some of the questions that this chapter addresses.

18.1 Electrical Charges, Conservation of Charge, and Transfer of Charge

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe positive and negative electric charges
- Use conservation of charge to calculate quantities of charge transferred between objects
- Characterize materials as conductors or insulators based on their electrical properties
- Describe electric polarization and charging by induction

Section Key Terms

conduction	conductor	electron	induction
insulator	law of conservation of charge	polarization	proton

Electric Charge

You may know someone who has an *electric* personality, which usually means that other people are attracted to this person. This saying is based on electric charge, which is a property of matter that causes objects to attract or repel each other. Electric charge comes in two varieties, which we call *positive* and *negative.* Like charges repel each other, and unlike charges attract each other. Thus, two positive charges repel each other, as do two negative charges. A positive charge and a negative charge attract each other.

How do we know there are two types of electric charge? When various materials are rubbed together in controlled ways, certain combinations of materials always result in a net charge of one type on one material and a net charge of the opposite type on the other material. By convention, we call one type of charge positive and the other type negative. For example, when glass is rubbed with silk, the glass becomes positively charged and the silk negatively charged. Because the glass and silk have opposite charges, they attract one another like clothes that have rubbed together in a dryer. Two glass rods rubbed with silk in this manner will repel one another, because each rod has positive charge on it. Similarly, two silk cloths rubbed in this manner will repel each other, because both cloths have negative charge. Figure 18.2 shows how these simple materials can be used to explore the nature of the force between charges.

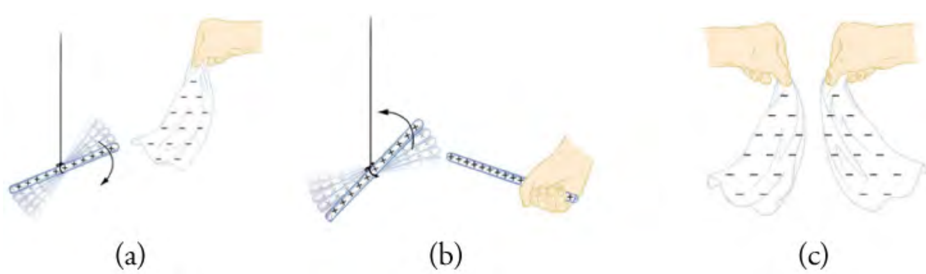


Figure 18.2 A glass rod becomes positively charged when rubbed with silk, whereas the silk becomes negatively charged. (a) The glass rod is attracted to the silk, because their charges are opposite. (b) Two similarly charged glass rods repel. (c) Two similarly charged silk cloths repel.

It took scientists a long time to discover what lay behind these two types of charges. The word *electric* itself comes from the Greek word *elektron* for amber, because the ancient Greeks noticed that amber, when rubbed by fur, attracts dry straw. Almost 2,000 years later, the English physicist William Gilbert proposed a model that explained the effect of electric charge as being due to a mysterious electrical fluid that would pass from one object to another. This model was debated for several hundred years, but it was finally put to rest in 1897 by the work of the English physicist J. J. Thomson and French physicist Jean Perrin. Along with many others, Thomson and Perrin were studying the mysterious *cathode rays* that were known at the time to consist of particles smaller than the smallest atom. Perrin showed that cathode rays actually carried negative electrical charge. Later, Thomson's work led him to declare, "I can see no escape from the conclusion that [cathode rays] are charges of negative electricity carried by particles of matter."

It took several years of further experiments to confirm Thomson's interpretation of the experiments, but science had in fact discovered the particle that carries the fundamental unit of negative electrical charge. We now know this particle as the **electron**.

Atoms, however, were known to be electrically neutral, which means that they carry the same amount of positive and negative charge, so their net charge is zero. Because electrons are negative, some other part of the atom must contain positive charge. Thomson put forth what is called the *plum pudding model*, in which he described atoms as being made of thousands of electrons swimming around in a nebulous mass of positive charge, as shown by the left-side image of Figure 18.3. His student, Ernest Rutherford, originally believed that this model was correct and used it (along with other models) to try to understand the results of his experiments bombarding gold foils with *alpha* particles (i.e., helium atoms stripped of their electrons). The results, however, did not confirm Thomson's model but rather destroyed it! Rutherford found that most of the space occupied by the gold atoms was actually empty and that almost all of the matter of each atom was concentrated into a tiny, extremely dense nucleus, as shown by the right-side image of Figure 18.3. The atomic nucleus was later found to contain particles called **protons**, each of which carries a unit of positive electric charge.[1]

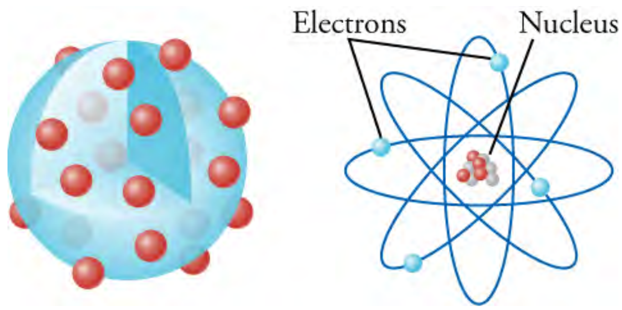


Figure 18.3 The left drawing shows Thompson's plum-pudding model, in which the electrons swim around in a nebulous mass of positive charge. The right drawing shows Rutherford's model, in which the electrons orbit around a tiny, massive nucleus. Note that the size of the nucleus is vastly exaggerated in this drawing. Were it drawn to scale with respect to the size of the electron orbits, the nucleus would not be visible to the naked eye in this drawing. Also, as far as science can currently detect, electrons are point particles, which means that they have no size at all!

Protons and electrons are thus the fundamental particles that carry electric charge. Each proton carries one unit of positive charge, and each electron carries one unit of negative charge. To the best precision that modern technology can provide, the charge carried by a proton is *exactly* the opposite of that carried by an electron. The SI unit for electric charge is the *coulomb* (abbreviated as "C"), which is named after the French physicist Charles Augustin de Coulomb, who studied the force between charged objects. The proton carries $+1.602 \times 10^{-19}\text{C}$. and the electron carries $-1.602 \times 10^{-19}\text{C}$, . The number *n* of protons required to make +1.00 C is

$$n = 1.00\text{ C} \times \frac{1\text{ proton}}{1.602 \times 10^{-19}\text{C}} = 6.25 \times 10^{18}\text{protons.} \tag{18.1}$$

The same number of electrons is required to make –1.00 C of electric charge. The fundamental unit of charge is often represented as *e*. Thus, the charge on a proton is *e*, and the charge on an electron is –*e*. Mathematically, $e = +1.602 \times 10^{-19}\text{C}$.

LINKS TO PHYSICS

Measuring the Fundamental Electric Charge

The American physicist Robert Millikan (1868–1953) and his student Harvey Fletcher (1884–1981) were the first to make a relatively accurate measurement of the fundamental unit of charge on the electron. They designed what is now a classic

1Protons were later found to contain sub particles called *quarks*, which have fractional electric charge. But that is another story that we leave for subsequent physics courses.

experiment performed by students. The Millikan oil-drop experiment is shown in Figure 18.4. The experiment involves some concepts that will be introduced later, but the basic idea is that a fine oil mist is sprayed between two plates that can be charged with a known amount of opposite charge. Some oil drops accumulate some excess negative charge when being sprayed and are attracted to the positive charge of the upper plate and repelled by the negative charge on the lower plate. By tuning the charge on these plates until the weight of the oil drop is balanced by the electric forces, the net charge on the oil drop can be determined quite precisely.

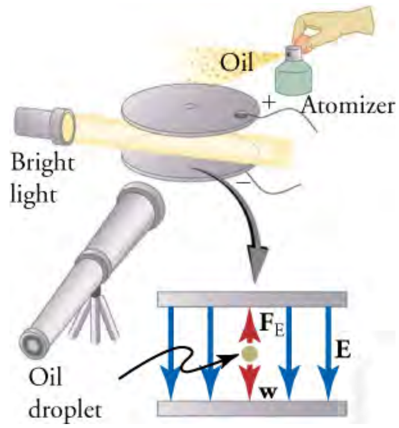


Figure 18.4 The oil-drop experiment involved spraying a fine mist of oil between two metal plates charged with opposite charges. By knowing the mass of the oil droplets and adjusting the electric charge on the plates, the charge on the oil drops can be determined with precision.

Millikan and Fletcher found that the drops would accumulate charge in discrete units of about $-1.59 \times 10^{-19}\,\mathrm{C}$, which is within 1 percent of the modern value of $-1.60 \times 10^{-19}\,\mathrm{C}$. Although this difference may seem quite small, it is actually five times greater than the possible error Millikan reported for his results!

Because the charge on the electron is a fundamental constant of nature, determining its precise value is very important for all of science. This created pressure on Millikan and others after him that reveals some equally important aspects of human nature.

First, Millikan took sole credit for the experiment and was awarded the 1923 Nobel Prize in physics for this work, although his student Harvey Fletcher apparently contributed in significant ways to the work. Just before his death in 1981, Fletcher divulged that Millikan coerced him to give Millikan sole credit for the work, in exchange for which Millikan promoted Fletcher's career at Bell Labs.

Another great scientist, Richard Feynman, points out that many scientists who measured the fundamental charge after Millikan were reluctant to report values that differed much from Millikan's value. History shows that later measurements slowly crept up from Millikan's value until settling on the modern value. Why did they not immediately find the error and correct the value, asks Feynman. Apparently, having found a value higher than the much-respected value found by Millikan, scientists would look for possible mistakes that might lower their value to make it agree better with Millikan's value. This reveals the important psychological weight carried by preconceived notions and shows how hard it is to refute them. Scientists, however devoted to logic and data they may be, are apparently just as vulnerable to this aspect of human nature as everyone else. The lesson here is that, although it is good to be skeptical of new results, you should not discount them just because they do not agree with conventional wisdom. If your reasoning is sound and your data are reliable, the conclusion demanded by the data must be seriously considered, even if that conclusion disagrees with the commonly accepted *truth*.

GRASP CHECK

Suppose that Millikan observed an oil drop carrying three fundamental units of charge. What would be the net charge on this oil drop?

a. -4.81×10^{-19} C
b. -1.602×10^{-19} C
c. 1.602×10^{-19} C
d. 4.81×10^{-19} C

Snap Lab

Like and Unlike Charges

This activity investigates the repulsion and attraction caused by static electrical charge.

- Adhesive tape
- Nonconducting surface, such as a plastic table or chair

Instructions

Procedure for Part (a)

1. Prepare two pieces of tape about 4 cm long. To make a handle, double over about 0.5 cm at one end so that the sticky side sticks together.
2. Attach the pieces of tape side by side onto a nonmetallic surface, such as a tabletop or the seat of a chair, as shown in Figure 18.5(a).
3. Peel off both pieces of tape and hang them downward, holding them by the handles, as shown in Figure 18.5(b). If the tape bends upward and sticks to your hand, try using a shorter piece of tape, or simply shake the tape so that it no longer sticks to your hand.
4. Now slowly bring the two pieces of tape together, as shown in Figure 18.5(c). What happens?

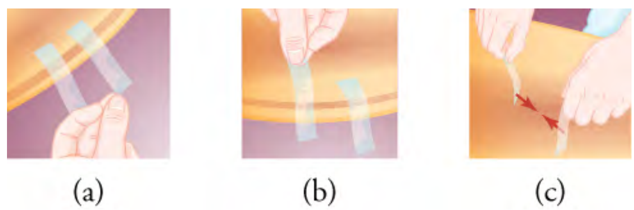


Figure 18.5

Procedure for Part (b)

5. Stick one piece of tape on the nonmetallic surface, and stick the second piece of tape on top of the first piece, as shown in Figure 18.6(a).
6. Slowly peel off the two pieces by pulling on the handle of the bottom piece.
7. Gently stroke your finger along the top of the second piece of tape (i.e., the nonsticky side), as shown in Figure 18.6(b).
8. Peel the two pieces of tape apart by pulling on their handles, as shown in Figure 18.6(c).
9. Slowly bring the two pieces of tape together. What happens?

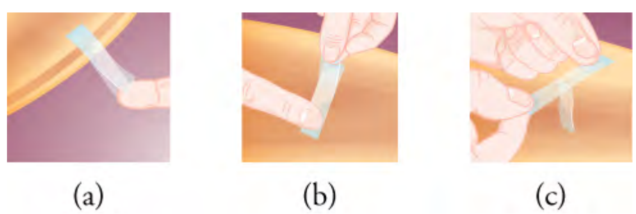


Figure 18.6

GRASP CHECK

In step 4, why did the two pieces of tape repel each other? In step 9, why did they attract each other?

a. Like charges attract, while unlike charges repel each other.
b. Like charges repel, while unlike charges attract each other.
c. Tapes having positive charge repel, while tapes having negative charge attract each other.
d. Tapes having negative charge repel, while tapes having positive charge attract each other.

Conservation of Charge

Because the fundamental positive and negative units of charge are carried on protons and electrons, we would expect that the total charge cannot change in any system that we define. In other words, although we might be able to move charge around, we cannot create or destroy it. This should be true provided that we do not create or destroy protons or electrons in our system. In the twentieth century, however, scientists learned how to create and destroy electrons and protons, but they found that charge is still conserved. Many experiments and solid theoretical arguments have elevated this idea to the status of a law. The **law of conservation of charge** says that electrical charge cannot be created or destroyed.

The law of conservation of charge is very useful. It tells us that the net charge in a system is the same before and after any interaction within the system. Of course, we must ensure that no external charge enters the system during the interaction and that no internal charge leaves the system. Mathematically, conservation of charge can be expressed as

$$q_{initial} = q_{final}. \tag{18.2}$$

where q_{initial} is the net charge of the system before the interaction, and q_{final}, is the net charge after the interaction.

WORKED EXAMPLE

What is the missing charge?

Figure 18.7 shows two spheres that initially have +4 C and +8 C of charge. After an interaction (which could simply be that they touch each other), the blue sphere has +10 C of charge, and the red sphere has an unknown quantity of charge. Use the law of conservation of charge to find the final charge on the red sphere.

Strategy

The net initial charge of the system is $q_{\text{initial}} = +4\text{ C} + 8\text{ C} = +12\text{ C}$. The net final charge of the system is $q_{\text{final}} = +10\text{ C} + q_{\text{red}}$, where q_{red} is the final charge on the red sphere. Conservation of charge tells us that $q_{\text{initial}} = q_{\text{final}}$, so we can solve for q_{red}.

Solution

Equating q_{initial} and q_{final} and solving for q_{red} gives

$$\begin{aligned} q_{\text{initial}} &= q_{\text{final}} \\ +12\text{ C} &= +10\text{ C} + q_{\text{red.}} \\ +2\text{ C} &= q_{\text{red.}} \end{aligned} \tag{18.3}$$

The red sphere has +2 C of charge.

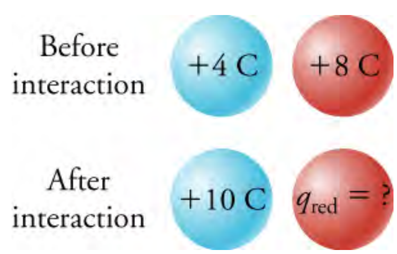


Figure 18.7 Two spheres, one blue and one red, initially have +4 C and +8 C of charge, respectively. After the two spheres interact, the blue sphere has a charge of +10 C. The law of conservation of charge allows us to find the final charge q_{red} on the red sphere.

Discussion

Like all conservation laws, conservation of charge is an accounting scheme that helps us keep track of electric charge.

Practice Problems

1. Which equation describes conservation of charge?
   a. $q_{\text{initial}} = q_{\text{final}} = \text{constant}$
   b. $q_{\text{initial}} = q_{\text{final}} = 0$
   c. $q_{\text{initial}} - q_{\text{final}} = 0$

d. $q_{\text{initial}}/q_{\text{final}} = \text{constant}$

2. An isolated system contains two objects with charges q_1 and q_2. If object 1 loses half of its charge, what is the final charge on object 2?
   a. $\frac{q_2}{2}$
   b. $\frac{3q_2}{2}$
   c. $q_2 - \frac{q_1}{2}$
   d. $q_2 + \frac{q_1}{2}$

Conductors and Insulators

Materials can be classified depending on whether they allow charge to move. If charge can easily move through a material, such as metals, then these materials are called **conductors**. This means that charge can be conducted (i.e., move) through the material rather easily. If charge cannot move through a material, such as rubber, then this material is called an **insulator**.

Most materials are insulators. Their atoms and molecules hold on more tightly to their electrons, so it is difficult for electrons to move between atoms. However, it is not impossible. With enough energy, it is possible to force electrons to move through an insulator. However, the insulator is often physically destroyed in the process. In metals, the outer electrons are loosely bound to their atoms, so not much energy is required to make electrons move through metal. Such metals as copper, silver, and aluminum are good conductors. Insulating materials include plastics, glass, ceramics, and wood.

The conductivity of some materials is intermediate between conductors and insulators. These are called *semiconductors*. They can be made conductive under the right conditions, which can involve temperature, the purity of the material, and the force applied to push electrons through them. Because we can control whether semiconductors are conductors or insulators, these materials are used extensively in computer chips. The most commonly used semiconductor is silicon. Figure 18.8 shows various materials arranged according to their ability to conduct electrons.

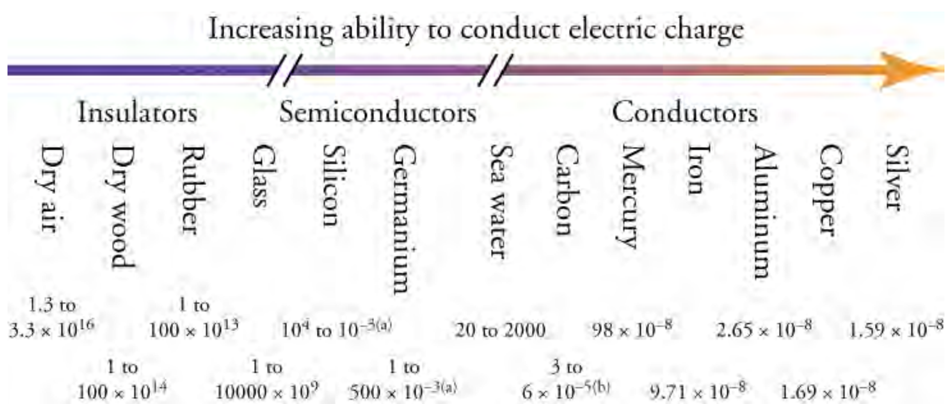


Figure 18.8 Materials can be arranged according to their ability to conduct electric charge. The slashes on the arrow mean that there is a very large gap in conducting ability between conductors, semiconductors, and insulators, but the drawing is compressed to fit on the page. The numbers below the materials give their *resistivity* in Ω•m (which you will learn about below). The resistivity is a measure of how hard it is to make charge move through a given material.

What happens if an excess negative charge is placed on a conducting object? Because like charges repel each other, they will push against each other until they are as far apart as they can get. Because the charge can move in a conductor, it moves to the outer surfaces of the object. Figure 18.9(a) shows schematically how an excess negative charge spreads itself evenly over the outer surface of a metal sphere.

What happens if the same is done with an insulating object? The electrons still repel each other, but they are not able to move, because the material is an insulator. Thus, the excess charge stays put and does not distribute itself over the object. Figure 18.9(b) shows this situation.

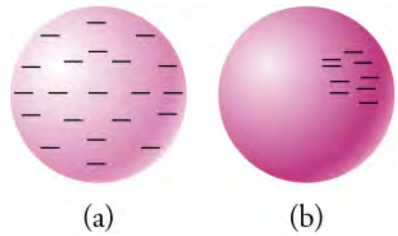


Figure 18.9 (a) A conducting sphere with excess negative charge (i.e., electrons). The electrons repel each other and spread out to cover the outer surface of the sphere. (b) An insulating sphere with excess negative charge. The electrons cannot move, so they remain in their original positions.

Transfer and Separation of Charge

Most objects we deal with are electrically neutral, which means that they have the same amount of positive and negative charge. However, transferring negative charge from one object to another is fairly easy to do. When negative charge is transferred from one object to another, an excess of positive charge is left behind. How do we know that the negative charge is the mobile charge? The positive charge is carried by the proton, which is stuck firmly in the nucleus of atoms, and the atoms are stuck in place in solid materials. Electrons, which carry the negative charge, are much easier to remove from their atoms or molecules and can therefore be transferred more easily.

Electric charge can be transferred in several manners. One of the simplest ways to transfer charge is charging by contact, in which the surfaces of two objects made of different materials are placed in close contact. If one of the materials holds electrons more tightly than the other, then it takes some electrons with it when the materials are separated. Rubbing two surfaces together increases the transfer of electrons, because it creates a closer contact between the materials. It also serves to present *fresh* material with a full supply of electrons to the other material. Thus, when you walk across a carpet on a dry day, your shoes rub against the carpet, and some electrons are removed from the carpet by your shoes. The result is that you have an excess of negative charge on your shoes. When you then touch a doorknob, some of your excess of electrons transfer to the neutral doorknob, creating a small spark.

Touching the doorknob with your hand demonstrates a second way to transfer electric charge, which is charging by **conduction**. This transfer happens because like charges repel, and so the excess electrons that you picked up from the carpet want to be as far away from each other as possible. Some of them move to the doorknob, where they will distribute themselves over the outer surface of the metal. Another example of charging by conduction is shown in the top row of Figure 18.10. A metal sphere with 100 excess electrons touches a metal sphere with 50 excess electrons, so 25 electrons from the first sphere transfer to the second sphere. Each sphere finishes with 75 excess electrons.

The same reasoning applies to the transfer of positive charge. However, because positive charge essentially cannot move in solids, it is transferred by moving negative charge in the opposite direction. For example, consider the bottom row of Figure 18.10. The first metal sphere has 100 excess protons and touches a metal sphere with 50 excess protons, so the second sphere transfers 25 electrons to the first sphere. These 25 extra electrons will electrically cancel 25 protons so that the first metal sphere is left with 75 excess protons. This is shown in the bottom row of Figure 18.10. The second metal sphere lost 25 electrons so it has 25 more excess protons, for a total of 75 excess protons. The end result is the same if we consider that the first ball transferred a net positive charge equal to that of 25 protons to the first ball.

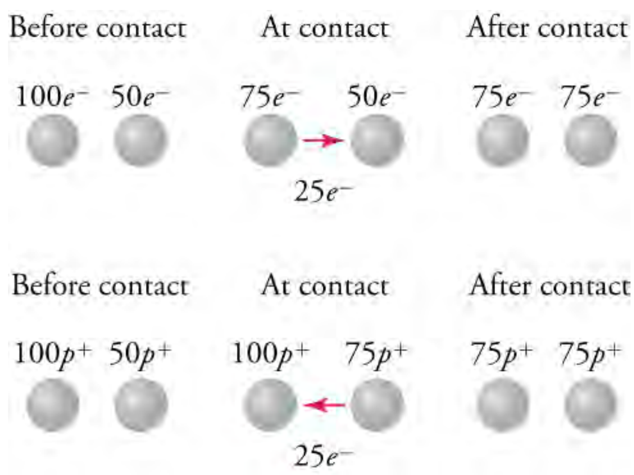


Figure 18.10 In the top row, a metal sphere with 100 excess electrons transfers 25 electrons to a metal sphere with an excess of 50 electrons. After the transfer, both spheres have 75 excess electrons. In the bottom row, a metal sphere with 100 excess protons receives 25 electrons from a ball with 50 excess protons. After the transfer, both spheres have 75 excess protons.

In this discussion, you may wonder how the excess electrons originally got from your shoes to your hand to create the spark when you touched the doorknob. The answer is that *no* electrons actually traveled from your shoes to your hands. Instead, because like charges repel each other, the excess electrons on your shoe simply pushed away some of the electrons in your feet. The electrons thus dislodged from your feet moved up into your leg and in turn pushed away some electrons in your leg. This process continued through your whole body until a distribution of excess electrons covered the extremities of your body. Thus your head, your hands, the tip of your nose, and so forth all received their doses of excess electrons that had been pushed out of their normal positions. All this was the result of electrons being pushed out of your feet by the excess electrons on your shoes.

This type of charge separation is called **polarization**. As soon as the excess electrons leave your shoes (by rubbing off onto the floor or being carried away in humid air), the distribution of electrons in your body returns to normal. Every part of your body is again electrically neutral (i.e., zero excess charge).

The phenomenon of polarization is seen in . The child has accumulated excess positive charge by sliding on the slide. This excess charge repels itself and so becomes distributed over the extremities of the child's body, notably in his hair. As a result, the hair stands on end, because the excess negative charge on each strand repels the excess positive charge on neighboring strands.

Polarization can be used to charge objects. Consider the two metallic spheres shown in Figure 18.11. The spheres are electrically neutral, so they carry the same amounts of positive and negative charge. In the top picture (Figure 18.11(a)), the two spheres are touching, and the positive and negative charge is evenly distributed over the two spheres. We then approach a glass rod that carries an excess positive charge, which can be done by rubbing the glass rod with silk, as shown in Figure 18.11(b). Because opposite charges attract each other, the negative charge is attracted to the glass rod, leaving an excess positive charge on the opposite side of the right sphere. This is an example of charging by **induction**, whereby a charge is created by approaching a charged object with a second object to create an unbalanced charge in the second object. If we then separate the two spheres, as shown in Figure 18.11(c), the excess charge is stuck on each sphere. The left sphere now has an excess negative charge, and the right sphere has an excess positive charge. Finally, in the bottom picture, the rod is removed, and the opposite charges attract each other, so they move as close together as they can get.

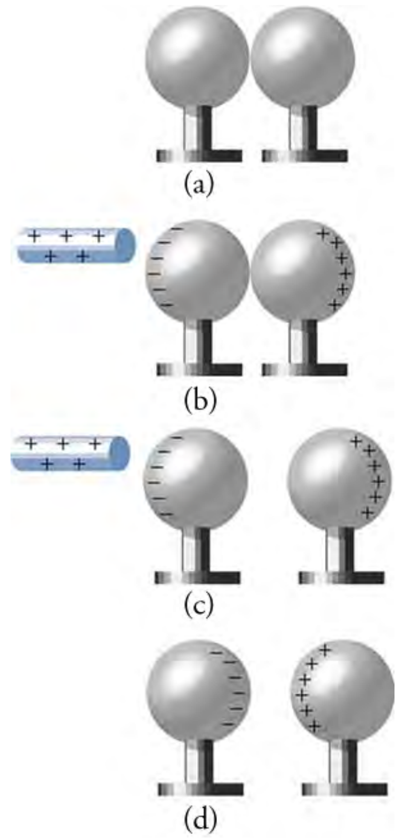


Figure 18.11 (a) Two neutral conducting spheres are touching each other, so the charge is evenly spread over both spheres. (b) A positively charged rod approaches, which attracts negative charges, leaving excess positive charge on the right sphere. (c) The spheres are separated. Each sphere now carries an equal magnitude of excess charge. (d) When the positively charged rod is removed, the excess negative charge on the left sphere is attracted to the excess positive charge on the right sphere.

FUN IN PHYSICS

Create a Spark in a Science Fair

Van de Graaff generators are devices that are used not only for serious physics research but also for demonstrating the physics of static electricity at science fairs and in classrooms. Because they deliver relatively little electric current, they can be made safe for use in such environments. The first such generator was built by Robert Van de Graaff in 1931 for use in nuclear physics research. Figure 18.12 shows a simplified sketch of a Van de Graaff generator.

Van de Graaff generators use smooth and pointed surfaces and conductors and insulators to generate large static charges. In the version shown in Figure 18.12, electrons are “sprayed” from the tips of the lower comb onto a moving belt, which is made of an insulating material like, such as rubber. This technique of charging the belt is akin to charging your shoes with electrons by walking across a carpet. The belt raises the charges up to the upper comb, where they transfer again, akin to your touching the doorknob and transferring your charge to it. Because like charges repel, the excess electrons all rush to the outer surface of the globe, which is made of metal (a conductor). Thus, the comb itself never accumulates too much charge, because any charge it gains is quickly depleted by the charge moving to the outer surface of the globe.

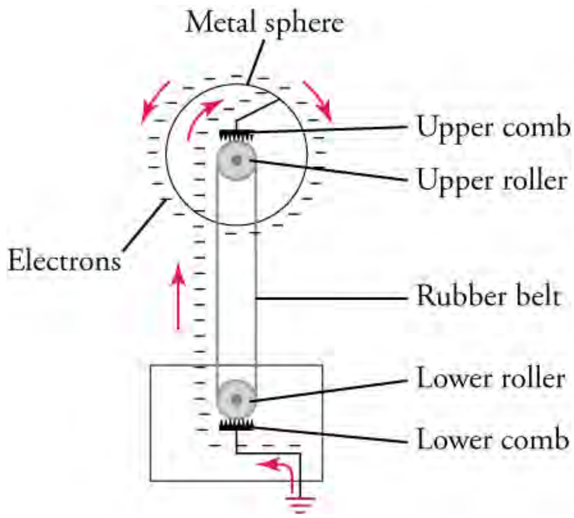


Figure 18.12 Van de Graaff generators transfer electrons onto a metallic sphere, where the electrons distribute themselves uniformly over the outer surface.

Van de Graaff generators are used to demonstrate many interesting effects caused by static electricity. By touching the globe, a person gains excess charge, so his or her hair stands on end, as shown in Figure 18.13. You can also create mini lightning bolts by moving a neutral conductor toward the globe. Another favorite is to pile up aluminum muffin tins on top of the uncharged globe, then turn on the generator. Being made of conducting material, the tins accumulate excess charge. They then repel each other and fly off the globe one by one. A quick Internet search will show many examples of what you can do with a Van de Graaff generator.

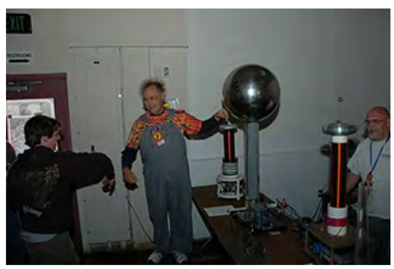


Figure 18.13 The man touching the Van de Graaff generator has excess charge, which spreads over his hair and repels hair strands from his neighbors. (credit: Jon “ShakataGaNai” Davis)

GRASP CHECK

Why don’t the electrons stay on the rubber belt when they reach the upper comb?

a. The upper comb has no excess electrons, and the excess electrons in the rubber belt get transferred to the comb by contact.
b. The upper comb has no excess electrons, and the excess electrons in the rubber belt get transferred to the comb by conduction.
c. The upper comb has excess electrons, and the excess electrons in the rubber belt get transferred to the comb by conduction.
d. The upper comb has excess electrons, and the excess electrons in the rubber belt get transferred to the comb by contact.

Virtual Physics

Balloons and Static Electricity

Click to view content (http://www.openstax.org/l/28balloons)

This simulation allows you to observe negative charge accumulating on a balloon as you rub it against a sweater. You can then observe how two charged balloons interact and how they cause polarization in a wall.

GRASP CHECK

Click the reset button, and start with two balloons. Charge a first balloon by rubbing it on the sweater, and then move it toward the second balloon. Why does the second balloon not move?

a. The second balloon has an equal number of positive and negative charges.
b. The second balloon has more positive charges than negative charges.
c. The second balloon has more negative charges than positive charges.
d. The second balloon is positively charged and has polarization.

Snap Lab

Polarizing Tap Water

This lab will demonstrate how water molecules can easily be polarized.

- Plastic object of small dimensions, such as comb or plastic stirrer
- Source of tap water

Instructions

Procedure

1. Thoroughly rub the plastic object with a dry cloth.
2. Open the faucet just enough to let a smooth filament of water run from the tap.
3. Move an edge of the charged plastic object toward the filament of running water.

What do you observe? What happens when the plastic object touches the water filament? Can you explain your observations?

GRASP CHECK

Why does the water curve around the charged object?

a. The charged object induces uniform positive charge on the water molecules.
b. The charged object induces uniform negative charge on the water molecules.
c. The charged object attracts the polarized water molecules and ions that are dissolved in the water.
d. The charged object depolarizes the water molecules and the ions dissolved in the water.

WORKED EXAMPLE

Charging Ink Droplets

Electrically neutral ink droplets in an ink-jet printer pass through an electron beam created by an electron gun, as shown in Figure 18.14. Some electrons are captured by the ink droplet, so that it becomes charged. After passing through the electron beam, the net charge of the ink droplet is $q_{\text{inkdrop}} = -1 \times 10^{-10}\,\text{C}$. How many electrons are captured by the ink droplet?

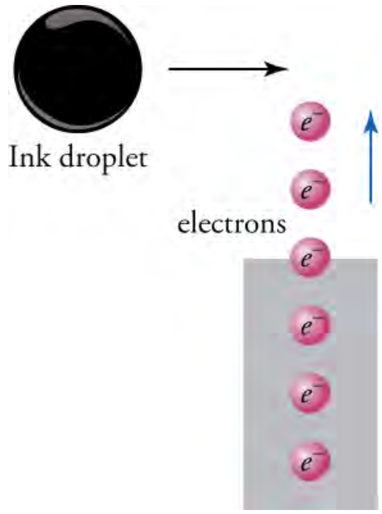


Figure 18.14 Electrons from an *electron gun* charge a passing ink droplet.

STRATEGY
A single electron carries a charge of $q_{e^-} = -1.602 \times 10^{-19}\,\text{C}$. Dividing the net charge of the ink droplet by the charge q_{e^-} of a single electron will give the number of electrons captured by the ink droplet.

Solution
The number n of electrons captured by the ink droplet are

$$n = \frac{q_{\text{inkdrop}}}{q_{e^-}} = \frac{-1 \times 10^{-10}\,\text{C}}{-1.602 \times 10^{-19}\,\text{C}} = 6 \times 108. \quad (18.4)$$

Discussion
This is almost a billion electrons! It seems like a lot, but it is quite small compared to the number of atoms in an ink droplet, which number about 10^{16}. Thus, each extra electron is shared between about $10^{16} / \left(6 \times 10^{8}\right) \approx 10^{7}$ atoms.

Practice Problems

3. How many protons are needed to make 1 nC of charge? 1 nC = 10–9 C
   a. 1.6×10^{-28}
   b. 1.6×10^{-10}
   c. 3×10^{9}
   d. 6×10^{9}

4. In a physics lab, you charge up three metal spheres, two with $+3\text{ nC}$ and one with -5 nC. When you bring all three spheres together so that they all touch one another, what is the total charge on the three spheres?
   a. $+1\text{ nC}$
   b. $+3\text{ nC}$
   c. $+5\text{ nC}$
   d. $+6\text{ nC}$

Check Your Understanding

5. How many types of electric charge exist?
   a. one type
   b. two types
   c. three types
   d. four types

6. Which are the two main electrical classifications of materials based on how easily charges can move through them?

a. conductor and insulator
b. semiconductor and insulator
c. conductor and superconductor
d. conductor and semiconductor

7. True or false—A polarized material must have a nonzero net electric charge.
a. true
b. false

8. Describe the force between two positive point charges that interact.
a. The force is attractive and acts along the line joining the two point charges.
b. The force is attractive and acts tangential to the line joining the two point charges.
c. The force is repulsive and acts along the line joining the two point charges.
d. The force is repulsive and acts tangential to the line joining the two point charges.

9. How does a conductor differ from an insulator?
a. Electric charges move easily in an insulator but not in a conducting material.
b. Electric charges move easily in a conductor but not in an insulator.
c. A conductor has a large number of electrons.
d. More charges are in an insulator than in a conductor.

10. True or false—Charging an object by polarization requires touching it with an object carrying excess charge.
a. true
b. false

18.2 Coulomb's law

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Coulomb's law verbally and mathematically
- Solve problems involving Coulomb's law

Section Key Terms

Coulomb's law	inverse-square law

More than 100 years before Thomson and Rutherford discovered the fundamental particles that carry positive and negative electric charges, the French scientist Charles-Augustin de Coulomb mathematically described the force between charged objects. Doing so required careful measurements of forces between charged spheres, for which he built an ingenious device called a *torsion balance*.

This device, shown in Figure 18.15, contains an insulating rod that is hanging by a thread inside a glass-walled enclosure. At one end of the rod is the metallic sphere A. When no charge is on this sphere, it touches sphere B. Coulomb would touch the spheres with a third metallic ball (shown at the bottom of the diagram) that was charged. An unknown amount of charge would distribute evenly between spheres A and B, which would then repel each other, because like charges repel. This force would cause sphere A to rotate away from sphere B, thus twisting the wire until the torsion in the wire balanced the electrical force. Coulomb then turned the knob at the top, which allowed him to rotate the thread, thus bringing sphere A closer to sphere B. He found that bringing sphere A twice as close to sphere B required increasing the torsion by a factor of four. Bringing the sphere three times closer required a ninefold increase in the torsion. From this type of measurement, he deduced that the electrical force between the spheres was inversely proportional to the distance squared between the spheres. In other words,

$$F \propto \frac{1}{r^{2},} \tag{18.5}$$

where r is the distance between the spheres.

An electrical charge distributes itself equally between two conducting spheres of the same size. Knowing this allowed Coulomb to divide an unknown charge in half. Repeating this process would produce a sphere with one quarter of the initial charge, and so on. Using this technique, he measured the force between spheres A and B when they were charged with different amounts of charge. These measurements led him to deduce that the force was proportional to the charge on each sphere, or

$$F \propto q_A q_B, \tag{18.6}$$

where q_A is the charge on sphere A, and q_B is the charge on sphere B.

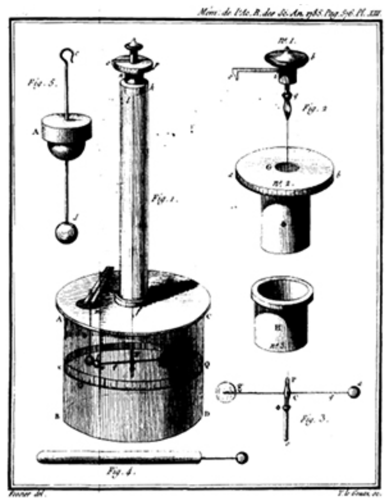


Figure 18.15 A drawing of Coulomb's torsion balance, which he used to measure the electrical force between charged spheres. (credit: Charles-Augustin de Coulomb)

Combining these two proportionalities, he proposed the following expression to describe the force between the charged spheres.

$$F = \frac{kq_1 q_2}{r^2} \tag{18.7}$$

This equation is known as **Coulomb's law**, and it describes the electrostatic force between charged objects. The constant of proportionality k is called *Coulomb's constant.* In SI units, the constant k has the value $k = 8.99 \times 10^9 \mathrm{N \cdot m^2/C^2}$.

The direction of the force is along the line joining the centers of the two objects. If the two charges are of opposite signs, Coulomb's law gives a negative result. This means that the force between the particles is attractive. If the two charges have the same signs, Coulomb's law gives a positive result. This means that the force between the particles is repulsive. For example, if both q_1 and q_2 are negative or if both are positive, the force between them is repulsive. This is shown in Figure 18.16(a). If q_1 is a negative charge and q_2 is a positive charge (or vice versa), then the charges are different, so the force between them is attractive. This is shown in Figure 18.16(b).

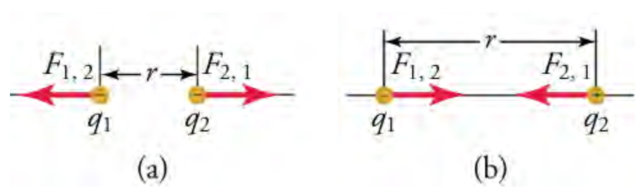


Figure 18.16 The magnitude of the electrostatic force F between point charges q_1 and q_2 separated by a distance r is given by Coulomb's law. Note that Newton's third law (every force exerted creates an equal and opposite force) applies as usual—the force ($F_{1,2}$) on q_1 is equal in magnitude and opposite in direction to the force ($F_{2,1}$) it exerts on q_2. (a) Like charges. (b) Unlike charges.

Note that Coulomb's law applies only to charged objects that are not moving with respect to each other. The law says that the force is proportional to the amount of charge on each object and inversely proportional to the square of the distance between the objects. If we double the charge q_1, for instance, then the force is doubled. If we double the distance between the objects, then the force between them *decreases* by a factor of $2^2 = 4$. Although Coulomb's law is true in general, it is easiest to apply to spherical objects or to objects that are much smaller than the distance between the objects (in which case, the objects can be approximated as spheres).

Coulomb's law is an example of an **inverse-square law**, which means the force depends on the square of the denominator. Another inverse-square law is Newton's law of universal gravitation, which is $F = Gm_1 m_2/r^2$. Although these laws are similar, they differ in two important respects: (i) The gravitational constant *G* is much, much smaller than *k* ($G = 6.67 \times 10^{-11}\,\text{m}^3/\text{kg} \cdot \text{s}^2$); and (ii) only one type of mass exists, whereas two types of electric charge exist. These two differences explain why gravity is so much weaker than the electrostatic force and why gravity is only attractive, whereas the electrostatic force can be attractive or repulsive.

Finally, note that Coulomb measured the distance between the spheres from the centers of each sphere. He did not explain this assumption in his original papers, but it turns out to be valid. From *outside* a uniform spherical distribution of charge, it can be treated as if all the charge were located at the center of the sphere.

WATCH PHYSICS

Electrostatics (part 1): Introduction to charge and Coulomb's law

This video explains the basics of Coulomb's law. Note that the lecturer uses *d* for the distance between the center of the particles instead of *r*.

Click to view content (https://www.openstax.org/l/28coulomb)

GRASP CHECK

True or false—If one particle carries a positive charge and another carries a negative charge, then the force between them is attractive.

a. true
b. false

Snap Lab

Hovering plastic

In this lab, you will use electrostatics to hover a thin piece of plastic in the air.

- Balloon
- Light plastic bag (e.g., produce bag from grocery store)

Instructions

Procedure

1. Cut the plastic bag to make a plastic loop about 2 inches wide.
2. Inflate the balloon.
3. Charge the balloon by rubbing it on your clothes.
4. Charge the plastic loop by placing it on a nonmetallic surface and rubbing it with a cloth.
5. Hold the balloon in one hand, and in the other hand hold the plastic loop above the balloon. If the loop clings too much to your hand, recruit a friend to hold the strip above the balloon with both hands. Now let go of the plastic loop, and maneuver the balloon under the plastic loop to keep it hovering in the air above the balloon.

GRASP CHECK

How does the balloon keep the plastic loop hovering?

a. The balloon and the loop are both negatively charged. This will help the balloon keep the plastic loop hovering.
b. The balloon is charged, while the plastic loop is neutral.This will help the balloon keep the plastic loop hovering.
c. The balloon and the loop are both positively charged. This will help the balloon keep the plastic loop hovering.
d. The balloon is positively charged, while the plastic loop is negatively charged. This will help the balloon keep the plastic loop hovering.

WORKED EXAMPLE

Using Coulomb's law to find the force between charged objects

Suppose Coulomb measures a force of 20×10^{-6}N between the two charged spheres when they are separated by 5.0 cm. By turning the dial at the top of the torsion balance, he approaches the spheres so that they are separated by 3.0 cm. Which force does he measure now?

STRATEGY

Apply Coulomb's law to the situation before and after the spheres are brought closer together. Although we do not know the charges on the spheres, we do know that they remain the same. We call these unknown but constant charges q_1 and q_2 . Because these charges appear as a product in Coulomb's law, they form a single unknown. We thus have two equations and two unknowns, which we can solve. The first unknown is the force (which we call F_{f}) when the spheres are 3.0 cm apart, and the second is $q_1 q_2$.

Use the following notation: When the charges are 5.0 cm apart, the force is $F_{\text{i}} = 20 \times 10^{-6}$N and $r_{\text{i}} = 5.0 \text{ cm} = 0.050 \text{ m}$, where the subscript *i* means *initial*. Once the charges are brought closer together, we know $r_{\text{f}} = 3.0 \text{ cm} = 0.030 \text{ m}$, where the subscript *f* means *final*.

Solution

Coulomb's law applied to the spheres in their initial positions gives

$$F_{\text{i}} = \frac{kq_1 q_2}{r_{\text{i}}^2}. \tag{18.8}$$

Coulomb's law applied to the spheres in their final positions gives

$$F_{\text{f}} = \frac{kq_1 q_2}{r_{\text{f}}^2}. \tag{18.9}$$

Dividing the second equation by the first and solving for the final force F_{f} leads to

$$\begin{aligned} \frac{F_{\text{f}}}{F_{\text{i}}} &= \frac{kq_1 q_2 / r_{\text{f}}^2}{kq_1 q_2 / r_{\text{i}}^2}. \\ &= \frac{r_{\text{i}}^2}{r_{\text{f}}^2} \\ F_{\text{f}} &= F_{\text{i}} \frac{r_{\text{i}}^2}{r_{\text{f}}^2}. \end{aligned} \tag{18.10}$$

Inserting the known quantities yields

$$\begin{aligned} F_{\text{f}} &= F_{\text{i}} \frac{r_{\text{i}}^2}{r_{\text{f}}^2} \\ &= \left(20 \times 10^{-6}\text{N}\right) \frac{(0.050 \text{ m})^2}{(0.030 \text{ m})^2} \\ &= 56 \times 10^{-6}\text{N} \\ &= 5.6 \times 10^{-5}\text{N}. \end{aligned} \tag{18.11}$$

The force acts along the line joining the centers of the spheres. Because the same type of charge is on each sphere, the force is repulsive.

Discussion

As expected, the force between the charges is greater when they are 3.0 cm apart than when they are 5.0 cm apart. Note that although it is a good habit to convert cm to m (because the constant *k* is in SI units), it is not necessary in this problem, because the distances cancel out.

We can also solve for the second unknown $|q_1 q_2|$. By using the first equation, we find

$$\begin{aligned} F_{\mathrm{f}} &= \frac{kq_1q_2}{r_{\mathrm{i}}^2} \\ q_1q_2 &= \frac{F_{\mathrm{i}}r_{\mathrm{i}}^2}{k} \\ &= \frac{(20 \times 10^{-6}\,\mathrm{N})(0.050\ \mathrm{m})^2}{8.99 \times 10^9\,\mathrm{N{\cdot}m^2/C^2}} \\ &= 5.6 \times 10^{-18}\,\mathrm{C}^2 \end{aligned} \tag{18.12}$$

Note how the units cancel in the second-to-last line. Had we not converted cm to m, this would not occur, and the result would be incorrect. Finally, because the charge on each sphere is the same, we can further deduce that

$$q_1 = q_2 = \pm\sqrt{5.6 \times 10^{-18}\,\mathrm{C}^2} = \pm 2.4\ \mathrm{nC}. \tag{18.13}$$

WORKED EXAMPLE

Using Coulomb's law to find the distance between charged objects

An engineer measures the force between two ink drops by measuring their acceleration and their diameter. She finds that each member of a pair of ink drops exerts a repulsive force of $F = 5.5\ \mathrm{mN}$ on its partner. If each ink drop carries a charge $q_{\mathrm{inkdrop}} = -1 \times 10^{-10}\,\mathrm{C}$, how far apart are the ink drops?

STRATEGY

We know the force and the charge on each ink drop, so we can solve Coulomb's law for the distance r between the ink drops. Do not forget to convert the force into SI units: $F = 5.5\ \mathrm{mN} = 5.5 \times 10^{-3}\,\mathrm{N}$.

Solution

The charges in Coulomb's law are $q_1 = q_2 = q_{\mathrm{inkdrop}}$, so the numerator in Coulomb's law takes the form $q_1q_2 = q_{\mathrm{inkdrop}}^2$. Inserting this into Coulomb's law and solving for the distance r gives

$$\begin{aligned} F &= \frac{kq_{\mathrm{inkdrop}}^2}{r^2} \\ r &= \pm\sqrt{\frac{kq_{\mathrm{inkdrop}}^2}{F}} \\ &= \pm\sqrt{\frac{(8.99 \times 10^9\,\mathrm{N{\cdot}m^2/C^2})(-1 \times 10^{-10}\,\mathrm{C})^2}{5.5{\times}10^{-3}\,\mathrm{N}}} \\ &= \pm 1.3 \times 10^{-4}\,\mathrm{m} \end{aligned} \tag{18.14}$$

or 130 microns (about one-tenth of a millimeter).

Discussion

The plus-minus sign means that we do not know which ink drop is to the right and which is to the left, but that is not important, because both ink drops are the same.

Practice Problems

11. A charge of -4×10^{-9} C is a distance of 3 cm from a charge of 3×10^{-9} C . What is the magnitude and direction of the force between them?
    a. 1.2×10^{-4} N, and the force is attractive
    b. 1.2×10^{14} N, and the force is attractive
    c. 6.74×10^{23} N, and the force is attractive
    d. $-\hat{y}$, and the force is attractive

12. Two charges are repelled by a force of 2.0 N. If the distance between them triples, what is the force between the charges?
    a. 0.22 N
    b. 0.67 N
    c. 2.0 N
    d. 18.0 N

Check Your Understanding

13. How are electrostatic force and charge related?
   a. The force is proportional to the product of two charges.
   b. The force is inversely proportional to the product of two charges.
   c. The force is proportional to any one of the charges between which the force is acting.
   d. The force is inversely proportional to any one of the charges between which the force is acting.

14. Why is Coulomb's law called an inverse-square law?
   a. because the force is proportional to the inverse of the distance squared between charges
   b. because the force is proportional to the product of two charges
   c. because the force is proportional to the inverse of the product of two charges
   d. because the force is proportional to the distance squared between charges

18.3 Electric Field

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Calculate the strength of an electric field
- Create and interpret drawings of electric fields

Section Key Terms

electric field	test charge

You may have heard of a *force field* in science fiction movies, where such fields apply forces at particular positions in space to keep a villain trapped or to protect a spaceship from enemy fire. The concept of a *field* is very useful in physics, although it differs somewhat from what you see in movies.

A *field* is a way of conceptualizing and mapping the force that surrounds any object and acts on another object at a distance without apparent physical connection. For example, the gravitational field surrounding Earth and all other masses represents the gravitational force that would be experienced if another mass were placed at a given point within the field. Michael Faraday, an English physicist of the nineteenth century, proposed the concept of an **electric field**. If you know the electric field, then you can easily calculate the force (magnitude and direction) applied to any electric charge that you place in the field.

An electric field is generated by electric charge and tells us the force per unit charge at all locations in space around a charge distribution. The charge distribution could be a single point charge; a distribution of charge over, say, a flat plate; or a more complex distribution of charge. The electric field extends into space around the charge distribution. Now consider placing a test charge in the field. A **test charge** is a positive electric charge whose charge is so small that it does not significantly disturb the charges that create the electric field. The electric field exerts a force on the test charge in a given direction. The force exerted is proportional to the charge of the test charge. For example, if we double the charge of the test charge, the force exerted on it doubles. Mathematically, saying that electric field is the force per unit charge is written as

$$\vec{E} = \frac{\vec{F}}{q_{\text{test}}} \tag{18.15}$$

where we are considering only electric forces. Note that the electric field is a vector field that points in the same direction as the force on the positive test charge. The units of electric field are N/C.

If the electric field is created by a point charge or a sphere of uniform charge, then the magnitude of the force between this point charge Q and the test charge is given by Coulomb's law

$$F = \frac{k\,|Qq_{\text{test}}|}{r^2}$$

where the absolute value is used, because we only consider the magnitude of the force. The magnitude of the electric field is then

$$E = \frac{F}{q_{\text{test}}} = \frac{k\,|Q|}{r^2}. \tag{18.16}$$

This equation gives the magnitude of the electric field created by a point charge Q. The distance r in the denominator is the distance from the point charge, Q, or from the center of a spherical charge, to the point of interest.

If the test charge is removed from the electric field, the electric field still exists. To create a three-dimensional map of the electric field, imagine placing the test charge in various locations in the field. At each location, measure the force on the charge, and use the vector equation $\vec{E} = \vec{F}/q_{\text{test}}$ to calculate the electric field. Draw an arrow at each point where you place the test charge to represent the strength and the direction of the electric field. The length of the arrows should be proportional to the strength of the electric field. If you join together these arrows, you obtain lines. Figure 18.17 shows an image of the three-dimensional electric field created by a positive charge.

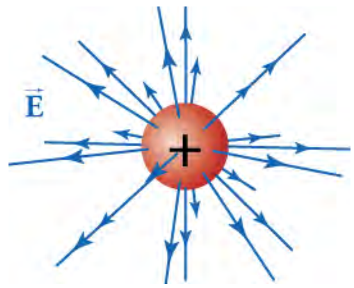


Figure 18.17 Three-dimensional representation of the electric field generated by a positive charge.

Just drawing the electric field lines in a plane that slices through the charge gives the two-dimensional electric-field maps shown in Figure 18.18. On the left is the electric field created by a positive charge, and on the right is the electric field created by a negative charge.

Notice that the electric field lines point away from the positive charge and toward the negative charge. Thus, a positive test charge placed in the electric field of the positive charge will be repelled. This is consistent with Coulomb's law, which says that like charges repel each other. If we place the positive charge in the electric field of the negative charge, the positive charge is attracted to the negative charge. The opposite is true for negative test charges. Thus, the direction of the electric field lines is consistent with what we find by using Coulomb's law.

The equation $E = k\,|Q|\,/r^2$ says that the electric field gets stronger as we approach the charge that generates it. For example, at 2 cm from the charge Q ($r = 2$ cm), the electric field is four times stronger than at 4 cm from the charge ($r = 4$ cm). Looking at Figure 18.17 and Figure 18.18 again, we see that the electric field lines become denser as we approach the charge that generates it. In fact, the density of the electric field lines is proportional to the strength of the electric field!

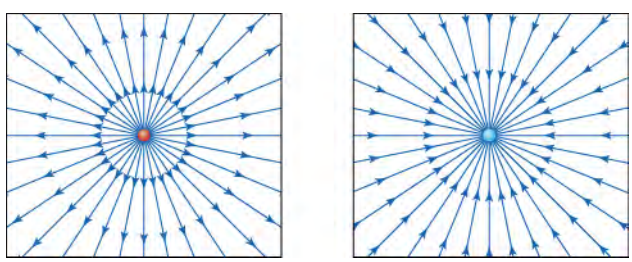


Figure 18.18 Electric field lines from two point charges. The red point on the left carries a charge of +1 nC, and the blue point on the right carries a charge of −1 nC. The arrows point in the direction that a positive test charge would move. The field lines are denser as you approach the point charge.

Electric-field maps can be made for several charges or for more complicated charge distributions. The electric field due to multiple charges may be found by adding together the electric field from each individual charge. Because this sum can only be a single number, we know that only a single electric-field line can go through any given point. In other words, electric-field lines *cannot* cross each other.

Figure 18.19(a) shows a two-dimensional map of the electric field generated by a charge of $+q$ and a nearby charge of $-q$. The three-dimensional version of this map is obtained by rotating this map about the axis that goes through both charges. A positive test charge placed in this field would experience a force in the direction of the field lines at its location. It would thus be repelled from the positive charge and attracted to the negative charge. Figure 18.19(b) shows the electric field generated by two charges of $-q$. Note how the field lines tend to repel each other and do not overlap. A positive test charge placed in this field would be attracted to both charges. If you are far from these two charges, where far means much farther than the distance between the charges, the electric field looks like the electric field from a single charge of $-2q$.

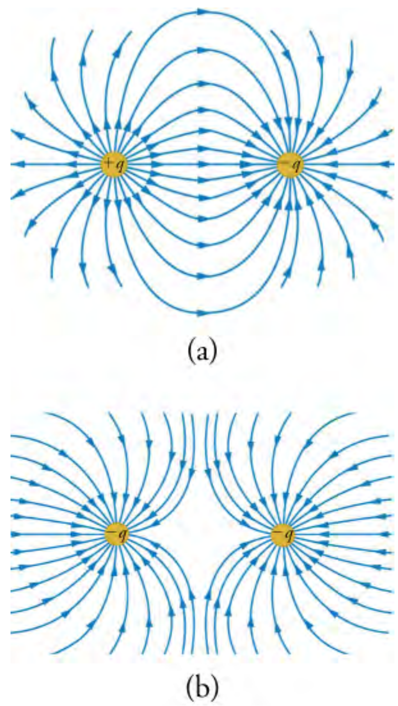


Figure 18.19 (a) The electric field generated by a positive point charge (left) and a negative point charge of the same magnitude (right). (b) The electric field generated by two equal negative charges.

Virtual Physics

Probing an Electric Field

Click to view content (http://www.openstax.org/l/28charge-field)

This simulation shows you the electric field due to charges that you place on the screen. Start by clicking the top checkbox in the options panel on the right-hand side to show the electric field. Drag charges from the buckets onto the screen, move them around, and observe the electric field that they form. To see more precisely the magnitude and direction of the electric field, drag an electric-field sensor, or *E-field* sensor from the bottom bucket, and move it around the screen.

GRASP CHECK

Two positive charges are placed on a screen. Which statement describes the electric field produced by the charges?

a. It is constant everywhere.
b. It is zero near each charge.
c. It is zero halfway between the charges.
d. It is strongest halfway between the charges.

WATCH PHYSICS

Electrostatics (part 2): Interpreting electric field

This video explains how to calculate the electric field of a point charge and how to interpret electric-field maps in general. Note that the lecturer uses d for the distance between particles instead of r. Note that the point charges are infinitesimally small, so all their charges are focused at a point. When larger charged objects are considered, the distance between the objects must be measured between the center of the objects.

Click to view content (https://www.youtube.com/embed/0YOGrTNgGhE)

GRASP CHECK

True or false—If a point charge has electric field lines that point into it, the charge must be ositive.

a. true
b. false

WORKED EXAMPLE

What is the charge?

Look at the drawing of the electric field in Figure 18.20. What is the relative strength and sign of the three charges?

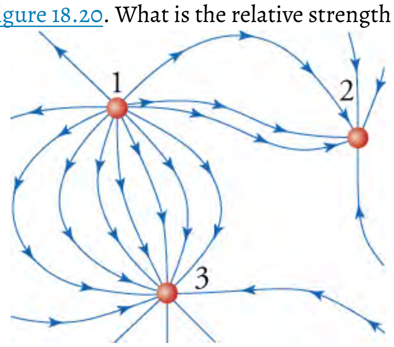


Figure 18.20 Map of electric field due to three charged particles.

STRATEGY

We know the electric field extends out from positive charge and terminates on negative charge. We also know that the number of electric field lines that touch a charge is proportional to the charge. Charge 1 has 12 fields coming out of it. Charge 2 has six field lines going into it. Charge 3 has 12 field lines going into it.

Solution

The electric-field lines come out of charge 1, so it is a positive charge. The electric-field lines go into charges 2 and 3, so they are negative charges. The ratio of the charges is $q_1 : q_2 : q_3 = +12 : -6 : -12$. Thus, magnitude of charges 1 and 3 is twice that of charge 2.

Discussion

Although we cannot determine the precise charge on each particle, we can get a lot of information from the electric field regarding the magnitude and sign of the charges and where the force on a test charge would be greatest (or least).

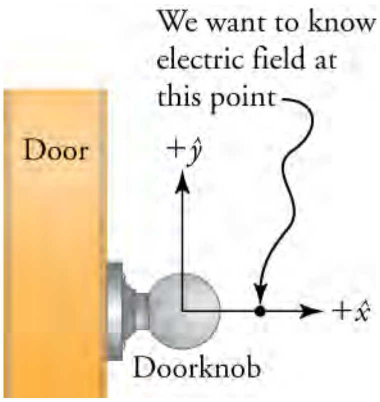
WORKED EXAMPLE

Electric field from doorknob

A doorknob, which can be taken to be a spherical metal conductor, acquires a static electricity charge of $q = -1.5\ \text{nC}$. What is the electric field 1.0 cm in front of the doorknob? The diameter of the doorknob is 5.0 cm.

STRATEGY

Because the doorknob is a conductor, the entire charge is distributed on the outside surface of the metal. In addition, because the doorknob is assumed to be perfectly spherical, the charge on the surface is uniformly distributed, so we can treat the doorknob as if all the charge were located at the center of the doorknob. The validity of this simplification will be proved in a later physics course. Now sketch the doorknob, and define your coordinate system. Use $+x$ to indicate the outward direction perpendicular to the door, with $x = 0$ at the center of the doorknob (as shown in the figure below).



If the diameter of the doorknob is 5.0 cm, its radius is 2.5 cm. We want to know the electric field 1.0 cm from the surface of the doorknob, which is a distance $r = 2.5\ \text{cm} + 1.0\ \text{cm} = 3.5\ \text{cm}$ from the center of the doorknob. We can use the equation $E = \frac{k|Q|}{r^2}$ to find the magnitude of the electric field. The direction of the electric field is determined by the sign of the charge, which is negative in this case.

Solution

Inserting the charge $Q = -1.5\ \text{nC} = -1.5 \times 10^{-9}\text{C}$ and the distance $r = 3.5\ \text{cm} = 0.035\ \text{m}$ into the equation $E = \frac{k|Q|}{r^2}$ gives

$$\begin{aligned} E &= \frac{k|Q|}{r^2} \\ &= \frac{(8.99 \times 10^9 \text{N·m}^2/\text{C}^2)\left|-1.5 \times 10^{-9}\text{C}\right|}{(0.035\ \ \text{m})^2} \\ &= 1.1 \times 10^4 \text{N/C}. \end{aligned} \tag{18.17}$$

Because the charge is negative, the electric-field lines point toward the center of the doorknob. Thus, the electric field at $x = 3.5\ \text{cm}$ is $(-1.1 \times 10^4 \text{N/C})\hat{x}$.

Discussion

This seems like an enormous electric field. Luckily, it takes an electric field roughly 100 times stronger ($3 \times 10^6 \text{N/C}$) to cause air to break down and conduct electricity. Also, the weight of an adult is about $70\ \text{kg} \times 9.8 \text{m/s}^2 \approx 700\ \text{N}$, so why don't you feel a force on the protons in your hand as you reach for the doorknob? The reason is that your hand contains an equal amount of negative charge, which repels the negative charge in the doorknob. A very small force might develop from polarization in your hand, but you would never notice it.

Practice Problems

15. What is the magnitude of the electric field from 20 cm from a point charge of $q = 33$ nC?

a. 7.4×10^3 N/C
b. 1.48×10^3 N / C
c. 7.4×10^{12} N / C
d. 0

16. A –10 nC charge is at the origin. In which direction does the electric field from the charge point at $x + 10$ cm ?

a. The electric field points away from negative charges.
b. The electric field points toward negative charges.
c. The electric field points toward positive charges.
d. The electric field points away from positive charges.

Check Your Understanding

17. When electric field lines get closer together, what does that tell you about the electric field?

a. The electric field is inversely proportional to the density of electric field lines.
b. The electric field is directly proportional to the density of electric field lines.
c. The electric field is not related to the density of electric field lines.
d. The electric field is inversely proportional to the square root of density of electric field lines.

18. If five electric-field lines come out of a +5 nC charge, how many electric-field lines should come out of a +20 nC charge?
a. five field lines
b. 10 field lines
c. 15 field lines
d. 20 field lines

18.4 Electric Potential

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain the similarities and differences between electric potential energy and gravitational potential energy
- Calculate the electric potential difference between two point charges and in a uniform electric field

Section Key Terms

electric potential	electric potential energy

As you learned in studying gravity, a mass in a gravitational field has potential energy, which means it has the potential to accelerate and thereby increase its kinetic energy. This kinetic energy can be used to do work. For example, imagine you want to use a stone to pound a nail into a piece of wood. You first lift the stone high above the nail, which increases the potential energy of the stone-Earth system—because Earth is so large, it does not move, so we usually shorten this by saying simply that the potential energy of the stone increases. When you drop the stone, gravity converts the potential energy into kinetic energy. When the stone hits the nail, it does work by pounding the nail into the wood. The gravitational potential energy is the work that a mass can potentially do by virtue of its position in a gravitational field. Potential energy is a very useful concept, because it can be used with conservation of energy to calculate the motion of masses in a gravitational field.

Electric potential energy works much the same way, but it is based on the electric field instead of the gravitational field. By virtue of its position in an electric field, a charge has an electric potential energy. If the charge is free to move, the force due to the electric field causes it to accelerate, so its potential energy is converted to kinetic energy, just like a mass that falls in a gravitational field. This kinetic energy can be used to do work. The electric potential energy is the work that a charge can do by virtue of its position in an electric field.

The analogy between gravitational potential energy and electric potential energy is depicted in Figure 18.21. On the left, the ball-Earth system gains gravitational potential energy when the ball is higher in Earth's gravitational field. On the right, the two-charge system gains electric potential energy when the positive charge is farther from the negative charge.

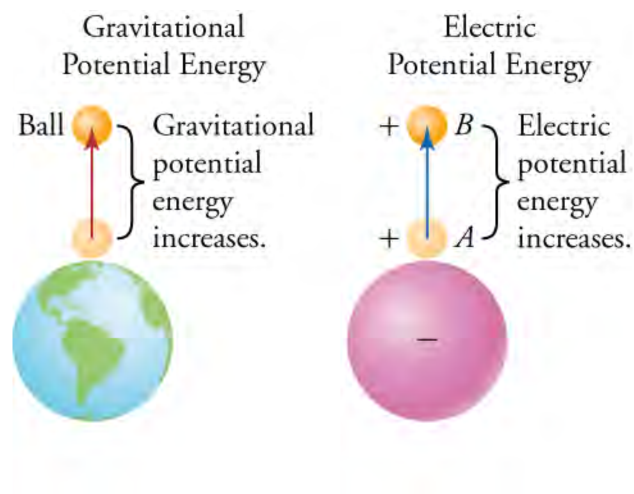


Figure 18.21 On the left, the gravitational field points toward Earth. The higher the ball is in the gravitational field, the higher the potential energy is of the Earth-ball system. On the right, the electric field points toward a negative charge. The farther the positive charge is from the negative charge, the higher the potential energy is of the two-charge system.

Let's use the symbol U_G to denote gravitational potential energy. When a mass falls in a gravitational field, its gravitational potential energy decreases. Conservation of energy tells us that the work done by the gravitational field to make the mass accelerate must equal the loss of potential energy of the mass. If we use the symbol $W_{\text{donebygravity}}$ to denote this work, then

$$-\Delta U_{\text{G}} = W_{\text{donebygravity}}, \tag{18.18}$$

where the minus sign reflects the fact that the potential energy of the ball decreases.

The work done by gravity on the mass is

$$W_{\text{donebygravity}} = -F(y_{\text{f}} - y_{\text{i}}), \tag{18.19}$$

where F is the force due to gravity, and y_{i} and y_{f} are the initial and final positions of the ball, respectively. The negative sign is because gravity points down, which we consider to be the negative direction. For the *constant* gravitational field near Earth's surface, $F = mg$. The change in gravitational potential energy of the mass is

$$-\Delta U_{\text{G}} = W_{\text{donebygravity}} = -F(y_{\text{f}} - y_{\text{i}}) = -mg(y_{\text{f}} - y_{\text{i}}), \text{ or} \Delta U_{\text{G}} = mg(y_{\text{f}} - y_{\text{i}}). \tag{18.20}$$

Note that $y_{\text{f}} - y_{\text{i}}$ is just the negative of the height h from which the mass falls, so we usually just write $\Delta U_{\text{G}} = -mgh$.

We now apply the same reasoning to a charge in an electric field to find the electric potential energy. The change ΔU_{E} in electric potential energy is the work done by the electric field to move a charge q from an initial position x_{i} to a final position x_{f} ($-\Delta U_{\text{E}} = W_{\text{donebyE-field}}$). The definition of work does not change, except that now the work is done by the electric field: $W_{\text{donebyE-field}} = F(x_{\text{f}} - x_{\text{i}})$. For a charge that falls through a *constant* electric field E, the force applied to the charge by the electric field is $F = qE$. The change in electric potential energy of the charge is thus

$$-\Delta U_{\text{E}} = W_{\text{donebyE-field}} = Fd = qE(x_{\text{f}} - x_{\text{i}}) \tag{18.21}$$

or

$$\Delta U_{\text{E}} = -qE(x_{\text{f}} - x_{\text{i}}). \tag{18.22}$$

This equation gives the change in electric potential energy of a charge q when it moves from position x_{i} to position x_{f} in a *constant* electric field E.

Figure 18.22 shows how this analogy would work if we were close to Earth's surface, where gravity is constant. The top image shows a charge accelerating due to a constant electric field. Likewise, the round mass in the bottom image accelerates due to a constant gravitation field. In both cases, the potential energy of the particle decreases, and its kinetic energy increases.

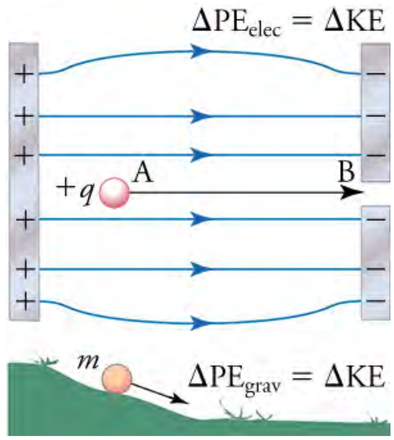


Figure 18.22 In the top picture, a mass accelerates due to a constant electric field. In the bottom picture, the mass accelerates due to a constant gravitational field.

WATCH PHYSICS

Analogy between Gravity and Electricity

This video discusses the analogy between gravitational potential energy and electric potential energy. It reviews the concepts of work and potential energy and shows the connection between a mass in a uniform gravitation field, such as on Earth's surface, and an electric charge in a uniform electric field.

Click to view content (https://www.openstax.org/l/28grav-elec)

If the electric field is not constant, then the equation $\Delta U_E = -qE(x_f - x_i)$ is not valid, and deriving the electric potential energy becomes more involved. For example, consider the electric potential energy of an assembly of two point charges q_1 and q_2 of the same sign that are initially very far apart. We start by placing charge q_1 at the origin of our coordinate system. This takes no electrical energy, because there is no electric field at the origin (because charge q_2 is very far away). We then bring charge q_2 in from very far away to a distance *r* from the center of charge q_1 . This requires some effort, because the electric field of charge q_1 applies a repulsive force on charge q_2 . The energy it takes to assemble these two charges can be recuperated if we let them fly apart again. Thus, the charges have potential energy when they are a distance *r* apart. It turns out that the electric potential energy of a pair of point charges q_1 and q_2 a distance *r* apart is

$$U_E = \frac{kq_1q_2}{r} \qquad (18.23)$$

To recap, if charges q_1 and q_2 are free to move, they can accumulate kinetic energy by flying apart, and this kinetic energy can be used to do work. The maximum amount of work the two charges can do (if they fly infinitely far from each other) is given by the equation above.

Notice that if the two charges have opposite signs, then the potential energy is negative. This means that the charges have more potential to do work when they are *far* apart than when they are at a distance *r* apart. This makes sense: Opposite charges attract, so the charges can gain more kinetic energy if they attract each other from far away than if they start at only a short distance apart. Thus, they have more potential to do work when they are far apart. Figure 18.23 summarizes how the electric potential energy depends on charge and separation.

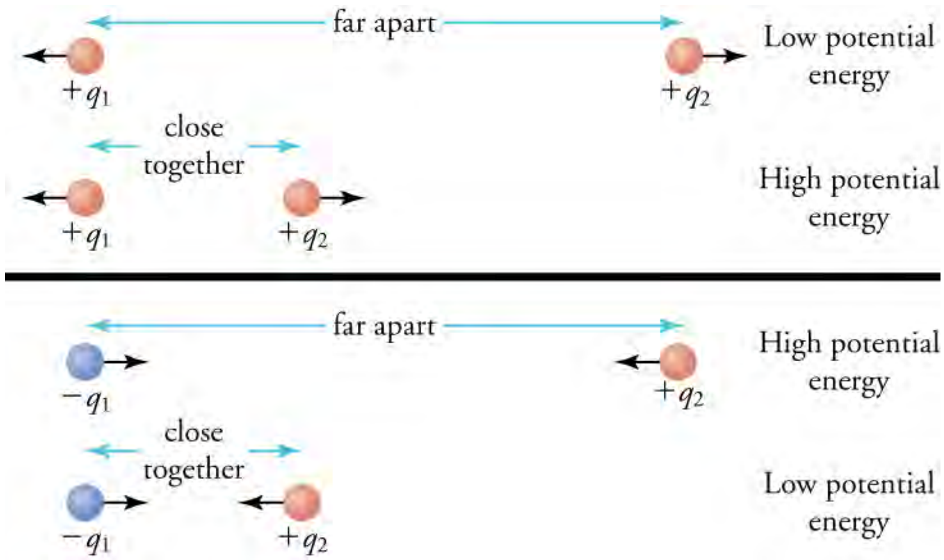


Figure 18.23 The potential energy depends on the sign of the charges and their separation. The arrows on the charges indicate the direction in which the charges would move if released. When charges with the same sign are far apart, their potential energy is low, as shown in the top panel for two positive charges. The situation is the reverse for charges of opposite signs, as shown in the bottom panel.

Electric Potential

Recall that to find the force applied by a fixed charge Q on any arbitrary test charge q, it was convenient to define the electric field, which is the force per unit charge applied by Q on any test charge that we place in its electric field. The same strategy is used here with electric potential energy: We now define the **electric potential** V, which is the electric potential energy per unit charge.

$$V = \frac{U_E}{q} \tag{18.24}$$

Normally, the electric potential is simply called the *potential* or *voltage*. The units for the potential are J/C, which are given the name *volt* (V) after the Italian physicist Alessandro Volta (1745–1827). From the equation $U_E = kq_1 q_2 / r$, the electric potential a distance r from a point charge q_1 is

$$V = \frac{U_E}{q_2} = \frac{kq_1}{r}. \tag{18.25}$$

This equation gives the energy required per unit charge to bring a charge q_2 from infinity to a distance r from a point charge q_1. Mathematically, this is written as

$$V = \left.\frac{U_E}{q_2}\right|_{R=r} - \left.\frac{U_E}{q_2}\right|_{R=\infty}. \tag{18.26}$$

Note that this equation actually represents a *difference* in electric potential. However, because the second term is zero, it is normally not written, and we speak of the electric potential instead of the electric potential difference, or we just say the potential difference, or voltage). Below, when we consider the electric potential energy per unit charge between two points not infinitely far apart, we speak of electric potential *difference* explicitly. Just remember that electric potential and electric potential difference are really the same thing; the former is used just when the electric potential energy is zero in either the initial or final charge configuration.

Coming back now to the electric potential a distance r from a point charge q_1, note that q_1 can be any arbitrary point charge, so we can drop the subscripts and simply write

$$V = \frac{kq}{r}. \tag{18.27}$$

Now consider the electric potential near a group of charges q_1, q_2, and q_3, as drawn in Figure 18.24. The electric potential is derived by considering the electric field. Electric fields follow the principle of superposition and can be simply added together, so the electric potential from different charges also add together. Thus, the electric potential of a point near a group of charges is

$$V = \frac{kq_1}{r_1} + \frac{kq_2}{r_2} + \frac{kq_3}{r_3} + \cdots. \tag{18.28}$$

where $r_1, r_2, r_3, \ldots$, are the distances from the center of charges $q_1, q_2, q_3, \ldots$ to the point of interest, as shown in Figure 18.24.

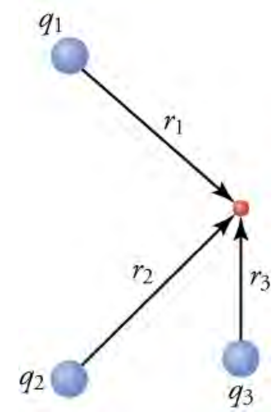


Figure 18.24 The potential at the red point is simply the sum of the potentials due to each individual charge.

Now let's consider the electric potential in a uniform electric field. From the equation $\Delta U_E = -qE\,(x_f - x_i)$, we see that the potential difference in going from x_i to x_f in a uniform electric field E is

$$\Delta V = \frac{\Delta U_E}{q} = -E\,(x_f - x_i)\,. \tag{18.29}$$

TIPS FOR SUCCESS

Notice from the equation $\Delta V = -E\,(x_f - x_i)$ that the electric field can be written as

$$E = \frac{\Delta V}{x_f - x_i} \tag{18.30}$$

which means that the electric field has units of V/m. Thus, if you know the potential difference between two points, calculating the electric field is very simple—you simply divide the potential difference by the distance!

Notice that a positive charge in a region with high potential will experience a force pushing it toward regions of lower potential. In this sense, potential is like pressure for fluids. Imagine a pipe containing fluid, with the fluid at one end of the pipe under high pressure and the fluid at the other end of the pipe under low pressure. If nothing prevents the fluid from flowing, it will flow from the high-pressure end to the low-pressure end. Likewise, a positive charge that is free to move will move from a region with high potential to a region with lower potential.

WATCH PHYSICS

Voltage

This video starts from electric potential energy and explains how this is related to electric potential (or voltage). The lecturer calculates the electric potential created by a uniform electric field.

Click to view content (https://www.openstax.org/l/28voltage)

GRASP CHECK

What is the voltage difference between the positions $x_f = 11\text{ m}$ and $x_i = 5.0\text{ m}$ in an electric field of $(2.0\text{ N/C})\,\hat{x}$?

a. 6 V

b. 12 V
c. 24 V
d. 32 V

LINKS TO PHYSICS

Electric Animals

Many animals generate and/or detect electric fields. This is useful for activities such as hunting, defense, navigation, communication, and mating. Because salt water is a relatively good conductor, electric fish have evolved in all the world's oceans. These fish have intrigued humans since the earliest times. In the nineteenth century, parties were even organized where the main attraction was getting a jolt from an electric fish! Scientists also studied electric fish to learn about electricity. Alessandro Volta based his research that led to batteries in 1799 on electric fish. He even referred to batteries as artificial electric organs, because he saw them as imitations of the electric organs of electric fish.

Animals that generate electricity are called *electrogenic* and those that detect electric fields are called *electroreceptive*. Most fish that are electrogenic are also electroreceptive. One of the most well-known electric fish is the electric eel (see Figure 18.25), which is both electrogenic and electroreceptive. These fish have three pairs of organs that produce the electric charge: the main organ, Hunter's organ, and Sach's organ. Together, these organs account for more than 80percent of the fish's body.

Electric eels can produce electric discharges of much greater voltage than what you would get from a standard wall socket. These discharges can stun or even kill their prey. They also use low-intensity discharges to navigate. The electric fields they generate reflect off nearby obstacles or animals and are then detected by electroreceptors in the eel's skin. The three organs that produce electricity contain electrolytes, which are substances that ionize when dissolved in water (or other liquids). An ionized atom or molecule is one that has lost or gained at least one electron, so it carries a net charge. Thus, a liquid solution containing an electrolyte conducts electricity, because the ions in the solution can move if an electric field is applied.

To produce large discharges, the main organ is used. It contains approximately 6,000 rows of electroplaques connected in a long chain. Connected this way, the voltage between electroplaques adds up, creating a large final voltage. Each electroplaque consists of a column of cells controlled by an excitor nerve. When triggered by the excitor nerve, the electroplaques allow ionized sodium to flow through them, creating a potential difference between electroplaques. These potentials add up, and a large current can flow through the electrolyte.

This geometry is reflected in batteries, which also use stacks of plates to produce larger potential differences.

Figure 18.25 An electric eel in its natural environment. (credit: Steven G. Johnson)

GRASP CHECK

If an electric eel produces 1,000 V, which voltage is produced by each electroplaque in the main organ?

a. 0.17 mV
b. 1.7 mV
c. 17 mV
d. 170 mV

WORKED EXAMPLE

X-ray Tube

Dentists use X-rays to image their patients' teeth and bones. The X-ray tubes that generate X-rays contain an electron source separated by about 10 cm from a metallic target. The electrons are accelerated from the source to the target by a uniform electric field with a magnitude of about 100 kN/C, as drawn in Figure 18.26. When the electrons hit the target, X-rays are produced. (a) What is the potential difference between the electron source and the metallic target? (b) What is the kinetic energy of the electrons when they reach the target, assuming that the electrons start at rest?

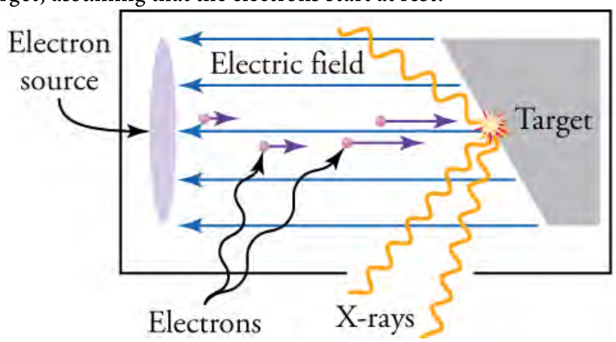


Figure 18.26 In an X-ray tube, a large current flows through the electron source, causing electrons to be ejected from the electron source. The ejected electrons are accelerated toward the target by the electric field. When they strike the target, X-rays are produced.

STRATEGY FOR (A)

Use the equation $\Delta V = -E\left(x_{\mathrm{f}} - x_{\mathrm{i}}\right)$ to find the potential difference given a constant electric field. Define the source position as $x_{\mathrm{i}} = 0$ and the target position as $x_{\mathrm{f}} = 10\ \mathrm{cm}$. To accelerate the electrons in the positive *x* direction, the electric field must point in the negative *x* direction. This way, the force $F = qE$ on the electrons will point in the positive *x* direction, because both *q* and *E* are negative. Thus, $E = -100 \times 10^3\,\mathrm{N/C}$.

Solution for (a)

Using $x_{\mathrm{i}} = 0$ and $x_{\mathrm{f}} = 10\ \mathrm{cm} = 0.10\ \mathrm{m}$, the equation $\Delta V = -E\left(x_{\mathrm{f}} - x_{\mathrm{i}}\right)$ tells us that the potential difference between the electron source and the target is

$$\Delta V = -E\left(x_{\mathrm{f}} - x_{\mathrm{i}}\right) = -\left(-100 \times 10^3\,\mathrm{N/C}\right)(0.10\ \mathrm{m} - 0) = +10\ \mathrm{kV}. \tag{18.31}$$

Discussion for (a)

The potential difference is positive, so the energy per unit positive charge is higher at the target than at the source. This means that free positive charges would fall from the target to the source. However, electrons are negative charges, so they accelerate from the source toward the target, gaining kinetic energy as they go.

STRATEGY FOR (B)

Apply conservation of energy to find the final kinetic energy of the electrons. In going from the source to the target, the change in electric potential energy plus the change in kinetic energy of the electrons must be zero, so $\Delta U_E + \Delta K = 0$. The change in electric potential energy for moving through a constant electric field is given by the equation

$$\Delta U_{\mathrm{E}} = -qE\left(x_{\mathrm{f}} - x_{\mathrm{i}}\right),$$

where the electric field is $E = -100 \times 10^3\,\mathrm{N/C}$. Because the electrons start at rest, their initial kinetic energy is zero. Thus, the change in kinetic energy is simply their final kinetic energy, so $\Delta K = K_{\mathrm{f}}$.

Solution for (b)

Again $x_{\mathrm{i}} = 0$ and $x_{\mathrm{f}} = 10\ \mathrm{cm} = 0.10\ \mathrm{m}$. The charge of an electron is $q = -1.602 \times 10^{-19}\,\mathrm{C}$. Conservation of energy gives

$$\begin{aligned} \Delta U_E + \Delta K &= 0. \\ -qE\left(x_{\mathrm{f}} - x_{\mathrm{i}}\right) + K_{\mathrm{f}} &= 0. \\ K_{\mathrm{f}} &= qE(x_{\mathrm{f}} - x_{\mathrm{i}}). \end{aligned} \tag{18.32}$$

Inserting the known values into the right-hand side of this equation gives

$$\begin{aligned} K_f &= \left(-1.60 \times 10^{-19}\text{C}\right)\left(-100 \times 10^{3}\text{N/C}\right)(0.10 \text{ m} - 0) \\ &= 1.6 \times 10^{-15}\text{J}. \end{aligned} \quad (18.33)$$

Discussion for (b)

This is a very small energy. However, electrons are very small, so they are easy to accelerate, and this energy is enough to make an electron go extremely fast. You can find their speed by using the definition of kinetic energy, $K = \frac{1}{2}mv^2$. The result is that the electrons are moving at more than 100 million miles per hour!

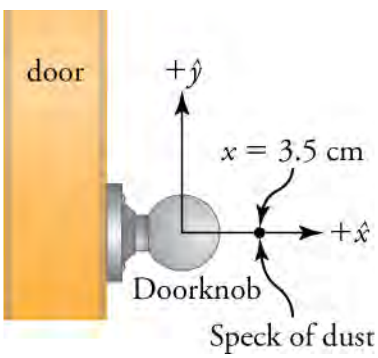
WORKED EXAMPLE

Electric Potential Energy of Doorknob and Dust Speck

Consider again the doorknob from the example in the previous section. The doorknob is treated as a spherical conductor with a uniform static charge $q_1 = -1.5$ nC on its surface. What is the electric potential energy between the doorknob and a speck of dust carrying a charge $q_2 = 0.20$ nC at 1.0 cm from the front surface of the doorknob? The diameter of the doorknob is 5.0 cm.

STRATEGY

As we did in the previous section, we treat the charge as if it were concentrated at the center of the doorknob. Again, as you will be able to validate in later physics classes, we can make this simplification, because the charge is uniformly distributed over the surface of the spherical object. Make a sketch of the situation and define a coordinate system, as shown in the image below. We use $+x$ to indicate the outward direction perpendicular to the door, with $x = 0$ at the center of the doorknob. If the diameter of the doorknob is 5.0 cm, its radius is 2.5 cm. Thus, the speck of dust 1.0 cm from the surface of the doorknob is a distance $r = 2.5 \text{ cm} + 1.0 \text{ cm} = 3.5 \text{ cm}$ from the center of the doorknob. To solve this problem, use the equation $U_E = kq_1q_2/r$.



Solution

The charge on the doorknob is $q_1 = -1.5 \text{ nC} = -1.5 \times 10^{-9}\text{C}$, and the charge on the speck of dust is $q_2 = 0.20 \text{ nC} = 2.0 \times 10^{-10}\text{C}$. The distance $r = 3.5 \text{ cm} = 0.035 \text{ m}$. Inserting these values into the equation $U_E = kq_1q_2/r$ gives

$$\begin{aligned} U_E &= \frac{kq_1q_2}{r} \\ &= \frac{\left(8.99 \times 10^9 \text{N}\cdot\text{m}^2/\text{C}^2\right)\left(-1.5 \times 10^{-9}\text{C}\right)\left(2.0 \times 10^{-10}\text{C}\right)}{(0.035 \text{ m})} \\ &= -7.7 \times 10^{-8}\text{J}. \end{aligned} \quad (18.34)$$

Discussion

The energy is negative, which means that the energy will decrease that is, get even *more* negative as the speck of dust approaches the doorknob. This helps explain why dust accumulates on objects that carry a static charge. However, note that insulators normally collect more static charge than conductors, because any charge that accumulates on insulators cannot move about on the insulator to find a way to escape. They must simply wait to be removed by some passing moist speck of dust or other host.

Practice Problems

19. What is the electric potential 10 cm from a –10 nC charge?

a. 9.0×10^2 V
b. 9.0×10^3 V
c. 9.0×10^4 V
d. 9.0×10^5 V

20. An electron accelerates from 0 to 10×10^4 m/s in an electric field. Through what potential difference did the electron travel? The mass of an electron is 9.11×10^{-31} kg, and its charge is -1.60×10^{-19} C.

a. 29 mV
b. 290 mV
c. 2,900 mV
d. 29 V

Check Your Understanding

21. Gravitational potential energy is the $(-10\ \mathrm{N/C})\hat{x}$ potential for two masses to do work by virtue of their positions with respect to each other. What is the analogous definition of electric potential energy?

a. Electric potential energy is the potential for two charges to do work by virtue of their positions with respect to the origin point.
b. Electric potential energy is the potential for two charges to do work by virtue of their positions with respect to infinity.
c. Electric potential energy is the potential for two charges to do work by virtue of their positions with respect to each other.
d. Electric potential energy is the potential for single charges to do work by virtue of their positions with respect to their final positions.

22. A negative charge is 10 m from a positive charge. Where would you have to move the negative charge to increase the potential energy of the system?

a. The negative charge should be moved closer to the positive charge.
b. The negative charge should be moved farther away from the positive charge.
c. The negative charge should be moved to infinity.
d. The negative charge should be placed just next to the positive charge.

18.5 Capacitors and Dielectrics

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Calculate the energy stored in a charged capacitor and the capacitance of a capacitor
- Explain the properties of capacitors and dielectrics

Section Key Terms

capacitor dielectric

Capacitors

Consider again the X-ray tube discussed in the previous sample problem. How can a uniform electric field be produced? A single positive charge produces an electric field that points away from it, as in . This field is not uniform, because the space between the lines increases as you move away from the charge. However, if we combine a positive and a negative charge, we obtain the electric field shown in (a). Notice that, between the charges, the electric field lines are more equally spaced.

What happens if we place, say, five positive charges in a line across from five negative charges, as in Figure 18.27? Now the region between the lines of charge contains a fairly uniform electric field.

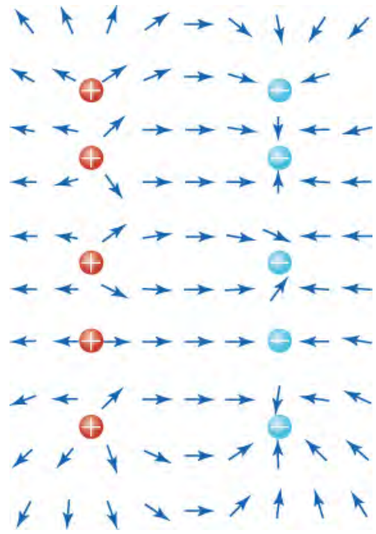


Figure 18.27 The red dots are positive charges, and the blue dots are negative charges. The electric-field direction is shown by the red arrows. Notice that the electric field between the positive and negative dots is fairly uniform.

We can extend this idea even further and into two dimensions by placing two metallic plates face to face and charging one with positive charge and the other with an equal magnitude of negative charge. This can be done by connecting one plate to the positive terminal of a battery and the other plate to the negative terminal, as shown in Figure 18.28. The electric field between these charged plates will be extremely uniform.

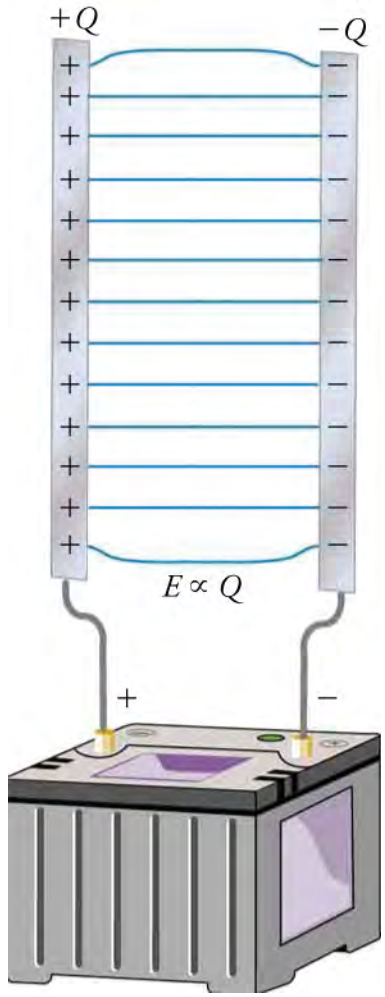


Figure 18.28 Two parallel metal plates are charged with opposite charge, by connecting the plates to the opposite terminals of a battery. The magnitude of the charge on each plate is the same.

Let's think about the work required to charge these plates. Before the plates are connected to the battery, they are neutral—that is, they have zero net charge. Placing the first positive charge on the left plate and the first negative charge on the right plate requires very little work, because the plates are neutral, so no opposing charges are present. Now consider placing a second positive charge on the left plate and a second negative charge on the right plate. Because the first two charges repel the new arrivals, a force must be applied to the two new charges over a distance to put them on the plates. This is the definition of work, which means that, compared with the first pair, more work is required to put the second pair of charges on the plates. To place the third positive and negative charges on the plates requires yet more work, and so on. Where does this work come from? The battery! Its chemical potential energy is converted into the work required to separate the positive and negative charges.

Although the battery does work, this work remains within the battery-plate system. Therefore, conservation of energy tells us that, if the potential energy of the battery decreases to separate charges, the energy of another part of the system must increase by the same amount. In fact, the energy from the battery is stored in the electric field between the plates. This idea is analogous to considering that the potential energy of a raised hammer is stored in Earth's gravitational field. If the gravitational field were to disappear, the hammer would have no potential energy. Likewise, if no electric field existed between the plates, no energy would be stored between them.

If we now disconnect the plates from the battery, they will hold the energy. We could connect the plates to a lightbulb, for example, and the lightbulb would light up until this energy was used up. These plates thus have the capacity to store energy. For this reason, an arrangement such as this is called a **capacitor**. A capacitor is an arrangement of objects that, by virtue of their geometry, can store energy an electric field.

Various real capacitors are shown in Figure 18.29. They are usually made from conducting plates or sheets that are separated by an insulating material. They can be flat or rolled up or have other geometries.

Figure 18.29 Some typical capacitors. (credit: Windell Oskay)

The capacity of a capacitor is defined by its capacitance C, which is given by

$$C = \frac{Q}{V}, \tag{18.35}$$

where Q is the *magnitude* of the charge on each capacitor plate, and V is the potential difference in going from the negative plate to the positive plate. This means that both Q and V are always positive, so the capacitance is always positive. We can see from the equation for capacitance that the units of capacitance are C/V, which are called farads (F) after the nineteenth-century English physicist Michael Faraday.

The equation $C = Q/V$ makes sense: A parallel-plate capacitor (like the one shown in Figure 18.28) the size of a football field could hold a lot of charge without requiring too much work per unit charge to push the charge into the capacitor. Thus, Q would be large, and V would be small, so the capacitance C would be very large. Squeezing the same charge into a capacitor the size of a fingernail would require much more work, so V would be very large, and the capacitance would be much smaller.

Although the equation $C = Q/V$ makes it seem that capacitance depends on voltage, in fact it does not. For a given capacitor, the ratio of the charge stored in the capacitor to the voltage difference between the plates of the capacitor always remains the same. Capacitance is determined by the geometry of the capacitor and the materials that it is made from. For a parallel-plate capacitor with nothing between its plates, the capacitance is given by

$$C_0 = \varepsilon_0 \frac{A}{d}, \tag{18.36}$$

where A is the area of the plates of the capacitor and d is their separation. We use C_0 instead of C, because the capacitor has nothing between its plates (in the next section, we'll see what happens when this is not the case). The constant ε_0, read *epsilon zero* is called the permittivity of free space, and its value is

$$\varepsilon_0 = 8.85 \times 10^{-12}\ \text{F/m} \tag{18.37}$$

Coming back to the energy stored in a capacitor, we can ask exactly how much energy a capacitor stores. If a capacitor is charged by putting a voltage V across it for example, by connecting it to a battery with voltage V—the electrical potential energy stored in the capacitor is

$$U_E = \frac{1}{2} C V^2. \tag{18.38}$$

Notice that the form of this equation is similar to that for kinetic energy, $K = \frac{1}{2} m v^2$.

WATCH PHYSICS

Where does Capacitance Come From?

This video shows how capacitance is defined and why it depends only on the geometric properties of the capacitor, not on voltage or charge stored. In so doing, it provides a good review of the concepts of work and electric potential.

Click to view content (https://www.openstax.org/l/28capacitance)

GRASP CHECK

If you increase the distance between the plates of a capacitor, how does the capacitance change?

a. Doubling the distance between capacitor plates will reduce the capacitance four fold.
b. Doubling the distance between capacitor plates will reduce the capacitance two fold.
c. Doubling the distance between capacitor plates will increase the capacitance two times.
d. Doubling the distance between capacitor plates will increase the capacitance four times.

Virtual Physics

Charge your Capacitor

[Click to view content (http://www.openstax.org/l/28charge-cap)](http://www.openstax.org/l/28charge-cap)

For this simulation, choose the tab labeled *Introduction* at the top left of the screen. You are presented with a parallel-plate capacitor connected to a variable-voltage battery. The battery is initially at zero volts, so no charge is on the capacitor. Slide the battery slider up and down to change the battery voltage, and observe the charges that accumulate on the plates. Display the capacitance, top-plate charge, and stored energy as you vary the battery voltage. You can also display the electric-field lines in the capacitor. Finally, probe the voltage between different points in this circuit with the help of the voltmeter, and probe the electric field in the capacitor with the help of the electric-field detector.

GRASP CHECK

True or false— In a capacitor, the stored energy is always positive, regardless of whether the top plate is charged with negative or positive charge.

a. false
b. true

WORKED EXAMPLE

Capacitance and Charge Stored in a Parallel Plate Capacitor

(a) What is the capacitance of a parallel-plate capacitor with metal plates, each of area 1.00 m^2, separated by 0.0010 m? (b) What charge is stored in this capacitor if a voltage of 3.00×10^3 V is applied to it?

STRATEGY FOR (A)

Use the equation $C_0 = \varepsilon_0 \frac{A}{d}$.

Solution for (a)

Entering the given values into this equation for the capacitance of a parallel-plate capacitor yields

$$\begin{aligned} C &= \varepsilon_0 \frac{A}{d} \\ &= \left(8.85 \times 10^{-12} \text{ F/m}\right) \frac{1.00\text{m}^2}{0.0010 \text{ m}} \\ &= 8.9 \times 10^{-9} \text{ F} \\ &= 8.9\text{nF}. \end{aligned} \tag{18.39}$$

Discussion for (a)

This small value for the capacitance indicates how difficult it is to make a device with a large capacitance. Special techniques help, such as using very-large-area thin foils placed close together or using a dielectric (to be discussed below).

STRATEGY FOR (B)

Knowing C, find the charge stored by solving the equation $C = Q/V$, for the charge Q.

Solution for (b)

The charge Q on the capacitor is

$$\begin{aligned} Q &= CV \\ &= \left(8.9 \times 10^{-9} \text{F}\right)\left(3.00 \times 10^{3} \text{V}\right) \\ &= 2.7 \times 10^{-5} \text{C}. \end{aligned} \tag{18.40}$$

Discussion for (b)

This charge is only slightly greater than typical static electricity charges. More charge could be stored by using a dielectric between the capacitor plates.

WORKED EXAMPLE

What battery is needed to charge a capacitor?

Your friend provides you with a $10\mu\text{F}$ capacitor. To store $120\mu\text{C}$ on this capacitor, what voltage battery should you buy?

STRATEGY

Use the equation $C = Q/V$ to find the voltage needed to charge the capacitor.

Solution

Solving $C = Q/V$ for the voltage gives $V = Q/C$. Inserting $C = 10\mu\text{F} = 10 \times 10^{-6}\text{F}$ and $Q = 120\mu\text{C} = 120 \times 10^{-6}\text{C}$ gives

$$V = \frac{Q}{C} = \frac{120 \times 10^{-6}\text{C}}{10 \times 10^{-6}\text{F}} = 12 \text{ V} \tag{18.41}$$

Discussion

Such a battery should be easy to procure. There is still a question of whether the battery contains enough energy to provide the desired charge. The equation $U_E = \frac{1}{2}CV^2$ allows us to calculate the required energy.

$$U_E = \frac{1}{2}CV^2 = \frac{1}{2}\left(10 \times 10^{-6}\text{F}\right)(12 \text{ V})^2 = 72 \text{ mJ} \tag{18.42}$$

A typical commercial battery can easily provide this much energy.

Practice Problems

23. What is the voltage on a 35 µF with 25 nC of charge?
   a. 8.75×10^{-13} V
   b. 0.71×10^{-3} V
   c. 1.4×10^{-3} V
   d. 1.4×10^{3} V

24. Which voltage is across a 100 µF capacitor that stores 10 J of energy?
   a. -4.5×10^{2} V
   b. 4.5×10^{2} V
   c. $\pm4.5 \times 10^{2}$ V
   d. $\pm9 \times 10^{2}$ V

Dielectrics

Before working through some sample problems, let's look at what happens if we put an insulating material between the plates of a capacitor that has been charged and then disconnected from the charging battery, as illustrated in Figure 18.30. Because the material is insulating, the charge cannot move through it from one plate to the other, so the charge Q on the capacitor does not change. An electric field exists between the plates of a charged capacitor, so the insulating material becomes polarized, as shown in the lower part of the figure. An electrically insulating material that becomes polarized in an electric field is called a **dielectric**.

Figure 18.30 shows that the negative charge in the molecules in the material shifts to the left, toward the positive charge of the capacitor. This shift is due to the electric field, which applies a force to the left on the electrons in the molecules of the dielectric. The right sides of the molecules are now missing a bit of negative charge, so their net charge is positive.

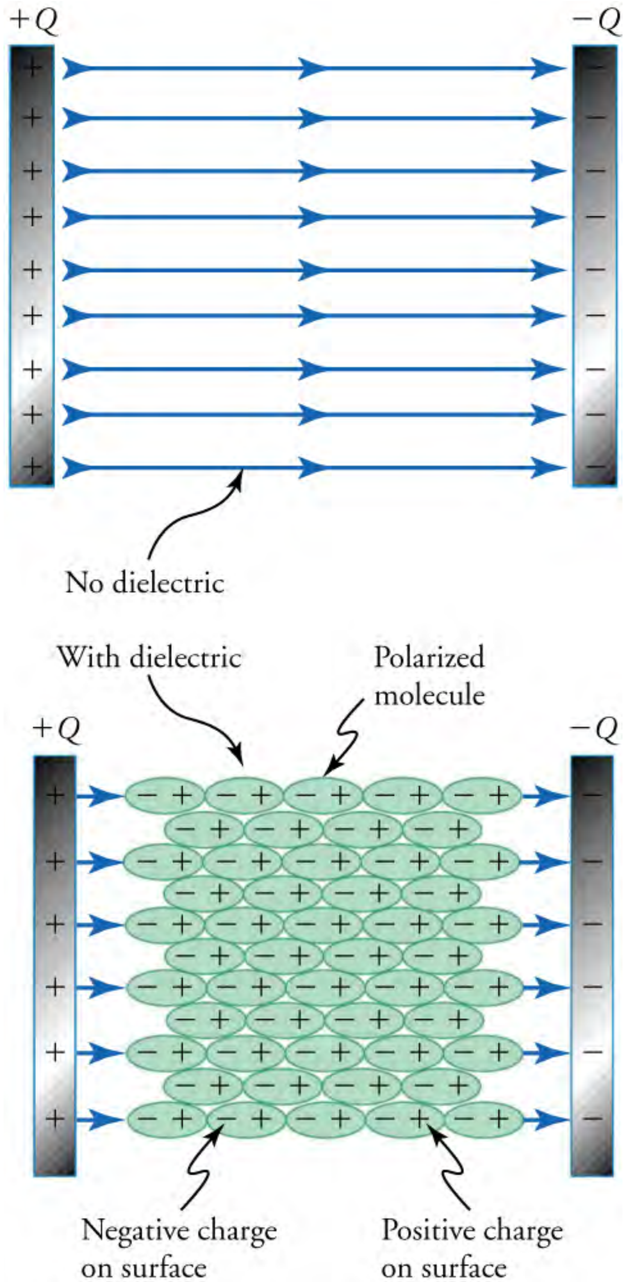


Figure 18.30 The top and bottom capacitors carry the same charge *Q*. The top capacitor has no dielectric between its plates. The bottom capacitor has a dielectric between its plates. The molecules in the dielectric are polarized by the electric field of the capacitor.

All electrically insulating materials are dielectrics, but some are *better* dielectrics than others. A good dielectric is one whose molecules allow their electrons to shift strongly in an electric field. In other words, an electric field pulls their electrons a fair bit away from their atom, but they do not escape completely from their atom (which is why they are insulators).

Figure 18.31 shows a macroscopic view of a dielectric in a charged capacitor. Notice that the electric-field lines in the capacitor with the dielectric are spaced farther apart than the electric-field lines in the capacitor with no dielectric. This means that the electric field in the dielectric is weaker, so it stores less electrical potential energy than the electric field in the capacitor with no dielectric.

Where has this energy gone? In fact, the molecules in the dielectric act like tiny springs, and the energy in the electric field goes into stretching these springs. With the electric field thus weakened, the voltage difference between the two sides of the capacitor is smaller, so it becomes easier to put more charge on the capacitor. Placing a dielectric in a capacitor before charging it therefore allows more charge and potential energy to be stored in the capacitor. A parallel plate with a dielectric has a capacitance of

$$C = \kappa\varepsilon_0 \frac{A}{d} = \kappa C_0, \tag{18.43}$$

where κ (*kappa*) is a dimensionless constant called the *dielectric constant*. Because κ is greater than 1 for dielectrics, the capacitance increases when a dielectric is placed between the capacitor plates. The dielectric constant of several materials is shown in Table 18.1.

Material	Dielectric Constant (κ)
Vacuum	1.00000
Air	1.00059
Fused quartz	3.78
Neoprene rubber	6.7
Nylon	3.4
Paper	3.7
Polystyrene	2.56
Pyrex glass	5.6
Silicon oil	2.5
Strontium titanate	233
Teflon	2.1
Water	80

Table 18.1 Dielectric Constants for Various Materials at 20 °C

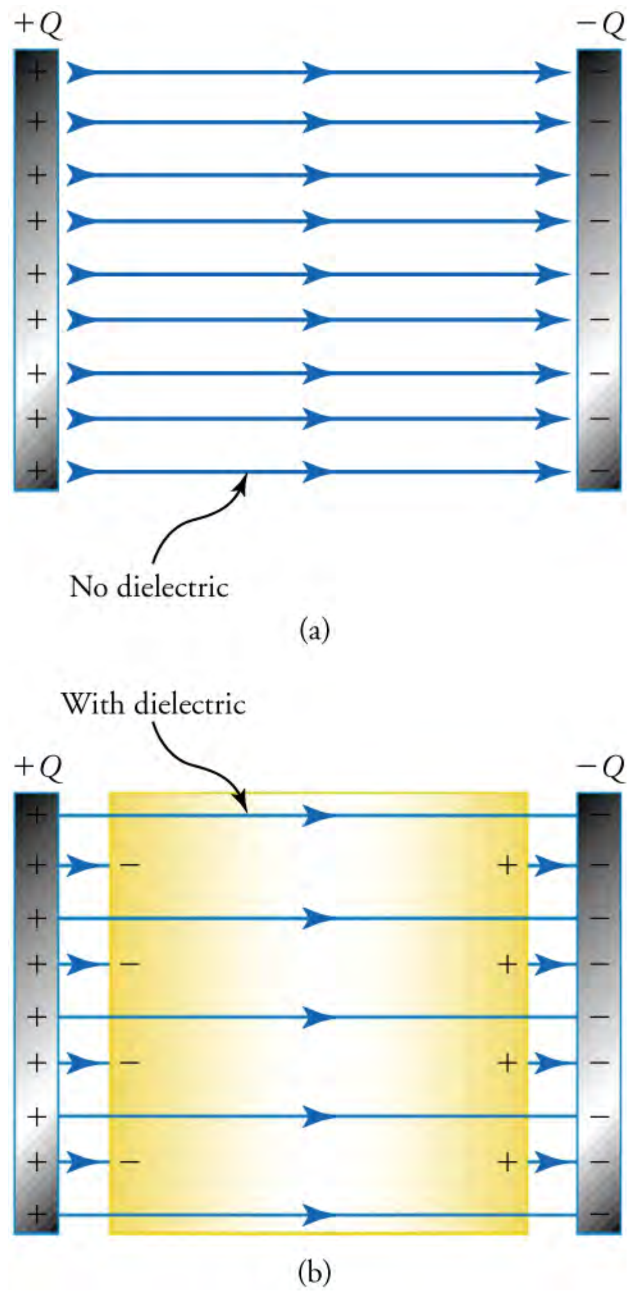


Figure 18.31 The top and bottom capacitors carry the same charge Q. The top capacitor has no dielectric between its plates. The bottom capacitor has a dielectric between its plates. Because some electric-field lines terminate and start on polarization charges in the dielectric, the electric field is less strong in the capacitor. Thus, for the same charge, a capacitor stores less energy when it contains a dielectric.

WORKED EXAMPLE

Capacitor for Camera Flash

A typical flash for a point-and-shoot camera uses a capacitor of about $200\ \mu\text{F}$. (a) If the potential difference between the capacitor plates is 100 V—that is, 100 V is placed "across the capacitor," how much energy is stored in the capacitor? (b) If the dielectric used in the capacitor were a 0.010-mm-thick sheet of nylon, what would be the surface area of the capacitor plates?

STRATEGY FOR (A)

Given that $V = 100\text{ V}$ and $C = 200 \times 10^{-6}\text{F}$, we can use the equation $U_E = \frac{1}{2}CV^2$, to find the electric potential energy stored in the capacitor.

Solution for (a)

Inserting the given quantities into $U_E = \frac{1}{2}CV^2$ gives

$$\begin{aligned} U_E &= \tfrac{1}{2}CV^2 \\ &= \tfrac{1}{2}\left(200 \times 10^{-6}\text{F}\right)(100\ \text{V})^2 \\ &= 1.0\ \text{J}. \end{aligned} \tag{18.44}$$

Discussion for (a)

This is enough energy to lift a 1-kg ball about 1 m up from the ground. The flash lasts for about 0.001 s, so the power delivered by the capacitor during this brief time is $P = \frac{U_E}{t} = \frac{1.0\ \text{J}}{0.001\ \text{s}} = 1\ \text{kW}$. Considering that a car engine delivers about 100 kW of power, this is not bad for a little capacitor!

STRATEGY FOR (B)

Because the capacitor plates are in contact with the dielectric, we know that the spacing between the capacitor plates is $d = 0.010\ \text{mm} = 1.0 \times 10^{-5}\text{m}$. From the previous table, the dielectric constant of nylon is $\kappa = 3.4$. We can now use the equation $C = \kappa\varepsilon_0 \frac{A}{d}$ to find the area *A* of the capacitor.

Solution (b)

Solving the equation for the area *A* and inserting the known quantities gives

$$\begin{aligned} C &= \kappa\varepsilon_0 \frac{A}{d} \\ A &= \frac{Cd}{\kappa\varepsilon_0} \\ &= \frac{\left(200 \times 10^{-6}\text{F}\right)\left(1.0 \times 10^{-5}\text{m}\right)}{(3.4)\left(8.85 \times 10^{-12}\ \text{F/m}\right)} \\ &= 66\text{m}^2. \end{aligned} \tag{18.45}$$

Discussion for (b)

This is much too large an area to roll into a capacitor small enough to fit in a handheld camera. This is why these capacitors don't use simple dielectrics but a more advanced technology to obtain a high capacitance.

Practice Problems

25. With 12 V across a capacitor, it accepts 10 mC of charge. What is its capacitance?

a. 0.83 μF
b. 83 μF
c. 120 μF
d. 830 μF

26. A parallel-plate capacitor has an area of 10 cm^2 and the plates are separated by 100 μm. If the capacitor contains paper between the plates, what is its capacitance?

a. 3.3×10^{-10} F
b. 3.3×10^{-8} F
c. 3.3×10^{-6} F
d. 3.3×10^{-4} F

Check Your Understanding

27. If the area of a parallel-plate capacitor doubles, how is the capacitance affected?

a. The capacitance will remain same.
b. The capacitance will double.
c. The capacitance will increase four times.
d. The capacitance will increase eight times.

28. If you double the area of a parallel-plate capacitor and reduce the distance between the plates by a factor of four, how is the capacitance affected?

a. It will increase by a factor of two.
b. It will increase by a factor of four.
c. It will increase by a factor of six.
d. It will increase by a factor of eight.

KEY TERMS

capacitor arrangement of objects that can store electrical energy by virtue of their geometry

conductor material through which electric charge can easily move, such as metals

Coulomb's law describes the electrostatic force between charged objects, which is proportional to the charge on each object and inversely proportional to the square of the distance between the objects

dielectric electrically insulating material that becomes polarized in an electric field

electric field defines the force per unit charge at all locations in space around a charge distribution

electric potential the electric potential energy per unit charge

electric potential energy the work that a charge can do by virtue of its position in an electric field

electron subatomic particle that carries one indivisible unit of negative electric charge

induction creating an unbalanced charge distribution in an object by moving a charged object toward it (but without touching)

insulator material through which a charge does not move, such as rubber

inverse-square law law that has the form of a ratio, with the denominator being the distance squared

law of conservation of charge states that total charge is constant in any process

polarization separation of charge induced by nearby excess charge

proton subatomic particle that carries the same magnitude charge as the electron, but its charge is positive

test charge positive electric charge whose with a charge magnitude so small that it does not significantly perturb any nearby charge distribution

SECTION SUMMARY

18.1 Electrical Charges, Conservation of Charge, and Transfer of Charge

- Electric charge is a conserved quantity, which means it can be neither created nor destroyed.
- Electric charge comes in two varieties, which are called *positive* and *negative*.
- Charges with the same sign repel each other. Charges with opposite signs attract each other.
- Charges can move easily in conducting material. Charges cannot move easily in an insulating material.
- Objects can be charged in three ways: by contact, by conduction, and by induction.
- Although a polarized object may be neutral, its electrical charge is unbalanced, so one side of the object has excess negative charge and the other side has an equal magnitude of excess positive charge.

18.2 Coulomb's law

- Coulomb's law is an inverse square law and describes the electrostatic force between particles.
- The electrostatic force between charged objects is proportional to the charge on each object and inversely proportional to the distance squared between the objects.
- If Coulomb's law gives a negative result, the force is attractive; if the result is positive, the force is repulsive.

18.3 Electric Field

- The electric field defines the force per unit charge in the space around a charge distribution.
- For a point charge or a sphere of uniform charge, the electric field is inversely proportional to the distance from the point charge or from the center of the sphere.
- Electric-field lines never cross each other.
- More force is applied to a charge in a region with many electric field lines than in a region with few electric field lines.
- Electric field lines start at positive charges and point away from positive charges. They end at negative charges and point toward negative charges.

18.4 Electric Potential

- Electric potential energy is a concept similar to gravitational potential energy: It is the potential that charges have to do work by virtue of their positions relative to each other.
- Electric potential is the electric potential energy per unit charge.
- The potential is always measured between two points, where one point may be at infinity.
- Positive charges move from regions of high potential to regions of low potential.
- Negative charges move from regions of low potential to regions of high potential.

18.5 Capacitors and Dielectrics

- The capacitance of a capacitor depends only on the geometry of the capacitor and the materials from which it is made. It does not depend on the voltage across the capacitor.
- Capacitors store electrical energy in the electric field between their plates.

- A dielectric material is an insulator that is polarized in an electric field.
- Putting a dielectric between the plates of a capacitor increases the capacitance of the capacitor.

KEY EQUATIONS

18.2 Coulomb's law

Coulomb's law	$F = \frac{kq_1 q_2}{r^2}$

18.3 Electric Field

electric field	$\vec{E} = \frac{\vec{F}}{q_{\text{test}}}$
magnitude of electric field of point charge	$E = \frac{k\lvert Q \rvert}{r^2}$

18.4 Electric Potential

change in electric potential energy for a charge that moves in a constant electric field	$\Delta U_{\text{E}} = -qE\left(x_{\text{f}} - x_{\text{i}}\right)$
electric potential energy of a charge a distance r from a point charge or sphere of uniform charge	$U_{\text{E}} = \frac{kQq}{r}$
definition of electric potential	$V = \frac{U_{\text{E}}}{q}$
change in electric potential for a charge that moves in a constant electric field	$\Delta V = -E\left(x_{\text{f}} - x_{\text{i}}\right)$
electric potential of a charge a distance r from a point charge or sphere of uniform charge	$V = \frac{kQ}{r}$

18.5 Capacitors and Dielectrics

capacitance	$C = \frac{Q}{V}$
energy stored in a capacitor	$U_E = \frac{1}{2}CV^2$
capacitance of a parallel-plate capacitor	$C = \kappa\varepsilon_0 \frac{A}{d}$

CHAPTER REVIEW

Concept Items

18.1 Electrical Charges, Conservation of Charge, and Transfer of Charge

1. There are very large numbers of charged particles in most objects. Why, then, don't most objects exhibit static electric effects?
   a. Most objects are neutral.
   b. Most objects have positive charge only.
   c. Most objects have negative charge only.
   d. Most objects have excess protons.

2. Can an insulating material be used to charge a conductor? If so, how? If not, why not?
   a. No, an insulator cannot charge a conductor by induction.
   b. No, an insulating material cannot charge a conductor.
   c. Yes, an uncharged insulator can charge a conductor by induction.
   d. Yes, a charged insulator can charge a conductor upon contact.

3. True or false—A liquid can be an insulating material.
   a. true
   b. false

18.2 Coulomb's law

4. Two plastic spheres with uniform charge repel each other with a force of 10 N . If you remove the charge from one sphere, what will be the force between the spheres?
   a. The force will be 15 N.
   b. The force will be 10 N.
   c. The force will be 5 N.
   d. The force will be zero.

5. What creates a greater magnitude of force, two charges

$+q$ a distance r apart or two charges $-q$ the same distance apart?

a. Two charges $+q$ a distance r away
b. Two charges $-q$ a distance r away
c. The magnitudes of forces are equal.

6. In Newton's law of universal gravitation, the force between two masses is proportional to the product of the two masses. What plays the role of mass in Coulomb's law?

a. the electric charge
b. the electric dipole
c. the electric monopole
d. the electric quadruple

18.3 Electric Field

7. Why can electric fields not cross each other?

a. Many electric-field lines can exist at any given point in space.
b. No electric-field lines can exist at any given point in space.
c. Only a single electric-field line can exist at any given point in space.
d. Two electric-field lines can exist at the same point in space.

8. A constant electric field is $(4.5 \times 10^5\ \mathrm{N/C})\hat{y}$. In which direction is the force on a –20 nC charge placed in this field?

a. The direction of the force is in the $+\hat{x}$ direction.
b. The direction of the force is in the $+\hat{x}$ direction.
c. The direction of the force is in the $-\hat{y}$ direction.
d. The direction of the force is in the $+\hat{y}$ direction.

18.4 Electric Potential

9. True or false—The potential from a group of charges is the sum of the potentials from each individual charge.

a. false
b. true

10. True or false—The characteristics of an electric field make it analogous to the gravitational field near the surface of Earth.

a. false
b. true

11. An electron moves in an electric field. Does it move toward regions of higher potential or lower potential? Explain.

a. It moves toward regions of higher potential because its charge is negative.
b. It moves toward regions of lower potential because its charge is negative
c. It moves toward regions of higher potential because its charge is positive.
d. It moves toward regions of lower potential because its charge is positive.

18.5 Capacitors and Dielectrics

12. You insert a dielectric into an air-filled capacitor. How does this affect the energy stored in the capacitor?

a. Energy stored in the capacitor will remain same.
b. Energy stored in the capacitor will decrease.
c. Energy stored in the capacitor will increase.
d. Energy stored in the capacitor will increase first, and then it will decrease.

13. True or false— Placing a dielectric between the plates of a capacitor increases the energy of the capacitor.

a. false
b. true

14. True or false— The electric field in an air-filled capacitor is reduced when a dielectric is inserted between the plates.

a. false
b. true

Critical Thinking Items

18.1 Electrical Charges, Conservation of Charge, and Transfer of Charge

15. If you dive into a pool of seawater through which an equal amount of positively and negatively charged particles is moving, will you receive an electric shock?

a. Yes, because negatively charged particles are moving.
b. No, because positively charged particles are moving.
c. Yes, because positively and negatively charged particles are moving.
d. No, because equal amounts of positively and negatively charged particles are moving.

16. True or false—The high-voltage wires that you see connected to tall metal-frame towers are held aloft by insulating connectors, and these wires are wrapped in an insulating material.

a. true
b. false

17. By considering the molecules of an insulator, explain how an insulator can be overall neutral but carry a surface charge when polarized.

a. Inside the insulator, the oppositely charged ends of the molecules cancel each other.

b. Inside the insulator, the oppositely charged ends of the molecules do not cancel each other.
c. The electron distribution in all the molecules shifts in every possible direction, leaving an excess of positive charge on the opposite end of each molecule.
d. The electron distribution in all the molecules shifts in a given direction, leaving an excess of negative charge on the opposite end of each molecule.

18.2 Coulomb's law

18. In terms of Coulomb's law, why are water molecules attracted by positive and negative charges?

a. Water molecules are neutral.
b. Water molecules have a third type of charge that is attracted by positive as well as negative charges.
c. Water molecules are polar.
d. Water molecule have either an excess of electrons or an excess of protons.

19. A negative lightning strike occurs when a negatively charged cloud discharges its excess electrons to the positively charged ground. If you observe a cloud-to-cloud lightning strike, what can you say about the charge on the area of the cloud struck by lightning?

a. The area of the cloud that was struck by lightning had a positive charge.
b. The area of the cloud that was struck by lightning had a negative charge.
c. The area of the cloud that was struck by lightning is neutral.
d. The area of the cloud that was struck by lightning had a third type of charge.

18.3 Electric Field

20. An arbitrary electric field passes through a box-shaped volume. There are no charges in the box. If 11 electric-field lines enter the box, how many electric-field lines must exit the box?

a. nine electric field lines
b. 10 electric field lines
c. 11 electric field lines
d. 12 electric field lines

21. In a science-fiction movie, a villain emits a radial electric field to repulse the hero. Knowing that the hero is electrically neutral, is this possible? Explain your reasoning.

a. No, because an electrically neutral body cannot be repelled or attracted.
b. No, because an electrically neutral body can be attracted but not repelled.
c. Yes, because an electrically neutral body can be repelled or attracted.
d. Yes, because an electrically neutral body can be repelled.

18.4 Electric Potential

22. What is the relationship between voltage and energy? More precisely, what is the relationship between potential difference and electric potential?

a. Voltage is the energy per unit mass at some point in space.
b. Voltage is the energy per unit length in space.
c. Voltage is the energy per unit charge at some point in space.
d. Voltage is the energy per unit area in space.

23. Three parallel plates are stacked above each other, with a separation between each plate. If the potential difference between the first two plates is ΔV_1 and the potential between the second two plates is ΔV_2, what is the potential difference between the first and the third plates?

a. $\Delta V_3 = \Delta V_2 + \Delta V_1$
b. $\Delta V_3 = \Delta V_2 - \Delta V_1$
c. $\Delta V_3 = \Delta V_2 / \Delta V_1$
d. $\Delta V_3 = \Delta V_2 \times \Delta V_1$

18.5 Capacitors and Dielectrics

24. When you insert a dielectric into a capacitor, the energy stored in the capacitor decreases. If you take the dielectric out, the energy increases again. Where does this energy go in the former case, and where does the energy come from in the latter case?

a. Energy is utilized to remove the dielectric and is released when the dielectric is introduced between the plates.
b. Energy is released when the dielectric is added and is utilized when the dielectric is introduced between the plates.
c. Energy is utilized to polarize the dielectric and is released when the dielectric is introduced between the plates.
d. Energy is released to polarize the dielectric and is utilized when dielectric is introduced between the plates.

Problems

18.1 Electrical Charges, Conservation of Charge, and Transfer of Charge

25. A dust particle acquires a charge of –13 nC. How many excess electrons does it carry?

a. 20.8×10^{-28} electrons
b. $20.8 \times {}^{-19}$ electrons
c. 8.1×10^{10} electrons
d. 8.1×10^{19} electrons

26. Two identical conducting spheres are charged with a net charge of +5.0 q on the first sphere and a net charge of –8.0 q on the second sphere. The spheres are brought together, allowed to touch, and then separated. What is the net charge on each sphere now?

a. $-3.0q$
b. $-1.5q$
c. $+1.5q$
d. $+3.0q$

18.2 Coulomb's law

27. Two particles with equal charge experience a force of 10 nN when they are 30 cm apart. What is the magnitude of the charge on each particle?

a. -5.8×10^{-10} C
b. -3.2×10^{-10} C
c. $+3.2 \times 10^{-10}$ C
d. $+1.4 \times 10^{-5}$ C

28. Three charges are on a line. The left charge is $q_1 = 2.0$ nC . The middle charge is $q_2 = 5.0$ nC . The right charge is $q_3 = -3.0$ nC . The left and right charges are 2.0 cm from the middle charge. What is the force on the middle charge?

a. -5.6×10^{-4} N to the left
b. -1.12×10^{-4} N to the left
c. $+1.12 \times 10^{-4}$ N to the right
d. 5.6×10^{-4} N to the right

18.3 Electric Field

29. An electric field (15 N/C)$\hat{z}$ applies a force $(-3 \times 10^{-6}\ \text{N})\hat{z}$ on a particle. What is the charge on the particle?

a. -2.0×10^{-7} C
b. 2.0×10^{-7} C
c. 2.0×10^{-8} C
d. 2.0×10^{-9} C

30. Two uniform electric fields are superimposed. The first electric field is $\vec{E}_1 = (14\ \text{N/C})\hat{x}$. The second electric field is $\vec{E}_2 = (7.0\ \text{N/C})\hat{y}$. With respect to the positive x axis, at which angle will a positive test charge accelerate in this combined field?

a. 27°
b. 54°
c. 90°
d. 108°

18.4 Electric Potential

31. You move a charge q from $r_i = 20$ cm to $r_f = 40$ cm from a fixed charge $Q = 10$ nC. What is the difference in potential for these two positions?

a. -2.2×10^{2} V
b. -1.7×10^{3} V
c. -2.2×10^{4} V
d. -1.7×10^{2} V

32. How much work is required from an outside agent to move an electron from $x_i = 0$ to $x_f = 20$ cm in an electric field $(50\text{N/C})\hat{x}$?

a. 1.6×10^{-15} J
b. 1.6×10^{-16} J
c. 1.6×10^{-20} J
d. 1.6×10^{-18} J

18.5 Capacitors and Dielectrics

33. A 4.12 μF parallel-plate capacitor has a plate area of 2,000 cm^2 and a plate separation of 10 μm . What dielectric is between the plates?

a. 1, the dielectric is strontium titanate
b. 466, the dielectric is strontium
c. 699, the dielectric is strontium nitrate
d. 1,000, the dielectric is strontium chloride

34. What is the capacitance of a metal sphere of radius R?

a. $C = \frac{R}{k}$
b. $C = \frac{k}{R}$
c. $C = \frac{V}{Q}$
d. $C = QV$

Performance Task

18.5 Capacitors and Dielectrics

35. Newton's law of universal gravitation is

$$F = \frac{Gm_1 m_2,}{r^2,} \tag{18.46}$$

where $G = 6.67 \times 10^{-11}\,\text{m}^3/\text{kg}\cdot\text{s}^2$. This describes the gravitational force between two point masses m_1 and m_2.

Coulomb's law is

$$F = \frac{kq_1 q_2}{r^2} \tag{18.47}$$

where $k = 8.99 \times 10^9 \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2$. This describes the electric force between two point charges q_1 and q_2.
(a) Describe how the force in each case depends on the distance r between the objects. How do the forces change if the distance is reduced by half? If the distance is doubled?
(b) Describe the similarities and differences between the two laws. Consider the signs of the quantities that create the interaction (i.e., mass and charge), the constants G and k, and their dependence on separation r.
(c) Given that the electric force is much stronger than the gravitational force, discuss why the law for gravitational force was discovered much earlier than the law for electric force.
(d) Consider a hydrogen atom, which is a single proton orbited by a single electron. The electric force holds the electron and proton together so that the hydrogen atom has a radius of about $0.5 \times 10^{-10} \mathrm{m}$. Assuming the force between electron and proton does not change, what would be the approximate radius of the hydrogen atom if $k = 6.67 \times 10^{-11} \mathrm{N} \cdot \mathrm{m}^2/\mathrm{C}^2$?

TEST PREP

Multiple Choice

18.1 Electrical Charges, Conservation of Charge, and Transfer of Charge

36. A neutral hydrogen atom has one proton and one electron. If you remove the electron, what will be the leftover sign of the charge?
- a. negative
- b. positive
- c. zero
- d. neutral

37. What is the charge on a proton?
- a. $+8.99 \times 10^{-9}$ C
- b. -8.99×10^{-9} C
- c. $+1.60 \times 10^{-19}$ C
- d. -1.60×10^{-19} C

38. True or false—Carbon is more conductive than pure water.
- a. true
- b. false

39. True or false—Two insulating objects are polarized. To cancel the polarization, it suffices to touch them together.
- a. true
- b. false

40. How is the charge of the proton related to the charge of the electron?
- a. The magnitudes of charge of the proton and the electron are equal, but the charge of the proton is positive, whereas the charge of the electron is negative.
- b. The magnitudes of charge of the proton and the electron are unequal, but the charge of the proton is positive, whereas the charge of the electron is negative.
- c. The magnitudes of charge of the proton and the electron are equal, but the charge of the proton is negative, whereas the charge of the electron is positive.
- d. The magnitudes of charge of the proton and the electron are unequal, but the charge of the proton is negative, whereas the charge of the electron is positive.

18.2 Coulomb's law

41. If you double the distance between two point charges, by which factor does the force between the particles change?
- a. 1/2
- b. 2
- c. 4
- d. 1/4

42. The combined charge of all the electrons in a dime is hundreds of thousands of coulombs. Because like charges repel, what keeps the dime from exploding?
- a. The dime has an equal number of protons, with positive charge.
- b. The dime has more protons than electrons, with positive charge.
- c. The dime has fewer protons than electrons, with positive charge.
- d. The dime is polarized, with electrons on one side and protons on the other side.

43. How can you modify the charges on two particles to quadruple the force between them without moving them?
- a. Increase the distance between the charges by a factor of two.
- b. Increase the distance between the charges by a factor of four.
- c. Increase the product of the charges by a factor of two
- d. Increase the product of the charges by a factor of four.

18.3 Electric Field

44. What is the magnitude of the electric field 12 cm from a charge of 1.5 nC ?
   a. 9.4×10^7 N/ C
   b. 1.1×10^2 N/C
   c. 9.4×10^2 N/C
   d. 9.4×10^{-2} N/C

45. A charge distribution has electric field lines pointing into it. What sign is the net charge?
   a. positive
   b. neutral
   c. final
   d. negative

46. If five electric field lines come out of point charge q_1 and 10 electric-field lines go into point charge q_2, what is the ratio q_1/q_2?
   a. −2
   b. −1
   c. −1/2
   d. 0

47. True or false—The electric-field lines from a positive point charge spread out radially and point outward.
   a. false
   b. true

18.4 Electric Potential

48. What is the potential at 1.0 m from a point charge $Q = -$ 25 nC?
   a. 6.6×10^2 V
   b. -2.3×10^2 V
   c. -6.6×10^2 V
   d. 2.3×10^2 V

49. Increasing the distance by a factor of two from a point charge will change the potential by a factor of how much?
   a. 2
   b. 4
   c. 1/2
   d. 1/4

50. True or false—*Voltage* is the common word for potential difference, because this term is more descriptive than potential difference.
   a. false
   b. true

18.5 Capacitors and Dielectrics

51. Which magnitude of charge is stored on each plate of a 12 μF capacitor with 12 V applied across it?
   a. -1.0×10^{-6} C
   b. 1.0×10^{-6} C
   c. -1.4×10^{-4} C
   d. 1.4×10^{-4} C

52. What is the capacitance of a parallel-plate capacitor with an area of 200 cm^2, a distance of 0.20 mm between the plates, and polystyrene as a dielectric?
   a. 2.3 nC
   b. 0.89 nC
   c. 23 nC
   d. 8.9 nC

53. Which factors determine the capacitance of a device?
   a. Capacitance depends only on the materials that make up the device.
   b. Capacitance depends on the electric field surrounding the device.
   c. Capacitance depends on the geometric and material parameters of the device.
   d. Capacitance depends only on the mass of the capacitor

Short Answer

18.1 Electrical Charges, Conservation of Charge, and Transfer of Charge

54. Compare the mass of the electron with the mass of the proton.
   a. The mass of the electron is about 1,000 times that of the proton.
   b. The mass of the proton is about 1,000 times that of the electron.
   c. The mass of the electron is about 1,836 times that of the proton.
   d. The mass of the proton is about 1,836 times that of the electron.

55. The positive terminal of a battery is connected to one connection of a lightbulb, and the other connection of the lightbulb is connected to the negative terminal of the battery. The battery pushes charge through the circuit but does not become charged itself. Does this violate the law of conservation of charge? Explain.
   a. No, because this is a closed circuit.
   b. No, because this is an open circuit.
   c. Yes, because this is a closed circuit.
   d. Yes, because this is an open circuit.

56. Two flat pieces of aluminum foil lay one on top of the other. What happens if you add charge to the top piece of aluminum foil?
   a. The charge will distribute over the top of the top

piece.

b. The charge will distribute to the bottom of the bottom piece.
c. The inner surfaces will have excess charge of the opposite sign.
d. The inner surfaces will have excess charge of the same sign.

57. The students in your class count off consecutively so each student has a number. The odd-numbered students are told to act as negative charge, and the even-numbered students are told to act as positive charge. How would you organize them to represent a polarized material?

a. The even-numbered and odd-numbered students will be arranged one after the other.
b. Two even-numbered will be followed by two odd-numbered, and so on.
c. Even-numbered students will be asked to come to the front, whereas odd-numbered students will be asked to go to the back of the class.
d. Half even-numbered and odd-numbered will come to the front, whereas half even-numbered and odd-numbered will go to the back.

58. An ion of iron contains 56 protons. How many electrons must it contain if its net charge is $+5e$?

a. five electrons
b. 51 electrons
c. 56 electrons
d. 61 electrons

59. An insulating rod carries $+2.0\ \mathrm{nC}$ of charge. After rubbing it with a material, you find it carries $-3\ \mathrm{nC}$ of charge. How much charge was transferred to it?

a. $-5\ \mathrm{nC}$
b. $-3\ \mathrm{nC}$
c. $-1\ \mathrm{nC}$
d. $+2.0\ \mathrm{nC}$

60. A solid cube carries a charge of $+8e$. You measure the charge on each face of the cube and find that each face carries $+0.5e$ of charge. Is the cube made of conducting or insulating material? Explain.

a. The cube is made of insulating material, because all the charges are on the surface of the cube.
b. The cube is made of conducting material, because some of the charges are inside the cube.
c. The cube is made of insulating material, because all the charges are on the surface of the cube.
d. The cube is made of insulating material, because some of the charges are inside the cube.

61. You have four neutral conducting spheres and a charging device that allows you to place charge q on any neutral object. You want to charge one sphere with a charge $q/2$ and the other three with a charge $q6$. How do you proceed?

a. Charge one sphere with charge q. Touch it simultaneously to the three remaining neutral spheres.
b. Charge one sphere with charge q. Touch it to one other sphere to produce two spheres with charge $\frac{q}{2}$. Touch one of these spheres to one other neutral sphere.
c. Charge one sphere with charge q. Touch it to one other sphere to produce two spheres with charge $\frac{q}{2}$. Touch one of these spheres simultaneously to the two remaining neutral spheres.
d. Charge one sphere with charge q. Touch it simultaneously to two other neutral spheres to produce three spheres with charge $q/3$. Touch one of these spheres to one other neutral sphere.

18.2 Coulomb's law

62. Why does dust stick to the computer screen?

a. The dust is neutral.
b. The dust is polarized.
c. The dust is positively charged.
d. The dust is negatively charged.

63. The force between two charges is 4×10^{-9} N . If the magnitude of one charge is reduced by a factor of two and the distance between the charges is reduced by a factor of two, what is the new force between the charges?

a. 2×10^{-9} N
b. 4×10^{-9} N
c. 6×10^{-9} N
d. 8×10^{-9} N

64. True or false—Coulomb's constant is $k = 8.99 \times 10^9$ $\mathrm{N{\cdot}m^2/C^2}$. Newton's gravitational constant is $G = 6.67 \times 10^{-11}$ $\mathrm{m^3/kg{\cdot}s^2}$. This tells you about the relative strength of the electrostatic force versus that of gravity.

a. true
b. false

65. An atomic nucleus contains 56 protons, for iron. Which force would this nucleus apply on an electron at a distance of $10{\times}10^{-12}$ m?

a. 0.65×10^{-4} N
b. 0.02×10^{-4} N
c. 1.3×10^{-4} N
d. 72.8×10^{-4} N

18.3 Electric Field

66. The electric field a distance of 10 km from a storm cloud is 1,000 N/C . What is the approximate charge in the

cloud?
a. 0.0011 C
b. 11 C
c. 110 C
d. 1,100 C

67. Which electric field would produce a 10 N force in the +*x*- direction on a charge of – 10 nC ?
a. -1.0×10^9 N/C
b. 1.0×10^9 N/C
c. 1.0×10^{10} N/C
d. 1.0×10^{11} N/C

68. A positive charge is located at $x = 0$. When a negative charge is placed at $x = 10$ cm, what happens to the electric field lines between the charges?
a. The electric field lines become denser between the charges.
b. The electric field lines become denser between the charges.
c. The electric field lines remains same between the charges.
d. The electric field lines will be zero between the charges.

18.4 Electric Potential

69. The energy required to bring a charge $q = -8.8$ nC from far away to 5.5 cm from a point charge Q is 13 mJ. What is the potential at the final position of *q*?
a. −112 MV
b. −1.5 MV
c. −0.66 MV
d. +1.5 MV

70. How is electric potential related to electric potential energy?
a. Electric potential is the electric potential energy per unit mass at a given position in space.
b. Electric potential is the electric potential energy per unit length at a given position in space. This relation is not dimensionally correct.
c. Electric potential is the electric potential energy per unit area in space.
d. Electric potential is the electric potential energy per unit charge at a given position in space.

71. If it takes 10 mJ to move a charge *q* from $x_i = 25$ cm to $x_f =$ – 25 cm in an electric field of $(-20\text{N/C})\hat{x}$, what is the charge *q*?
a. −1.0 mC
b. +0.25 mC
c. + 1.0 mC
d. +400 mC

72. Given the potential difference between two points and the distance between the points, explain how to obtain the electric field between the points.
a. Add the electric potential to the distance to obtain the electric field.
b. Divide the electric potential by the distance to obtain the electric field.
c. Multiply the electric potential and the distance to obtain the electric field.
d. Subtract the electric potential from the distance to obtain the electric field.

18.5 Capacitors and Dielectrics

73. If you double the voltage across the plates of a capacitor, how is the stored energy affected?
a. Stored energy will decrease two times.
b. Stored energy will decrease four times.
c. Stored energy will increase two times.
d. Stored energy will increase four times.

74. A capacitor with neoprene rubber as the dielectric stores 0.185 mJ of energy with a voltage of 50 V across the plates. If the area of the plates is 500 cm^2, what is the plate separation?
a. 20 µm
b. 20 m
c. 80 µm
d. 80 m

75. Explain why a storm cloud before a lightning strike is like a giant capacitor.
a. The storm cloud acts as a giant charged capacitor, as it can store a large amount of charge.
b. The storm cloud acts as a giant charged capacitor, as it contains a high amount of excess charges.
c. The storm cloud acts as a giant charged capacitor, as it splits in two capacitor plates with equal and opposite charge.
d. The storm cloud acts as a giant charged capacitor, as it splits in two capacitor plates with unequal and opposite charges.

76. A storm cloud is 2 km above the surface of Earth. The lower surface of the cloud is approximately 2 km^2 in area. What is the approximate capacitance of this storm cloud-Earth system?
a. 9×10^{-15} F
b. 9×10^{-9} F
c. 17.7×10^{-15} F
d. 17.7×10^{-9} F

Extended Response

18.1 Electrical Charges, Conservation of Charge, and Transfer of Charge

77. Imagine that the magnitude of the charge on the electron differed very slightly from that of the proton. How would this affect life on Earth and physics in general?

a. Many macroscopic objects would be charged, so we would experience the enormous force of electricity on a daily basis.
b. Many macroscopic objects would be charged, so we would experience the small force of electricity on a daily basis.
c. Many macroscopic objects would be charged, but it would not affect life on Earth and physics in general.
d. Macroscopic objects would remain neutral, so it would not affect life on Earth and physics in general.

78. True or false—Conservation of charge is like balancing a budget.

a. true
b. false

79. True or false—Although wood is an insulator, lightning can travel through a tree to reach Earth.

a. true
b. false

80. True or false—An eccentric inventor attempts to levitate by first placing a large negative charge on himself and then putting a large positive charge on the ceiling of his workshop. Instead, while he attempts to place a large negative charge on himself, his clothes fly off.

a. true
b. false

18.2 Coulomb's law

81. Electrostatic forces are enormous compared to gravitational force. Why do you not notice electrostatic forces in everyday life, whereas you do notice the force due to gravity?

a. Because there are two types of charge, but only one type of mass exists.
b. Because there is only one type of charge, but two types of mass exist.
c. Because opposite charges cancel each other, while gravity does not cancel out.
d. Because opposite charges do not cancel each other, while gravity cancels out.

82. A small metal sphere with a net charge of 3.0 nC is touched to a second small metal sphere that is initially neutral. The spheres are then placed 20 cm apart. What is the force between the spheres?

a. 1.02×10^{-7} N
b. 2.55×10^{-7} N
c. 5.1×10^{-7} N
d. 20.4×10^{-7} N

18.3 Electric Field

83. Point charges are located at each corner of a square with sides of 5.0 cm . The top-left charge is $q_1 = 8.0$ nC The top right charge is $q_2 = 4.0$ nC. The bottom-right charge is $q_3 = 4.0$ nC. The bottom-left charge is $q_4 = 8.0$ nC. What is the electric field at the point midway between charges q_2 and q_3?

a. $(-2.1 \times 10^4 \text{ N/C})\hat{x}$
b. $(2.3 \times 10^4 \text{ N/C})\hat{x}$
c. $(4.1 \times 10^4 \text{ N/C})\hat{x}$
d. $(4.6 \times 10^4 \text{ N/C})\hat{x}$

84. A long straight wire carries a uniform positive charge distribution. Draw the electric field lines in a plane containing the wire at a location far from the ends of the wire. Do not worry about the magnitude of the charge on the wire.

a. Take the wire on the x-axis, and draw electric-field lines perpendicular to it.
b. Take the wire on the *x*-axis, and draw electric-field lines parallel to it.
c. Take the wire on the y-axis, and draw electric-field lines along it.
d. Take the wire on the z-axis, and draw electric-field lines along it.

18.4 Electric Potential

85. A square grid has charges of $Q = 10$ nC are each corner. The sides of the square at 10 cm . How much energy does it require to bring a $q = 1.0$ nC charge from very far away to the point at the center of this square?

a. 1.3×10^{-6} J
b. 2.5×10^{-6} J
c. 3.8×10^{-6} J
d. 5.1×10^{-6} J

86. How are potential difference and electric-field strength related for a constant electric field?

a. The magnitude of electric-field strength is equivalent to the potential divided by the distance.
b. The magnitude of electric-field strength is equivalent to the product of the electric potential and the distance.
c. The magnitude of electric-field strength is

equivalent to the difference between magnitude of the electric potential and the distance.

d. The magnitude of electric-field strength is equivalent to the sum of the magnitude of the electric potential and the distance.

18.5 Capacitors and Dielectrics

87. A 12 μF air-filled capacitor has 12 V across it. If the surface charge on each capacitor plate is $\sigma = 7.2\ \mathrm{mC/m^2}$, what is the attractive force of one capacitor plate toward the other?

a. 0.81×10^5 N
b. 0.81×10^6 N
c. 1.2×10^5 N
d. 1.2×10^6 N

88. Explain why capacitance should be inversely proportional to the separation between the plates of a capacitor.

a. Capacitance is directly proportional to the electric field, which is inversely proportional to the distance between the capacitor plates.
b. Capacitance is inversely proportional to the electric field, which is inversely proportional to the distance between the capacitor plates.
c. Capacitance is inversely proportional to the electric field, which is directly proportional to the distance between the capacitor plates.
d. Capacitance is directly proportional to the electric field, which is directly proportional to the distance between the capacitor plates.

CHAPTER 19

Electrical Circuits

Figure 19.1 Electric energy in massive quantities is transmitted from this hydroelectric facility, the Srisailam power station located along the Krishna River in India, by the movement of charge—that is, by electric current. (credit: Chintohere, Wikimedia Commons)

Chapter Outline

INTRODUCTION The flicker of numbers on a handheld calculator, nerve impulses carrying signals of vision to the brain, an ultrasound device sending a signal to a computer screen, the brain sending a message for a baby to twitch its toes, an electric train pulling into a station, a hydroelectric plant sending energy to metropolitan and rural users—these and many other examples of electricity involve electric current, which is the movement of charge. Humanity has harnessed electricity, the basis of this technology, to improve our quality of life. Whereas the previous chapter concentrated on static electricity and the fundamental force underlying its behavior, the next two chapters will be devoted to electric and magnetic phenomena involving current. In addition to exploring applications of electricity, we shall gain new insights into the workings of nature.

19.1 Ohm's law

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe how current is related to charge and time, and distinguish between direct current and alternating current
- Define resistance and verbally describe Ohm's law
- Calculate current and solve problems involving Ohm's law

Section Key Terms

alternating current	ampere	conventional current	direct current	electric current
nonohmic	ohmic	Ohm's law	resistance	

Direct and Alternating Current

Just as water flows from high to low elevation, electrons that are free to move will travel from a place with low potential to a place with high potential. A battery has two terminals that are at different potentials. If the terminals are connected by a conducting wire, an electric current (charges) will flow, as shown in Figure 19.2. Electrons will then move from the low-potential terminal of the battery (the *negative* end) through the wire and enter the high-potential terminal of the battery (the *positive* end).

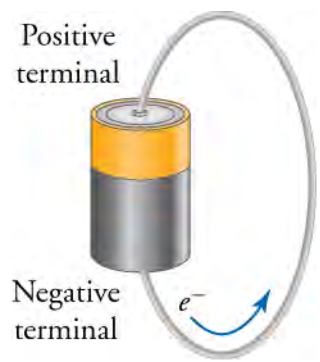


Figure 19.2 A battery has a wire connecting the positive and negative terminals, which allows electrons to move from the negative terminal to the positive terminal.

Electric current is the rate at which electric charge moves. A large current, such as that used to start a truck engine, moves a large amount very quickly, whereas a small current, such as that used to operate a hand-held calculator, moves a small amount of charge more slowly. In equation form, electric current I is defined as

$$I = \frac{\Delta Q}{\Delta t}$$

where ΔQ is the amount of charge that flows past a given area and Δt is the time it takes for the charge to move past the area. The SI unit for electric current is the ampere (A), which is named in honor of the French physicist André-Marie Ampère (1775–1836). One **ampere** is one coulomb per second, or

$$1 \text{ A} = 1 \text{ C/s}.$$

Electric current moving through a wire is in many ways similar to water current moving through a pipe. To define the flow of water through a pipe, we can count the water molecules that flow past a given section of the pipe. As shown in Figure 19.3, electric current is very similar. We count the number of electrical charges that flow past a section of a conductor; in this case, a wire.

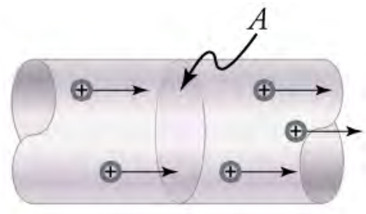


Figure 19.3 The electric current moving through this wire is the charge that moves past the cross-section A divided by the time it takes for this charge to move past the section *A*.

Assume each particle q in Figure 19.3 carries a charge $q = 1\ \text{nC}$, in which case the total charge shown would be $\Delta Q = 5q = 5\ \text{nC}$. If these charges move past the area *A* in a time $\Delta t = 1\ \text{ns}$, then the current would be

$$I = \frac{\Delta Q}{\Delta t} = \frac{5\ \text{nC}}{1\ \text{ns}} = 5\ \text{A}. \tag{19.1}$$

Note that we assigned a positive charge to the charges in Figure 19.3. Normally, negative charges—electrons—are the mobile charge in wires, as indicated in Figure 19.2. Positive charges are normally stuck in place in solids and cannot move freely. However, because a positive current moving to the right is the same as a negative current of equal magnitude moving to the left, as shown in Figure 19.4, we define **conventional current** to flow in the direction that a positive charge would flow if it could move. Thus, unless otherwise specified, an electric current is assumed to be composed of positive charges.

Also note that one Coulomb is a significant amount of electric charge, so 5 A is a very large current. Most often you will see current on the order of milliamperes (mA).

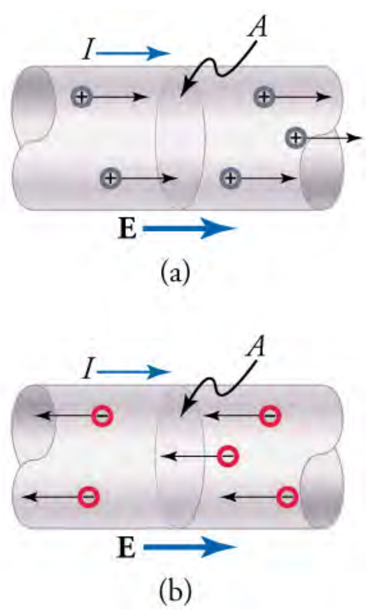


Figure 19.4 (a) The electric field points to the right, the current moves to the right, and positive charges move to the right. (b) The equivalent situation but with negative charges moving to the left. The electric field and the current are still to the right.

Snap Lab

Vegetable Current

This lab helps students understand how current works. Given that particles confined in a pipe cannot occupy the same space, pushing more particles into one end of the pipe will force the same number of particles out of the opposite end. This creates a current of particles.

Find a straw and dried peas that can move freely in the straw. Place the straw flat on a table and fill the straw with peas. When you push one pea in at one end, a different pea should come out of the other end. This demonstration is a model for an electric current. Identify the part of the model that represents electrons and the part of the model that represents the supply of electrical energy. For a period of 30 s, count the number of peas you can push through the straw. When finished, calculate the *pea current* by dividing the number of peas by the time in seconds.

Note that the flow of peas is based on the peas physically bumping into each other; electrons push each other along due to mutually repulsive electrostatic forces.

GRASP CHECK

Suppose four peas per second pass through a straw. If each pea carried a charge of $1\ \text{nC}$, what would the electric current be through the straw?

a. The electric current would be the pea charge multiplied by $1\ \text{nC/pea}$.
b. The electric current would be the pea current calculated in the lab multiplied by $1\ \text{nC/pea}$.
c. The electric current would be the pea current calculated in the lab.
d. The electric current would be the pea charge divided by time.

The direction of conventional current *is the direction that positive charge would flow*. Depending on the situation, positive charges, negative charges, or both may move. In metal wires, as we have seen, current is carried by electrons, so the negative charges move. In ionic solutions, such as salt water, both positively charged and negatively charged ions move. This is also true in nerve cells. Pure positive currents are relatively rare but do occur. History credits American politician and scientist Benjamin Franklin with describing current as the direction that positive charges flow through a wire. He named the type of charge associated with electrons negative long before they were known to carry current in so many situations.

As electrons move through a metal wire, they encounter obstacles such as other electrons, atoms, impurities, etc. The electrons scatter from these obstacles, as depicted in Figure 19.5. Normally, the electrons lose energy with each interaction. [1] To keep the electrons moving thus requires a force, which is supplied by an electric field. The electric field in a wire points from the end of the wire at the higher potential to the end of the wire at the lower potential. Electrons, carrying a negative charge, move on average (or *drift*) in the direction opposite the electric field, as shown in Figure 19.5.

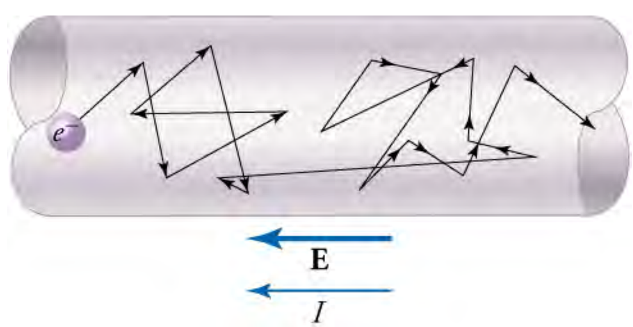


Figure 19.5 Free electrons moving in a conductor make many collisions with other electrons and atoms. The path of one electron is shown. The average velocity of free electrons is in the direction opposite to the electric field. The collisions normally transfer energy to the conductor, so a constant supply of energy is required to maintain a steady current.

So far, we have discussed current that moves constantly in a single direction. This is called **direct current**, because the electric charge flows in only one direction. Direct current is often called *DC* current.

Many sources of electrical power, such as the hydroelectric dam shown at the beginning of this chapter, produce **alternating current**, in which the current direction alternates back and forth. Alternating current is often called *AC current*. Alternating current moves back and forth at regular time intervals, as shown in Figure 19.6. The alternating current that comes from a normal wall socket does not suddenly switch directions. Rather, it increases smoothly up to a maximum current and then smoothly decreases back to zero. It then grows again, but in the opposite direction until it has reached the same maximum value. After that, it decreases smoothly back to zero, and the cycle starts over again.

1This energy is transferred to the wire and becomes thermal energy, which is what makes wires hot when they carry a lot of current.

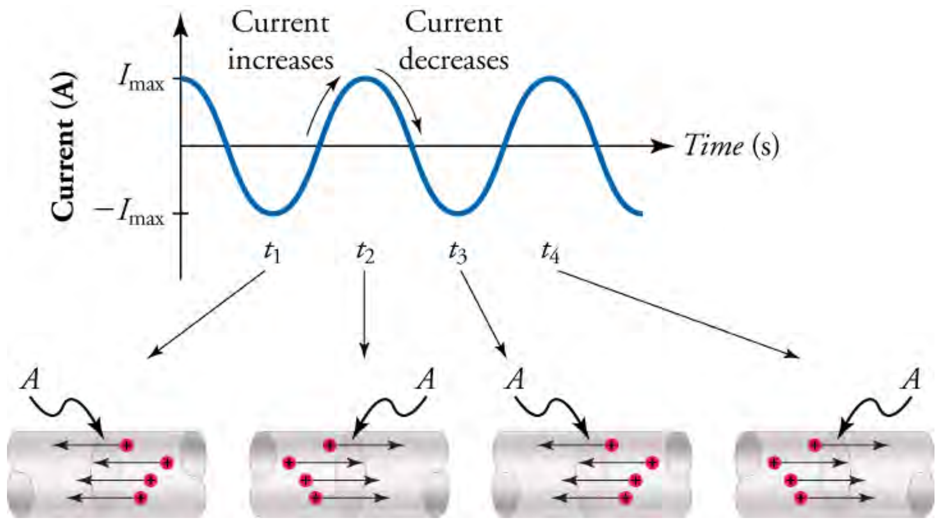


Figure 19.6 With alternating current, the direction of the current reverses at regular time intervals. The graph on the top shows the current versus time. The negative maxima correspond to the current moving to the left. The positive maxima correspond to current moving to the right. The current alternates regularly and smoothly between these two maxima.

Devices that use AC include vacuum cleaners, fans, power tools, hair dryers, and countless others. These devices obtain the power they require when you plug them into a wall socket. The wall socket is connected to the power grid that provides an alternating potential (AC potential). When your device is plugged in, the AC potential pushes charges back and forth in the circuit of the device, creating an alternating current.

Many devices, however, use DC, such as computers, cell phones, flashlights, and cars. One source of DC is a battery, which provides a constant potential (DC potential) between its terminals. With your device connected to a battery, the DC potential pushes charge in one direction through the circuit of your device, creating a DC current. Another way to produce DC current is by using a transformer, which converts AC potential to DC potential. Small transformers that you can plug into a wall socket are used to charge up your laptop, cell phone, or other electronic device. People generally call this a *charger* or a *battery*, but it is a transformer that transforms AC voltage into DC voltage. The next time someone asks to borrow your laptop charger, tell them that you don't have a laptop charger, but that they may borrow your converter.

WORKED EXAMPLE

Current in a Lightning Strike

A lightning strike can transfer as many as 10^{20} electrons from the cloud to the ground. If the strike lasts 2 ms, what is the average electric current in the lightning?

STRATEGY

Use the definition of current, $I = \frac{\Delta Q}{\Delta t}$. The charge ΔQ from 10^{20} electrons is $\Delta Q = ne$, where $n = 10^{20}$ is the number of electrons and $e = -1.60 \times 10^{-19}$ C is the charge on the electron. This gives

$$\Delta Q = 10^{20} \times \left(-1.60 \times 10^{-19}\text{C}\right) = -16.0 \text{ C}. \tag{19.2}$$

The time $\Delta t = 2 \times 10^{-3}$ s is the duration of the lightning strike.

Solution

The current in the lightning strike is

$$\begin{aligned} I &= \frac{\Delta Q}{\Delta t} \\ &= \frac{-16.0 \text{ C}}{2 \times 10^{-3}\text{s}} \\ &= -8 \text{ kA}. \end{aligned} \tag{19.3}$$

Discussion

The negative sign reflects the fact that electrons carry the negative charge. Thus, although the electrons flow from the cloud to the ground, the positive current is defined to flow from the ground to the cloud.

WORKED EXAMPLE

Average Current to Charge a Capacitor

In a circuit containing a capacitor and a resistor, it takes 1 min to charge a 16 µF capacitor by using a 9-V battery. What is the average current during this time?

STRATEGY

We can determine the charge on the capacitor by using the definition of capacitance: $C = \frac{Q}{V}$. When the capacitor is charged by a 9-V battery, the voltage across the capacitor will be $V = 9\ \text{V}$. This gives a charge of

$$\begin{aligned} C &= \frac{Q}{V} \\ Q &= CV. \end{aligned} \tag{19.4}$$

By inserting this expression for charge into the equation for current, $I = \frac{\Delta Q}{\Delta t}$, we can find the average current.

Solution

The average current is

$$\begin{aligned} I &= \frac{\Delta Q}{\Delta t} \\ &= \frac{CV}{\Delta t} \\ &= \frac{(16 \times 10^{-6}\text{F})(9\ \text{V})}{60\ \text{s}} \\ &= 2.4 \times 10^{-6}\text{A} \\ &= 2.4\ \mu\text{A}. \end{aligned} \tag{19.5}$$

Discussion

This small current is typical of the current encountered in circuits such as this.

Practice Problems

1. 10 nC of charge flows through a circuit in 3.0×10^{-6} s . What is the current during this time?
   a. The current passes through the circuit is 3.3×10^{-3} A.
   b. The current passes through the circuit is 30 A.
   c. The current passes through the circuit is 33 A.
   d. The current passes through the circuit is 0.3 A.

2. How long would it take a 10-mA current to charge a capacitor with 5.0 mC?
   a. 0.50 s
   b. 5 ns
   c. 0.50 ns
   d. 50 μs

Resistance and Ohm's Law

As mentioned previously, electrical current in a wire is in many ways similar to water flowing through a pipe. The water current that can flow through a pipe is affected by obstacles in the pipe, such as clogs and narrow sections in the pipe. These obstacles slow down the flow of current through the pipe. Similarly, electrical current in a wire can be slowed down by many factors, including impurities in the metal of the wire or collisions between the charges in the material. These factors create a resistance to the electrical current. **Resistance** is a description of how much a wire or other electrical component opposes the flow of charge through it. In the 19th century, the German physicist Georg Simon Ohm (1787–1854) found experimentally that current through a conductor is proportional to the voltage drop across a current-carrying conductor.

$$I \propto V$$

The constant of proportionality is the resistance R of the material, which leads to

$$V = IR(1.3).$$

This relationship is called **Ohm's law**. It can be viewed as a cause-and-effect relationship, with voltage being the cause and the current being the effect. Ohm's law is an empirical law like that for friction, which means that it is an experimentally observed phenomenon. The units of resistance are volts per ampere, or V/A. We call a V/A an *ohm*, which is represented by the uppercase Greek letter omega (Ω). Thus,

$$1\ \Omega = 1\ \text{V/A}(1.4).$$

Ohm's law holds for most materials and at common temperatures. At very low temperatures, resistance may drop to zero (superconductivity). At very high temperatures, the thermal motion of atoms in the material inhibits the flow of electrons, increasing the resistance. The many substances for which Ohm's law holds are called **ohmic**. Ohmic materials include good conductors like copper, aluminum, and silver, and some poor conductors under certain circumstances. The resistance of ohmic materials remains essentially the same for a wide range of voltage and current.

WATCH PHYSICS

Introduction to Electricity, Circuits, Current, and Resistance

This video presents Ohm's law and shows a simple electrical circuit. The speaker uses the analogy of pressure to describe how electric potential makes charge move. He refers to electric potential as *electric pressure*. Another way of thinking about electric potential is to imagine that lots of particles of the same sign are crowded in a small, confined space. Because these charges have the same sign (they are all positive or all negative), each charge repels the others around it. This means that lots of charges are constantly being pushed towards the outside of the space. A complete electric circuit is like opening a door in the small space: Whichever particles are pushed towards the door now have a way to escape. The higher the electric potential, the harder each particle pushes against the others.

GRASP CHECK

If, instead of a single resistor R, two resistors each with resistance R are drawn in the circuit diagram shown in the video, what can you say about the current through the circuit?

a. The amount of current through the circuit must decrease by half.
b. The amount of current through the circuit must increase by half.
c. The current must remain the same through the circuit.
d. The amount of current through the circuit would be doubled.

Virtual Physics

Ohm's Law

Click to view content (http://www.openstax.org/l/28ohms_law)

This simulation mimics a simple circuit with batteries providing the voltage source and a resistor connected across the batteries. See how the current is affected by modifying the resistance and/or the voltage. Note that the resistance is modeled as an element containing small *scattering centers*. These represent impurities or other obstacles that impede the passage of the current.

GRASP CHECK

In a circuit, if the resistance is left constant and the voltage is doubled (for example, from 3 V to 6 V), how does the current change? Does this conform to Ohm's law?

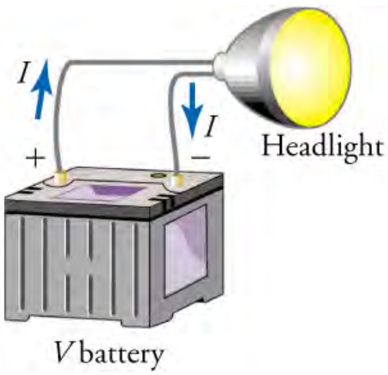
a. The current will get doubled. This conforms to Ohm's law as the current is proportional to the voltage.
b. The current will double. This does not conform to Ohm's law as the current is proportional to the voltage.

c. The current will increase by half. This conforms to Ohm's law as the current is proportional to the voltage.
d. The current will decrease by half. This does not conform to Ohm's law as the current is proportional to the voltage.

WORKED EXAMPLE

Resistance of a Headlight

What is the resistance of an automobile headlight through which 2.50 A flows when 12.0 V is applied to it?



STRATEGY

Ohm's law tells us $V_{\text{headlight}} = IR_{\text{headlight}}$. The voltage drop in going through the headlight is just the voltage rise supplied by the battery, $V_{\text{headlight}} = V_{\text{battery}}$. We can use this equation and rearrange Ohm's law to find the resistance $R_{\text{headlight}}$ of the headlight.

Solution

Solving Ohm's law for the resistance of the headlight gives

$$\begin{aligned} V_{\text{headlight}} &= IR_{\text{headlight}} \\ V_{\text{battery}} &= IR_{\text{headlight}} \\ R_{\text{headlight}} &= \frac{V_{\text{battery}}}{I} = \frac{12 \text{ V}}{2.5 \text{ A}} = 4.8 \ \Omega. \end{aligned} \tag{19.6}$$

Discussion

This is a relatively small resistance. As we will see below, resistances in circuits are commonly measured in kW or MW.

WORKED EXAMPLE

Determine Resistance from Current-Voltage Graph

Suppose you apply several different voltages across a circuit and measure the current that runs through the circuit. A plot of your results is shown in Figure 19.7. What is the resistance of the circuit?

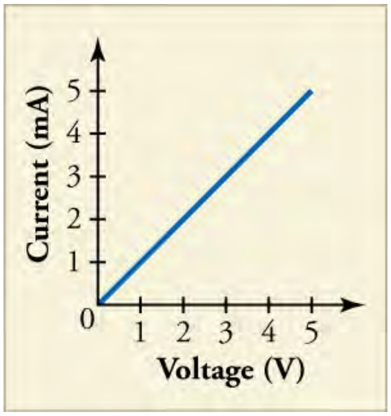


Figure 19.7 The line shows the current as a function of voltage. Notice that the current is given in milliamperes. For example, at 3 V, the current is 0.003 A, or 3 mA.

STRATEGY

The plot shows that current is proportional to voltage, which is Ohm's law. In Ohm's law ($V = IR$), the constant of proportionality is the resistance *R*. Because the graph shows current as a function of voltage, we have to rearrange Ohm's law in that form: $I = \frac{V}{R} = \frac{1}{R} \times V$. This shows that the slope of the line of *I* versus *V* is $\frac{1}{R}$. Thus, if we find the slope of the line in Figure 19.7, we can calculate the resistance *R*.

Solution

The slope of the line is the *rise* divided by the *run*. Looking at the lower-left square of the grid, we see that the line rises by 1 mA (0.001 A) and runs over a voltage of 1 V. Thus, the slope of the line is

$$\text{slope} = \frac{0.001\ \text{A}}{1\ \text{V}.} \tag{19.7}$$

Equating the slope with $\frac{1}{R}$ and solving for *R* gives

$$\begin{aligned} \frac{1}{R} &= \frac{0.001\ \text{A}}{1} \\ R &= \frac{1\ \text{V}}{0.001\ \text{A}} = 1{,}000\ \Omega \end{aligned} \tag{19.8}$$

or 1 k-ohm.

Discussion

This resistance is greater than what we found in the previous example. Resistances such as this are common in electric circuits, as we will discover in the next section. Note that if the line in Figure 19.7 were not straight, then the material would not be ohmic and we would not be able to use Ohm's law. Materials that do not follow Ohm's law are called **nonohmic**.

Practice Problems

3. If you double the voltage across an ohmic resistor, how does the current through the resistor change?
   a. The current will double.
   b. The current will increase by half.
   c. The current will decrease by half.
   d. The current will decrease by a factor of two.

4. The current through a $10\ \Omega$ resistor is $0.025\ \text{A}$. What is the voltage drop across the resistor?
   a. $2.5\ \text{mV}$
   b. $0.25\ \text{V}$
   c. $2.5\ \text{V}$
   d. $0.25\ \text{mV}$

Check Your Understanding

5. What is electric current?
   a. Electric current is the electric charge that is at rest.
   b. Electric current is the electric charge that is moving.
   c. Electric current is the electric charge that moves only from the positive terminal of a battery to the negative terminal.
   d. Electric current is the electric charge that moves only from a region of lower potential to higher potential.

6. What is an ohmic material?
   a. An ohmic material is a material that obeys Ohm's law.
   b. An ohmic material is a material that does not obey Ohm's law.
   c. An ohmic material is a material that has high resistance.
   d. An ohmic material is a material that has low resistance.

7. What is the difference between direct current and alternating current?
   a. Direct current flows continuously in every direction whereas alternating current flows in one direction.
   b. Direct current flows continuously in one direction whereas alternating current reverses its direction at regular time intervals.
   c. Both direct and alternating current flow in one direction but the magnitude of direct current is fixed whereas the magnitude of alternating current changes at regular intervals of time.
   d. Both direct and alternating current changes its direction of flow but the magnitude of direct current is fixed whereas the magnitude of alternating current changes at regular intervals of time.

19.2 Series Circuits

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Interpret circuit diagrams and diagram basic circuit elements
- Calculate equivalent resistance of resistors in series and apply Ohm's law to resistors in series and apply Ohm's law to resistors in series

Section Key Terms

circuit diagram	electric circuit	equivalent resistance
in series	resistor	steady state

Electric Circuits and Resistors

Now that we understand the concept of electric current, let's see what we can do with it. As you are no doubt aware, the modern lifestyle relies heavily on electrical devices. These devices contain ingenious **electric circuits**, which are complete, closed pathways through which electric current flows. Returning to our water analogy, an electric circuit is to electric charge like a network of pipes is to water: The electric circuit guides electric charge from one point to the next, running the charge through various devices along the way to extract work or information.

Electric circuits are made from many materials and cover a huge range of sizes, as shown in Figure 19.8. Computers and cell phones contain electric circuits whose features can be as small as roughly a billionth of a meter (a nanometer, or $10^{-9}\,\mathrm{m}$). The pathways that guide the current in these devices are made by ultraprecise chemical treatments of silicon or other semiconductors. Large power systems, on the other hand, contain electric circuits whose features are on the scale of meters. These systems carry such large electric currents that their physical dimensions must be relatively large.

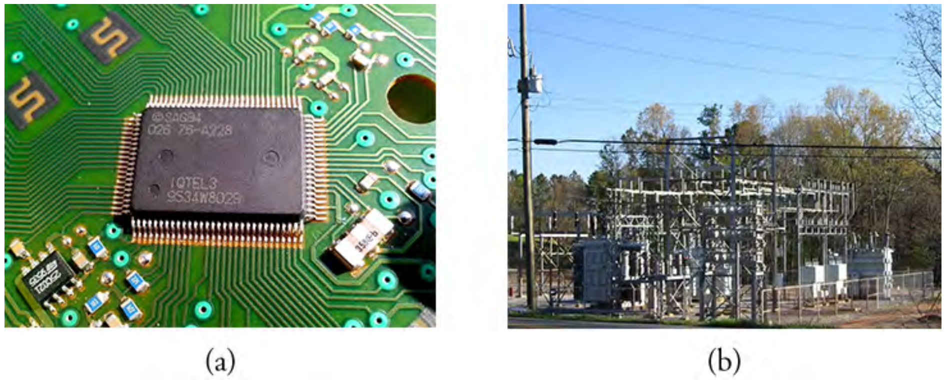


Figure 19.8 The photo on the left shows a *chip* that contains complex integrated electric circuitry. Chips such as this are at the heart of devices such as computers and cell phones. The photograph on the right shows some typical electric circuitry required for high-power electric power transmission.

The pathways that form electric circuits are made from a conducting material, normally a metal in macroscopic circuits. For example, copper wires inside your school building form the electrical circuits that power lighting, projectors, screens, speakers, etc. To represent an electric circuit, we draw **circuit diagrams**. We use lines and symbols to represent the elements in the circuit. A simple electric circuit diagram is shown on the left side of Figure 19.9. On the right side is an analogous water circuit, which we discuss below.

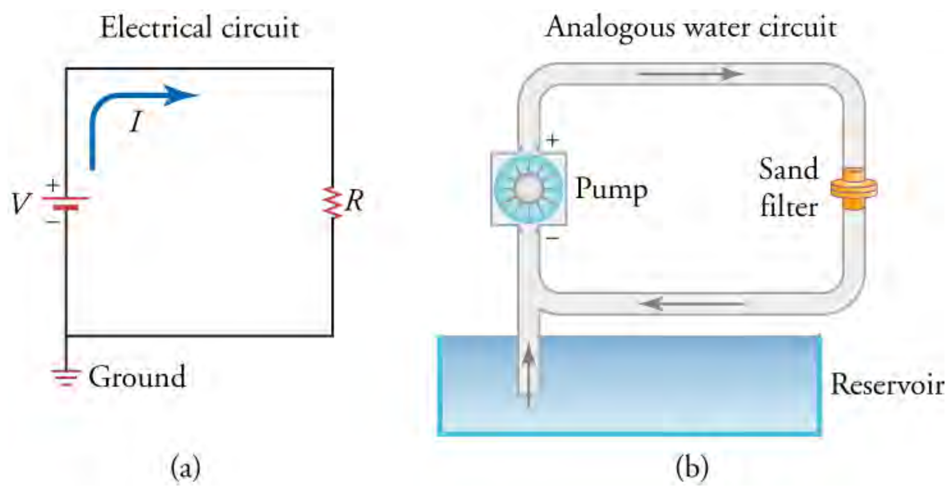


Figure 19.9 On the left is a circuit diagram showing a battery (in red), a resistor (black zigzag element), and the current *I*. On the right is the analogous water circuit. The pump is like the battery, the sand filter is like the resistor, the water current is like the electrical current, and the reservoir is like the ground.

There are many different symbols that scientists and engineers use in circuit diagrams, but we will focus on four main symbols: the wire, the battery or voltage source, resistors, and the ground. The thin black lines in the electric circuit diagram represent the pathway that the electric charge must follow. These pathways are assumed to be perfect conductors, so electric charge can move along these pathways without losing any energy. In reality, the wires in circuits are not perfect, but they come close enough for our purposes.

The zigzag element labeled *R* is a **resistor**, which is a circuit element that provides a known resistance. Macroscopic resistors are often color coded to indicate their resistance, as shown in Figure 19.10.

The red element in Figure 19.9 is a battery, with its positive and negative terminals indicated; the longer line represents the positive terminal of the battery, and the shorter line represents the negative terminal. Note that the battery icon is not always colored red; this is done in Figure 19.9 just to make it easy to identify.

Finally, the element labeled *ground* on the lower left of the circuit indicates that the circuit is connected to Earth, which is a large, essentially neutral object containing an infinite amount of charge. Among other things, the ground determines the potential of the negative terminal of the battery. Normally, the potential of the ground is defined to be zero: $V_{\text{ground}} \equiv 0$. This means that the entire lower wire in Figure 19.10 is at a voltage of zero volts.

Figure 19.10 Some typical resistors. The color bands indicate the value of the resistance of each resistor.

The electric current in Figure 19.9 is indicted by the blue line labeled I. The arrow indicates the direction in which positive charge would flow in this circuit. Recall that, in metals, electrons are mobile charge carriers, so negative charges actually flow in the opposite direction around this circuit (i.e., counterclockwise). However, we draw the current to show the direction in which positive charge would move.

On the right side of Figure 19.9 is an analogous water circuit. Water at a higher pressure leaves the top of the pump, which is like charges leaving the positive terminal of the battery. The water travels through the pipe, like the charges traveling through the wire. Next, the water goes through a sand filter, which heats up as the water squeezes through. This step is like the charges going through the resistor. When charges flow through a resistor, they do work to heat up the resistor. After flowing through the sand filter, the water has converted its potential energy into heat, so it is at a lower pressure. Likewise, the charges exiting the resistor have converted their potential energy into heat, so they are at a lower voltage. Recall that voltage is just potential energy per charge. Thus, water pressure is analogous to electric potential energy (i.e., voltage). Coming back to the water circuit again, we see that the water returns to the bottom of the pump, which is like the charge returning to the negative terminal of the battery. The water pump uses a source of energy to pump the water back up to a high pressure again, giving it the pressure required to go through the circuit once more. The water pump is like the battery, which uses chemical energy to increase the voltage of the charge up to the level of the positive terminal.

The potential energy per charge at the positive terminal of the battery is the voltage rating of the battery. This voltage is like water pressure in the upper pipe. Just like a higher pressure forces water to move toward a lower pressure, a higher voltage forces electric charge to flow toward a lower voltage. The pump takes water at low pressure and does work on it, ejecting water at a higher pressure. Likewise, a battery takes charge at a low voltage, does work on it, and ejects charge at a higher voltage.

Note that the current in the water circuit of Figure 19.9 is the same throughout the circuit. In other words, if we measured the number of water molecules passing a cross-section of the pipe per unit time at any point in the circuit, we would get the same answer no matter where in the circuit we measured. The same is true of the electrical circuit in the same figure. The electric current is the same at all points in this circuit, including inside the battery and in the resistor. The electric current neither speeds up in the wires nor slows down in the resistor. This would create points where too much or too little charge would be bunched up. Thus, the current is the same at all points in the circuit shown in Figure 19.9.

Although the current is the same everywhere in both the electric and water circuits, the voltage or water pressure changes as you move through the circuits. In the water circuit, the water pressure at the pump outlet stays the same until the water goes through the sand filter, assuming no energy loss in the pipe. Likewise, the voltage in the electrical circuit is the same at all points in a given wire, because we have assumed that the wires are perfect conductors. Thus, as indicated by the constant red color of the upper wire in Figure 19.11, the voltage throughout this wire is constant at $V = V_{\text{battery}}$. The voltage then drops as you go through the resistor, but once you reach the blue wire, the voltage stays at its new level of $V = 0$ all the way to the negative terminal of the battery (i.e., the blue terminal of the battery).

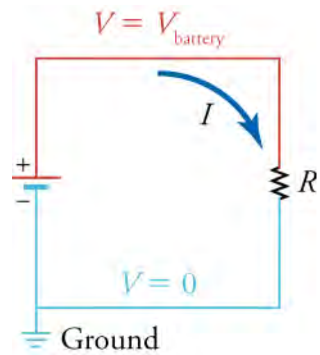


Figure 19.11 The voltage in the red wire is constant at $V = V_{\text{battery}}$ from the positive terminal of the battery to the top of the resistor. The voltage in the blue wire is constant at $V = V_{\text{ground}} = 0$ from the bottom of the resistor to the negative terminal of the battery.

If we go from the blue wire through the battery to the red wire, the voltage increases from $V = 0$ to $V = V_{\text{battery}}$. Likewise, if we go from the blue wire up through the resistor to the red wire, the voltage also goes from $V = 0$ to $V = V_{\text{battery}}$. Thus, using Ohm's law, we can write

$$V_{\text{resistor}} = V_{\text{battery}} = IR.$$

Note that V_{resistor} is measured from the bottom of the resistor to the top, meaning that the top of the resistor is at a higher voltage than the bottom of the resistor. Thus, current flows from the top of the resistor or higher voltage to the bottom of the resistor or lower voltage.

Virtual Physics

Battery-Resistor Circuit

Click to view content (http://www.openstax.org/l/21batteryresist)
Use this simulation to better understand how resistance, voltage, and current are related. The simulation shows a battery with a resistor connected between the terminals of the battery, as in the previous figure. You can modify the battery voltage and the resistance. The simulation shows how electrons react to these changes. It also shows the atomic cores in the resistor and how they are excited and heat up as more current goes through the resistor.

Draw the circuit diagram for the circuit, being sure to draw an arrow indicating the direction of the current. Now pick three spots along the wire. Without changing the settings, allow the simulation to run for 20 s while you count the number of electrons passing through that spot. Record the number on the circuit diagram. Now do the same thing at each of the other two spots in the circuit. What do you notice about the number of charges passing through each spot in 20 s? Remember that that current is defined as the rate that charges flow through the circuit. What does this mean about the current through the entire circuit?

GRASP CHECK

With the voltage slider, give the battery a positive voltage. Notice that the electrons are spaced farther apart in the left wire than they are in the right wire. How does this reflect the voltage in the two wires?

a. The voltage between static charges is directly proportional to the distance between them.
b. The voltage between static charges is directly proportional to square of the distance between them.
c. The voltage between static charges is inversely proportional to the distance between them.
d. The voltage between static charges is inversely proportional to square of the distance between them.

Other possible circuit elements include capacitors and switches. These are drawn as shown on the left side of Figure 19.12. A switch is a device that opens and closes the circuit, like a light switch. It is analogous to a valve in a water circuit, as shown on the right side of Figure 19.12. With the switch open, no current passes through the circuit. With the switch closed, it becomes part of the wire, so the current passes through it with no loss of voltage.

The capacitor is labeled C on the left of Figure 19.12. A capacitor in an electrical circuit is analogous to a flexible membrane in a water circuit. When the switch is closed in the circuit of Figure 19.12, the battery forces electrical current to flow toward the capacitor, charging the upper capacitor plate with positive charge. As this happens, the voltage across the capacitor plates increases. This is like the membrane in the water circuit: When the valve is opened, the pump forces water to flow toward the membrane, making it stretch to store the excess water. As this happens, the pressure behind the membrane increases.

Now if we open the switch, the capacitor holds the voltage between its plates because the charges have nowhere to go. Likewise, if we close the valve, the water has nowhere to go and the membrane maintains the water pressure in the pipe between itself and the valve.

If the switch is closed for a long time in the electric circuit or if the valve is open for a long time in the water circuit, the current will eventually stop flowing because the capacitor or the membrane will have become completely charged. Each circuit is now in the **steady state**, which means that its characteristics do not change over time. In this case, the steady state is characterized by zero current, and this does not change as long as the switch or valve remains in the same position. In the steady state, no electrical current passes through the capacitor, and no water current passes through the membrane. The voltage difference between the capacitor plates will be the same as the battery voltage. In the water circuit, the pressure behind the membrane will be the same as the pressure created by the pump.

Although the circuit in Figure 19.12 may seem a bit pointless because all that happens when the switch is closed is that the capacitor charges up, it does show the capacitor's ability to store charge. Thus, the capacitor serves as a reservoir for charge. This property of capacitors is used in circuits in many ways. For example, capacitors are used to power circuits while batteries are being charged. In addition, capacitors can serve as filters. To understand this, let's go back to the water analogy. Suppose you have a water hose and are watering your garden. Your friend thinks he's funny, and *kinks* the hose. While the hose is kinked, you experience no water flow. When he lets go, the water starts flowing again. If he does this really fast, you experience water-no water-water-no water, and that's really no way to water your garden. Now imagine that the hose is filling up a big bucket, and you are watering from the bottom of the bucket. As long as you had water in your bucket to begin with and your friend doesn't kink the water hose for too long, you would be able to water your garden without the interruptions. Your friend kinking the water hose is *filtered* by the big bucket's supply of water, so it does not impact your ability to water the garden. We can think of the interruptions in the current (be it water or electrical current) as *noise*. Capacitors act in an analogous way as the water bucket to help filter out the noise. Capacitors have so many uses that it is very rare to find an electronic circuit that does not include some capacitors.

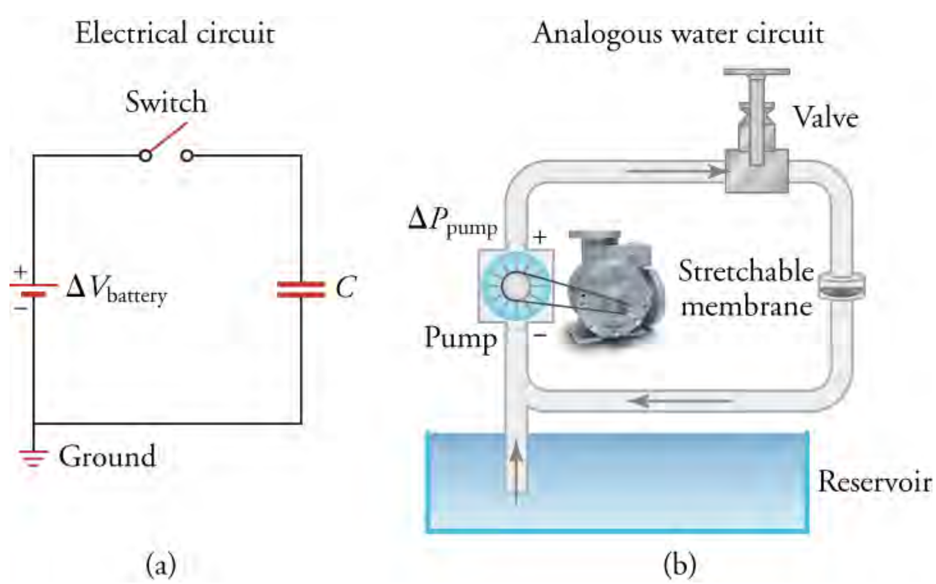


Figure 19.12 On the left is an electrical circuit containing a battery, a switch, and a capacitor. On the left is the analogous water circuit with a pump, a valve, and a stretchable membrane. The pump is like the battery, the valve is like the switch, and the stretchable membrane is like the capacitor. When the switch is closed, electrical current flows as the capacitor charges and its voltage increases. Likewise in the water circuit, when the valve is open, water current flows as the stretchable membrane stretches and the water pressure behind it increases.

WORK IN PHYSICS

What It Takes to be an Electrical Engineer

Physics is used in a wide variety of fields. One field that requires a very thorough knowledge of physics is electrical engineering. An electrical engineer can work on anything from the large-scale power systems that provide power to big cities to the nanoscale electronic circuits that are found in computers and cell phones (Figure 19.13).

In working with power companies, you can be responsible for maintaining the power grid that supplies electrical power to large areas. Although much of this work is done from an office, it is common to be called in for overtime duty after storms or other natural events. Many electrical engineers enjoy this part of the job, which requires them to race around the countryside repairing high-voltage transformers and other equipment. However, one of the more unpleasant aspects of this work is to remove the carcasses of unfortunate squirrels or other animals that have wandered into the transformers.

Other careers in electrical engineering can involve designing circuits for cell phones, which requires cramming some 10 billion transistors into an electronic chip the size of your thumbnail. These jobs can involve much work with computer simulations and can also involve fields other than electronics. For example, the 1-m-diameter lenses that are used to make these circuits (as of 2015) are so precise that they are shipped from the manufacture to the chip fabrication plant in temperature-controlled trucks to ensure that they are held within a certain temperature range. If they heat up or cool down too much, they deform ever so slightly, rendering them useless for the ultrahigh precision photolithography required to manufacture these chips.

In addition to a solid knowledge of physics, electrical engineers must above all be practical. Consider, for example, how one corporation managed to launch some anti-ballistic missiles at the White Sands Missile Test Range in New Mexico in the 1960s. Before launch, the skin of the missile had to be at the same voltage as the rail from which it was launched. The rail was connected to the ground by a large copper wire connected to a stake driven into the sandy earth. The missile, however, was connected by an umbilical cord to the equipment in the control shed a few meters away, which was grounded via a different grounding circuit. Before launching the missile, the voltage difference between the missile skin and the rail had to be less than 2.5 V. After an especially dry spell of weather, the missile could not be launched because the voltage difference stood at 5 V. A group of electrical engineers, including the father of your author, stood around pondering how to reduce the voltage difference. The situation was resolved when one of the engineers realized that urine contains electrolytes and conducts electricity quite well. With that, the four engineers quickly resolved the problem by urinating on the rail spike. The voltage difference immediately dropped to below 2.5 V and the missile was launched on schedule.

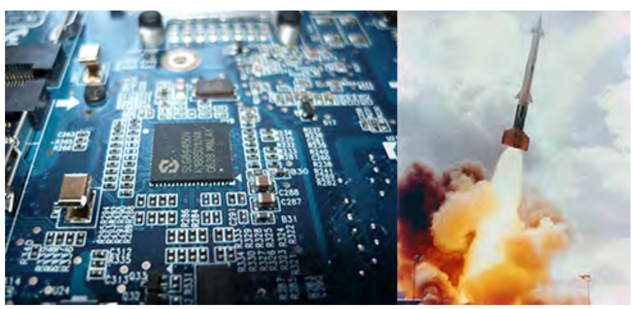


Figure 19.13 The systems that electrical engineers work on range from microprocessor circuits (left)] to missile systems (right).

Virtual Physics

Click to view content (http://www.openstax.org/l/21phetcirconstr)
Amuse yourself by building circuits of all different shapes and sizes. This simulation provides you with various standard circuit elements, such as batteries, AC voltage sources, resistors, capacitors, light bulbs, switches, etc. You can connect these in any configuration you like and then see the result.

Build a circuit that starts with a resistor connected to a capacitor. Connect the free side of the resistor to the positive terminal of a battery and the free side of the capacitor to the negative terminal of the battery. Click the *reset dynamics* button to see how the current flows starting with no charge on the capacitor. Now right click on the resistor to change its value. When you increase the resistance, does the circuit reach the steady state more rapidly or more slowly?

GRASP CHECK

When the circuit has reached the steady state, how does the voltage across the capacitor compare to the voltage of the battery? What is the voltage across the resistor?

a. The voltage across the capacitor is greater than the voltage of the battery. In the steady state, no current flows through this circuit, so the voltage across the resistor is zero.
b. The voltage across the capacitor is smaller than the voltage of the battery. In the steady state, finite current flows through this circuit, so the voltage across the resistor is finite.
c. The voltage across the capacitor is the same as the voltage of the battery. In the steady state, no current flows through this circuit, so the voltage across the resistor is zero.
d. The voltage across the capacitor is the same as the voltage of the battery. In the steady state, finite current flows through this circuit, so the voltage across the resistor is finite.

Resistors in Series and Equivalent Resistance

Now that we have a basic idea of how electrical circuits work, let's see what happens in circuits with more than one circuit element. In this section, we look at resistors in series. Components connected **in series** are connected one after the other in the same branch of a circuit, such as the resistors connected in series on the left side of Figure 19.14.

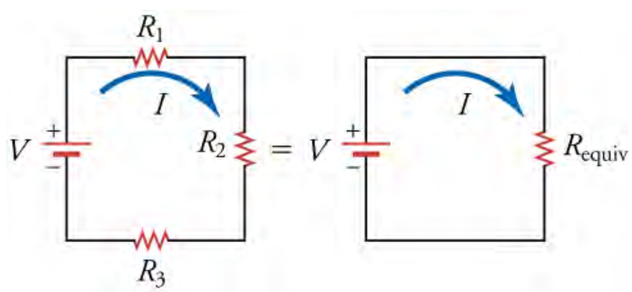


Figure 19.14 On the left is an electric circuit with three resistors R_1, R_2, and R_3 connected in series. On the right is an electric circuit with one resistor R_{equiv} that is equivalent to the combination of the three resistors R_1, R_2, and R_3.

We will now try to find a single resistance that is equivalent to the three resistors in series on the left side of Figure 19.14. An **equivalent resistor** is a resistor that has the same resistance as the combined resistance of a set of other resistors. In other words, the same current will flow through the left and right circuits in Figure 19.14 if we use the equivalent resistor in the right circuit.

According to Ohm's law, the voltage drop V across a resistor when a current flows through it is $V = IR$ where I is the current in amperes (A) and R is the resistance in ohms (Ω). Another way to think of this is that V is the voltage necessary to make a current I flow through a resistance R. Applying Ohm's law to each resistor on the left circuit of Figure 19.14, we find that the voltage drop across R_1 is $V_1 = IR_1$, that across R_2 is $V_2 = IR_2$, and that across R_3 is $V_3 = IR_3$. The sum of these voltages equals the voltage output of the battery, that is

$$V_{\text{battery}} = V_1 + V_2 + V_3. \tag{19.9}$$

You may wonder why voltages must add up like this. One way to understand this is to go once around the circuit and add up the successive changes in voltage. If you do this around a loop and get back to the starting point, the total change in voltage should be zero, because you end up at the same place that you started. To better understand this, consider the analogy of going for a stroll through some hilly countryside. If you leave your car and walk around, then come back to your car, the total height you gained in your stroll must be the same as the total height you lost, because you end up at the same place as you started. Thus, the gravitational potential energy you gain must be the same as the gravitational potential energy you lose. The same reasoning holds for voltage in going around an electric circuit. Let's apply this reasoning to the left circuit in Figure 19.14. We start just below the battery and move up through the battery, which contributes a voltage *gain* of V_{battery}. Next, we got through the resistors. The voltage *drops* by V_1 in going through resistor R_1, by V_2 in going through resistor R_2, and by V_3 in going through resistor R_3 . After going through resistor R_3, we arrive back at the starting point, so we add up these four changes in voltage and set the sum equal to zero. This gives

$$0 = V_{\text{battery}} - V_1 - V_2 - V_3. \tag{19.10}$$

which is the same as the previous equation. Note that the minus signs in front of V_1 ,V_2 , and V_3 are because these are voltage *drops*, whereas V_{battery} is a voltage *rise*.

Ohm's law tells us that $V_1 = IR_1$, $V_2 = IR_2$, and $V_3 = IR_3$. Inserting these values into equation $V_{\text{battery}} = V_1 + V_2 + V_3$ gives

$$\begin{aligned} V_{\text{battery}} &= IR_1 + IR_2 + IR_3 \\ &= I\left(R_1 + R_2 + R_3\right). \end{aligned} \tag{19.11}$$

Applying this same logic to the right circuit in Figure 19.14 gives

$$V_{\text{battery}} = IR_{\text{equiv}}. \tag{19.12}$$

Dividing the equation $V_{\text{battery}} = I\left(R_1 + R_2 + R_3\right)$ by $V_{\text{battery}} = IR_{\text{equiv}}$, we get

$$\begin{aligned} \frac{V_{\text{battery}}}{V_{\text{battery}}} &= \frac{I(R_1+R_2+R_3)}{IR_{\text{equiv}}} \\ R_{\text{equiv}} &= R_1 + R_2 + R_3. \end{aligned} \tag{19.13}$$

This shows that the equivalent resistance for a series of resistors is simply the sum of the resistances of each resistor. In general, N resistors connected in series can be replaced by an equivalent resistor with a resistance of

$$R_{\text{equiv}} = R_1 + R_2 + \cdots + R_N.$$

WATCH PHYSICS

Resistors in Series

This video discusses the basic concepts behind interpreting circuit diagrams and then shows how to calculate the equivalent resistance for resistors in series.

Click to view content (https://www.openstax.org/l/02resistseries)

GRASP CHECK

True or false—In a circuit diagram, we can assume that the voltage is the same at every point in a given wire.

a. false
b. true

WORKED EXAMPLE

Calculation of Equivalent Resistance

In the left circuit of the previous figure, suppose the voltage rating of the battery is 12 V, and the resistances are $R_1 = 1.0\ \Omega, R_2 = 6.0\ \Omega,$ and $R_3 = 13\ \Omega$. (a) What is the equivalent resistance? (b) What is the current through the circuit?

STRATEGY FOR (A)

Use the equation for the equivalent resistance of resistors connected in series. Because the circuit has three resistances, we only need to keep three terms, so it takes the form

$$R_{\text{equiv}} = R_1 + R_2 + R_3. \tag{19.14}$$

Solution for (a)

Inserting the given resistances into the equation above gives

$$\begin{aligned} R_{\text{equiv}} &= R_1 + R_2 + R_3 \\ &= 1.0\ \Omega + 6.0\ \Omega + 13\ \Omega \\ &= 20\ \Omega. \end{aligned} \tag{19.15}$$

Discussion for (a)

We can thus replace the three resistors R_1, R_2, andR_3 with a single 20- Ω resistor.

STRATEGY FOR (B)

Apply Ohm's law to the circuit on the right side of the previous figure with the equivalent resistor of 20 Ω .

Solution for (b)

The voltage drop across the equivalent resistor must be the same as the voltage rise in the battery. Thus, Ohm's law gives

$$\begin{aligned} V_{\text{battery}} &= IR_{\text{equiv}} \\ I &= \frac{V_{\text{battery}}}{R_{\text{equiv}}} \\ &= \frac{12\ \text{V}}{20\ \Omega} \\ &= 0.60\ \text{A}. \end{aligned} \tag{19.16}$$

Discussion for (b)

To check that this result is reasonable, we calculate the voltage drop across each resistor and verify that they add up to the voltage rating of the battery. The voltage drop across each resistor is

$$\begin{aligned} V_1 &= IR_1 = (0.60\ \text{A})(1.0\Omega) = 0.60\ \text{V} \\ V_2 &= IR_2 = (0.60\ \text{A})(6.0\Omega) = 3.6\ \text{V} \\ V_3 &= IR_3 = (0.60\ \text{A})(13\Omega) = 7.8\ \text{V}. \end{aligned} \tag{19.17}$$

Adding these voltages together gives

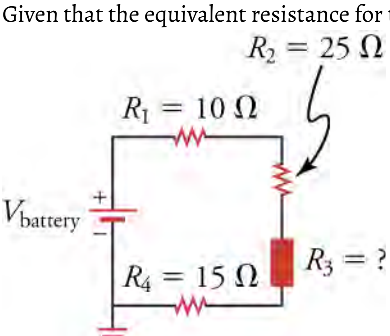
$$V_1 + V_2 + V_3 = 0.60\ \text{V} + 3.6\ \text{V} + 7.8\ \text{V} = 12\ \text{V}. \tag{19.18}$$

which is the voltage rating of the battery.

WORKED EXAMPLE

Determine the Unknown Resistance

The circuit shown in figure below contains three resistors of known value and a third element whose resistance R_3 is unknown. Given that the equivalent resistance for the entire circuit is 150 Ω, what is the resistance R_3 ?



STRATEGY

The four resistances in this circuit are connected in series, so we know that they must add up to give the equivalent resistance. We can use this to find the unknown resistance R_3 .

Solution

For four resistances in series, the equation for the equivalent resistance of resistors in series takes the form

$$R_{\text{equiv}} = R_1 + R_2 + R_3 + R_4. \tag{19.19}$$

Solving for R_3 and inserting the known values gives

$$\begin{aligned} R_3 &= R_{\text{equiv}} - R_1 - R_2 - R_4 \\ &= 150\ \Omega - 10\ \Omega - 25\ \Omega - 15\ \Omega \\ &= 100\ \Omega. \end{aligned} \tag{19.20}$$

Discussion

The equivalent resistance of a circuit can be measured with an ohmmeter. This is sometimes useful for determining the effective resistance of elements whose resistance is not marked on the element.

Check your Understanding

8.

Figure 19.15

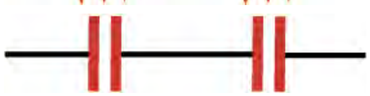
What circuit element is represented in the figure below?

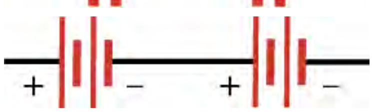
a. a battery
b. a resistor
c. a capacitor
d. an inductor

9. How would a diagram of two resistors connected in series appear?

a. 

b. 

c. 

d. 

19.3 Parallel Circuits

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Interpret circuit diagrams with parallel resistors
- Calculate equivalent resistance of resistor combinations containing series and parallel resistors

Section Key Terms

in parallel

Resistors in Parallel

In the previous section, we learned that resistors in series are resistors that are connected one after the other. If we instead combine resistors by connecting them next to each other, as shown in Figure 19.16, then the resistors are said to be connected *in parallel*. Resistors are **in parallel** when both ends of each resistor are connected directly together.

Note that the tops of the resistors are all connected to the same wire, so the voltage at the top of the each resistor is the same. Likewise, the bottoms of the resistors are all connected to the same wire, so the voltage at the bottom of each resistor is the same. This means that the voltage drop across each resistor is the same. In this case, the voltage drop is the voltage rating V of the battery, because the top and bottom wires connect to the positive and negative terminals of the battery, respectively.

Although the voltage drop across each resistor is the same, we cannot say the same for the current running through each resistor. Thus, I_1, I_2, and I_3 are not necessarily the same, because the resistors R_1, R_2, and R_3 do not necessarily have the same resistance.

Note that the three resistors in Figure 19.16 provide three different paths through which the current can flow. This means that the equivalent resistance for these three resistors must be less than the smallest of the three resistors. To understand this, imagine that the smallest resistor is the only path through which the current can flow. Now add on the alternate paths by connecting other resistors in parallel. Because the current has more paths to go through, the overall resistance (i.e., the equivalent resistance) will decrease. Therefore, the equivalent resistance must be less than the smallest resistance of the parallel resistors.

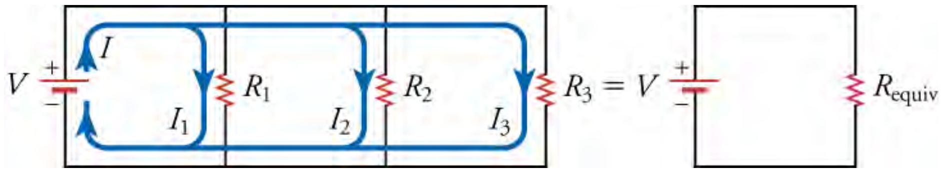


Figure 19.16 The left circuit diagram shows three resistors in parallel. The voltage *V* of the battery is applied across all three resistors. The currents that flow through each branch are not necessarily equal. The right circuit diagram shows an equivalent resistance that replaces the three parallel resistors.

To find the equivalent resistance R_{equiv} of the three resistors R_1, R_2, and R_3, we apply Ohm's law to each resistor. Because the voltage drop across each resistor is *V*, we obtain

$$V = I_1 R_1, \quad V = I_2 R_2, \quad V = I_3 R_3 \tag{19.21}$$

or

$$I_1 = \frac{V}{R_1}, \quad I_2 = \frac{V}{R_2}, \quad I_3 = \frac{V}{R_3}. \tag{19.22}$$

We also know from conservation of charge that the three currents I_1, I_2, and I_3 must add up to give the current *I* that goes through the battery. If this were not true, current would have to be mysteriously created or destroyed somewhere in the circuit, which is physically impossible. Thus, we have

$$I = I_1 + I_2 + I_3. \tag{19.23}$$

Inserting the expressions for I_1, I_2, and I_3 into this equation gives

$$I = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} = V\left(\frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}\right) \tag{19.24}$$

or

$$V = I\left(\frac{1}{1/R_1 + 1/R_2 + 1/R_3}\right). \tag{19.25}$$

This formula is just Ohm's law, with the factor in parentheses being the equivalent resistance.

$$V = I\left(\frac{1}{1/R_1 + 1/R_2 + 1/R_3}\right) = IR_{\text{equiv}}. \tag{19.26}$$

Thus, the equivalent resistance for three resistors in parallel is

$$R_{\text{equiv}} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}. \tag{19.27}$$

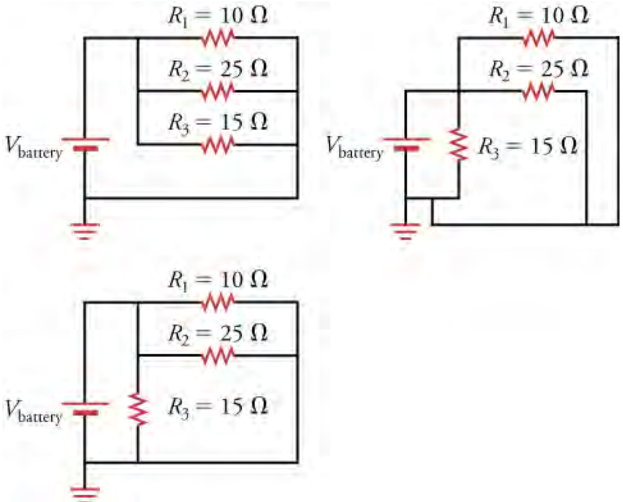
The same logic works for any number of resistors in parallel, so the general form of the equation that gives the equivalent resistance of *N* resistors connected in parallel is

$$R_{\text{equiv}} = \frac{1}{1/R_1 + 1/R_2 + \cdots + 1/R_N}. \tag{19.28}$$

WORKED EXAMPLE

Find the Current through Parallel Resistors

The three circuits below are equivalent. If the voltage rating of the battery is $V_{\text{battery}} = 3\text{ V}$, what is the equivalent resistance of the circuit and what current runs through the circuit?

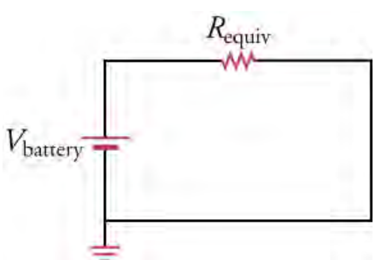


STRATEGY

The three resistors are connected in parallel and the voltage drop across them is V_{battery}. Thus, we can apply the equation for the equivalent resistance of resistors in parallel, which takes the form

$$R_{\text{equiv}} = \frac{1}{1/R_1 + 1/R_2 + 1/R_3}. \tag{19.29}$$

The circuit with the equivalent resistance is shown below. Once we know the equivalent resistance, we can use Ohm's law to find the current in the circuit.



Solution

Inserting the given values for the resistance into the equation for equivalent resistance gives

$$\begin{aligned} R_{\text{equiv}} &= \frac{1}{1/R_1+1/R_2+1/R_3} \\ &= \frac{1}{1/10\ \Omega+1/25\ \Omega+1/15\ \Omega} \\ &= 4.84\ \Omega. \end{aligned} \tag{19.30}$$

The current through the circuit is thus

$$\begin{aligned} V &= IR \\ I &= \frac{V}{R} \\ &= \frac{3\text{ V}}{4.84\ \Omega} \\ &= 0.62\text{ A}. \end{aligned} \tag{19.31}$$

Discussion

Although 0.62 A flows through the entire circuit, note that this current does not flow through each resistor. However, because electric charge must be conserved in a circuit, the sum of the currents going through each branch of the circuit must add up to the current going through the battery. In other words, we cannot magically create charge somewhere in the circuit and add this new charge to the current. Let's check this reasoning by using Ohm's law to find the current through each resistor.

$$\begin{aligned} I_1 &= \frac{V}{R_1} = \frac{3\ \text{V}}{10\ \Omega} = 0.30\ \text{A} \\ I_2 &= \frac{V}{R_2} = \frac{3\ \text{V}}{25\ \Omega} = 0.12\ \text{A} \\ I_3 &= \frac{V}{R_3} = \frac{3\ \text{V}}{15\ \Omega} = 0.20\ \text{A} \end{aligned} \tag{19.32}$$

As expected, these currents add up to give 0.62 A, which is the total current found going through the equivalent resistor. Also, note that the smallest resistor has the largest current flowing through it, and vice versa.

WORKED EXAMPLE

Reasoning with Parallel Resistors

Without doing any calculation, what is the equivalent resistance of three identical resistors R in parallel?

STRATEGY

Three identical resistors R in parallel make three identical paths through which the current can flow. Thus, it is three times easier for the current to flow through these resistors than to flow through a single one of them.

Solution

If it is three times easier to flow through three identical resistors R than to flow through a single one of them, the equivalent resistance must be three times less: $R/3$.

Discussion

Let's check our reasoning by calculating the equivalent resistance of three identical resistors R in parallel. The equation for the equivalent resistance of resistors in parallel gives

$$\begin{aligned} R_{\text{equiv}} &= \frac{1}{1/R+1/R+1/R} \\ &= \frac{1}{3/R} \\ &= \frac{R}{3}. \end{aligned} \tag{19.33}$$

Thus, our reasoning was correct. In general, when more paths are available through which the current can flow, the equivalent resistance decreases. For example, if we have identical resistors R in parallel, the equivalent resistance would be $R/10$.

Practice Problems

10. Three resistors, 10, 20, and 30 Ω, are connected in parallel. What is the equivalent resistance?

a. The equivalent resistance is 5.5 Ω
b. The equivalent resistance is 60 Ω
c. The equivalent resistance is 6 × 103 Ω
d. The equivalent resistance is 6 × 104 Ω

11. If a 5-V drop occurs across R_1, and R_1 is connected in parallel to R_2, what is the voltage drop across R_2?

a. Voltage drop across is $0\ \text{V}$.
b. Voltage drop across is $2.5\ \text{V}$.
c. Voltage drop across is $5\ \text{V}$.
d. Voltage drop across is $10\ \text{V}$.

Resistors in Parallel and in Series

More complex connections of resistors are sometimes just combinations of series and parallel. Combinations of series and parallel resistors can be reduced to a single equivalent resistance by using the technique illustrated in Figure 19.17. Various parts are identified as either series or parallel, reduced to their equivalents, and further reduced until a single resistance is left. The process is more time consuming than difficult.

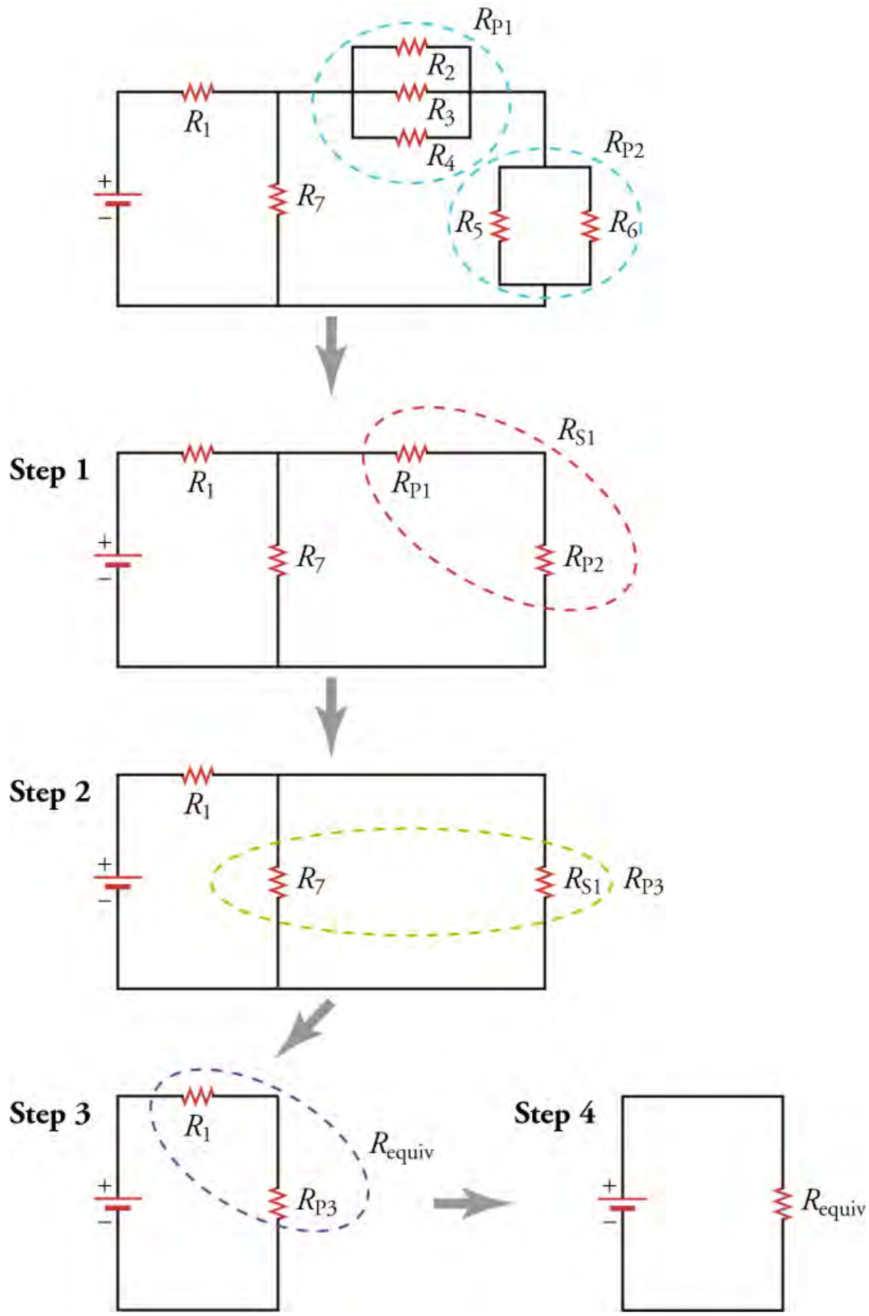


Figure 19.17 This combination of seven resistors has both series and parallel parts. Each is identified and reduced to an equivalent resistance, and these are further reduced until a single equivalent resistance is reached.

Let's work through the four steps in Figure 19.17 to reduce the seven resistors to a single equivalent resistor. To avoid distracting algebra, we'll assume each resistor is 10 Ω . In step 1, we reduce the two sets of parallel resistors circled by the blue dashed loop. The upper set has three resistors in parallel and will be reduced to a single equivalent resistor R_{P1} . The lower set has two resistors in parallel and will be reduced to a single equivalent resistor R_{P2} . Using the equation for the equivalent resistance of resistors in parallel, we obtain

$$\begin{aligned} R_{P1} &= \frac{1}{1/R_2+1/R_3+1/R_4} &= \frac{1}{1/10\ \Omega+1/10\ \Omega+1/10\ \Omega} &= \frac{10}{3}\Omega \\ R_{P2} &= \frac{1}{1/R_5+1/R_6} &= \frac{1}{1/10\ \Omega+1/10\ \Omega} &= 5\ \Omega. \end{aligned} \tag{19.34}$$

These two equivalent resistances are encircled by the red dashed loop following step 1. They are in series, so we can use the equation for the equivalent resistance of resistors in series to reduce them to a single equivalent resistance R_{S1} . This is done in step 2, with the result being

$$R_{S1} = R_{P1} + R_{P2} = \frac{10}{3}\Omega + 5\ \Omega = \frac{25}{3}\Omega. \tag{19.35}$$

The equivalent resistor R_{S1} appears in the green dashed loop following step 2. This resistor is in parallel with resistor R_7 , so the pair can be replaced by the equivalent resistor R_{P3} , which is given by

$$R_{P3} = \frac{1}{1/R_{S1} + 1/R_7} = \frac{1}{3/25\ \Omega + 1/10\ \Omega} = \frac{50}{11}\Omega. \tag{19.36}$$

This is done in step 3. The resistor R_{P3} is in series with the resistor R_1 , as shown in the purple dashed loop following step 3. These two resistors are combined in the final step to form the final equivalent resistor R_{equiv} , which is

$$R_{\text{equiv}} = R_1 + R_{P3} = 10\ \Omega + \frac{50}{11}\Omega = \frac{160}{11}\Omega. \tag{19.37}$$

Thus, the entire combination of seven resistors may be replaced by a single resistor with a resistance of about 14.5 Ω .

That was a lot of work, and you might be asking why we do it. It's important for us to know the equivalent resistance of the entire circuit so that we can calculate the current flowing through the circuit. Ohm's law tells us that the current flowing through a circuit depends on the resistance of the circuit and the voltage across the circuit. But to know the current, we must first know the equivalent resistance.

Here is a general approach to find the equivalent resistor for any arbitrary combination of resistors:

1. Identify a group of resistors that are only in parallel or only in series.
2. For resistors in series, use the equation for the equivalent resistance of resistors in series to reduce them to a single equivalent resistance. For resistors in parallel, use the equation for the equivalent resistance of resistors in parallel to reduce them to a single equivalent resistance.
3. Draw a new circuit diagram with the resistors from step 1 replaced by their equivalent resistor.
4. If more than one resistor remains in the circuit, return to step 1 and repeat. Otherwise, you are finished.

FUN IN PHYSICS

Robot

Robots have captured our collective imagination for over a century. Now, this dream of creating clever machines to do our dirty work, or sometimes just to keep us company, is becoming a reality. *Robotics* has become a huge field of research and development, with some technology already being commercialized. Think of the small autonomous vacuum cleaners, for example.

Figure 19.18 shows just a few of the multitude of different forms robots can take. The most advanced humanoid robots can walk, pour drinks, even dance (albeit not very gracefully). Other robots are bio-inspired, such as the *dogbot* shown in the middle photograph of Figure 19.18. This robot can carry hundreds of pounds of load over rough terrain. The photograph on the right in Figure 19.18 shows the inner workings of an *M-block*, developed by the Massachusetts Institute of Technology. These simple-looking blocks contain inertial wheels and electromagnets that allow them to spin and flip into the air and snap together in a variety of shapes. By communicating wirelessly between themselves, they self-assemble into a variety of shapes, such as desks, chairs, and someday maybe even buildings.

All robots involve an immense amount of physics and engineering. The simple act of pouring a drink has only recently been mastered by robots, after over 30 years of research and development! The balance and timing that we humans take for granted is in fact a very tricky act to follow, requiring excellent balance, dexterity, and feedback. To master this requires sensors to detect balance, computing power to analyze the data and communicate the appropriate compensating actions, and joints and actuators to implement the required actions.

In addition to sensing gravity or acceleration, robots can contain multiple different sensors to detect light, sound, temperature, smell, taste, etc. These devices are all based on the physical principles that you are studying in this text. For example, the optics used for robotic vision are similar to those used in your digital cameras: pixelated semiconducting detectors in which light is converted into electrical signals. To detect temperature, simple thermistors may be used, which are resistors whose resistance changes depending on temperature.

Building a robot today is much less arduous than it was a few years ago. Numerous companies now offer kits for building robots. These range in complexity something suitable for elementary school children to something that would challenge the best professional engineers. If interested, you may find these easily on the Internet and start making your own robot today.

Figure 19.18 Robots come in many shapes and sizes, from the classic *humanoid* type to *dogbots* to small cubes that self-assemble to perform a variety of tasks.

WATCH PHYSICS

Resistors in Parallel

This video shows a lecturer discussing a simple circuit with a battery and a pair of resistors in parallel. He emphasizes that electrons flow in the direction opposite to that of the positive current and also makes use of the fact that the voltage is the same at all points on an ideal wire. The derivation is quite similar to what is done in this text, but the lecturer goes through it well, explaining each step.

Click to view content (https://www.openstax.org/l/28resistors)

GRASP CHECK

True or false—In a circuit diagram, we can assume that the voltage is the same at every point in a given wire.

a. false
b. true

WATCH PHYSICS

Resistors in Series and in Parallel

This video shows how to calculate the equivalent resistance of a circuit containing resistors in parallel and in series. The lecturer uses the same approach as outlined above for finding the equivalent resistance.

Click to view content (https://www.openstax.org/l/28resistorssp)

GRASP CHECK

Imagine connected N identical resistors in parallel. Each resistor has a resistance of R. What is the equivalent resistance for this group of parallel resistors?

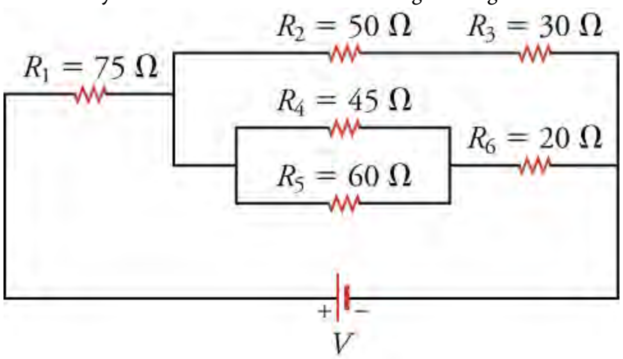
a. The equivalent resistance is $(R)^N$.

b. The equivalent resistance is NR.
c. The equivalent resistance is $\frac{R}{N}$.
d. The equivalent resistance is $\frac{N}{R}$.

WORKED EXAMPLE

Find the Current through a Complex Resistor Circuit

The battery in the circuit below has a voltage rating of 10 V. What current flows through the circuit and in what direction?



STRATEGY

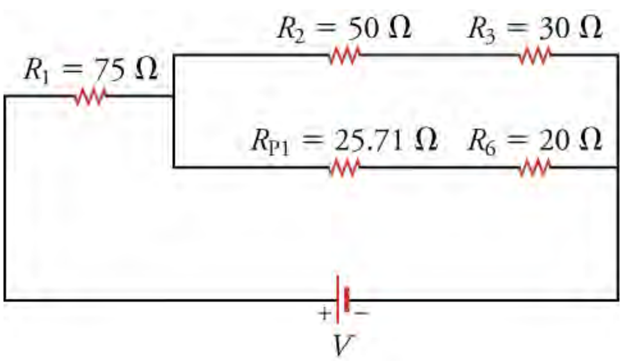
Apply the strategy for finding equivalent resistance to replace all the resistors with a single equivalent resistance, then use Ohm's law to find the current through the equivalent resistor.

Solution

The resistor combination R_4 and R_5 can be reduced to an equivalent resistance of

$$R_{\mathrm{P1}} = \frac{1}{1/R_4 + 1/R_5} = \frac{1}{1/45\ \Omega + 1/60\ \Omega} = 25.71\ \Omega\mathrm{R}. \tag{19.38}$$

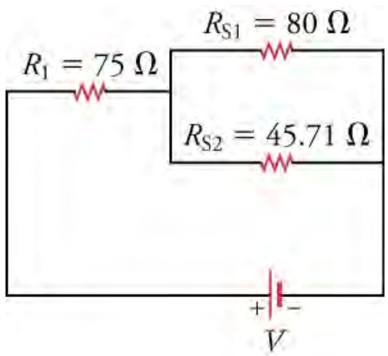
Replacing R_4 and R_5 with this equivalent resistance gives the circuit below.



We now replace the two upper resistors R_2 and R_3 by the equivalent resistor R_{S1} and the two lower resistors R_{P1} and R_6 by their equivalent resistor R_{S2} . These resistors are in series, so we add them together to find the equivalent resistance.

$$\begin{aligned} R_{\mathrm{S1}} &= R_2 + R_3 &&= 50\ \Omega + 30\ \Omega &&= 80\ \Omega \\ R_{\mathrm{S2}} &= R_{\mathrm{P1}} + R_6 &&= 25.71\ \Omega + 20\ \Omega &&= 45.71\ \Omega \end{aligned} \tag{19.39}$$

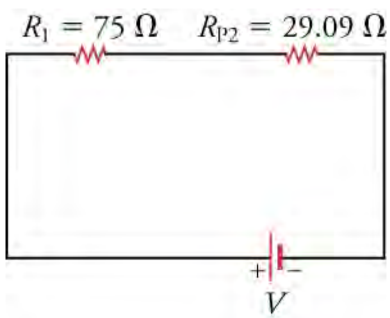
Replacing the relevant resistors with their equivalent resistor gives the circuit below.



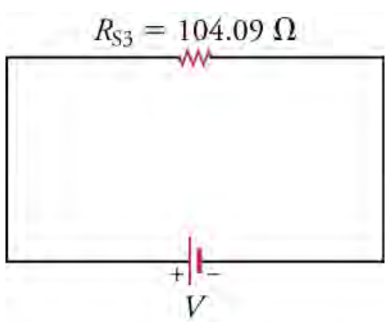
Now replace the two resistors R_{S1} and R_{S2}, which are in parallel, with their equivalent resistor R_{P2}. The resistance of R_{P2} is

$$R_{P2} = \frac{1}{1/R_{S1} + 1/R_{S2}} = \frac{1}{1/80\ \Omega + 1/45.71\ \Omega} = 29.09\ \Omega. \quad (19.40)$$

Updating the circuit diagram by replacing R_{S1} and R_{S2} with this equivalent resistance gives the circuit below.



Finally, we combine resistors R_1 and R_{P2}, which are in series. The equivalent resistance is $R_{S3} = R_1 + R_{P2} = 75\ \Omega + 29.09\ \Omega = 104.09\ \Omega$. The final circuit is shown below.



We now use Ohm's law to find the current through the circuit.

$$\begin{aligned} V &= IR_{S3} \\ I &= \frac{V}{R_{S3}} = \frac{10\ \text{V}}{104.09\ \Omega} = 0.096\ \text{A} \end{aligned} \quad (19.41)$$

The current goes from the positive terminal of the battery to the negative terminal of the battery, so it flows clockwise in this circuit.

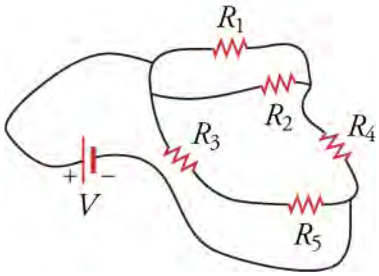
Discussion

This calculation may seem rather long, but with a little practice, you can combine some steps. Note also that extra significant digits were carried through the calculation. Only at the end was the final result rounded to two significant digits.

WORKED EXAMPLE

Strange-Looking Circuit Diagrams

Occasionally, you may encounter circuit diagrams that are not drawn very neatly, such as the diagram shown below. This circuit diagram looks more like how a real circuit might appear on the lab bench. What is the equivalent resistance for the resistors in this diagram, assuming each resistor is 10 Ω and the voltage rating of the battery is 12 V.

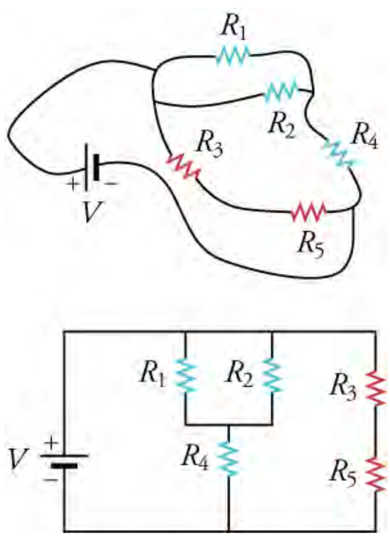


STRATEGY

Let's redraw this circuit diagram to make it clearer. Then we'll apply the strategy outlined above to calculate the equivalent resistance.

Solution

To redraw the diagram, consider the figure below. In the upper circuit, the blue resistors constitute a path from the positive terminal of the battery to the negative terminal. In parallel with this circuit are the red resistors, which constitute another path from the positive to negative terminal of the battery. The blue and red paths are shown more cleanly drawn in the lower circuit diagram. Note that, in both the upper and lower circuit diagrams, the blue and red paths connect the positive terminal of the battery to the negative terminal of the battery.



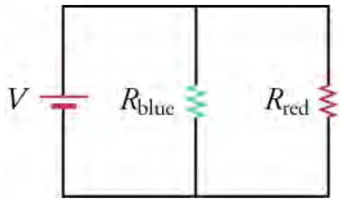
Now it is easier to see that R_1 and R_2 are in parallel, and the parallel combination is in series with R_4 . This combination in turn is in parallel with the series combination of R_3 and R_5 . First, we calculate the blue branch, which contains R_1 , R_2, and R_4 . The equivalent resistance is

$$R_{\text{blue}} = \frac{1}{1/R_1 + 1/R_2} + R_4 = \frac{1}{1/10\Omega + 1/10\ \Omega} + 10\ \Omega = 15\ \Omega. \tag{19.42}$$

where we show the contribution from the parallel combination of resistors and from the series combination of resistors. We now calculate the equivalent resistance of the red branch, which is

$$R_{\text{red}} = R_3 + R_5 = 10\ \Omega + 10\ \Omega = 20\ \Omega. \tag{19.43}$$

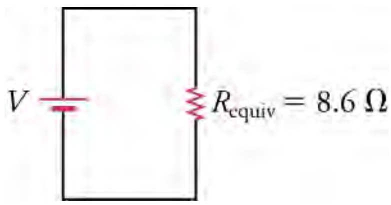
Inserting these equivalent resistors into the circuit gives the circuit below.



These two resistors are in parallel, so they can be replaced by a single equivalent resistor with a resistance of

$$R_{\mathrm{equiv}} = \frac{1}{1/R_{\mathrm{blue}} + 1/R_{\mathrm{red}}} = \frac{1}{1/15\ \Omega + 1/20\ \Omega} = 8.6\ \Omega. \tag{19.44}$$

The final equivalent circuit is show below.



Discussion

Finding the equivalent resistance was easier with a clear circuit diagram. This is why we try to make clear circuit diagrams, where the resistors in parallel are lined up parallel to each other and at the same horizontal position on the diagram.

We can now use Ohm's law to find the current going through each branch to this circuit. Consider the circuit diagram with R_{blue} and R_{red} . The voltage across each of these branches is 12 V (i.e., the voltage rating of the battery). The current in the blue branch is

$$I_{\mathrm{blue}} = \frac{V}{R_{\mathrm{blue}}} = \frac{12\ \mathrm{V}}{15\ \Omega} = 0.80\ \mathrm{A}. \tag{19.45}$$

The current across the red branch is

$$I_{\mathrm{red}} = \frac{V}{R_{\mathrm{red}}} = \frac{12\ \mathrm{V}}{20\ \Omega} = 0.60\ \mathrm{A}. \tag{19.46}$$

The current going through the battery must be the sum of these two currents (can you see why?), or 1.4 A.

Practice Problems

12. What is the formula for the equivalent resistance of two parallel resistors with resistance R_1 and R_2?

a. Equivalent resistance of two parallel resistors $R_{\mathrm{eqv}} = R_1 + R_2$
b. Equivalent resistance of two parallel resistors $R_{\mathrm{eqv}} = R_1 \times R_2$
c. Equivalent resistance of two parallel resistors $R_{\mathrm{eqv}} = R_1 - R_2$
d. Equivalent resistance of two parallel resistors $R_{\mathrm{eqv}} = \frac{1}{1/R_1 + 1/R_2}$

13.

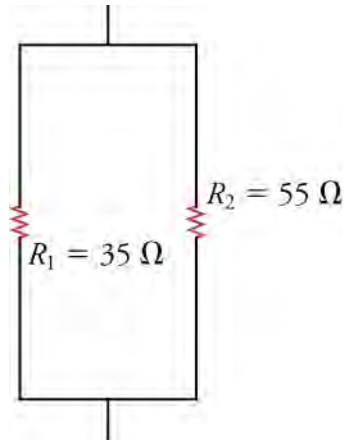


Figure 19.19

What is the equivalent resistance for the two resistors shown below?

a. The equivalent resistance is 20 Ω
b. The equivalent resistance is 21 Ω
c. The equivalent resistance is 90 Ω
d. The equivalent resistance is 1,925 Ω

Check Your Understanding

14. The voltage drop across parallel resistors is ________.
    a. the same for all resistors
    b. greater for the larger resistors
    c. less for the larger resistors
    d. greater for the smaller resistors

15. Consider a circuit of parallel resistors. The smallest resistor is 25 Ω . What is the upper limit of the equivalent resistance?
    a. The upper limit of the equivalent resistance is 2.5 Ω.
    b. The upper limit of the equivalent resistance is 25 Ω.
    c. The upper limit of the equivalent resistance is 100 Ω.
    d. There is no upper limit.

19.4 Electric Power

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Define electric power and describe the electric power equation
- Calculate electric power in circuits of resistors in series, parallel, and complex arrangements

Section Key Terms

electric power

Power is associated by many people with electricity. Every day, we use electric power to run our modern appliances. Electric power transmission lines are visible examples of electricity providing power. We also use electric power to start our cars, to run our computers, or to light our homes. Power is the rate at which energy of any type is transferred; **electric power** is the rate at which electric energy is transferred in a circuit. In this section, we'll learn not only what this means, but also what factors determine electric power.

To get started, let's think of light bulbs, which are often characterized in terms of their power ratings in watts. Let us compare a 25-W bulb with a 60-W bulb (see Figure 19.20). Although both operate at the same voltage, the 60-W bulb emits more light intensity than the 25-W bulb. This tells us that something other than voltage determines the power output of an electric circuit.

Incandescent light bulbs, such as the two shown in Figure 19.20, are essentially resistors that heat up when current flows through them and they get so hot that they emit visible and invisible light. Thus the two light bulbs in the photo can be considered as two different resistors. In a simple circuit such as a light bulb with a voltage applied to it, the resistance determines the current by Ohm's law, so we can see that current as well as voltage must determine the power.

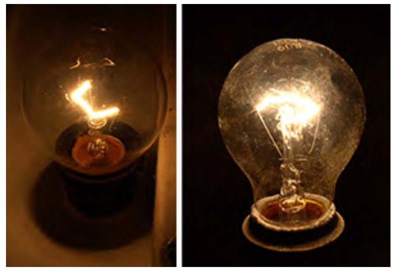


Figure 19.20 On the left is a 25-W light bulb, and on the right is a 60-W light bulb. Why are their power outputs different despite their operating on the same voltage?

The formula for power may be found by dimensional analysis. Consider the units of power. In the SI system, power is given in watts (W), which is energy per unit time, or J/s

$$\mathrm{W} = \frac{\mathrm{J}}{\mathrm{s}}. \tag{19.47}$$

Recall now that a voltage is the potential energy per unit charge, which means that voltage has units of J/C

$$\mathrm{V} = \frac{\mathrm{J}}{\mathrm{C}}. \tag{19.48}$$

We can rewrite this equation as $\mathrm{J} = \mathrm{V} \times \mathrm{C}$ and substitute this into the equation for watts to get

$$\mathrm{W} = \frac{\mathrm{J}}{\mathrm{s}} = \frac{\mathrm{V} \times \mathrm{C}}{\mathrm{s}} = \mathrm{V} \times \frac{\mathrm{C}}{\mathrm{s}}.$$

But a Coulomb per second (C/s) is an electric current, which we can see from the definition of electric current, $I = \frac{\Delta Q}{\Delta t}$, where ΔQ is the charge in coulombs and Δt is time in seconds. Thus, equation above tells us that electric power is voltage times current, or

$$P = IV.$$

This equation gives the electric power consumed by a circuit with a voltage drop of V and a current of I.

For example, consider the circuit in Figure 19.21. From Ohm's law, the current running through the circuit is

$$I = \frac{V}{R} = \frac{12\ \mathrm{V}}{100\ \Omega} = 0.12\ \mathrm{A}. \tag{19.49}$$

Thus, the power consumed by the circuit is

$$P = VI = (12\ \mathrm{V})(0.12\ \mathrm{A}) = 1.4\ \mathrm{W}. \tag{19.50}$$

Where does this power go? In this circuit, the power goes primarily into heating the resistor in this circuit.

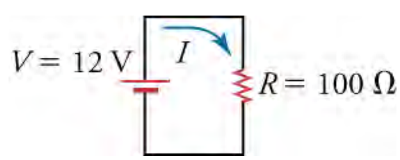


Figure 19.21 A simple circuit that consumes electric power.

In calculating the power in the circuit of Figure 19.21, we used the resistance and Ohm's law to find the current. Ohm's law gives the current: $I = V/R$, which we can insert into the equation for electric power to obtain

$$P = IV = \left(\frac{V}{R}\right)V = \frac{V^2}{R}.$$

This gives the power in terms of only the voltage and the resistance.

We can also use Ohm's law to eliminate the voltage in the equation for electric power and obtain an expression for power in terms of just the current and the resistance. If we write Ohm's law as $V = IR$ and use this to eliminate V in the equation $P = IV$, we obtain

$$P = IV = I\,(IR) = I^2R.$$

This gives the power in terms of only the current and the resistance.

Thus, by combining Ohm's law with the equation $P = IV$ for electric power, we obtain two more expressions for power: one in terms of voltage and resistance and one in terms of current and resistance. Note that only resistance (not capacitance or anything else), current, and voltage enter into the expressions for electric power. This means that the physical characteristic of a circuit that determines how much power it dissipates is its resistance. Any capacitors in the circuit do not dissipate electric power—on the contrary, capacitors either store electric energy or release electric energy back to the circuit.

To clarify how voltage, resistance, current, and power are all related, consider Figure 19.22, which shows the *formula wheel*. The quantities in the center quarter circle are equal to the quantities in the corresponding outer quarter circle. For example, to express a potential V in terms of power and current, we see from the formula wheel that $V = P/I$.

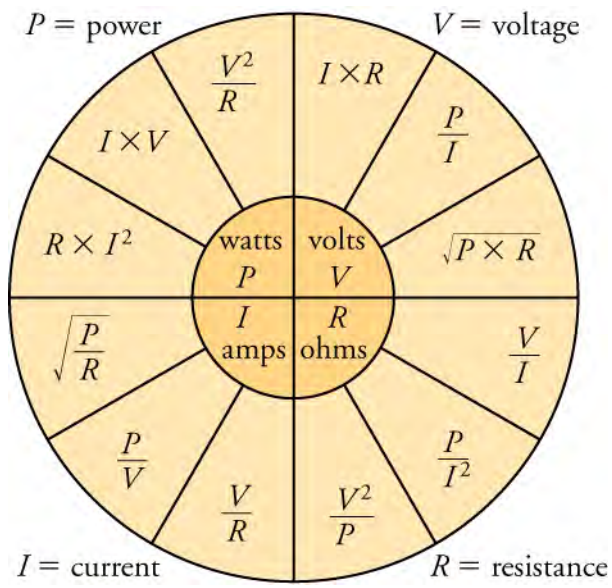


Figure 19.22 The formula wheel shows how volts, resistance, current, and power are related. The quantities in the inner quarter circles equal the quantities in the corresponding outer quarter circles.

WORKED EXAMPLE

Find the Resistance of a Lightbulb

A typical older incandescent lightbulb was 60 W. Assuming that 120 V is applied across the lightbulb, what is the current through the lightbulb?

STRATEGY

We are given the voltage and the power output of a simple circuit containing a lightbulb, so we can use the equation $P = IV$ to find the current I that flows through the lightbulb.

Solution

Solving $P = IV$ for the current and inserting the given values for voltage and power gives

$$\begin{aligned} P &= IV \\ I &= \tfrac{P}{V} = \tfrac{60\ \text{W}}{120\ \text{V}} = 0.50\ \text{A}. \end{aligned} \qquad (19.51)$$

Thus, a half ampere flows through the lightbulb when 120 V is applied across it.

Discussion

This is a significant current. Recall that household power is AC and not DC, so the 120 V supplied by household sockets is an alternating power, not a constant power. The 120 V is actually the time-averaged power provided by such sockets. Thus, the average current going through the light bulb over a period of time longer than a few seconds is 0.50 A.

WORKED EXAMPLE

Boot Warmers

To warm your boots on cold days, you decide to sew a circuit with some resistors into the insole of your boots. You want 10 W of heat output from the resistors in each insole, and you want to run them from two 9-V batteries (connected in series). What total resistance should you put in each insole?

STRATEGY

We know the desired power and the voltage (18 V, because we have two 9-V batteries connected in series), so we can use the equation $P = V^2/R$ to find the requisite resistance.

Solution

Solving $P = V^2/R$ for the resistance and inserting the given voltage and power, we obtain

$$\begin{aligned} P &= \frac{V^2}{R} \\ R &= \frac{V^2}{P} = \frac{(18\ \text{V})^2}{10\ \text{W}} = 32\ \Omega. \end{aligned} \tag{19.52}$$

Thus, the total resistance in each insole should be 32 Ω.

Discussion

Let's see how much current would run through this circuit. We have 18 V applied across a resistance of 32 Ω, so Ohm's law gives

$$I = \frac{V}{R} = \frac{18\ \text{V}}{32\ \Omega} = 0.56\ \text{A}. \tag{19.53}$$

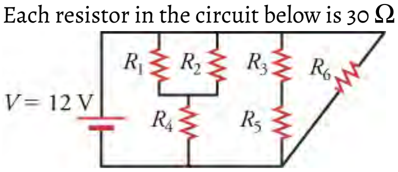
All batteries have labels that say how much charge they can deliver (in terms of a current multiplied by a time). A typical 9-V alkaline battery can deliver a charge of 565 $\text{mA} \cdot \text{h}$ (so two 9 V batteries deliver 1,130 $\text{mA} \cdot \text{h}$), so this heating system would function for a time of

$$t = \frac{1130\ \times\ 10^{-3}\,\text{A} \cdot \text{h}}{0.56\ \text{A}} = 2.0\ \text{h}. \tag{19.54}$$

WORKED EXAMPLE

Power through a Branch of a Circuit

Each resistor in the circuit below is 30 Ω. What power is dissipated by the middle branch of the circuit?



STRATEGY

The middle branch of the circuit contains resistors R_3 and R_5 in series. The voltage across this branch is 12 V. We will first find the equivalent resistance in this branch, and then use $P = V^2/R$ to find the power dissipated in the branch.

Solution

The equivalent resistance is $R_{\text{middle}} = R_3 + R_5 = 30\ \Omega + 30\ \Omega = 60\ \Omega$. The power dissipated by the middle branch of the circuit is

$$P_{\text{middle}} = \frac{V^2}{R_{\text{middle}}} = \frac{(12\ \text{V})^2}{60\ \Omega} = 2.4\ \text{W}. \quad (19.55)$$

Discussion

Let's see if energy is conserved in this circuit by comparing the power dissipated in the circuit to the power supplied by the battery. First, the equivalent resistance of the left branch is

$$R_{\text{left}} = \frac{1}{1/R_1 + 1/R_2} + R_4 = \frac{1}{1/30\ \Omega + 1/30\ \Omega} + 30\ \Omega = 45\ \Omega. \quad (19.56)$$

The power through the left branch is

$$P_{\text{left}} = \frac{V^2}{R_{\text{left}}} = \frac{(12\ \text{V})^2}{45\ \Omega} = 3.2\ \text{W}. \quad (19.57)$$

The right branch contains only R_6, so the equivalent resistance is $R_{\text{right}} = R_6 = 30\ \Omega$. The power through the right branch is

$$P_{\text{right}} = \frac{V^2}{R_{\text{right}}} = \frac{(12\ \text{V})^2}{30\ \Omega} = 4.8\ \text{W}. \quad (19.58)$$

The total power dissipated by the circuit is the sum of the powers dissipated in each branch.

$$P = P_{\text{left}} + P_{\text{middle}} + P_{\text{right}} = 2.4\ \text{W} + 3.2\ \text{W} + 4.8\ \text{W} = 10.4\ \text{W} \quad (19.59)$$

The power provided by the battery is

$$P = IV. \quad (19.60)$$

where I is the total current flowing through the battery. We must therefore add up the currents going through each branch to obtain I. The branches contributes currents of

$$\begin{aligned} I_{\text{left}} &= \frac{V}{R_{\text{left}}} = \frac{12\ \text{V}}{45\ \Omega} = 0.2667\ \text{A} \\ \text{I}_{\text{middle}} &= \frac{V}{R_{\text{middle}}} = \frac{12\ \text{V}}{60\ \Omega} = 0.20\ \text{A} \\ \text{I}_{\text{right}} &= \frac{V}{R_{\text{right}}} = \frac{12\ \text{V}}{30\ \Omega} = 0.40\ \text{A}. \end{aligned} \quad (19.61)$$

The total current is

$$I = I_{\text{left}} + I_{\text{middle}} + I_{\text{right}} = 0.2667\ \text{A} + 0.20\ \text{A} + 0.40\ \text{A} = 0.87\ \text{A}. \quad (19.62)$$

and the power provided by the battery is

$$P = IV = (0.87\ \text{A})(12\ \text{V}) = 10.4\ \text{W}. \quad (19.63)$$

This is the same power as is dissipated in the resistors of the circuit, which shows that energy is conserved in this circuit.

Practice Problems

16. What is the formula for the power dissipated in a resistor?

a. The formula for the power dissipated in a resistor is $P = \frac{I}{V}$.
b. The formula for the power dissipated in a resistor is $P = \frac{V}{I}$.
c. The formula for the power dissipated in a resistor is $P = IV$.
d. The formula for the power dissipated in a resistor is $P = I^2V$.

17. What is the formula for power dissipated by a resistor given its resistance and the voltage across it?

a. The formula for the power dissipated in a resistor is $P = \frac{R}{V^2}$
b. The formula for the power dissipated in a resistor is $P = V^2R$
c. The formula for the power dissipated in a resistor is $P = \frac{V^2}{R}$
d. The formula for the power dissipated in a resistor is $P = I^2R$

Check your Understanding

18. Which circuit elements dissipate power?

a. capacitors
b. inductors
c. ideal switches
d. resistors

19. Explain in words the equation for power dissipated by a given resistance.

a. Electric power is proportional to current through the resistor multiplied by the square of the voltage across the resistor.
b. Electric power is proportional to square of current through the resistor multiplied by the voltage across the resistor.
c. Electric power is proportional to current through the resistor divided by the voltage across the resistor.
d. Electric power is proportional to current through the resistor multiplied by the voltage across the resistor.

KEY TERMS

alternating current electric current whose direction alternates back and forth at regular intervals
ampere unit for electric current; one ampere is one coulomb per second ($1\ \text{A} = 1\ \text{C/s}$)
circuit diagram schematic drawing of an electrical circuit including all circuit elements, such as resistors, capacitors, batteries, and so on
conventional current flows in the direction that a positive charge would flow if it could move
direct current electric current that flows in a single direction
electric circuit physical network of paths through which electric current can flow
electric current electric charge that is moving
electric power rate at which electric energy is transferred in a circuit
equivalent resistor resistance of a single resistor that is the same as the combined resistance of a group of resistors
in parallel when a group of resistors are connected side by side, with the top ends of the resistors connected together by a wire and the bottom ends connected together by a different wire
in series when elements in a circuit are connected one after the other in the same branch of the circuit
nonohmic material that does not follow Ohm's law
Ohm's law electric current is proportional to the voltage applied across a circuit or other path
ohmic material that obeys Ohm's law
resistance how much a circuit element opposes the passage of electric current; it appears as the constant of proportionality in Ohm's law
resistor circuit element that provides a known resistance
steady state when the characteristics of a system do not change over time

SECTION SUMMARY

19.1 Ohm's law

- Direct current is constant over time; alternating current alternates smoothly back and forth over time.
- Electrical resistance causes materials to extract work from the current that flows through them.
- In ohmic materials, voltage drop along a path is proportional to the current that runs through the path.

19.2 Series Circuits

- Circuit diagrams are schematic representations of electric circuits.
- Resistors in series are resistors that are connected head to tail.
- The same current runs through all resistors in series; however, the voltage drop across each resistor can be different.
- The voltage is the same at every point in a given wire.

19.3 Parallel Circuits

- The equivalent resistance of a group of N identical resistors R connected in parallel is R/N.
- Connecting resistors in parallel provides more paths for the current to go through, so the equivalent resistance is always less than the smallest resistance of the parallel resistors.
- The same voltage drop occurs across all resistors in parallel; however, the current through each resistor can differ.

19.4 Electric Power

- Electric power is dissipated in the resistances of a circuit. Capacitors do not dissipate electric power.
- Electric power is proportional to the voltage and the current in a circuit.
- Ohm's law provides two extra expressions for electric power: one that does not involve current and one that does not involve voltage.

KEY EQUATIONS

19.1 Ohm's law

electric current I is the charge ΔQ that passes a plane per unit time Δt	$I = \frac{\Delta Q}{\Delta t}$
an ampere is the coulombs per unit time that pass a plane	$1\ \text{A} = 1\ \text{C/s}$
Ohm's law: the current I is proportional to the voltage V, with the resistance R being the constant of proportionality	$V = IR$

19.2 Series Circuits

the equivalent resistance of N resistors connected in series	$R_{\text{equiv}} = R_1 + R_2 + \cdots + R_N$

19.3 Parallel Circuits

the equivalent resistance of N resistors connected in parallel	$R_{\text{equiv}} = \frac{1}{1/R_1 + 1/R_2 + \cdots + 1/R_N}$

19.4 Electric Power

for a given current I flowing through a potential difference V, the electric power dissipated	$P = IV$
for a given current I flowing through a resistance R, the electric power dissipated	$P = I^2 R$
for a given voltage difference V across a resistor R, the electric power dissipated	$P = \frac{V^2}{R}$

CHAPTER REVIEW

Concept Items

19.1 Ohm's law

1. You connect a resistor across a battery. In which direction do the electrons flow?

a. The electrons flow from the negative terminal of the battery to the positive terminal of the battery.
b. The electrons flow from the positive terminal of the battery to the negative terminal of the battery.

2. How does current depend on resistance in Ohm's law?

a. Current is directly proportional to the resistance.
b. Current is inversely proportional to the resistance.
c. Current is proportional to the square of the resistance.
d. Current is inversely proportional to the square of the resistance.

3. In the context of electricity, what is resistance?

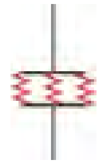
a. Resistance is the property of materials to resist the passage of voltage.
b. Resistance is the property of materials to resist the passage of electric current.
c. Resistance is the property of materials to increase the passage of voltage.
d. Resistance is the property of materials to increase the passage of electric current.

4. What is the mathematical formula for Ohm's law?

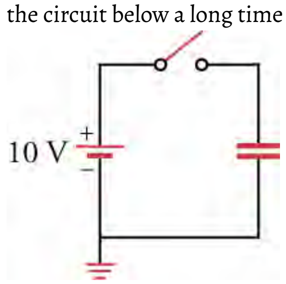
a. $V = I^2 R$
b. $V = \frac{R}{I}$
c. $V = \frac{I}{R}$
d. $V = IR$

19.2 Series Circuits

5. In which circuit are all the resistors connected in series?

a. 
b. 
c. 
d. 

6. What is the voltage and current through the capacitor in the circuit below a long time after the switch is closed?



a. 0 V, 0 A
b. 0 V, 10 A
c. 10 V, 0 A
d. 10 V, 10 A

19.3 Parallel Circuits

7. If you remove resistance from a circuit, does the total resistance of the circuit always decrease? Explain.
   a. No, because for parallel combination of resistors, the resistance through the remaining circuit increases.
   b. Yes, because for parallel combination of resistors, the resistance through the remaining circuit increases.

8. Explain why the equivalent resistance of a parallel combination of resistors is always less than the smallest of the parallel resistors.
   a. Adding resistors in parallel gives the current a shorter path through which it can flow hence decreases the overall resistance.
   b. Adding resistors in parallel gives the current another path through which it can flow hence decreases the overall resistance.
   c. Adding resistors in parallel reduce the number of paths through which the current can flow hence decreases the overall resistance.
   d. Adding resistors in parallel gives the current longer path through which it can flow hence decreases the overall resistance.

19.4 Electric Power

9. To draw the most power from a battery, should you connect a small or a large resistance across its terminals? Explain.
   a. Small resistance, because smaller resistance will lead to the largest power
   b. Large resistance, because smaller resistance will lead to the largest power

10. If you double the current through a resistor, by what factor does the power dissipated by the resistor change?
    a. Power increases by a factor of two.
    b. Power increases by a factor of four.
    c. Power increases by a factor of eight.
    d. Power increases by a factor of 16.

Critical Thinking Items

19.1 Ohm's law

11. An accelerator accelerates He nuclei (change = $2e$) to a speed of $v = 2 \times 10^6$ m/s. What is the current if the linear density of He nuclei is $\lambda = 108\ \text{m}^{-1}$?
    a. $I = 9.6 \times 10^{-5}$ A
    b. $I = 3.2 \times 10^{-5}$ A
    c. $I = 12.8 \times 10^{-5}$ A
    d. $I = 6.4 \times 10^{-5}$ A

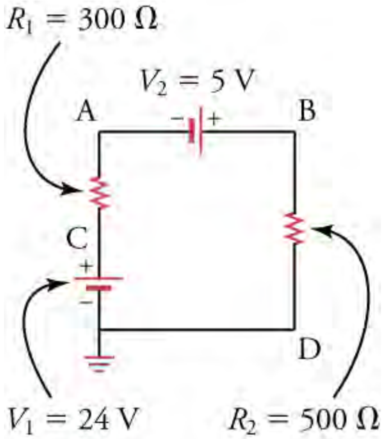
12. How can you verify whether a certain material is ohmic?
    a. Make a resistor from this material and measure the current going through this resistor for several different voltages. If the current is proportional to the voltage, then the material is ohmic.
    b. Make a resistor from this material and measure the current going through this resistor for several different voltages. If the current is inversely proportional to the voltage, then the material is ohmic.
    c. Make a resistor from this material and measure the current going through this resistor for several different voltages. If the current is proportional to the square of the voltage, then the material is ohmic.
    d. Make a resistor from this material and measure the current going through this resistor for several different voltages. If the current is inversely proportional to the square of the voltage, then the material is ohmic.

19.2 Series Circuits

13. Given three batteries (5V, 9V, 12V) and five resistors (10, 20, 30, 40, 50Ω) to choose from, what can you choose to form a circuit diagram with a current of 0.175A? You do not need to use all of the components.
    a. Batteries (5V, 9V) and resistors (30Ω, 50Ω) connected in series
    b. Batteries (5V4, 12V) and resistors (10Ω, 20Ω, 40Ω, and 50Ω) connected in series.
    c. Batteries (5V, 9V, and 12V) and resistors (10Ω, 20Ω, and 30Ω) connected in series.

14. What is the maximum resistance possible given a resistor of 100 and a resistor of $40\ \Omega$?
    a. $100\ \Omega$
    b. $140\ \Omega$
    c. $180\ \Omega$
    d. $240\ \Omega$

15. Rank the points A, B, C, and D in the circuit diagram from lowest voltage to highest voltage.



a. A, B, C, D
b. B, C, A, D
c. C, B, A, D
d. D, A, B, C

19.3 Parallel Circuits

16. Can all resistor combinations be reduced to series and parallel combinations?
  a. No, all practical resistor circuits cannot be reduced to series and parallel combinations.
  b. Yes, all practical resistor circuits can be reduced to series and parallel combinations.

17. What is the equivalent resistance of the circuit shown below?

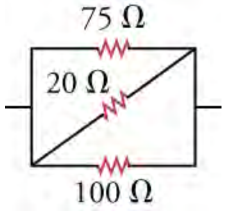


Figure 19.23

  a. The equivalent resistance of the circuit 14 Ω.
  b. The equivalent resistance of the circuit 16.7 Ω.
  c. The equivalent resistance of the circuit 140 Ω.
  d. The equivalent resistance of the circuit 195 Ω.

19.4 Electric Power

18. Two lamps have different resistances. (a) If the lamps are connected in parallel, which one is brighter, the lamp with greater resistance or the lamp with less resistance? (b) If the lamps are connected in series, which one is brighter? Note that the brighter lamp dissipates more power.
  a. (a) lamp with greater resistance; (b) lamp with less resistance
  b. (a) lamp with greater resistance; (b) lamp with greater resistance
  c. (a) lamp with less resistance; (b) lamp with less resistance
  d. (a) lamp with less resistance; (b) lamp with greater resistance

19. To measure the power consumed by your laptop computer, you place an ammeter (a device that measures electric current) in series with its DC power supply. When the screen is off, the computer draws 0.40 A of current.
When the screen is on at full brightness, it draws 0.90 A of current. Knowing the DC power supply delivers 16 V, how much power is used by the screen?
  a. The power used by the screen is –8.0 W.
  b. The power used by the screen is 0.3 W.
  c. The power used by the screen is 3.2 W.
  d. The power used by the screen is 8.0 W.

Problems

19.1 Ohm's law

20. What voltage is needed to make 6 C of charge traverse a 100-Ω resistor in 1 min?
  a. The required voltage is 1×10^{-3} V.
  b. The required voltage is 10 V.
  c. The required voltage is 1,000 V.
  d. The required voltage is 10,000 V.

21. Resistors typically obey Ohm's law at low currents, but show deviations at higher currents because of heating. Suppose you were to conduct an experiment measuring the voltage, *V*, across a resistor as a function of current, *I*, including currents whose deviations from Ohm's law start to become apparent. For a data plot of *V* versus *I*, which of the following functions would be best to fit the data? Assume that *a*, *b*, and *c* are nonzero constants adjusted to fit the data.
  a. $V = aI$
  b. $V = aI + b$
  c. $V = aI + bI^2$
  d. $V = aI + bI^2 + c$

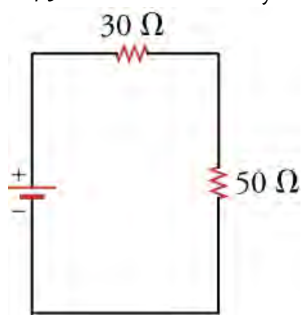
22. A battery of unknown voltage V_1 is attached across a resistor R_1. You add a second battery with $V_2 = 9.0\ \text{V}$ in series with V_1 so that the voltage across R_1 is now $V_1 + V_2$ and measure $0.3\ \text{A}$ of current through resistor R_1. You add a third battery with $V_3 = 9.0\ \text{V}$ in series with the first two batteries so that the voltage across R_3 is $V_1 + V_2 + V_3$ and measure $0.4\ \text{A}$ of current through R_1. What is the resistance of R_1?

a. $23.25\,\Omega$
b. $21.75\,\Omega$
c. $31.33\,\Omega$
d. $13.0\,\Omega$

19.2 Series Circuits

23. What is the voltage drop across two 80-Ω resistors connected in series with 0.15 A flowing through them?
a. 12 V
b. 24 V
c. 36 V
d. 48 V

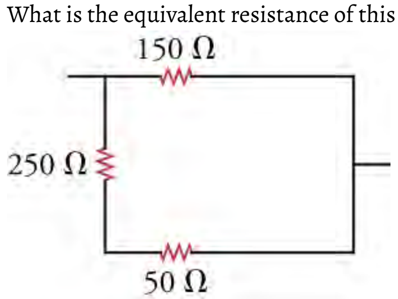
24. In this circuit, the voltage drop across the upper resistor is 4.5 V. What is the battery voltage?



a. 4.5V
b. 7.5V
c. 12V
d. 18V

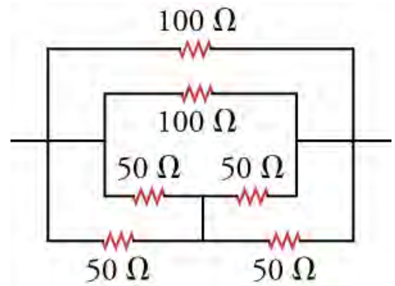
19.3 Parallel Circuits

25. What is the equivalent resistance of this circuit?



a. The equivalent resistance of the circuit is 32.7 Ω.
b. The equivalent resistance of the circuit is 100 Ω.
c. The equivalent resistance of the circuit is 327 Ω.
d. The equivalent resistance of the circuit is 450 Ω.

26. What is the equivalent resistance of the circuit shown below?



a. The equivalent resistance is 25 Ω.
b. The equivalent resistance is 50 Ω.
c. The equivalent resistance is 75 Ω.
d. The equivalent resistance is 100 Ω.

19.4 Electric Power

27. When 12 V are applied across a resistor, it dissipates 120 W of power. What is the current through the resistor?
a. The current is 1,440 A.
b. The current is 10 A.
c. The current is 0.1 A.
d. The current is 0.01 A.

28. Warming 1 g of water requires 1 J of energy per . How long would it take to warm 1 L of water from 20 to 40 °C if you immerse in the water a 1-kW resistor connected across a 9.0-V batteries aligned in series?
a. 10 min
b. 20 min
c. 30 min
d. 40 min

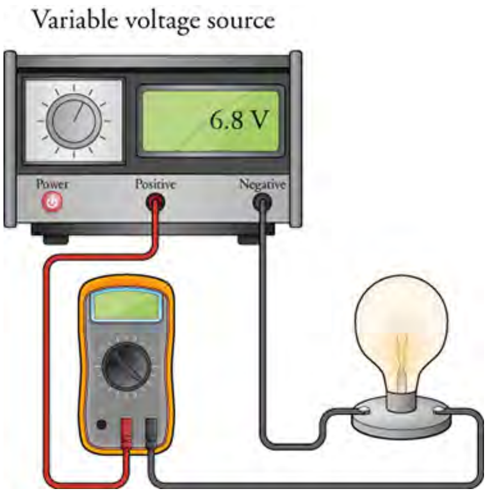
Performance Task

19.4 Electric Power

29.
1. An incandescent light bulb (i.e., an old-fashioned light bulb with a little wire in it)
2. A lightbulb socket to hold the light bulb
3. A variable voltage source
4. An ammeter

Procedure

- Screw the lightbulb into its socket. Connect the positive terminal of the voltage source to the input of the ammeter. Connect the output of the ammeter to one connection of the socket. Connect the other connection of the socket to the negative terminal of the voltage source. Ensure that the voltage source is set to supply DC voltage and that the ammeter is set to measure DC amperes. The desired circuit is shown below.



- On a piece of paper, make a two-column table with 10 rows. Label the left column *volts* and the right column *current*. Adjust the voltage source so that it supplies from between 1 and 10 volts DC. For each voltage, write the voltage in the volts column and the corresponding amperage measured by the ammeter in the current column. Make a plot of volts versus current, that is, a plot with volts on the vertical axis and current on the horizontal axis. Use this data and the plot to answer the following questions:

1. What is the resistance of the lightbulb?
2. What is the range of possible error in your result for the resistance?
3. In a single word, how would you describe the curve formed by the data points?

TEST PREP

Multiple Choice

19.1 Ohm's law

30. What are the SI units for electric current?

a. C/s
b. e/s
c. −e/s
d. C/s^2

31. What is the SI unit for resistance?

a. C/m
b. C/s
c. Ω
d. Ψ

32. The equivalent unit for an ohm is a ________.

a. V/A
b. C/m
c. $\frac{A}{V}$
d. V/s

33. You put $9.0\ \mathrm{V}$ DC across resistor R_1 and measure the current through it. With the same voltage across resistor R_2, you measure twice the current. What is the ratio $\frac{R_1}{R_2}$?

a. 1
b. $\frac{1}{2}$
c. 4
d. 2

19.2 Series Circuits

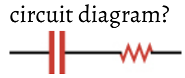
34. What does the circuit element shown represent?

a. a battery
b. a capacitor
c. the ground
d. a switch

35. How many 10-Ω resistors must be connected in series to make an equivalent resistance of 80 Ω?

a. 80
b. 8
c. 20
d. 40

36. Which two circuit elements are represented in the circuit diagram?



a. a battery connected in series with an inductor
b. a capacitor connected in series with a resistor
c. a resistor connected in series with a battery
d. an inductor connected in series with a resistor

37. How much current will flow through a 10-V battery with a 100-Ω resistor connected across its terminals?

a. 0.1 A
b. 1.0 A
c. 0
d. 1,000 A

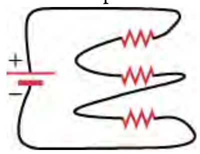
19.3 Parallel Circuits

38. A 10-Ω resistor is connected in parallel to another resistor R . The equivalent resistance of the pair is 8 Ω. What is the resistance R?

a. 10 Ω
b. 20 Ω
c. 30 Ω

d. 40 Ω

39. Are the resistors shown connected in parallel or in series? Explain.



a. The resistors are connected in parallel because the same current flows through all three resistors.
b. The resistors are connected in parallel because different current flows through all three resistors.
c. The resistors are connected in series because the same current flows through all three resistors.
d. The resistors are connected in series because different current flows through all three resistors.

19.4 Electric Power

40. Which equation below for electric power is incorrect?
a. $P = I^2R$
b. $P = \frac{V}{R^2}$
c. $P = IV$
d. $P = \frac{V^2}{R}$

41. What power is dissipated in a circuit through which $0.12\ \mathrm{A}$ flows across a potential drop of $3.0\ \mathrm{V}$?
a. $0.36\ \mathrm{W}$
b. $0.011\ \mathrm{W}$
c. Voltage drop across is $5\ \mathrm{V}$.
d. $2.5\ \mathrm{W}$

42. How does a resistor dissipate power?
a. A resistor dissipates power in the form of heat.
b. A resistor dissipates power in the form of sound.
c. A resistor dissipates power in the form of light.
d. A resistor dissipates power in the form of charge.

43. What power is dissipated in a circuit through which 0.12 A flows across a potential drop of 3.0 V?
a. 0.36 W
b. 0.011 W
c. 5 V
d. 2.5 W

Short Answer

19.1 Ohm's law

44. True or false—it is possible to produce nonzero DC current by adding together AC currents.
a. false
b. true

45. What type of current is used in cars?
a. alternating current
b. indirect current
c. direct current
d. straight current

46. If current were represented by C, voltage by B, and resistance by g, what would the mathematical expression be for Ohm's law?
a. $C = Bg$
b. $g = BC$
c. $\frac{B}{C} = \frac{C}{g}$
d. $B = Cg$

47. Give a verbal expression for Ohm's law.
a. Ohm's law says that the current through a resistor equals the voltage across the resistor multiplied by the resistance of the resistor.
b. Ohm's law says that the voltage across a resistor equals the current through the resistor multiplied by the resistance of the resistor.
c. Ohm's law says that the resistance of the resistor equals the current through the resistor multiplied by the voltage across a resistor.
d. Ohm's law says that the voltage across a resistor equals the square of the current through the resistor multiplied by the resistance of the resistor.

48. What is the current through a 100-Ω resistor with 12 V across it?
a. 0
b. 0.12 A
c. 8.33 A
d. 1,200 A

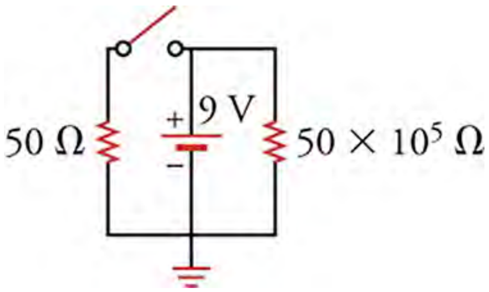
49. What resistance is required to produce $0.15\ \mathrm{A}$ from a $9.0\ \mathrm{V}$ battery?
a. 0.017
b. 1
c. 60
d. 120

19.2 Series Circuits

50. Given a circuit with one 9-V battery and with its negative terminal connected to ground. The two paths are connected to ground from the positive terminal: the right path with a 20-Ω and a 100-Ω resistor and the left path with a 50-Ω resistor. How much current will flow in the right branch?
a. $\frac{9}{120}$
b. $\frac{9}{100}$

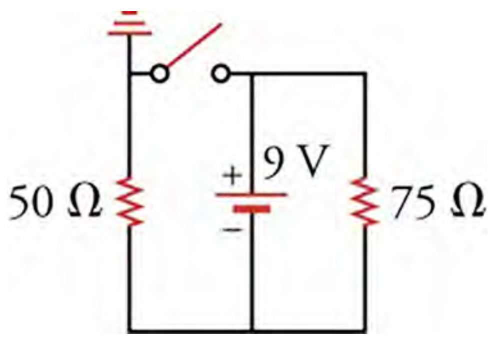
c. $\frac{9}{50}$
d. $\frac{9}{20}$

51. Through which branch in the circuit below does the most current flow?



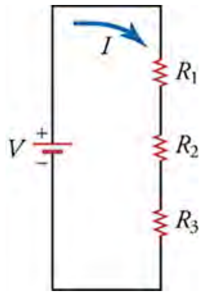
a. All of the current flows through the left branch due to the open switch.
b. All of the current flows through the right branch due to the open switch in the left branch.
c. All of the current flows through the middle branch due to the open switch in the left branch
d. There will be no current in any branch of the circuit due to the open switch.

52. What current flows through the $75\text{-}\Omega$ resistor in the circuit below?



a. 0.072 A
b. 0.12 A
c. 0.18 A
d. 0.3 A

53. What is the equivalent resistance for the circuit below if $V = 9.0\text{ V}$ and $I = 0.25\text{ A}$?



a. $0.028\,\Omega$
b. $2.25\,\Omega$
c. $36\,\Omega$
d. $72\,\Omega$

19.3 Parallel Circuits

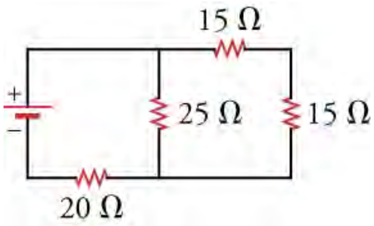
54. Ten 100-Ω resistors are connected in series. How can you increase the total resistance of the circuit by about 40 percent?

a. Adding two 10-Ω resistors increases the total resistance of the circuit by about 40 percent.
b. Removing two 10-Ω resistors increases the total resistance of the circuit by about 40 percent.
c. Adding four 10-Ω resistors increases the total resistance of the circuit by about 40 percent.
d. Removing four 10-Ω resistors increases the total resistance of the circuit by about 40 percent.

55. Two identical resistors are connected in parallel across the terminals of a battery. If you increase the resistance of one of the resistors, what happens to the current through and the voltage across the other resistor?

a. The current and the voltage remain the same.
b. The current decreases and the voltage remains the same.
c. The current and the voltage increases.
d. The current increases and the voltage remains the same.

56.



In the circuit below, through which resistor(s) does the most current flow? Through which does the least flow? Explain.

a. The most current flows through the 15-Ω resistor because all the current must pass through this resistor.
b. The most current flows through the 20-Ω resistor because all the current must pass through this resistor.
c. The most current flows through the 25-Ω resistor because it is the highest resistance.
d. The same current flows through the all the resistor because all the current must pass through each of the resistors.

19.4 Electric Power

57. You want to increase the power dissipated in a circuit.

You have the choice between doubling the current or doubling the resistance, with the voltage remaining constant. Which one would you choose?
  a. doubling the resistance
  b. doubling the current

58. You want to increase the power dissipated in a circuit. You have the choice between reducing the voltage or reducing the resistance, with the current remaining constant. Which one would you choose?
  a. reduce the voltage to increase the power
  b. reduce the resistance to increase the power

59. What power is dissipated in the circuit consisting of 310-Ω resistors connected in series across a 9.0-V battery?
  a. The power dissipated is 2430 W.
  b. The power dissipated is 270 W.
  c. The power dissipated is 2.7 W.
  d. The power dissipated is 0.37 W.

60. What power is dissipated in a circuit consisting of three 10-Ω resistors connected in parallel across a 9.0-V battery?
  a. The power dissipated is 270 W.
  b. The power dissipated is 30 W.
  c. The power dissipated is 24 W.
  d. The power dissipated is 1/24 W.

Extended Response

19.1 Ohm's law

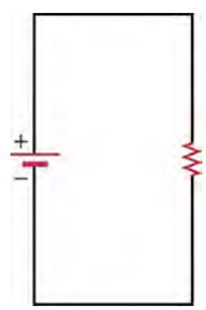
61. Describe the relationship between current and charge. Include an explanation of how the direction of the current is defined.
  a. Electric current is the charge that passes through a conductor per unit time. The direction of the current is defined to be the direction in which positive charge would flow.
  b. Electric current is the charges that move in a conductor. The direction of the current is defined to be the direction in which positive charge would flow.
  c. Electric current is the charge that passes through a conductor per unit time. The direction of the current is defined to be the direction in which negative charge would flow.
  d. Electric current is the charges that move in a conductor. The direction of the current is defined to be the direction in which negative charge would flow.

62. What could cause Ohm's law to break down?
  a. If small amount of current flows through a resistor, the resistor will heat up so much that it will change state, in violation of Ohm's law.
  b. If excessive amount of current flows through a resistor, the resistor will heat up so much that it will change state, in violation of Ohm's law.
  c. If small amount of current flows through a resistor, the resistor will not heat up so much and it will not change its state, in violation of Ohm's law.
  d. If excessive amount of current flows through a resistor, the resistor will heat up so much that it will not change its state, in violation of Ohm's law.

63. You connect a single resistor R across a 10-V battery and find that 0.01 A flows through the circuit. You add another resistor R after the first resistor and find that 0.005 A flows through the circuit. If you have 10 resistors R connected in a line one after the other, what would be their total resistance?
  a. $\frac{R}{10}$
  b. 5 R
  c. $\frac{10}{R}$
  d. 10 R

19.2 Series Circuits

64. Explain why the current is the same at all points in the circuit below.

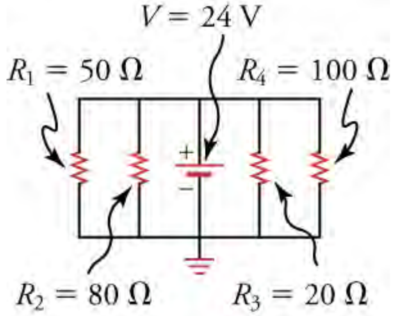


  a. If the current were not constant, the mobile charges would bunch up in places, which means that the voltage would decrease at that point. A lower voltage at some point would push the current in the direction that further decreases the voltage.
  b. If the current were not constant, the mobile charges would bunch up in places, which means that the voltage would increase at that point. But a higher voltage at some point would push the current in the direction that decreases the voltage.
  c. If the current were not constant, the mobile charges would bunch up in places, which mean that the voltage would increase at that point. A higher voltage at some point would push the current in the direction that further increases the

voltage.

d. If the current were not constant, the mobile charges would bunch up in places, which mean that the voltage would decrease at that point. But a lower voltage at some point would push the current in the direction that increases the voltage.

65. What is the current through each resistor in the circuit?



a. Current through resistors R_1, R_2, R_3, and R_4 is 0.48 A, 0.30 A, 1.2 A, and 0.24 A, respectively.
b. Current through resistors R_1, R_2, R_3, and R_4 is 1200 A, 1920 A, 480 A, and 2400 A, respectively.
c. Current through resistors R_1, R_2, R_3, and is R_4 2.08 A, 3.34 A, 0.833 A, and 4.17 A, respectively.
d. The same amount of current, 0.096 A, flows through all of the resistors.

19.3 Parallel Circuits

66. In a house, a single incoming wire at a high potential with respect to the ground provides electric power. How are the appliances connected between this wire and the ground, in parallel or in series? Explain.

a. The appliances are connected in parallel to provide different voltage differences across each appliance.
b. The appliances are connected in parallel to provide the same voltage difference across each appliance.
c. The appliances are connected in series to provide the same voltage difference across each appliance.
d. The appliances are connected in series to provide different voltage differences across each appliance.

19.4 Electric Power

67. A single resistor is connected across the terminals of a battery When you attach a second resistor in parallel with the first, does the power dissipated by the system change?

a. No, the power dissipated remain same.
b. Yes, the power dissipated increases.
c. Yes, the power dissipated decreases.

68. In a flashlight, the batteries are normally connected in series. Why are they not connected in parallel?

a. Batteries are connected in series for higher voltage and power output.
b. Batteries are connected in series for lower voltage and power output.
c. Batteries are connected in series so that power output is a much lower for the same amount of voltage.
d. Batteries are connected in series to reduce the overall loss of energy from the circuit.

CHAPTER 20
Magnetism

Figure 20.1 The magnificent spectacle of the Aurora Borealis, or northern lights, glows in the northern sky above Bear Lake near Eielson Air Force Base, Alaska. Shaped by Earth's magnetic field, this light is produced by radiation spewed from solar storms. (credit: Senior Airman Joshua Strang, Flickr)

Chapter Outline

INTRODUCTION You may have encountered magnets for the first time as a small child playing with magnetic toys or refrigerator magnets. At the time, you likely noticed that two magnets that repulse each other will attract each other if you flip one of them around. The force that acts across the air gaps between magnets is the same force that creates wonders such as the Aurora Borealis. In fact, magnetic effects pervade our lives in myriad ways, from electric motors to medical imaging and computer memory. In this chapter, we introduce magnets and learn how they work and how magnetic fields and electric currents interact.

20.1 Magnetic Fields, Field Lines, and Force

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Summarize properties of magnets and describe how some nonmagnetic materials can become magnetized
- Describe and interpret drawings of magnetic fields around permanent magnets and current-carrying wires
- Calculate the magnitude and direction of magnetic force in a magnetic field and the force on a current-carrying wire in a magnetic field

Section Key Terms

Curie temperature	domain	electromagnet	electromagnetism	ferromagnetic
magnetic dipole	magnetic field	magnetic pole	magnetized	north pole
permanent magnet	right-hand rule	solenoid	south pole	

Magnets and Magnetization

People have been aware of magnets and magnetism for thousands of years. The earliest records date back to ancient times, particularly in the region of Asia Minor called Magnesia—the name of this region is the source of words like *magnet*. Magnetic rocks found in Magnesia, which is now part of western Turkey, stimulated interest during ancient times. When humans first discovered magnetic rocks, they likely found that certain parts of these rocks attracted bits of iron or other magnetic rocks more strongly than other parts. These areas are called the *poles* of a magnet. A **magnetic pole** is the part of a magnet that exerts the strongest force on other magnets or magnetic material, such as iron. For example, the poles of the bar magnet shown in Figure 20.2 are where the paper clips are concentrated.

Figure 20.2 A bar magnet with paper clips attracted to the two poles.

If a bar magnet is suspended so that it rotates freely, one pole of the magnet will always turn toward the north, with the opposite pole facing south. This discovery led to the compass, which is simply a small, elongated magnet mounted so that it can rotate freely. An example of a compass is shown Figure 20.3. The pole of the magnet that orients northward is called the **north pole**, and the opposite pole of the magnet is called the **south pole**.

Figure 20.3 A compass is an elongated magnet mounted in a device that allows the magnet to rotate freely.

The discovery that one particular pole of a magnet orients northward, whereas the other pole orients southward allowed people to identify the north and south poles of any magnet. It was then noticed that the north poles of two different magnets repel each other, and likewise for the south poles. Conversely, the north pole of one magnet attracts the south pole of other magnets. This situation is analogous to that of electric charge, where like charges repel and unlike charges attract. In magnets, we simply replace charge with *pole*: Like poles repel and unlike poles attract. This is summarized in Figure 20.4, which shows how the force between magnets depends on their relative orientation.

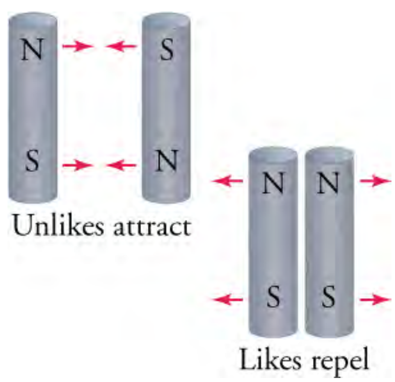


Figure 20.4 Depending on their relative orientation, magnet poles will either attract each other or repel each other.

Consider again the fact that the pole of a magnet that orients northward is called the north pole of the magnet. If unlike poles attract, then the magnetic pole of Earth that is close to the geographic North Pole must be a magnetic south pole! Likewise, the magnetic pole of Earth that is close to the geographic South Pole must be a magnetic north pole. This situation is depicted in Figure 20.5, in which Earth is represented as containing a giant internal bar magnet with its magnetic south pole at the geographic North Pole and vice versa. If we were to somehow suspend a giant bar magnet in space near Earth, then the north pole of the space magnet would be attracted to the south pole of Earth's internal magnet. This is in essence what happens with a compass needle: Its magnetic north pole is attracted to the magnet south pole of Earth's internal magnet.

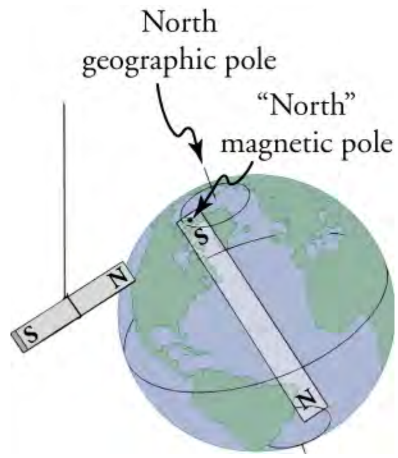


Figure 20.5 Earth can be thought of as containing a giant magnet running through its core. The magnetic south pole of Earth's magnet is at the geographic North Pole, so the north pole of magnets is attracted to the North Pole, which is how the north pole of magnets got their name. Likewise, the south pole of magnets is attracted to the geographic South Pole of Earth.

What happens if you cut a bar magnet in half? Do you obtain one magnet with two south poles and one magnet with two north poles? The answer is no: Each half of the bar magnet has a north pole and a south pole. You can even continue cutting each piece of the bar magnet in half, and you will always obtain a new, smaller magnet with two opposite poles. As shown in Figure 20.6, you can continue this process down to the atomic scale, and you will find that even the smallest particles that behave as magnets have two opposite poles. In fact, no experiment has ever found any object with a single magnetic pole, from the smallest subatomic particle such as electrons to the largest objects in the universe such as stars. Because magnets always have two poles, they are referred to as **magnetic dipoles**—*di* means *two*. Below, we will see that magnetic dipoles have properties that are analogous to electric dipoles.

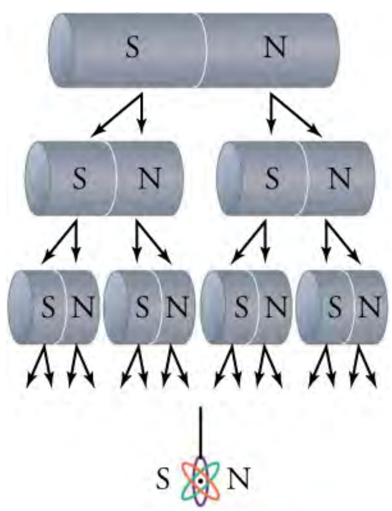


Figure 20.6 All magnets have two opposite poles, from the smallest, such as subatomic particles, to the largest, such as stars.

WATCH PHYSICS

Introduction to Magnetism

This video provides an interesting introduction to magnetism and discusses, in particular, how electrons around their atoms contribute to the magnetic effects that we observe.

Click to view content (https://www.openstax.org/l/28_intro_magn)

GRASP CHECK

Toward which magnetic pole of Earth is the north pole of a compass needle attracted?

a. The north pole of a compass needle is attracted to the north magnetic pole of Earth, which is located near the geographic North Pole of Earth.
b. The north pole of a compass needle is attracted to the south magnetic pole of Earth, which is located near the geographic North Pole of Earth.
c. The north pole of a compass needle is attracted to the north magnetic pole of Earth, which is located near the geographic South Pole of Earth.
d. The north pole of a compass needle is attracted to the south magnetic pole of Earth, which is located near the geographic South Pole of Earth.

Only certain materials, such as iron, cobalt, nickel, and gadolinium, exhibit strong magnetic effects. Such materials are called **ferromagnetic**, after the Latin word *ferrum* for iron. Other materials exhibit weak magnetic effects, which are detectable only with sensitive instruments. Not only do ferromagnetic materials respond strongly to magnets—the way iron is attracted to magnets—but they can also be **magnetized** themselves—that is, they can be induced to be magnetic or made into permanent magnets (Figure 20.7). A **permanent magnet** is simply a material that retains its magnetic behavior for a long time, even when exposed to demagnetizing influences.

Figure 20.7 An unmagnetized piece of iron is placed between two magnets, heated, and then cooled, or simply tapped when cold. The iron becomes a permanent magnet with the poles aligned as shown: Its south pole is adjacent to the north pole of the original magnet, and its north pole is adjacent to the south pole of the original magnet. Note that attractive forces are created between the central magnet and the outer magnets.

When a magnet is brought near a previously unmagnetized ferromagnetic material, it causes local magnetization of the material with unlike poles closest, as in the right side of Figure 20.7. This causes an attractive force, which is why unmagnetized iron is attracted to a magnet.

What happens on a microscopic scale is illustrated in Figure 7(a). Regions within the material called **domains** act like small bar magnets. Within domains, the magnetic poles of individual atoms are aligned. Each atom acts like a tiny bar magnet. Domains are small and randomly oriented in an unmagnetized ferromagnetic object. In response to an external magnetic field, the domains may grow to millimeter size, aligning themselves, as shown in Figure 7(b). This induced magnetization can be made permanent if the material is heated and then cooled, or simply tapped in the presence of other magnets.

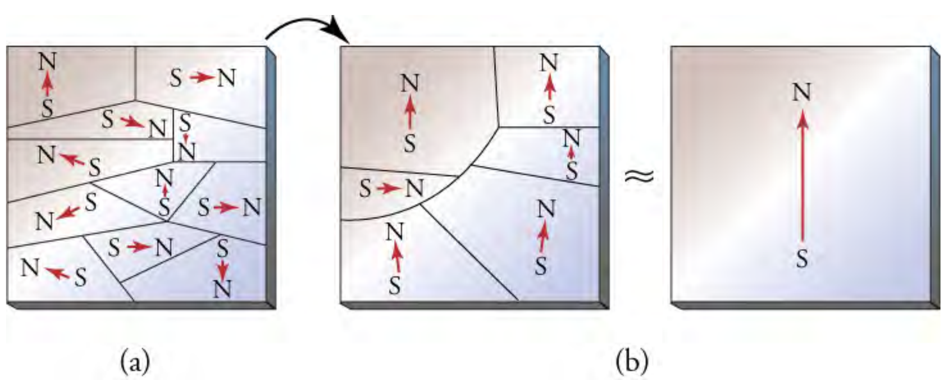


Figure 20.8 (a) An unmagnetized piece of iron—or other ferromagnetic material—has randomly oriented domains. (b) When magnetized by an external magnet, the domains show greater alignment, and some grow at the expense of others. Individual atoms are aligned within

domains; each atom acts like a tiny bar magnet.

Conversely, a permanent magnet can be demagnetized by hard blows or by heating it in the absence of another magnet. Increased thermal motion at higher temperature can disrupt and randomize the orientation and size of the domains. There is a well-defined temperature for ferromagnetic materials, which is called the **Curie temperature**, above which they cannot be magnetized. The Curie temperature for iron is 1,043 K (770 $^\circ\text{C}$), which is well above room temperature. There are several elements and alloys that have Curie temperatures much lower than room temperature and are ferromagnetic only below those temperatures.

Snap Lab

Refrigerator Magnets

We know that like magnetic poles repel and unlike poles attract. See if you can show this for two refrigerator magnets. Will the magnets stick if you turn them over? Why do they stick to the refrigerator door anyway? What can you say about the magnetic properties of the refrigerator door near the magnet? Do refrigerator magnets stick to metal or plastic spoons? Do they stick to all types of metal?

GRASP CHECK

You have one magnet with the north and south poles labeled. How can you use this magnet to identify the north and south poles of other magnets?

a. If the north pole of a known magnet is repelled by a pole of an unknown magnet on bringing them closer, that pole of unknown magnet is its north pole; otherwise, it is its south pole.
b. If the north pole of known magnet is attracted to a pole of an unknown magnet on bringing them closer, that pole of unknown magnet is its north pole; otherwise, it is its south pole.

Magnetic Fields

We have thus seen that forces can be applied between magnets and between magnets and ferromagnetic materials without any contact between the objects. This is reminiscent of electric forces, which also act over distances. Electric forces are described using the concept of the electric field, which is a force field around electric charges that describes the force on any other charge placed in the field. Likewise, a magnet creates a **magnetic field** around it that describes the force exerted on other magnets placed in the field. As with electric fields, the pictorial representation of magnetic field lines is very useful for visualizing the strength and direction of the magnetic field.

As shown in Figure 20.9, the direction of magnetic field lines is defined to be the direction in which the north pole of a compass needle points. If you place a compass near the north pole of a magnet, the north pole of the compass needle will be repelled and point away from the magnet. Thus, the magnetic field lines point away from the north pole of a magnet and toward its south pole.

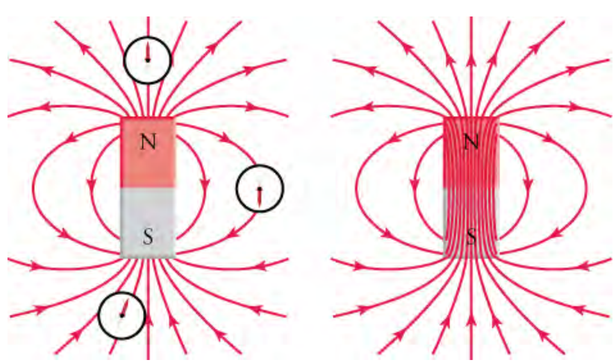


Figure 20.9 The black lines represent the magnetic field lines of a bar magnet. The field lines point in the direction that the north pole of a small compass would point, as shown at left. Magnetic field lines never stop, so the field lines actually penetrate the magnet to form complete loops, as shown at right.

Magnetic field lines can be mapped out using a small compass. The compass is moved from point to point around a magnet, and at each point, a short line is drawn in the direction of the needle, as shown in Figure 20.10. Joining the lines together then reveals the path of the magnetic field line. Another way to visualize magnetic field lines is to sprinkle iron filings around a magnet. The filings will orient themselves along the magnetic field lines, forming a pattern such as that shown on the right in Figure 20.10.

Virtual Physics

Using a Compass to Map Out the Magnetic Field

Click to view content (http://www.openstax.org/l/28magcomp)

This simulation presents you with a bar magnet and a small compass. Begin by dragging the compass around the bar magnet to see in which direction the magnetic field points. Note that the strength of the magnetic field is represented by the brightness of the magnetic field icons in the grid pattern around the magnet. Use the magnetic field meter to check the field strength at several points around the bar magnet. You can also flip the polarity of the magnet, or place Earth on the image to see how the compass orients itself.

GRASP CHECK

With the slider at the top right of the simulation window, set the magnetic field strength to 100 percent . Now use the magnetic field meter to answer the following question: Near the magnet, where is the magnetic field strongest and where is it weakest? Don't forget to check inside the bar magnet.

a. The magnetic field is strongest at the center and weakest between the two poles just outside the bar magnet. The magnetic field lines are densest at the center and least dense between the two poles just outside the bar magnet.
b. The magnetic field is strongest at the center and weakest between the two poles just outside the bar magnet. The magnetic field lines are least dense at the center and densest between the two poles just outside the bar magnet.
c. The magnetic field is weakest at the center and strongest between the two poles just outside the bar magnet. The magnetic field lines are densest at the center and least dense between the two poles just outside the bar magnet.
d. The magnetic field is weakest at the center and strongest between the two poles just outside the bar magnet and the magnetic field lines are least dense at the center and densest between the two poles just outside the bar magnet.

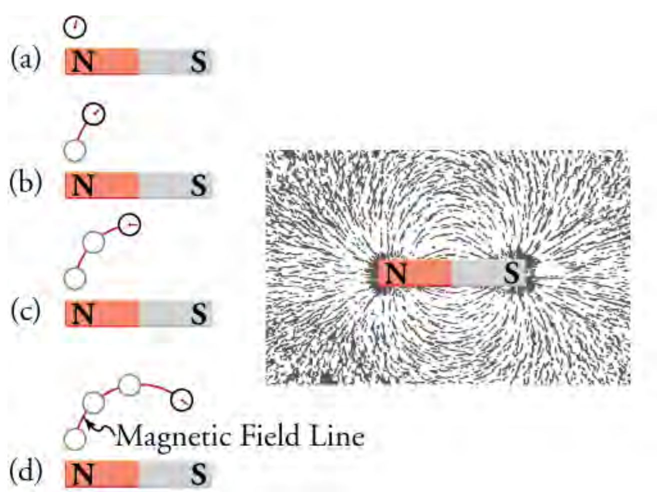


Figure 20.10 Magnetic field lines can be drawn by moving a small compass from point to point around a magnet. At each point, draw a short line in the direction of the compass needle. Joining the points together reveals the path of the magnetic field lines. Another way to visualize magnetic field lines is to sprinkle iron filings around a magnet, as shown at right.

When two magnets are brought close together, the magnetic field lines are perturbed, just as happens for electric field lines when two electric charges are brought together. Bringing two north poles together—or two south poles—will cause a repulsion, and the magnetic field lines will bend away from each other. This is shown in Figure 20.11, which shows the magnetic field lines created by the two closely separated north poles of a bar magnet. When opposite poles of two magnets are brought together, the magnetic field lines join together and become denser between the poles. This situation is shown in Figure 20.11.

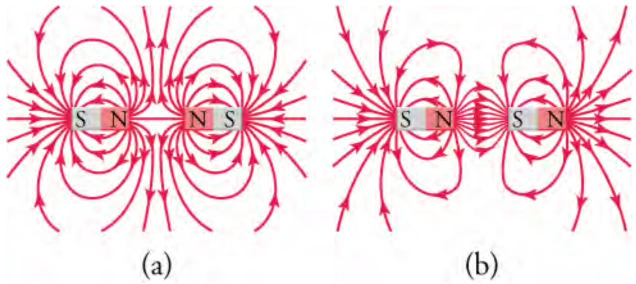


Figure 20.11 (a) When two north poles are approached together, the magnetic field lines repel each other and the two magnets experience a repulsive force. The same occurs if two south poles are approached together. (b) If opposite poles are approached together, the magnetic field lines become denser between the poles and the magnets experience an attractive force.

Like the electric field, the magnetic field is stronger where the lines are denser. Thus, between the two north poles in Figure 20.11, the magnetic field is very weak because the density of the magnetic field is almost zero. A compass placed at that point would essentially spin freely if we ignore Earth's magnetic field. Conversely, the magnetic field lines between the north and south poles in Figure 20.11 are very dense, indicating that the magnetic field is very strong in this region. A compass placed here would quickly align with the magnetic field and point toward the south pole on the right.

Note that magnets are not the only things that make magnetic fields. Early in the nineteenth century, people discovered that electrical currents cause magnetic effects. The first significant observation was by the Danish scientist Hans Christian Oersted (1777–1851), who found that a compass needle was deflected by a current-carrying wire. This was the first significant evidence that the movement of electric charges had any connection with magnets. An **electromagnet** is a device that uses electric current to make a magnetic field. These temporarily induced magnets are called electromagnets. Electromagnets are employed for everything from a wrecking yard crane that lifts scrapped cars to controlling the beam of a 90-km-circumference particle accelerator to the magnets in medical-imaging machines (see Figure 20.12).

Figure 20.12 Instrument for magnetic resonance imaging (MRI). The device uses a cylindrical-coil electromagnet to produce for the main magnetic field. The patient goes into the *tunnel* on the gurney. (credit: Bill McChesney, Flickr)

The magnetic field created by an electric current in a long straight wire is shown in Figure 20.13. The magnetic field lines form concentric circles around the wire. The direction of the magnetic field can be determined using the *right-hand rule*. This rule shows up in several places in the study of electricity and magnetism. Applied to a straight current-carrying wire, the **right-hand rule** says that, with your right thumb pointed in the direction of the current, the magnetic field will be in the direction in which your right fingers curl, as shown in Figure 20.13. If the wire is very long compared to the distance r from the wire, the strength B of the magnetic field is given by

$$B_{\text{straightwire}} = \frac{\mu_0 I}{2\pi r} \tag{20.1}$$

where I is the current in the wire in amperes. The SI unit for magnetic field is the tesla (T). The symbol μ_0 —read "mu-zero"—is a constant called the "permeability of free space" and is given by

$$\mu_0 = 4\pi \times 10^{-7}\,\text{T} \cdot \text{m/A}. \tag{20.2}$$

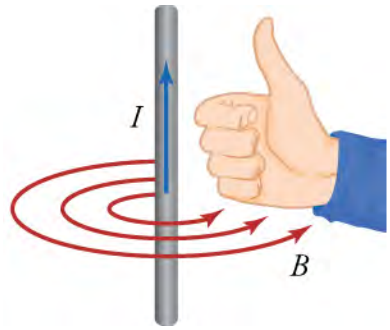


Figure 20.13 This image shows how to use the right-hand rule to determine the direction of the magnetic field created by current flowing through a straight wire. Point your right thumb in the direction of the current, and the magnetic field will be in the direction in which your fingers curl.

WATCH PHYSICS

Magnetic Field Due to an Electric Current

This video describes the magnetic field created by a straight current-carrying wire. It goes over the right-hand rule to determine the direction of the magnetic field, and presents and discusses the formula for the strength of the magnetic field due to a straight current-carrying wire.

Click to view content (https://www.openstax.org/l/28magfield)

GRASP CHECK

A long straight wire is placed on a table top and electric current flows through the wire from right to left. If you look at the wire end-on from the left end, does the magnetic field go clockwise or counterclockwise?

a. By pointing your right-hand thumb in the direction opposite of current, the right-hand fingers will curl counterclockwise, so the magnetic field will be in the counterclockwise direction.
b. By pointing your right-hand thumb in the direction opposite of current, the right-hand fingers will curl clockwise, so the magnetic field will be in the clockwise direction.
c. By pointing your right-hand thumb in the direction of current, the right-hand fingers will curl counterclockwise, so the magnetic field will be in the counterclockwise direction.
d. By pointing your right-hand thumb in the direction of current, the right-hand fingers will curl clockwise, so the magnetic field will be in the clockwise direction.

Now imagine winding a wire around a cylinder with the cylinder then removed. The result is a wire coil, as shown in Figure 20.14. This is called a **solenoid**. To find the direction of the magnetic field produced by a solenoid, apply the right-hand rule to several points on the coil. You should be able to convince yourself that, inside the coil, the magnetic field points from left to right. In fact, another application of the right-hand rule is to curl your right-hand fingers around the coil in the direction in which the current flows. Your right thumb then points in the direction of the magnetic field inside the coil: left to right in this case.

Figure 20.14 A wire coil with current running through as shown produces a magnetic field in the direction of the red arrow.

Each loop of wire contributes to the magnetic field inside the solenoid. Because the magnetic field lines must form closed loops, the field lines close the loop outside the solenoid. The magnetic field lines are much denser inside the solenoid than outside the solenoid. The resulting magnetic field looks very much like that of a bar magnet, as shown in Figure 20.15. The magnetic field strength deep inside a solenoid is

$$B_{\text{solenoid}} = \mu_0 \frac{NI}{\ell}, \tag{20.3}$$

where N is the number of wire loops in the solenoid and ℓ is the length of the solenoid.

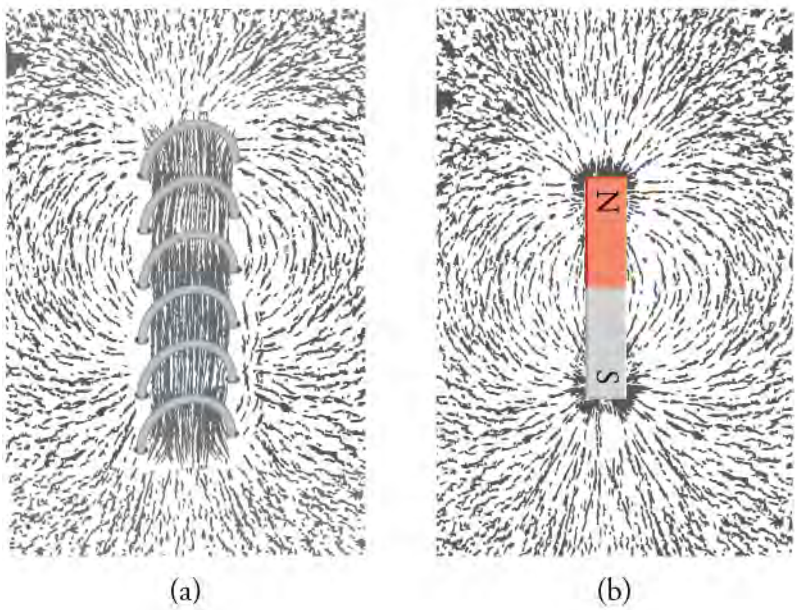


Figure 20.15 Iron filings show the magnetic field pattern around (a) a solenoid and (b) a bar magnet. The fields patterns are very similar, especially near the ends of the solenoid and bar magnet.

Virtual Physics

Electromagnets

[Click to view content (http://www.openstax.org/l/28elec_magnet)](http://www.openstax.org/l/28elec_magnet)

Use this simulation to visualize the magnetic field made from a solenoid. Be sure to click on the tab that says Electromagnet. You can drive AC or DC current through the solenoid by choosing the appropriate current source. Use the field meter to measure the strength of the magnetic field and then change the number of loops in the solenoid to see how this affects the magnetic field strength.

GRASP CHECK

Choose the battery as current source and set the number of wire loops to four. With a nonzero current going through the solenoid, measure the magnetic field strength at a point. Now decrease the number of wire loops to two. How does the magnetic field strength change at the point you chose?

a. There will be no change in magnetic field strength when number of loops reduces from four to two.
b. The magnetic field strength decreases to half of its initial value when number of loops reduces from four to two.
c. The magnetic field strength increases to twice of its initial value when number of loops reduces from four to two.
d. The magnetic field strength increases to four times of its initial value when number of loops reduces from four to two.

Magnetic Force

If a moving electric charge, that is electric current, produces a magnetic field that can exert a force on another magnet, then the reverse should be true by Newton's third law. In other words, a charge moving through the magnetic field produced by another object should experience a force—and this is exactly what we find. As a concrete example, consider Figure 20.16, which shows a charge q moving with velocity $\vec{v}$ through a magnetic field $\vec{B}$ between the poles of a permanent magnet. The magnitude F of the force experienced by this charge is

$$F = qvB \sin \theta, \tag{20.4}$$

where θ is the angle between the velocity of the charge and the magnetic field.

The direction of the force may be found by using another version of the right-hand rule: First, we join the tails of the velocity vector and a magnetic field vector, as shown in step 1 of Figure 20.16. We then curl our right fingers from $\vec{v}$ to $\vec{B}$, as indicated in step (2) of Figure 20.16. The direction in which the right thumb points is the direction of the force. For the charge in Figure 20.16, we find that the force is directed into the page.

Note that the factor $\sin \theta$ in the equation $F = qvB \sin \theta$ means that zero force is applied on a charge that moves parallel to a magnetic field because $\theta = 0$ and $\sin 0 = 0$. The maximum force a charge can experience is when it moves perpendicular to the magnetic field, because $\theta = 90°$ and $\sin 90° = 1$.

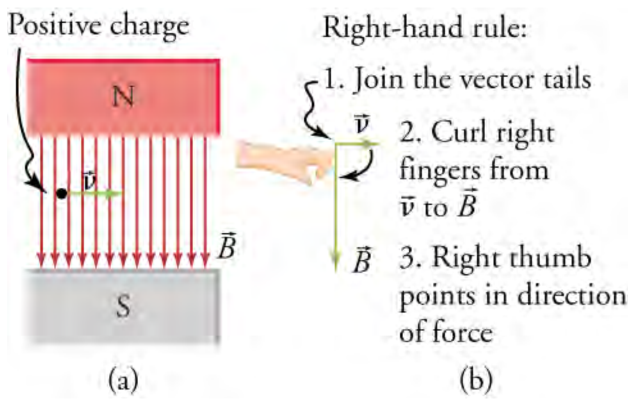


Figure 20.16 (a) An electron moves through a uniform magnetic field. (b) Using the right-hand rule, the force on the electron is found to be directed into the page.

LINKS TO PHYSICS

Magnetohydrodynamic Drive

In Tom Clancy's Cold War novel "The Hunt for Red October," the Soviet Union built a submarine (see Figure 20.17) with a magnetohydrodynamic drive that was so silent it could not be detected by surface ships. The only conceivable purpose to build such a submarine was to give the Soviet Union first-strike capability, because this submarine could sneak close to the coast of the United States and fire its ballistic missiles, destroying key military and government installations to prevent an American counterattack.

Figure 20.17 A Typhoon-class Russian ballistic-missile submarine on which the fictional submarine Red October was based.

A magnetohydrodynamic drive is supposed to be silent because it has no moving parts. Instead, it uses the force experienced by charged particles that move in a magnetic field. The basic idea behind such a drive is depicted in Figure 20.18. Salt water flows through a channel that runs from the front to the back of the submarine. A magnetic field is applied horizontally across the channel, and a voltage is applied across the electrodes on the top and bottom of the channel to force a downward electric current through the water. The charge carriers are the positive sodium ions and the negative chlorine ions of salt. Using the right-hand rule, the force on the charge carriers is found to be toward the rear of the vessel. The accelerated charges collide with water molecules and transfer their momentum, creating a jet of water that is propelled out the rear of the channel. By Newton's third law, the vessel experiences a force of equal magnitude, but in the opposite direction.

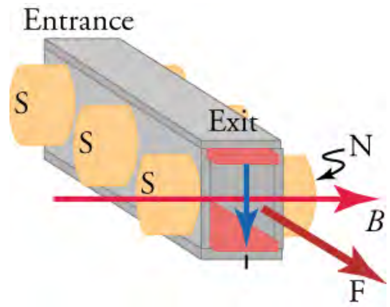


Figure 20.18 A schematic drawing of a magnetohydrodynamic drive showing the water channel, the current direction, the magnetic field direction, and the resulting force.

Fortunately for all involved, it turns out that such a propulsion system is not very practical. Some back-of-the-envelope calculations show that, to power a submarine, either extraordinarily high magnetic fields or extraordinarily high electric currents would be required to obtain a reasonable thrust. In addition, prototypes of magnetohydrodynamic drives show that they are anything but silent. Electrolysis caused by running a current through salt water creates bubbles of hydrogen and oxygen, which makes this propulsion system quite noisy. The system also leaves a trail of chloride ions and metal chlorides that can easily be detected to locate the submarine. Finally, the chloride ions are extremely reactive and very quickly corrode metal parts, such as the electrode or the water channel itself. Thus, the Red October remains in the realm of fiction, but the physics involved is quite real.

GRASP CHECK

If the magnetic field is downward, in what direction must the current flow to obtain rearward-pointing force?

a. The current must flow vertically from up to down when viewed from the rear of the boat.
b. The current must flow vertically from down to up when viewed from the rear of the boat.
c. The current must flow horizontally from left to right when viewed from the rear of the boat.
d. The current must flow horizontally from right to left when viewed from the rear of the boat.

Instead of a single charge moving through a magnetic field, consider now a steady current I moving through a straight wire. If we place this wire in a uniform magnetic field, as shown in Figure 20.19, what is the force on the wire or, more precisely, on the electrons in the wire? An electric current involves charges that move. If the charges q move a distance ℓ in a time t, then their speed is $v = \ell/t$. Inserting this into the equation $F = qvB\sin\theta$ gives

$$\begin{aligned} F &= q\left(\frac{\ell}{t}\right)B\sin\theta \\ &= \left(\frac{q}{t}\right)\ell B\sin\theta. \end{aligned} \tag{20.5}$$

The factor q/t in this equation is nothing more than the current in the wire. Thus, using $I = q/t$, we obtain

$$F = I\ell B\sin\theta(1.4). \tag{20.6}$$

This equation gives the force on a straight current-carrying wire of length ℓ in a magnetic field of strength B. The angle θ is the angle between the current vector and the magnetic field vector. Note that ℓ is the length of wire that is in the magnetic field and for which $\theta \neq 0$, as shown in Figure 20.19.

The direction of the force is determined in the same way as for a single charge. Curl your right fingers from the vector for I to the vector for B, and your right thumb will point in the direction of the force on the wire. For the wire shown in Figure 20.19, the force is directed into the page.

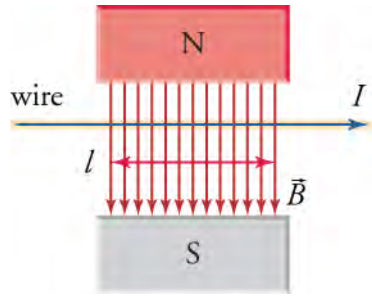


Figure 20.19 A straight wire carrying current I in a magnetic field B. The force exerted on the wire is directed into the page. The length ℓ is the length of the wire that is *in* the magnetic field.

Throughout this section, you may have noticed the symmetries between magnetic effects and electric effects. These effects all fall under the umbrella of **electromagnetism**, which is the study of electric and magnetic phenomena. We have seen that electric charges produce electric fields, and moving electric charges produce magnetic fields. A magnetic dipole produces a magnetic field, and, as we will see in the next section, moving magnetic dipoles produce an electric field. Thus, electricity and magnetism are two intimately related and symmetric phenomena.

WORKED EXAMPLE

Trajectory of Electron in Magnetic Field

A proton enters a region of constant magnetic field, as shown in Figure 20.20. The magnetic field is coming out of the page. If the electron is moving at $3.0 \times 10^6 \text{m/s}$ and the magnetic field strength is 2.0 T, what is the magnitude and direction of the force on the proton?

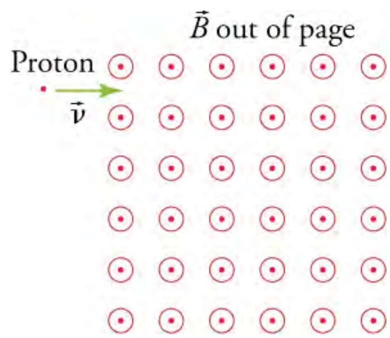


Figure 20.20 A proton enters a region of uniform magnetic field. The magnetic field is coming out of the page—the circles with dots represent vector arrow heads coming out of the page.

STRATEGY

Use the equation $F = qvB \sin \theta$ to find the magnitude of the force on the proton. The angle between the magnetic field vectors and the velocity vector of the proton is $90°$. The direction of the force may be found by using the right-hand rule.

Solution

The charge of the proton is $q = 1.60 \times 10^{-19} \text{C}$. Entering this value and the given velocity and magnetic field strength into the equation $F = qvB \sin \theta$ gives

$$\begin{aligned} F &= qvB \sin \theta \\ &= \left(1.60 \times 10^{-19} \text{C}\right)\left(3.0 \times 10^{6} \text{m/s}\right)(2.0 \text{ T}) \sin (90°) \\ &= 9.6 \times 10^{-13} \text{N}. \end{aligned} \tag{20.7}$$

To find the direction of the force, first join the velocity vector end to end with the magnetic field vector, as shown in Figure 20.21. Now place your right hand so that your fingers point in the direction of the velocity and curl them upward toward the magnetic field vector. The force is in the direction in which your thumb points. In this case, the force is downward in the plane of the paper in the $\hat{z}$ -direction, as shown in Figure 20.21.

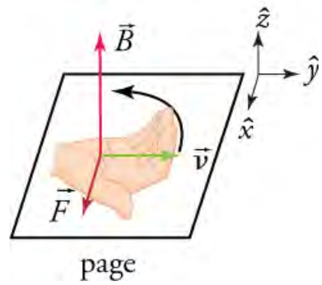


Figure 20.21 The velocity vector and a magnetic field vector from Figure 20.20 are placed end to end. A right hand is shown with the fingers curling up from the velocity vector toward the magnetic field vector. The thumb points in the direction of the resulting force, which is the $\hat{z}$ -direction in this case.

Thus, combining the magnitude and the direction, we find that the force on the proton is $\left(9.6 \times 10^{-13}\text{N}\right)\hat{z}$.

Discussion

This seems like a very small force. However, the proton has a mass of $1.67 \times 10^{-27}\text{kg}$, so its acceleration is $a = \frac{F}{m} = \frac{9.6 \times 10^{-13}\text{N}}{1.67 \times 10^{-27}\text{kg}} = 5.7 \times 10^{14}\ \text{m/s}^2$, or about ten thousand billion times the acceleration due to gravity!

We found that the proton's initial acceleration as it enters the magnetic field is downward in the plane of the page. Notice that, as the proton accelerates, its velocity remains perpendicular to the magnetic field, so the magnitude of the force does not change. In addition, because of the right-hand rule, the direction of the force remains perpendicular to the velocity. This force is nothing more than a centripetal force: It has a constant magnitude and is always perpendicular to the velocity. Thus, the magnitude of the velocity does not change, and the proton executes circular motion. The radius of this circle may be found by using the kinematics relationship.

$$\begin{aligned} F &= ma = m\frac{v^2}{r} \\ a &= \frac{v^2}{r} \\ r &= \frac{v^2}{a} = \frac{\left(3.0 \times 10^6\text{m/s}\right)^2}{5.7 \times 10^{14}\ \text{m/s}^2} = 1.6\ \text{cm} \end{aligned} \tag{20.8}$$

The path of the proton in the magnetic field is shown in Figure 20.22.

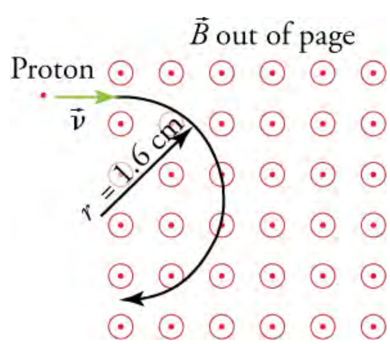
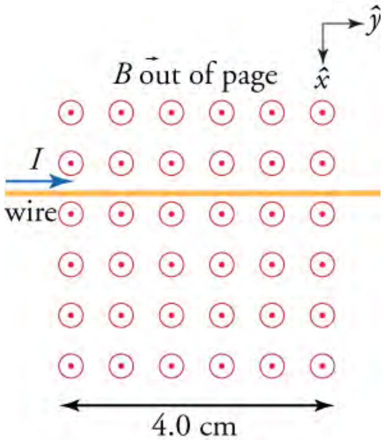


Figure 20.22 When traveling perpendicular to a constant magnetic field, a charged particle will execute circular motion, as shown here for a proton.

WORKED EXAMPLE

Wire with Current in Magnetic Field

Now suppose we run a wire through the uniform magnetic field from the previous example, as shown. If the wire carries a current of 1.0 A in the $\hat{y}$ -direction, and the region with magnetic field is 4.0 cm long, what is the force on the wire?



STRATEGY

Use equation $F = I\ell B \sin\theta$ to find the magnitude of the force on the wire. The length of the wire inside the magnetic field is 4.0 cm, and the angle between the current direction and the magnetic field direction is 90°. To find the direction of the force, use the right-hand rule as explained just after the equation $F = I\ell B \sin\theta$.

Solution

Insert the given values into equation $F = I\ell B \sin\theta$ to find the magnitude of the force

$$F = I\ell B \sin\theta = (1.5\ \text{A})\,(0.040\ \text{m})\,(2.0\ \text{T}) = 0.12\ \text{N}. \tag{20.9}$$

To find the direction of the force, begin by placing the current vector end to end with a vector for the magnetic field. The result is as shown in the figure in the previous Worked Example with $\vec{v}$ replaced by $\vec{I}$. Curl your right-hand fingers from $\vec{I}$ to $\vec{B}$ and your right thumb points down the page, again as shown in the figure in the previous Worked Example. Thus, the direction of the force is in the $\hat{x}$ -direction. The complete force is thus $(0.12\ \text{N})\,\hat{x}$.

Discussion

The direction of the force is the same as the initial direction of the force was in the previous example for a proton. However, because the current in a wire is confined to a wire, the direction in which the charges move does not change. Instead, the entire wire accelerates in the $\hat{x}$ -direction. The force on a current-carrying wire in a magnetic field is the basis of all electrical motors, as we will see in the upcoming sections.

Practice Problems

1. What is the magnitude of the force on an electron moving at 1.0 × 106 m/s perpendicular to a 1.0-T magnetic field?
   a. 0.8×10^{-13} N
   b. 1.6×10^{-14} N
   c. 0.8×10^{-14} N
   d. 1.6×10^{-13} N

2. A straight 10 cm wire carries 0.40 A and is oriented perpendicular to a magnetic field. If the force on the wire is 0.022 N, what is the magnitude of the magnetic field?
   a. 1.10×10^{-2} T
   b. 0.55×10^{-2} T
   c. 1.10 T
   d. 0.55 T

Check Your Understanding

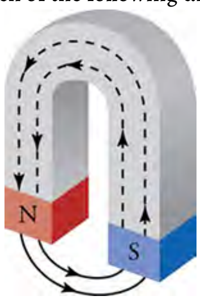
3. If two magnets repel each other, what can you conclude about their relative orientation?
   a. Either the south pole of magnet 1 is closer to the north pole of magnet 2 or the north pole of magnet 1 is closer to the south pole of magnet 2.
   b. Either the south poles of both the magnet 1 and magnet 2 are closer to each other or the north poles of both the magnet 1

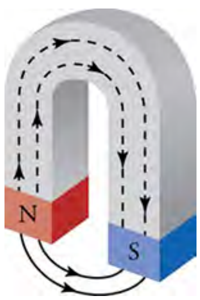
and magnet 2 are closer to each other.

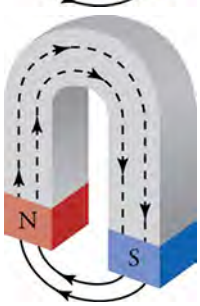
4. Describe methods to demagnetize a ferromagnet.
    a. by cooling, heating, or submerging in water
    b. by heating, hammering, and spinning it in external magnetic field
    c. by hammering, heating, and rubbing with cloth
    d. by cooling, submerging in water, or rubbing with cloth

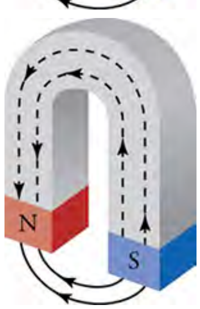
5. What is a magnetic field?
    a. The directional lines present inside and outside the magnetic material that indicate the magnitude and direction of the magnetic force.
    b. The directional lines present inside and outside the magnetic material that indicate the magnitude of the magnetic force.
    c. The directional lines present inside the magnetic material that indicate the magnitude and the direction of the magnetic force.
    d. The directional lines present outside the magnetic material that indicate the magnitude and the direction of the magnetic force.

6. Which of the following drawings is correct?

a. 

b. 

c. 

d. 

20.2 Motors, Generators, and Transformers

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain how electric motors, generators, and transformers work
- Explain how commercial electric power is produced, transmitted, and distributed

Section Key Terms

electric motor	generator	transformer

Electric Motors, Generators, and Transformers

As we learned previously, a current-carrying wire in a magnetic field experiences a force—recall $F = I\ell B \sin\theta$. **Electric motors**, which convert electrical energy into mechanical energy, are the most common application of magnetic force on current-carrying wires. Motors consist of loops of wire in a magnetic field. When current is passed through the loops, the magnetic field exerts a torque on the loops, which rotates a shaft. Electrical energy is converted to mechanical work in the process. Figure 20.23 shows a schematic drawing of an electric motor.

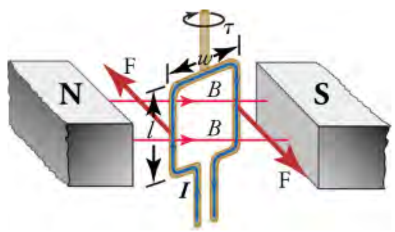


Figure 20.23 Torque on a current loop. A vertical loop of wire in a horizontal magnetic field is attached to a vertical shaft. When current is passed through the wire loop, torque is exerted on it, making it turn the shaft.

Let us examine the force on each segment of the loop in Figure 20.23 to find the torques produced about the axis of the vertical shaft—this will lead to a useful equation for the torque on the loop. We take the magnetic field to be uniform over the rectangular loop, which has width w and height ℓ, as shown in the figure. First, consider the force on the top segment of the loop. To determine the direction of the force, we use the right-hand rule. The current goes from left to right into the page, and the magnetic field goes from left to right in the plane of the page. Curl your right fingers from the current vector to the magnetic field vector and your right thumb points down. Thus, the force on the top segment is downward, which produces no torque on the shaft. Repeating this analysis for the bottom segment—neglect the small gap where the lead wires go out—shows that the force on the bottom segment is upward, again producing no torque on the shaft.

Consider now the left vertical segment of the loop. Again using the right-hand rule, we find that the force exerted on this segment is perpendicular to the magnetic field, as shown in Figure 20.23. This force produces a torque on the shaft. Repeating this analysis on the right vertical segment of the loop shows that the force on this segment is in the direction opposite that of the force on the left segment, thereby producing an equal torque on the shaft. The total torque on the shaft is thus twice the toque on one of the vertical segments of the loop.

To find the magnitude of the torque as the wire loop spins, consider Figure 20.24, which shows a view of the wire loop from above. Recall that torque is defined as $\tau = rF\sin\theta$, where F is the applied force, r is the distance from the pivot to where the force is applied, and θ is the angle between r and F. Notice that, as the loop spins, the current in the vertical loop segments is always perpendicular to the magnetic field. Thus, the equation $F = I\ell B \sin\theta$ gives the magnitude of the force on each vertical segment as $F = I\ell B$. The distance r from the shaft to where this force is applied is $w/2$, so the torque created by this force is

$$\tau_{\text{segment}} = rF\sin\theta = w/2I\ell B\sin\theta = (w/2)\,I\ell B\sin\theta. \tag{20.10}$$

Because there are two vertical segments, the total torque is twice this, or

$$\tau = wI\ell B\sin\theta. \tag{20.11}$$

If we have a multiple loop with N turns, we get N times the torque of a single loop. Using the fact that the area of the loop is $A = w\ell$; the expression for the torque becomes

$$\tau = NIAB \sin\theta. \tag{20.12}$$

This is the torque on a current-carrying loop in a uniform magnetic field. This equation can be shown to be valid for a loop of any shape.

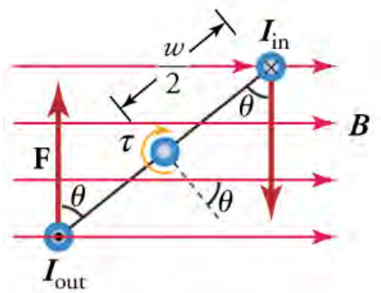


Figure 20.24 View from above of the wire loop from Figure 20.23. The magnetic field generates a force F on each vertical segment of the wire loop, which generates a torque on the shaft. Notice that the currents I_{in} and I_{out} have the same magnitude because they both represent the current flowing in the wire loop, but I_{in} flows into the page and I_{out} flows out of the page.

From the equation $\tau = NIAB \sin\theta$, we see that the torque is zero when $\theta = 0$. As the wire loop rotates, the torque increases to a maximum positive torque of $w\ell B$ when $\theta = 90°$. The torque then decreases back to zero as the wire loop rotates to $\theta = 180°$. From $\theta = 180°$ to $\theta = 360°$, the torque is negative. Thus, the torque changes sign every half turn, so the wire loop will oscillate back and forth.

For the coil to continue rotating in the same direction, the current is reversed as the coil passes through $\theta = 0$ and$\theta = 180°$ using automatic switches called *brushes*, as shown in Figure 20.25.

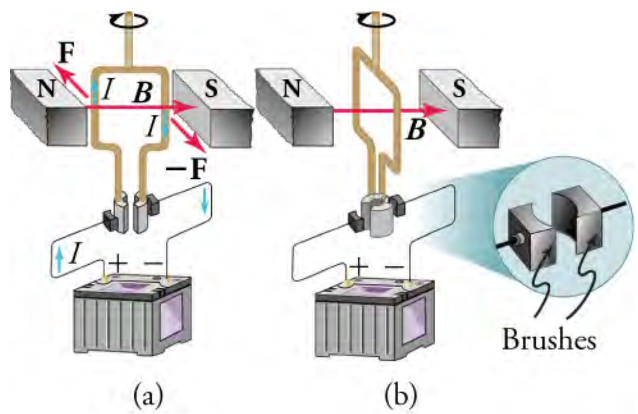


Figure 20.25 (a) As the angular momentum of the coil carries it through $\theta = 0$, the brushes reverse the current and the torque remains clockwise. (b) The coil rotates continuously in the clockwise direction, with the current reversing each half revolution to maintain the clockwise torque.

Consider now what happens if we run the motor in reverse; that is, we attach a handle to the shaft and mechanically force the coil to rotate within the magnetic field, as shown in Figure 20.26. As per the equation $F = qvB \sin\theta$—where θ is the angle between the vectors $\vec{v}$ and $\vec{B}$—charges in the wires of the loop experience a magnetic force because they are moving in a magnetic field. Again using the right-hand rule, where we curl our fingers from vector $\vec{v}$ to vector $\vec{B}$, we find that charges in the top and bottom segments feel a force perpendicular to the wire, which does not cause a current. However, charges in the vertical wires experience forces parallel to the wire, causing a current to flow through the wire and through an external circuit if one is connected. A device such as this that converts mechanical energy into electrical energy is called a **generator**.

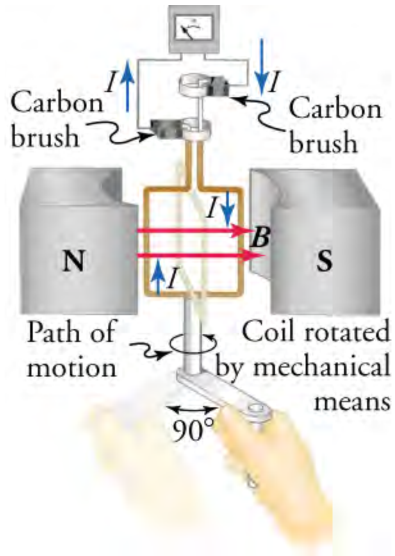


Figure 20.26 When this coil is rotated through one-fourth of a revolution, the magnetic flux Φ changes from its maximum to zero, inducing an emf, which drives a current through an external circuit.

Because current is induced only in the side wires, we can find the induced emf by only considering these wires. As explained in Induced Current in a Wire, motional emf in a straight wire moving at velocity v through a magnetic field B is $E = B\ell v$, where the velocity is perpendicular to the magnetic field. In the generator, the velocity makes an angle θ with B (see Figure 20.27), so the velocity component perpendicular to B is $v \sin\theta$. Thus, in this case, the emf induced on each vertical wire segment is $E = B\ell v \sin\theta$, and they are in the same direction. The total emf around the loop is then

$$E = 2B\ell v \sin\theta. \tag{20.13}$$

Although this expression is valid, it does not give the emf as a function of time. To find how the emf evolves in time, we assume that the coil is rotated at a constant angular velocity ω. The angle θ is related to the angular velocity by $\theta = \omega t$, so that

$$E = 2B\ell v \sin\omega t. \tag{20.14}$$

Recall that tangential velocity v is related to angular velocity ω by $v = r\omega$. Here, $r = w/2$, so that $v = (w/2)\,\omega$ and

$$E = 2B\ell\left(\frac{w}{2}\omega\right)\sin\omega t = B\ell w\omega\sin\omega t. \tag{20.15}$$

Noting that the area of the loop is $A = \ell w$ and allowing for N wire loops, we find that

$$E = NAB\omega\sin\omega t \tag{20.16}$$

is the emf induced in a generator coil of N turns and area A rotating at a constant angular velocity ω in a uniform magnetic field B. This can also be expressed as

$$E = E_0 \sin\omega t \tag{20.17}$$

where

$$E_0 = NAB\omega \tag{20.18}$$

is the maximum (peak) emf.

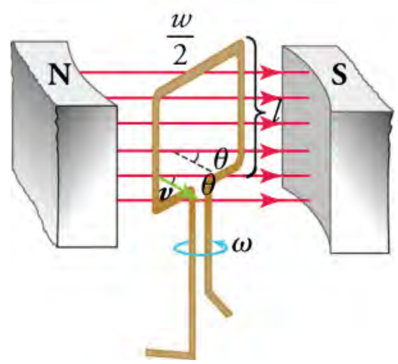


Figure 20.27 The instantaneous velocity of the vertical wire segments makes an angle θ with the magnetic field. The velocity is shown in the figure by the green arrow, and the angle θ is indicated.

Figure 20.28 shows a generator connected to a light bulb and a graph of the emf vs. time. Note that the emf oscillates from a positive maximum of E_0 to a negative maximum of $-E_0$. In between, the emf goes through zero, which means that zero current flows through the light bulb at these times. Thus, the light bulb actually flickers on and off at a frequency of 2*f*, because there are two zero crossings per period. Since alternating current such as this is used in homes around the world, why do we not notice the lights flickering on and off? In the United States, the frequency of alternating current is 60 Hz, so the lights flicker on and off at a frequency of 120 Hz. This is faster than the refresh rate of the human eye, so you don't notice the flicker of the lights. Also, other factors prevent various different types of light bulbs from switching on and off so fast, so the light output is *smoothed out* a bit.

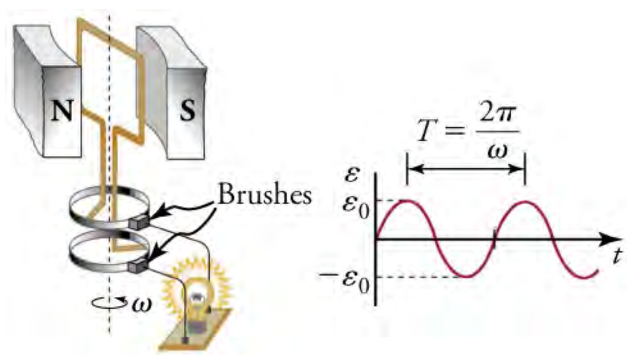


Figure 20.28 The emf of a generator is sent to a light bulb with the system of rings and brushes shown. The graph gives the emf of the generator as a function of time. E_0 is the peak emf. The period is $T = 1/f = 2\pi/\omega$, where *f* is the frequency at which the coil is rotated in the magnetic field.

Virtual Physics

Generator

Click to view content (http://www.openstax.org/l/28gen)

Use this simulation to discover how an electrical generator works. Control the water supply that makes a water wheel turn a magnet. This induces an emf in a nearby wire coil, which is used to light a light bulb. You can also replace the light bulb with a voltmeter, which allows you to see the polarity of the voltage, which changes from positive to negative.

GRASP CHECK

Set the number of wire loops to three, the bar-magnet strength to about 50 percent, and the loop area to 100 percent. Note the maximum voltage on the voltmeter. Assuming that one major division on the voltmeter is 5V, what is the maximum voltage when using only a single wire loop instead of three wire loops?

a. 5 V
b. 15 V

c. 125 V
d. 53 V

In real life, electric generators look a lot different than the figures in this section, but the principles are the same. The source of mechanical energy that turns the coil can be falling water—hydropower—steam produced by the burning of fossil fuels, or the kinetic energy of wind. Figure 20.29 shows a cutaway view of a steam turbine; steam moves over the blades connected to the shaft, which rotates the coil within the generator.

Figure 20.29 Steam turbine generator. The steam produced by burning coal impacts the turbine blades, turning the shaft which is connected to the generator. (credit: Nabonaco, Wikimedia Commons)

Another very useful and common device that exploits magnetic induction is called a **transformer**. Transformers do what their name implies—they transform voltages from one value to another; the term voltage is used rather than emf because transformers have internal resistance. For example, many cell phones, laptops, video games, power tools, and small appliances have a transformer built into their plug-in unit that changes 120 V or 240 V AC into whatever voltage the device uses. Figure 20.30 shows two different transformers. Notice the wire coils that are visible in each device. The purpose of these coils is explained below.

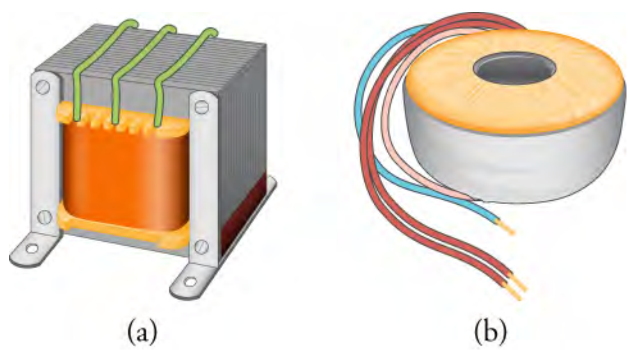


Figure 20.30 On the left is a common laminated-core transformer, which is widely used in electric power transmission and electrical appliances. On the right is a toroidal transformer, which is smaller than the laminated-core transformer for the same power rating but is more expensive to make because of the equipment required to wind the wires in the doughnut shape.

Figure 20.31 shows a laminated-coil transformer, which is based on Faraday's law of induction and is very similar in construction to the apparatus Faraday used to demonstrate that magnetic fields can generate electric currents. The two wire coils are called the primary and secondary coils. In normal use, the input voltage is applied across the primary coil, and the secondary produces the transformed output voltage. Not only does the iron core trap the magnetic field created by the primary coil, but also its magnetization increases the field strength, which is analogous to how a dielectric increases the electric field strength in a capacitor. Since the input voltage is AC, a time-varying magnetic flux is sent through the secondary coil, inducing an AC output voltage.

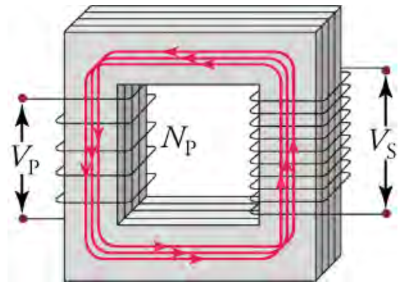


Figure 20.31 A typical construction of a simple transformer has two coils wound on a ferromagnetic core. The magnetic field created by the primary coil is mostly confined to and increased by the core, which transmits it to the secondary coil. Any change in current in the primary coil induces a current in the secondary coil.

LINKS TO PHYSICS

Magnetic Rope Memory

To send men to the moon, the Apollo program had to design an onboard computer system that would be robust, consume little power, and be small enough to fit onboard the spacecraft. In the 1960s, when the Apollo program was launched, entire buildings were regularly dedicated to housing computers whose computing power would be easily outstripped by today's most basic handheld calculator.

To address this problem, engineers at MIT and a major defense contractor turned to *magnetic rope memory*, which was an offshoot of a similar technology used prior to that time for creating random access memories. Unlike random access memory, magnetic rope memory was read-only memory that contained not only data but instructions as well. Thus, it was actually more than memory: It was a hard-wired computer program.

The components of magnetic rope memory were wires and iron rings—which were called *cores*. The iron cores served as transformers, such as that shown in the previous figure. However, instead of looping the wires multiple times around the core, individual wires passed only a single time through the cores, making these single-turn transformers. Up to 63 *word* wires could pass through a single core, along with a single *bit* wire. If a word wire passed through a given core, a voltage pulse on this wire would induce an emf in the bit wire, which would be interpreted as a *one*. If the word wire did not pass through the core, no emf would be induced on the bit wire, which would be interpreted as a *zero*.

Engineers would create programs that would be hard wired into these magnetic rope memories. The wiring process could take as long as a month to complete as workers painstakingly threaded wires through some cores and around others. If any mistakes were made either in the programming or the wiring, debugging would be extraordinarily difficult, if not impossible.

These modules did their job quite well. They are credited with correcting an astronaut mistake in the lunar landing procedure, thereby allowing Apollo 11 to land on the moon. It is doubtful that Michael Faraday ever imagined such an application for magnetic induction when he discovered it.

GRASP CHECK

If the bit wire were looped twice around each core, how would the voltage induced in the bit wire be affected?

a. If number of loops around the wire is doubled, the emf is halved.
b. If number of loops around the wire is doubled, the emf is not affected.
c. If number of loops around the wire is doubled, the emf is also doubled.
d. If number of loops around the wire is doubled, the emf is four times the initial value.

For the transformer shown in Figure 20.31, the output voltage V_S from the secondary coil depends almost entirely on the input voltage V_P across the primary coil and the number of loops in the primary and secondary coils. Faraday's law of induction for the secondary coil gives its induced output voltage V_S to be

$$V_S = -N_S \frac{\Delta \Phi}{\Delta t}, \tag{20.19}$$

where N_S is the number of loops in the secondary coil and $\Delta\Phi/\Delta t$ is the rate of change of magnetic flux. The output voltage equals the induced emf $(V_S = E_S)$, provided coil resistance is small—a reasonable assumption for transformers. The cross-sectional area of the coils is the same on each side, as is the magnetic field strength, and so $\Delta\Phi/\Delta t$ is the same on each side. The input primary voltage V_P is also related to changing flux by

$$V_P = -N_P \frac{\Delta\Phi}{\Delta t}. \tag{20.20}$$

Taking the ratio of these last two equations yields the useful relationship

$$\frac{V_S}{V_P} = \frac{N_S}{N_P}(3.07). \tag{20.21}$$

This is known as the transformer equation. It simply states that the ratio of the secondary voltage to the primary voltage in a transformer equals the ratio of the number of loops in secondary coil to the number of loops in the primary coil.

Transmission of Electrical Power

Transformers are widely used in the electric power industry to increase voltages—called *step-up* transformers—before long-distance transmission via high-voltage wires. They are also used to decrease voltages—called *step-down* transformers—to deliver power to homes and businesses. The overwhelming majority of electric power is generated by using magnetic induction, whereby a wire coil or copper disk is rotated in a magnetic field. The primary energy required to rotate the coils or disk can be provided by a variety of means. Hydroelectric power plants use the kinetic energy of water to drive electric generators. Coal or nuclear power plants create steam to drive steam turbines that turn the coils. Other sources of primary energy include wind, tides, or waves on water.

Once power is generated, it must be transmitted to the consumer, which often means transmitting power over hundreds of kilometers. To do this, the voltage of the power plant is increased by a step-up transformer, that is stepped up, and the current decreases proportionally because

$$P_{\text{transmitted}} = I_{\text{transmitted}} V_{\text{transmitted}}. \tag{20.22}$$

The lower current $I_{\text{transmitted}}$ in the transmission wires reduces the *Joule losses*, which is heating of the wire due to a current flow. This heating is caused by the small, but nonzero, resistance R_{wire} of the transmission wires. The power lost to the environment through this heat is

$$P_{\text{lost}} = I^2_{\text{transmitted}} R_{\text{wire}}, \tag{20.23}$$

which is proportional to the current *squared* in the transmission wire. This is why the transmitted current $I_{\text{transmitted}}$ must be as small as possible and, consequently, the voltage must be large to transmit the power $P_{\text{transmitted}}$.

Voltages ranging from 120 to 700 kV are used for transmitting power over long distances. The voltage is stepped up at the exit of the power station by a step-up transformer, as shown in Figure 20.32.

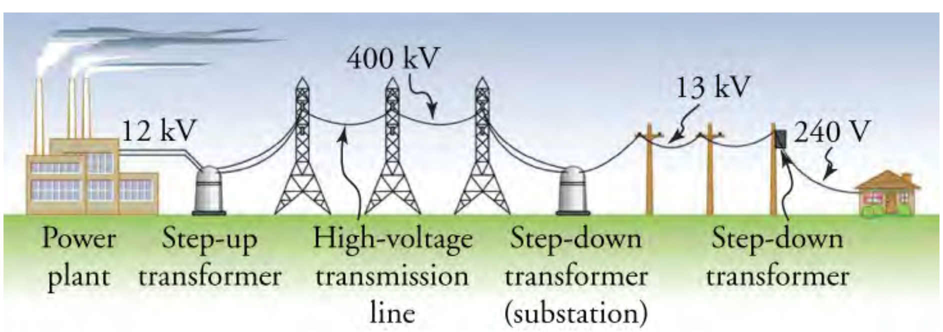


Figure 20.32 Transformers change voltages at several points in a power distribution system. Electric power is usually generated at greater than 10 kV, and transmitted long distances at voltages ranging from 120 kV to 700 kV to limit energy losses. Local power distribution to neighborhoods or industries goes through a substation and is sent short distances at voltages ranging from 5 to 13 kV. This is reduced to 120, 240, or 480 V for safety at the individual user site.

Once the power has arrived at a population or industrial center, the voltage is stepped down at a substation to between 5 and 30 kV. Finally, at individual homes or businesses, the power is stepped down again to 120, 240, or 480 V. Each step-up and step-down transformation is done with a transformer designed based on Faradays law of induction. We've come a long way since Queen Elizabeth asked Faraday what possible use could be made of electricity.

Check Your Understanding

7. What is an electric motor?
   a. An electric motor transforms electrical energy into mechanical energy.
   b. An electric motor transforms mechanical energy into electrical energy.
   c. An electric motor transforms chemical energy into mechanical energy.
   d. An electric motor transforms mechanical energy into chemical energy.

8. What happens to the torque provided by an electric motor if you double the number of coils in the motor?
   a. The torque would be doubled.
   b. The torque would be halved.
   c. The torque would be quadrupled.
   d. The torque would be tripled.

9. What is a step-up transformer?
   a. A step-up transformer decreases the current to transmit power over short distance with minimum loss.
   b. A step-up transformer increases the current to transmit power over short distance with minimum loss.
   c. A step-up transformer increases voltage to transmit power over long distance with minimum loss.
   d. A step-up transformer decreases voltage to transmit power over short distance with minimum loss.

10. What should be the ratio of the number of output coils to the number of input coil in a step-up transformer to increase the voltage fivefold?
   a. The ratio is five times.
   b. The ratio is 10 times.
   c. The ratio is 15 times.
   d. The ratio is 20 times.

20.3 Electromagnetic Induction

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain how a changing magnetic field produces a current in a wire
- Calculate induced electromotive force and current

Section Key Terms

emf	induction	magnetic flux

Changing Magnetic Fields

In the preceding section, we learned that a current creates a magnetic field. If nature is symmetrical, then perhaps a magnetic field can create a current. In 1831, some 12 years after the discovery that an electric current generates a magnetic field, English scientist Michael Faraday (1791–1862) and American scientist Joseph Henry (1797–1878) independently demonstrated that magnetic fields can produce currents. The basic process of generating currents with magnetic fields is called **induction**; this process is also called magnetic induction to distinguish it from charging by induction, which uses the electrostatic Coulomb force.

When Faraday discovered what is now called Faraday's law of induction, Queen Victoria asked him what possible use was electricity. "Madam," he replied, "What good is a baby?" Today, currents induced by magnetic fields are essential to our technological society. The electric generator—found in everything from automobiles to bicycles to nuclear power plants—uses magnetism to generate electric current. Other devices that use magnetism to induce currents include pickup coils in electric guitars, transformers of every size, certain microphones, airport security gates, and damping mechanisms on sensitive chemical balances.

One experiment Faraday did to demonstrate magnetic induction was to move a bar magnet through a wire coil and measure the resulting electric current through the wire. A schematic of this experiment is shown in Figure 20.33. He found that current is induced only when the magnet moves with respect to the coil. When the magnet is motionless with respect to the coil, no current is induced in the coil, as in Figure 20.33. In addition, moving the magnet in the opposite direction (compare Figure 20.33 with Figure 20.33) or reversing the poles of the magnet (compare Figure 20.33 with Figure 20.33) results in a current in the opposite direction.

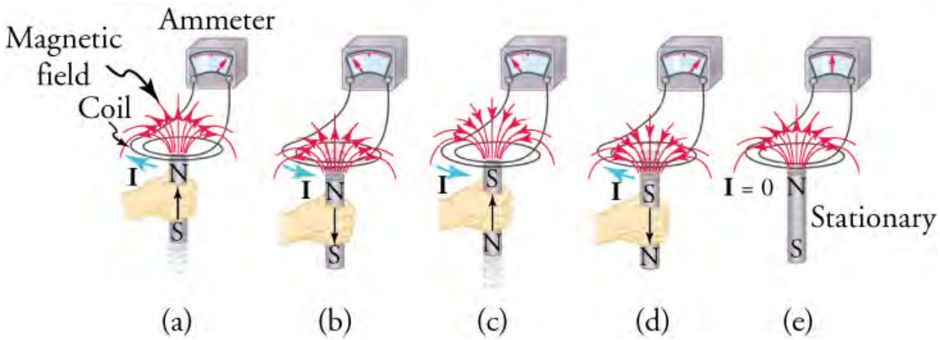


Figure 20.33 Movement of a magnet relative to a coil produces electric currents as shown. The same currents are produced if the coil is moved relative to the magnet. The greater the speed, the greater the magnitude of the current, and the current is zero when there is no motion. The current produced by moving the magnet upward is in the opposite direction as the current produced by moving the magnet downward.

Virtual Physics

Faraday's Law

Click to view content (http://www.openstax.org/l/faradays-law)

Try this simulation to see how moving a magnet creates a current in a circuit. A light bulb lights up to show when current is flowing, and a voltmeter shows the voltage drop across the light bulb. Try moving the magnet through a four-turn coil and through a two-turn coil. For the same magnet speed, which coil produces a higher voltage?

GRASP CHECK

With the north pole to the left and moving the magnet from right to left, a positive voltage is produced as the magnet enters the coil. What sign voltage will be produced if the experiment is repeated with the south pole to the left?

a. The sign of voltage will change because the direction of current flow will change by moving south pole of the magnet to the left.
b. The sign of voltage will remain same because the direction of current flow will not change by moving south pole of the magnet to the left.
c. The sign of voltage will change because the magnitude of current flow will change by moving south pole of the magnet to the left.
d. The sign of voltage will remain same because the magnitude of current flow will not change by moving south pole of the magnet to the left.

Induced Electromotive Force

If a current is induced in the coil, Faraday reasoned that there must be what he called an *electromotive force* pushing the charges through the coil. This interpretation turned out to be incorrect; instead, the external source doing the work of moving the magnet adds energy to the charges in the coil. The energy added per unit charge has units of volts, so the electromotive force is actually a potential. Unfortunately, the name electromotive force stuck and with it the potential for confusing it with a real force. For this reason, we avoid the term *electromotive force* and just use the abbreviation *emf*, which has the mathematical symbol ε. The **emf** may be defined as the rate at which energy is drawn from a source per unit current flowing through a circuit. Thus, emf is the energy per unit charge *added* by a source, which contrasts with voltage, which is the energy per unit charge *released* as the charges flow through a circuit.

To understand why an emf is generated in a coil due to a moving magnet, consider Figure 20.34, which shows a bar magnet moving downward with respect to a wire loop. Initially, seven magnetic field lines are going through the loop (see left-hand image). Because the magnet is moving away from the coil, only five magnetic field lines are going through the loop after a short time Δt (see right-hand image). Thus, when a change occurs in the number of magnetic field lines going through the area defined by the wire loop, an emf is induced in the wire loop. Experiments such as this show that the induced emf is proportional to the *rate of change* of the magnetic field. Mathematically, we express this as

$$\varepsilon \propto \frac{\Delta B}{\Delta t}, \tag{20.24}$$

where ΔB is the change in the magnitude in the magnetic field during time Δt and A is the area of the loop.

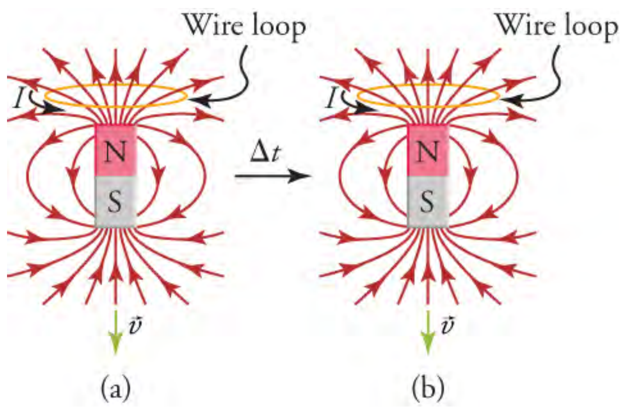


Figure 20.34 The bar magnet moves downward with respect to the wire loop, so that the number of magnetic field lines going through the loop decreases with time. This causes an emf to be induced in the loop, creating an electric current.

Note that magnetic field lines that lie in the plane of the wire loop do not actually pass through the loop, as shown by the left-most loop in Figure 20.35. In this figure, the arrow coming out of the loop is a vector whose magnitude is the area of the loop and whose direction is perpendicular to the plane of the loop. In Figure 20.35, as the loop is rotated from $\theta = 90°$ to $\theta = 0°$, the contribution of the magnetic field lines to the emf increases. Thus, what is important in generating an emf in the wire loop is the component of the magnetic field that is *perpendicular* to the plane of the loop, which is $B \cos \theta$.

This is analogous to a sail in the wind. Think of the conducting loop as the sail and the magnetic field as the wind. To maximize the force of the wind on the sail, the sail is oriented so that its surface vector points in the same direction as the winds, as in the right-most loop in Figure 20.35. When the sail is aligned so that its surface vector is perpendicular to the wind, as in the left-most loop in Figure 20.35, then the wind exerts no force on the sail.

Thus, taking into account the angle of the magnetic field with respect to the area, the proportionality $E \propto \Delta B/\Delta t$ becomes

$$E \propto \frac{\Delta B \cos \theta}{\Delta t}. \tag{20.25}$$

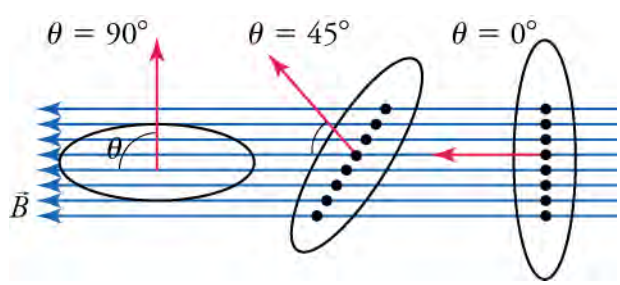


Figure 20.35 The magnetic field lies in the plane of the left-most loop, so it cannot generate an emf in this case. When the loop is rotated so that the angle of the magnetic field with the vector perpendicular to the area of the loop increases to $90°$ (see right-most loop), the magnetic field contributes maximally to the emf in the loop. The dots show where the magnetic field lines intersect the plane defined by the loop.

Another way to reduce the number of magnetic field lines that go through the conducting loop in Figure 20.35 is not to move the magnet but to make the loop smaller. Experiments show that changing the area of a conducting loop in a stable magnetic field induces an emf in the loop. Thus, the emf produced in a conducting loop is proportional to the rate of change of the *product* of the perpendicular magnetic field and the loop area

$$\varepsilon \propto \frac{\Delta\left[(B\cos\theta)A\right]}{\Delta t}, \tag{20.26}$$

where $B\cos\theta$ is the perpendicular magnetic field and A is the area of the loop. The product $BA\cos\theta$ is very important. It is proportional to the number of magnetic field lines that pass perpendicularly through a surface of area A. Going back to our sail analogy, it would be proportional to the force of the wind on the sail. It is called the **magnetic flux** and is represented by Φ .

$$\Phi = BA\cos\theta \tag{20.27}$$

The unit of magnetic flux is the weber (Wb), which is magnetic field per unit area, or T/m^2. The weber is also a volt second (Vs).

The induced emf is in fact proportional to the rate of change of the magnetic flux through a conducting loop.

$$\varepsilon \propto \frac{\Delta\Phi}{\Delta t} \tag{20.28}$$

Finally, for a coil made from N loops, the emf is N times stronger than for a single loop. Thus, the emf induced by a changing magnetic field in a coil of N loops is

$$\varepsilon \propto N\frac{\Delta B\cos\theta}{\Delta t}\mathrm{A}.$$

The last question to answer before we can change the proportionality into an equation is "In what direction does the current flow?" The Russian scientist Heinrich Lenz (1804–1865) explained that the current flows in the direction that creates a magnetic field that tries to keep the flux constant in the loop. For example, consider again Figure 20.34. The motion of the bar magnet causes the number of upward-pointing magnetic field lines that go through the loop to decrease. Therefore, an emf is generated in the loop that drives a current in the direction that creates more upward-pointing magnetic field lines. By using the right-hand rule, we see that this current must flow in the direction shown in the figure. To express the fact that the induced emf acts to counter the change in the magnetic flux through a wire loop, a minus sign is introduced into the proportionality $\varepsilon \propto \Delta\Phi/\Delta t.$, which gives Faraday's law of induction.

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} \tag{20.29}$$

Lenz's law is very important. To better understand it, consider Figure 20.36, which shows a magnet moving with respect to a wire coil and the direction of the resulting current in the coil. In the top row, the north pole of the magnet approaches the coil, so the magnetic field lines from the magnet point toward the coil. Thus, the magnetic field $\vec{B}_{\text{mag}} = B_{\text{mag}}\,(\hat{x})$ pointing to the right increases in the coil. According to Lenz's law, the emf produced in the coil will drive a current in the direction that creates a magnetic field $\vec{B}_{\text{coil}} = B_{\text{coil}}\,(-\hat{x})$ inside the coil pointing to the left. This will counter the increase in magnetic flux pointing to the right. To see which way the current must flow, point your right thumb in the desired direction of the magnetic field $\vec{B}_{\text{coil}}$, and the current will flow in the direction indicated by curling your right fingers. This is shown by the image of the right hand in the top row of Figure 20.36. Thus, the current must flow in the direction shown in Figure 4(a).

In Figure 4(b), the direction in which the magnet moves is reversed. In the coil, the right-pointing magnetic field $\vec{B}_{\text{mag}}$ due to the moving magnet decreases. Lenz's law says that, to counter this decrease, the emf will drive a current that creates an additional right-pointing magnetic field $\vec{B}_{\text{coil}}$ in the coil. Again, point your right thumb in the desired direction of the magnetic field, and the current will flow in the direction indicate by curling your right fingers (Figure 4(b)).

Finally, in Figure 4(c), the magnet is reversed so that the south pole is nearest the coil. Now the magnetic field $\vec{B}_{\text{mag}}$ points toward the magnet instead of toward the coil. As the magnet approaches the coil, it causes the left-pointing magnetic field in the coil to increase. Lenz's law tells us that the emf induced in the coil will drive a current in the direction that creates a magnetic field pointing to the right. This will counter the increasing magnetic flux pointing to the left due to the magnet. Using the right-hand rule again, as indicated in the figure, shows that the current must flow in the direction shown in Figure 4(c).

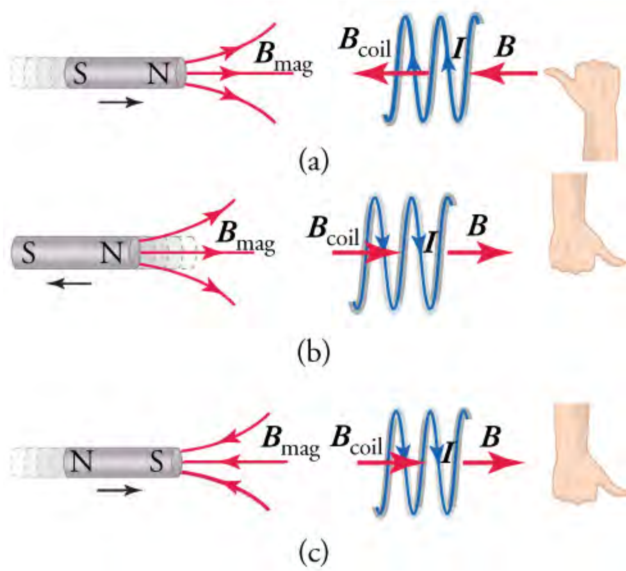


Figure 20.36 Lenz's law tells us that the magnetically induced emf will drive a current that resists the change in the magnetic flux through a circuit. This is shown in panels (a)–(c) for various magnet orientations and velocities. The right hands at right show how to apply the right-hand rule to find in which direction the induced current flows around the coil.

Virtual Physics

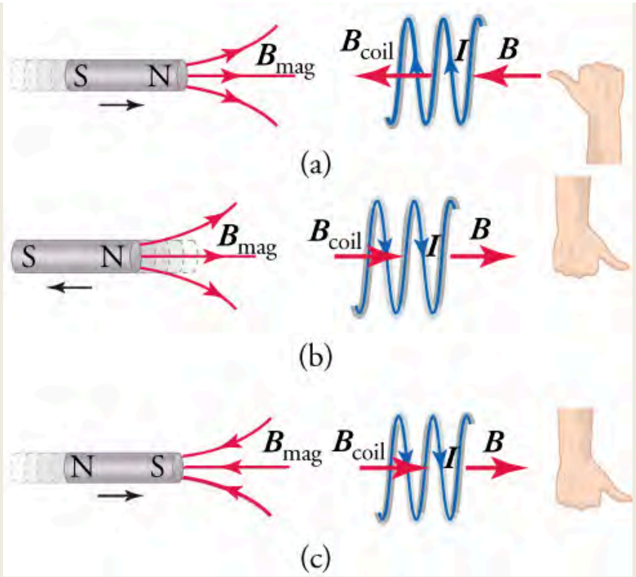
Faraday's Electromagnetic Lab

[Click to view content (http://www.openstax.org/l/Faraday-EM-lab)](http://www.openstax.org/l/Faraday-EM-lab)

This simulation proposes several activities. For now, click on the tab Pickup Coil, which presents a bar magnet that you can move through a coil. As you do so, you can see the electrons move in the coil and a light bulb will light up or a voltmeter will indicate the voltage across a resistor. Note that the voltmeter allows you to see the sign of the voltage as you move the magnet about. You can also leave the bar magnet at rest and move the coil, although it is more difficult to observe the results.

GRASP CHECK

Orient the bar magnet with the north pole facing to the right and place the pickup coil to the right of the bar magnet. Now move the bar magnet toward the coil and observe in which way the electrons move. This is the same situation as depicted below. Does the current in the simulation flow in the same direction as shown below? Explain why or why not.



a. Yes, the current in the simulation flows as shown because the direction of current is opposite to the direction of flow of electrons.
b. No, current in the simulation flows in the opposite direction because the direction of current is same to the direction of flow of electrons.

WATCH PHYSICS

Induced Current in a Wire

This video explains how a current can be induced in a straight wire by moving it through a magnetic field. The lecturer uses the *cross product*, which a type of vector multiplication. Don't worry if you are not familiar with this, it basically combines the right-hand rule for determining the force on the charges in the wire with the equation $F = qvB \sin \theta$.

Click to view content (https://www.openstax.org/l/induced-current)

GRASP CHECK

What emf is produced across a straight wire 0.50 m long moving at a velocity of (1.5 m/s) $\hat{x}$ through a uniform magnetic field (0.30 T)$\hat{z}$? The wire lies in the $\hat{y}$-direction. Also, which end of the wire is at the higher potential—let the lower end of the wire be at $y = 0$ and the upper end at $y = 0.5$ m)?

a. 0.15 V and the lower end of the wire will be at higher potential
b. 0.15 V and the upper end of the wire will be at higher potential
c. 0.075 V and the lower end of the wire will be at higher potential
d. 0.075 V and the upper end of the wire will be at higher potential

WORKED EXAMPLE

EMF Induced in Conducing Coil by Moving Magnet

Imagine a magnetic field goes through a coil in the direction indicated in Figure 20.37. The coil diameter is 2.0 cm. If the magnetic field goes from 0.020 to 0.010 T in 34 s, what is the direction and magnitude of the induced current? Assume the coil has a resistance of 0.1 Ω.

Figure 20.37 A coil through which passes a magnetic field *B*.

STRATEGY

Use the equation $\varepsilon = -N\Delta\Phi/\Delta t$ to find the induced emf in the coil, where $\Delta t = 34\ \text{s}$. Counting the number of loops in the solenoid, we find it has 16 loops, so $N = 16$. Use the equation $\Phi = BA\cos\theta$ to calculate the magnetic flux

$$\Phi = BA\cos\theta = B\pi\left(\frac{d}{2}\right)^2, \qquad 20.30$$

where d is the diameter of the solenoid and we have used $\cos 0° = 1$. Because the area of the solenoid does not vary, the change in the magnetic of the flux through the solenoid is

$$\Delta\Phi = \Delta B\pi\left(\frac{d}{2}\right)^2. \qquad 20.31$$

Once we find the emf, we can use Ohm's law, $\varepsilon = IR$, to find the current.

Finally, Lenz's law tells us that the current should produce a magnetic field that acts to oppose the decrease in the applied magnetic field. Thus, the current should produce a magnetic field to the right.

Solution

Combining equations $\varepsilon = -N\Delta\Phi/\Delta t$ and $\Phi = BA\cos\theta$ gives

$$\varepsilon = -N\frac{\Delta\Phi}{\Delta t} = -N\frac{\Delta B\pi d^2}{4\ \Delta t}. \qquad 20.32$$

Solving Ohm's law for the current and using this result gives

$$\begin{aligned} I &= \tfrac{\varepsilon}{R} = -N\tfrac{\Delta B\pi d^2}{4R\Delta t} \\ &= -16\tfrac{(-0.010\ \text{T})\pi(0.020\ \text{m})^2}{4(0.10\ \Omega)(34\ \text{s})} \\ &= 15\ \mu\text{A} \end{aligned} \qquad 20.33$$

Lenz's law tells us that the current must produce a magnetic field to the right. Thus, we point our right thumb to the right and curl our right fingers around the solenoid. The current must flow in the direction in which our fingers are pointing, so it enters at the left end of the solenoid and exits at the right end.

Discussion

Let's see if the minus sign makes sense in Faraday's law of induction. Define the direction of the magnetic field to be the positive direction. This means the change in the magnetic field is negative, as we found above. The minus sign in Faraday's law of induction negates the negative change in the magnetic field, leaving us with a positive current. Therefore, the current must flow in the direction of the magnetic field, which is what we found.

Now try defining the positive direction to be the direction opposite that of the magnetic field, that is positive is to the left in Figure 20.37. In this case, you will find a negative current. But since the positive direction is to the left, a negative current must flow to the right, which again agrees with what we found by using Lenz's law.

WORKED EXAMPLE

Magnetic Induction due to Changing Circuit Size

The circuit shown in Figure 20.38 consists of a U-shaped wire with a resistor and with the ends connected by a sliding conducting rod. The magnetic field filling the area enclosed by the circuit is constant at 0.01 T. If the rod is pulled to the right at speed $v = 0.50\ \text{m/s}$, what current is induced in the circuit and in what direction does the current flow?

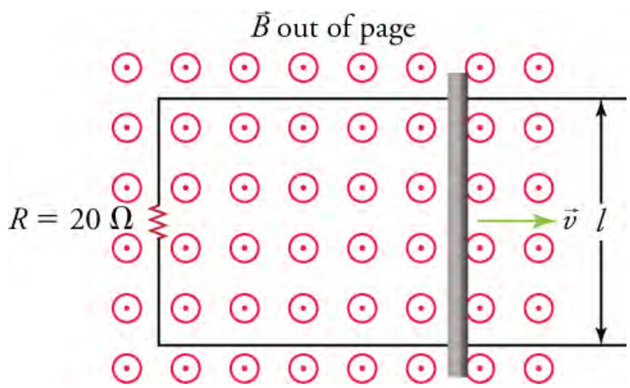


Figure 20.38 A slider circuit. The magnetic field is constant and the rod is pulled to the right at speed *v*. The changing area enclosed by the circuit induces an emf in the circuit.

STRATEGY

We again use Faraday's law of induction, $E = -N\frac{\Delta\Phi}{\Delta t}$, although this time the magnetic field is constant and the area enclosed by the circuit changes. The circuit contains a single loop, so $N = 1$. The rate of change of the area is $\frac{\Delta A}{\Delta t} = v\ell$. Thus the rate of change of the magnetic flux is

$$\frac{\Delta\Phi}{\Delta t} = \frac{\Delta\left(BA\cos\theta\right)}{\Delta t} = B\frac{\Delta A}{\Delta t} = Bv\ell, \tag{20.34}$$

where we have used the fact that the angle θ between the area vector and the magnetic field is 0°. Once we know the emf, we can find the current by using Ohm's law. To find the direction of the current, we apply Lenz's law.

Solution

Faraday's law of induction gives

$$E = -N\frac{\Delta\Phi}{\Delta t} = -Bv\ell. \tag{20.35}$$

Solving Ohm's law for the current and using the previous result for emf gives

$$I = \frac{E}{R} = \frac{-Bv\ell}{R} = \frac{-\left(0.010\ \text{T}\right)\left(0.50\ \text{m/s}\right)\left(0.10\ \text{m}\right)}{20\ \Omega} = 25\ \mu\text{A}. \tag{20.36}$$

As the rod slides to the right, the magnetic flux passing through the circuit increases. Lenz's law tells us that the current induced will create a magnetic field that will counter this increase. Thus, the magnetic field created by the induced current must be into the page. Curling your right-hand fingers around the loop in the clockwise direction makes your right thumb point into the page, which is the desired direction of the magnetic field. Thus, the current must flow in the clockwise direction around the circuit.

Discussion

Is energy conserved in this circuit? An external agent must pull on the rod with sufficient force to just balance the force on a current-carrying wire in a magnetic field—recall that $F = I\ell B\sin\theta$. The rate at which this force does work on the rod should be balanced by the rate at which the circuit dissipates power. Using $F = I\ell B\sin\theta$, the force required to pull the wire at a constant speed v is

$$F_{\text{pull}} = I\ell B\sin\theta = I\ell B, \tag{20.37}$$

where we used the fact that the angle θ between the current and the magnetic field is $90°$. Inserting our expression above for the current into this equation gives

$$F_{\text{pull}} = I\ell B = -\frac{Bv\ell}{R}(\ell B) = -\frac{B^2v\ell^2}{R}. \tag{20.38}$$

The power contributed by the agent pulling the rod is $F_{\text{pull}}v$, or

$$P_{\text{pull}} = F_{\text{pull}} v = -\frac{B^2 v^2 \ell^2}{R}. \tag{20.39}$$

The power dissipated by the circuit is

$$P_{\text{dissipated}} = I^2 R = \left(\frac{-Bv\ell}{R}\right)^2 R = \frac{B^2 v^2 \ell^2}{R}. \tag{20.40}$$

We thus see that $P_{\text{pull}} + P_{\text{dissipated}} = 0$, which means that power is conserved in the system consisting of the circuit and the agent that pulls the rod. Thus, energy is conserved in this system.

Practice Problems

11. The magnetic flux through a single wire loop changes from 3.5 Wb to 1.5 Wb in 2.0 s. What emf is induced in the loop?

a. −2.0 V
b. −1.0 V
c. +1.0 V
d. +2.0 V

12. What is the emf for a 10-turn coil through which the flux changes at 10 Wb/s?

a. −100 V
b. −10 V
c. +10 V
d. +100 V

Check Your Understanding

13. Given a bar magnet, how can you induce an electric current in a wire loop?

a. An electric current is induced if a bar magnet is placed near the wire loop.
b. An electric current is induced if wire loop is wound around the bar magnet.
c. An electric current is induced if a bar magnet is moved through the wire loop.
d. An electric current is induced if a bar magnet is placed in contact with the wire loop.

14. What factors can cause an induced current in a wire loop through which a magnetic field passes?

a. Induced current can be created by changing the size of the wire loop only.
b. Induced current can be created by changing the orientation of the wire loop only.
c. Induced current can be created by changing the strength of the magnetic field only.
d. Induced current can be created by changing the strength of the magnetic field, changing the size of the wire loop, or changing the orientation of the wire loop.

KEY TERMS

Curie temperature well-defined temperature for ferromagnetic materials above which they cannot be magnetized

domain region within a magnetic material in which the magnetic poles of individual atoms are aligned

electric motor device that transforms electrical energy into mechanical energy

electromagnet device that uses electric current to make a magnetic field

electromagnetism study of electric and magnetic phenomena

emf rate at which energy is drawn from a source per unit current flowing through a circuit

ferromagnetic material such as iron, cobalt, nickel, or gadolinium that exhibits strong magnetic effects

generator device that transforms mechanical energy into electrical energy

induction rate at which energy is drawn from a source per unit current flowing through a circuit

magnetic dipole term that describes magnets because they always have two poles: north and south

magnetic field directional lines around a magnetic material that indicates the direction and magnitude of the magnetic force

magnetic flux component of the magnetic field perpendicular to the surface area through which it passes and multiplied by the area

magnetic pole part of a magnet that exerts the strongest force on other magnets or magnetic material

magnetized material that is induced to be magnetic or that is made into a permanent magnet

north pole part of a magnet that orients itself toward the geographic North Pole of Earth

permanent magnet material that retains its magnetic behavior for a long time, even when exposed to demagnetizing influences

right-hand rule rule involving curling the right-hand fingers from one vector to another; the direction in which the right thumb points is the direction of the resulting vector

solenoid uniform cylindrical coil of wire through which electric current is passed to produce a magnetic field

south pole part of a magnet that orients itself toward the geographic South Pole of Earth

transformer device that transforms voltages from one value to another

SECTION SUMMARY

20.1 Magnetic Fields, Field Lines, and Force

- All magnets have two poles: a north pole and a south pole. If the magnet is free to move, its north pole orients itself toward the geographic North Pole of Earth, and the south pole orients itself toward the geographic South Pole of Earth.
- A repulsive force occurs between the north poles of two magnets and likewise for two south poles. However, an attractive force occurs between the north pole of one magnet and the south pole of another magnet.
- A charged particle moving through a magnetic field experiences a force whose direction is determined by the right-hand rule.
- An electric current generates a magnetic field.
- Electromagnets are magnets made by passing a current through a system of wires.

20.2 Motors, Generators, and Transformers

- Electric motors contain wire loops in a magnetic field. Current is passed through the wire loops, which forces them to rotate in the magnetic field. The current is reversed every half rotation so that the torque on the loop is always in the same direction.
- Electric generators contain wire loops in a magnetic field. An external agent provides mechanical energy to force the loops to rotate in the magnetic field, which produces an AC voltage that drives an AC current through the loops.
- Transformers contain a ring made of magnetic material and, on opposite sides of the ring, two windings of wire wrap around the ring. A changing current in one wire winding creates a changing magnetic field, which is trapped in the ring and thus goes through the second winding and induces an emf in the second winding. The voltage in the second winding is proportional to the ratio of the number of loops in each winding.
- Transformers are used to step up and step down the voltage for power transmission.
- Over long distances, electric power is transmitted at high voltage to minimize the current and thereby minimize the Joule losses due to resistive heating.

20.3 Electromagnetic Induction

- **Faraday's law of induction** states that a changing magnetic flux that occurs within an area enclosed by a conducting loop induces an electric current in the loop.
- **Lenz' law** states that an induced current flows in the direction such that it opposes the change that induced it.

KEY EQUATIONS

20.1 Magnetic Fields, Field Lines, and Force

the magnitude of the force on an electric charge	$F = qvB\sin\theta$
the force on a wire carrying current	$F = I\ell B\sin\theta$
the magnitude of the magnetic field created by a long, straight current-carrying wire	$B_{\text{straightwire}} = \frac{\mu_0 I}{2\pi r}$
the magnitude of the magnetic field inside a solenoid	$B_{\text{solenoid}} = \mu_0 \frac{NI}{\ell}$

20.3 Electromagnetic Induction

magnetic flux	$\Phi = BA\cos\theta$
emf	$E = -N\frac{\Delta\Phi}{\Delta t}$

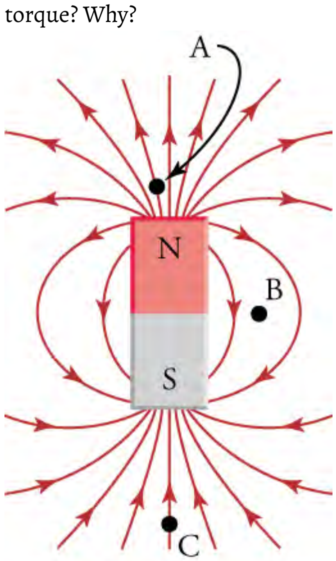
CHAPTER REVIEW

Concept Items

20.1 Magnetic Fields, Field Lines, and Force

1. If you place a small needle between the north poles of two bar magnets, will the needle become magnetized?
    a. Yes, the magnetic fields from the two north poles will point in the same directions.
    b. Yes, the magnetic fields from the two north poles will point in opposite directions.
    c. No, the magnetic fields from the two north poles will point in opposite directions.
    d. No, the magnetic fields from the two north poles will point in the same directions.

2. If you place a compass at the three points in the figure, at which point will the needle experience the greatest torque? Why?



    a. The density of the magnetic field is minimized at B, so the magnetic compass needle will experience the greatest torque at B.
    b. The density of the magnetic field is minimized at C, so the magnetic compass needle will experience the greatest torque at C.
    c. The density of the magnetic field is maximized at B, so the magnetic compass needle will experience the greatest torque at B.
    d. The density of the magnetic field is maximized at A, so the magnetic compass needle will experience the greatest torque at A.

3. In which direction do the magnetic field lines point near the south pole of a magnet?
    a. Outside the magnet the direction of magnetic field lines is towards the south pole of the magnet.
    b. Outside the magnet the direction of magnetic field lines is away from the south pole of the magnet.

20.2 Motors, Generators, and Transformers

4. Consider the angle between the area vector and the magnetic field in an electric motor. At what angles is the torque on the wire loop the greatest?
    a. $0°$ and $180°$
    b. $45°$ and $135°$
    c. $90°$ and $270°$
    d. $225°$ and $315°$

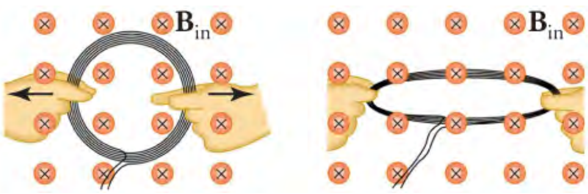
5. What is a voltage transformer?
    a. A transformer is a device that transforms current to voltage.
    b. A transformer is a device that transforms voltages from one value to another.
    c. A transformer is a device that transforms resistance of wire to voltage.

6. Why is electric power transmitted at high voltage?

a. To increase the current for the transmission
b. To reduce energy loss during transmission
c. To increase resistance during transmission
d. To reduce resistance during transmission

20.3 Electromagnetic Induction

7. Yes or no—Is an emf induced in the coil shown when it is stretched? If so, state why and give the direction of the induced current.



a. No, because induced current does not depend upon the area of the coil.
b. Yes, because area of the coil increases; the direction of the induced current is counterclockwise.
c. Yes, because area of the coil increases; the direction of the induced current is clockwise.
d. Yes, because the area of the coil does not change; the direction of the induced current is clockwise.

8. What is Lenz's law?

a. If induced current flows, its direction is such that it adds to the changes which induced it.
b. If induced current flows, its direction is such that it opposes the changes which induced it.
c. If induced current flows, its direction is always clockwise to the changes which induced it.
d. If induced current flows, its direction is always counterclockwise to the changes which induced it.

9. Explain how magnetic flux can be zero when the magnetic field is not zero.

a. If angle between magnetic field and area vector is 0°, then its sine is also zero, which means that there is zero flux.
b. If angle between magnetic field and area vector is 45°, then its sine is also zero, which means that there is zero flux.
c. If angle between magnetic field and area vector is 60°, then its cosine is also zero, which means that there is zero flux.
d. If the angle between magnetic field and area vector is 90°, then its cosine is also zero, which means that there is zero flux.

Critical Thinking Items

20.1 Magnetic Fields, Field Lines, and Force

10. True or false—It is not recommended to place credit cards with magnetic strips near permanent magnets.

a. false
b. true

11. True or false—A square magnet can have sides that alternate between north and south poles.

a. false
b. true

12. You move a compass in a circular plane around a planar magnet. The compass makes four complete revolutions. How many poles does the magnet have?

a. two poles
b. four poles
c. eight poles
d. 12 poles

20.2 Motors, Generators, and Transformers

13. How can you maximize the peak emf from a generator?

a. The peak emf from a generator can be maximized only by maximizing number of turns.
b. The peak emf from a generator can be maximized only by maximizing area of the wired loop.
c. The peak emf from a generator can be maximized only by maximizing frequency.
d. The peak emf from a generator can be maximized by maximizing number of turns, maximizing area of the wired loop or maximizing frequency.

14. Explain why power is transmitted over long distances at high voltages.

a. $P_{\text{lost}} = I_{\text{transmitted}} V_{\text{transmitted}}$, so to maximize current, the voltage must be maximized
b. $P_{\text{transmitted}} = I_{\text{transmitted}} V_{\text{transmitted}}$, so to maximize current, the voltage must be maximized
c. $P_{\text{lost}} = I_{\text{transmitted}} V_{\text{transmitted}}$, so to minimize current, the voltage must be maximized
d. $P_{\text{transmitted}} = I_{\text{transmitted}} V_{\text{transmitted}}$, so to minimize current, the voltage must be maximized

20.3 Electromagnetic Induction

15. To obtain power from the current in the wire of your vacuum cleaner, you place a loop of wire near it to obtain an induced emf. How do you place and orient the loop?

a. A loop of wire should be placed nearest to the vacuum cleaner wire to maximize the magnetic flux through the loop.
b. A loop of wire should be placed farthest to the vacuum cleaner wire to maximize the magnetic flux through the loop.

c. A loop of wire should be placed perpendicular to the vacuum cleaner wire to maximize the magnetic flux through the loop.
d. A loop of wire should be placed at angle greater than 90° to the vacuum cleaner wire to maximize the magnetic flux through the loop.

16. A magneto is a device that creates a spark across a gap by creating a large voltage across the gap. To do this, the device spins a magnet very quickly in front of a wire coil, with the ends of the wires forming the gap. Explain how this creates a sufficiently large voltage to produce a spark.
a. The electric field in the coil increases rapidly due to spinning of magnet which creates an emf in the coil that is proportional to the rate of change of the magnetic flux.
b. The magnetic field in the coil changes rapidly due to spinning of magnet which creates an emf in the coil that is proportional to the rate of change of the magnetic flux.

17. If you drop a copper tube over a bar magnet with its north pole up, is a current induced in the copper tube? If so, in what direction? Consider when the copper tube is approaching the bar magnet.
a. Yes, the induced current will be produced in the clockwise direction when viewed from above.
b. No, the induced current will not be produced.

Problems

20.1 Magnetic Fields, Field Lines, and Force

18. A straight wire segment carries 0.25 A. What length would it need to be to exert a 4.0-mN force on a magnet that produces a uniform magnetic field of 0.015 T that is perpendicular to the wire?
a. 0.55 m
b. 1.10 m
c. 2.20 m
d. 4.40 m

20.3 Electromagnetic Induction

19. What is the current in a wire loop of resistance 10 Ω through which the magnetic flux changes from zero to 10 Wb in 1.0 s?
a. −100 A
b. −2.0 A
c. −1.0 A
d. +1.0 A

20. An emf is induced by rotating a 1,000 turn, 20.0 cm diameter coil in Earth's 5.00×10^{-5} T magnetic field. What average emf is induced, given the plane of the coil is originally perpendicular to Earth's field and is rotated to be parallel to the field in 10.0 ms?
a. -1.6×10^{-4} V
b. $+1.6 \times 10^{-4}$ V
c. $+1.6 \times 10^{-1}$ V
d. -1.6×10^{-1} V

Performance Task

20.2 Motors, Generators, and Transformers

21. Your family takes a trip to Cuba, and rents an old car to drive into the countryside to see the sights. Unfortunately, the next morning you find yourself deep in the countryside and the car won't start because the battery is too weak. Wanting to jump-start the car, you open the hood and find that you can't tell which battery terminal is positive and which is negative. However, you do have a bar magnet with the north and south poles labeled and you manage to find a short wire. How do you use these to determine which terminal is which? For starters, how do you determine the direction of a magnetic field around a current-carrying wire? And in which direction will the force be on another magnet placed in this field? Do you need to worry about the sign of the mobile charge carriers in the wire?

TEST PREP

Multiple Choice

20.1 Magnetic Fields, Field Lines, and Force

22. For a magnet, a domain refers to ______.
a. the region between the poles of the magnet
b. the space around the magnet that is affected by the magnetic field
c. the region within the magnet in which the magnetic poles of individual atoms are aligned
d. the region from which the magnetic material is mined

23. In the region just outside the south pole of a magnet, the magnetic field lines ______.
a. point away from the south pole
b. go around the south pole
c. are less concentrated than at the north pole

d. point toward the south pole

24. Which equation gives the force for a charge moving through a magnetic field?
a. $F = qvB \sin\theta$
b. $F = I\ell B \sin\theta$
c. $F = I\ell B$
d. $F = qvB$

25. Can magnetic field lines cross each other? Explain why or why not.
a. Yes, magnetic field lines can cross each other because that point of intersection indicates two possible directions of magnetic field, which is possible.
b. No, magnetic field lines cannot cross each other because that point of intersection indicates two possible directions of magnetic field, which is not possible.

26. True or false—If a magnet shatters into many small pieces, all the pieces will have north and south poles
a. true
b. false

20.2 Motors, Generators, and Transformers

27. An electrical generator ________.
a. is a generator powered by electricity
b. must be turned by hand
c. converts other sources of power into electrical power
d. uses magnetism to create electrons

28. A step-up transformer increases the
a. voltage from power lines for use in homes
b. current from the power lines for use in homes
c. current from the electrical generator for transmission along power lines
d. voltage from the electrical power plant for transmission along power lines

29. What would be the effect on the torque of an electric motor of doubling the width of the current loop in the motor?
a. Torque remains the same.
b. Torque is doubled.
c. Torque is quadrupled.
d. Torque is halved.

30. Why are the coils of a transformer wrapped around a loop of ferrous material?
a. The magnetic field from the source coil is trapped and also increased in strength.
b. The magnetic field from the source coil is dispersed and also increased in strength.
c. The magnetic field from the source coil is trapped and also decreased in strength.
d. Magnetic field from the source coil is dispersed and also decreased in strength.

20.3 Electromagnetic Induction

31. What does *emf* stand for?
a. electromotive force
b. electro motion force
c. electromagnetic factor
d. electronic magnetic factor

32. Which formula gives magnetic flux?
a. $\frac{\mu_0 I}{2\pi r}$
b. $qvB \sin\ \theta$
c. $-N\frac{\Delta\Phi}{\Delta t}$
d. $BA\cos\theta$

33. What is the relationship between the number of coils in a solenoid and the emf induced in it by a change in the magnetic flux through the solenoid?
a. The induced emf is inversely proportional to the number of coils in a solenoid.
b. The induced emf is directly proportional to the number of coils in a solenoid.
c. The induced emf is inversely proportional to the square of the number of coils in a solenoid.
d. The induced emf is proportional to square of the number of coils in a solenoid.

34. True or false—If you drop a bar magnet through a copper tube, it induces an electric current in the tube.
a. false
b. true

Short Answer

20.1 Magnetic Fields, Field Lines, and Force

35. Given a bar magnet, a needle, a cork, and a bowl full of water, describe how to make a compass.
a. Magnetize the needle by holding it perpendicular to a bar magnet's north pole and pierce the cork along its longitudinal axis by the needle and place the needle-cork combination in the water. The needle now orients itself along the magnetic field lines of Earth.
b. Magnetize the needle by holding it perpendicular to a bar magnet's north pole and pierce the cork along its longitudinal axis by the needle and place the needle-cork combination in the water. The needle now orients itself perpendicular to the

magnetic field lines of Earth.

c. Magnetize the needle by holding its axis parallel to the axis of a bar magnet and pierce the cork along its longitudinal axis by the needle and place the needle-cork combination in the water. The needle now orients itself along the magnetic field lines of Earth.

d. Magnetize the needle by holding its axis parallel to the axis of a bar magnet and pierce the cork along its longitudinal axis by the needle and place the needle-cork combination in the water. The needle now orients itself perpendicular to the magnetic field lines of Earth.

36. Give two differences between electric field lines and magnetic field lines.

a. Electric field lines begin and end on opposite charges and the electric force on a charge is in the direction of field, while magnetic fields form a loop and the magnetic force on a charge is perpendicular to the field.

b. Electric field lines form a loop and the electric force on a charge is in the direction of field, while magnetic fields begin and end on opposite charge and the magnetic force on a charge is perpendicular to the field.

c. Electric field lines begin and end on opposite charges and the electric force on a charge is in the perpendicular direction of field, while magnetic fields form a loop and the magnetic force on a charge is in the direction of the field.

d. Electric field lines form a loop and the electric force on a charge is in the perpendicular direction of field, while magnetic fields begin and end on opposite charge and the magnetic force on a charge is in the direction of the field.

37. To produce a magnetic field of 0.0020 T, what current is required in a 500-turn solenoid that is 25 cm long?

a. 0.80 A

b. 1.60 A

c. 80 A

d. 160 A

38. You magnetize a needle by aligning it along the axis of a bar magnet and just outside the north pole of the magnet. Will the point of the needle that was closest to the bar magnet then be attracted to or repelled from the south pole of another magnet?

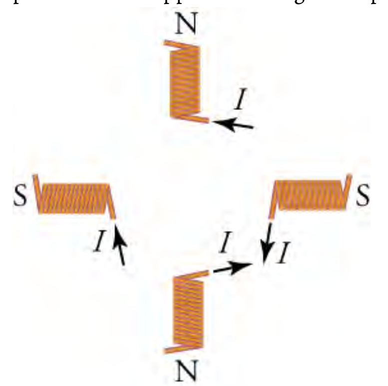
a. The needle will magnetize and the point of needle kept closer to the north pole will act as a south pole. Hence, it will repel the south pole of other magnet.

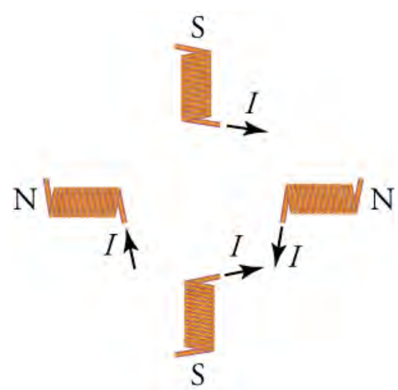
b. The needle will magnetize and the point of needle kept closer to the north pole will act as a south pole. Hence, it will attract the south pole of other magnet.

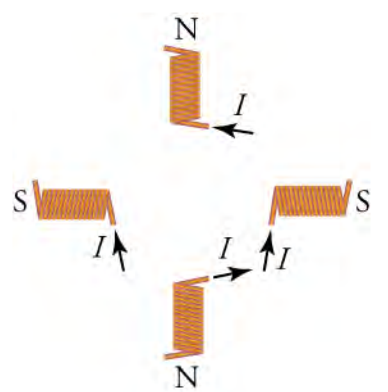
c. The needle will magnetize and the point of a needle kept closer to the north pole will act as a north pole. Hence, it will repel the south pole of the other magnet.

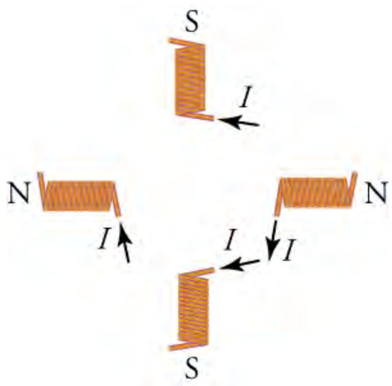
d. The needle will magnetize and the point of needle kept closer to the north pole will act as a north pole. Hence, it will attract the south pole of other magnet.

39. Using four solenoids of the same size, describe how to orient them and in which direction the current should flow to make a magnet with two opposite-facing north poles and two opposite-facing south poles.









40. How far from a straight wire carrying 0.45 A is the magnetic field strength 0.040 T?
   a. 0.23 µm
   b. 0.72 µm
   c. 2.3 µm
   d. 7.2 µm

20.2 Motors, Generators, and Transformers

41. A laminated-coil transformer has a wire coiled 12 times around one of its sides. How many coils should you wrap around the opposite side to get a voltage output that is one half of the input voltage? Explain.
   a. six output coils because the ratio of output to input voltage is the same as the ratio of number of output coils to input coils
   b. 12 output coils because the ratio of output to input voltage is the same as the ratio of number of output coils to input coils
   c. 24 output coils because the ratio of output to input voltage is half the ratio of the number of output coils to input coils
   d. 36 output coils because the ratio of output to input voltage is three times the ratio of the number of output coils to input coils

42. Explain why long-distance electrical power lines are designed to carry very high voltages.
   a. $P_{transmitted} = I_{transmitted>}^2 R_{wire}$ and $P_{lost} = I_{transmitted} V_{transmitted}$, so V must be low to make the current transmitted as high as possible.
   b. $P_{transmitted} = I_{transmitted>}^2 R_{wire}$ and $P_{lost} = I_{lost} V_{lost}$, so V must be low to make the current transmitted as high as possible.
   c. $P_{transmitted} = I_{transmitted>}^2 R_{wire}$ and $P_{lost} = I_{transmitted} V_{transmitted}$, so V must be high to make the current transmitted as low as possible
   d. $P_{lost} = I_{transmitted}^2 R_{wire}$ and $P_{transmitted} = I_{transmitted} V_{transmitted}$, so V must be high to make the current transmitted as low as possible.

43. How is the output emf of a generator affected if you double the frequency of rotation of its coil?
   a. The output emf will be doubled.
   b. The output emf will be halved.
   c. The output emf will be quadrupled.
   d. The output emf will be tripled.

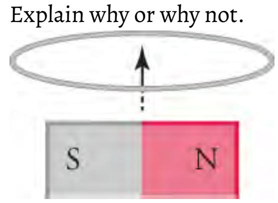
44. In a hydroelectric dam, what is used to power the electrical generators that provide electric power? Explain.
   a. The electric potential energy of stored water is used to produce emf with the help of a turbine.
   b. The electric potential energy of stored water is used to produce resistance with the help of a turbine.
   c. Gravitational potential energy of stored water is used to produce resistance with the help of a turbine.
   d. Gravitational potential energy of stored water is used to produce emf with the help of a turbine.

20.3 Electromagnetic Induction

45. A uniform magnetic field is perpendicular to the plane of a wire loop. If the loop accelerates in the direction of the field, will a current be induced in the loop? Explain why or why not.
   a. No, because magnetic flux through the loop remains constant.
   b. No, because magnetic flux through the loop changes continuously.
   c. Yes, because magnetic flux through the loop remains constant.
   d. Yes, because magnetic flux through the loop changes continuously.

46. The plane of a square wire circuit with side 4.0 cm long is at an angle of 45° with respect to a uniform magnetic field of 0.25 T. The wires have a resistance per unit length of 0.2. If the field drops to zero in 2.5 s, what magnitude current is induced in the square circuit?
   a. 35 µA
   b. 87.5 µA
   c. 3.5 mA
   d. 35 A

47. Yes or no—If a bar magnet moves through a wire loop as shown in the figure, is a current induced in the loop? Explain why or why not.



   a. No, because the net magnetic field passing through the loop is zero.
   b. No, because the net magnetic field passing through

the loop is nonzero.

c. Yes, because the net magnetic field passing through the loop is zero.
d. Yes, because the net magnetic field line passing through the loop is nonzero.

48. What is the magnetic flux through an equilateral triangle with side 60 cm long and whose plane makes a 60° angle with a uniform magnetic field of 0.33 T?

a. 0.045 Wb
b. 0.09 Wb
c. 0.405 Wb
d. 4.5 Wb

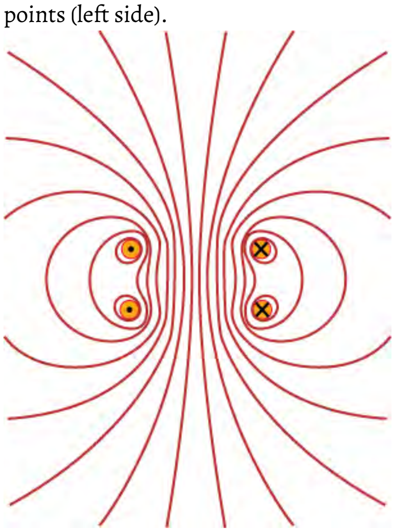
Extended Response

20.1 Magnetic Fields, Field Lines, and Force

49. Summarize the properties of magnets.

a. A magnet can attract metals like iron, nickel, etc., but cannot attract nonmetals like piece of plastic or wood, etc. If free to rotate, an elongated magnet will orient itself so that its north pole will face the magnetic south pole of Earth.
b. A magnet can attract metals like iron, nickel, etc., but cannot attract nonmetals like piece of plastic or wood, etc. If free to rotate, an elongated magnet will orient itself so that its north pole will face the magnetic north pole of Earth.
c. A magnet can attract metals like iron, nickel, etc., and nonmetals like piece of plastic or wood, etc. If free to rotate, an elongated magnet will orient itself so that its north pole will face the magnetic south pole of Earth.
d. A magnet can attract metals like iron, nickel, etc., and nonmetals like piece of plastic or wood, etc. If free to rotate, an elongated magnet will orient itself so that its north pole will face the magnetic north pole of Earth.

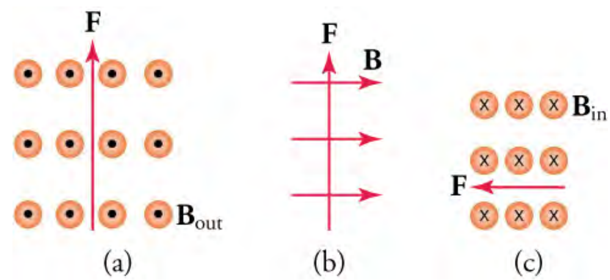
50. The magnetic field shown in the figure is formed by current flowing in two rings that intersect the page at the dots. Current flows into the page at the dots with crosses (right side) and out of the page at the dots with points (left side).



Where is the field strength the greatest and in what direction do the magnetic field lines point?

a. The magnetic field strength is greatest where the magnetic field lines are less dense; magnetic field lines points up the page.
b. The magnetic field strength is greatest where the magnetic field lines are most dense; magnetic field lines points up the page.
c. The magnetic field strength is greatest where the magnetic field lines are most dense; magnetic field lines points down the page.
d. The magnetic field strength is greatest where the magnetic field lines are less dense; magnetic field lines points down the page.

51. The forces shown below are exerted on an electron as it moves through the magnetic field. In each case, what direction does the electron move?



a. (a) left to right, (b) out of the page, (c) upwards
b. (a) left to right, (b) into the page, (c) downwards
c. (a) right to left, (b) out of the page, (c) upwards
d. (a) right to left, (b) into the page, (c) downwards

20.2 Motors, Generators, and Transformers

52. Explain why increasing the frequency of rotation of the coils in an electrical generator increases the output emf.

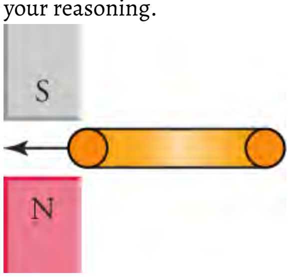
a. The induced emf is proportional to the rate of change of magnetic flux with respect to distance.
b. The induced emf is inversely proportional to the rate of change of magnetic flux with respect to distance.
c. The induced emf is inversely proportional to the rate of change of magnetic flux with respect to time.
d. The induced emf is proportional to the rate of change of magnetic flux with respect to time.

53. Your friend tells you that power lines must carry a maximum current because $P = I^2 R$, where R is the resistance of the transmission line. What do you tell her?

a. $P_{transmitted} = I_{transmitted}{}^2 R_{wire}$ and $P_{lost} = I_{transmitted} V_{transmitted}$, so I must be high to reduce power lost due to transmission.
b. $P_{lost} = I_{transmitted}{}^2 R_{wire}$ and $P_{lost} = I_{transmitted} V_{transmitted}$, so I must be high to reduce power lost due to transmission.
c. $P_{transmitted} = I_{transmitted}{}^2 R_{wire}$ and $P_{lost} = I_{transmitted} V_{transmitted}$, so I must be low to reduce power lost due to transmission.
d. $P_{lost} = I_{transmitted}{}^2 R_{wire}$ and $P_{lost} = I_{transmitted} V_{transmitted}$, so I must be low to reduce power lost due to transmission.

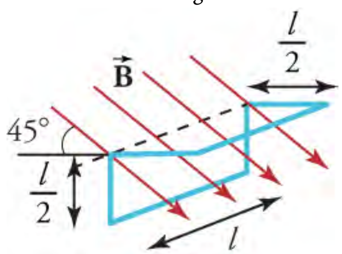
20.3 Electromagnetic Induction

54. When you insert a copper ring between the poles of two bar magnets as shown in the figure, do the magnets exert an attractive or repulsive force on the ring? Explain your reasoning.



a. Magnets exert an attractive force, because magnetic field due to induced current is repulsed by the magnetic field of the magnets.
b. Magnets exert an attractive force, because magnetic field due to induced current is attracted by the magnetic field of the magnets.
c. Magnets exert a repulsive force, because magnetic field due to induced current is repulsed by the magnetic field of the magnets.
d. Magnets exert a repulsive force, because magnetic field due to induced current is attracted by the magnetic field of the magnets.

55. The figure shows a uniform magnetic field passing through a closed wire circuit. The wire circuit rotates at an angular frequency of about the axis shown by the dotted line in the figure.



What is an expression for the magnetic flux through the circuit as a function of time?

a. expression for the magnetic flux through the circuit $\Phi(t) = BA\cos\omega t$
b. expression for the magnetic flux through the circuit $\Phi(t) = \sqrt{2}BA\cos\omega t$
c. expression for the magnetic flux through the circuit $\Phi(t) = \sqrt{3}BA\cos\omega t$
d. expression for the magnetic flux through the circuit $\Phi(t) = 2BA\cos\omega t$

CHAPTER 21

The Quantum Nature of Light

Figure 21.1 In Lewis Carroll's classic text Alice's Adventures in Wonderland, Alice follows a rabbit down a hole into a land of curiosity. While many of her interactions in Wonderland are of surprising consequence, they follow a certain inherent logic. (credit: modification of work by John Tenniel, Wikimedia Commons)

Chapter Outline

INTRODUCTION At first glance, the quantum nature of light can be a strange and bewildering concept. Between light acting as discrete chunks, massless particles providing momenta, and fundamental particles behaving like waves, it may often seem like something out of Alice in Wonderland.

For many, the study of this branch of physics can be as enthralling as Lewis Carroll's classic novel. Recalling the works of legendary characters and brilliant scientists such as Einstein, Planck, and Compton, the study of light's quantum nature will provide you an interesting tale of how a clever interpretation of some small details led to the most important discoveries of the past 150 years. From the electronics revolution of the twentieth century to our future progress in solar energy and space exploration, the quantum nature of light should yield a rabbit hole of curious consequence, within which lie some of the most fascinating truths of our time.

21.1 Planck and Quantum Nature of Light

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe blackbody radiation
- Define quantum states and their relationship to modern physics
- Calculate the quantum energy of lights
- Explain how photon energies vary across divisions of the electromagnetic spectrum

Section Key Terms

blackbody	quantized	quantum	ultraviolet catastrophe

Blackbodies

Our first story of curious significance begins with a T-shirt. You are likely aware that wearing a tight black T-shirt outside on a hot day provides a significantly less comfortable experience than wearing a white shirt. Black shirts, as well as all other black objects, will absorb and re-emit a significantly greater amount of radiation from the sun. This shirt is a good approximation of what is called a **blackbody**.

A perfect blackbody is one that absorbs and re-emits all radiated energy that is incident upon it. Imagine wearing a tight shirt that did this! This phenomenon is often modeled with quite a different scenario. Imagine carving a small hole in an oven that can be heated to very high temperatures. As the temperature of this container gets hotter and hotter, the radiation out of this dark hole would increase as well, re-emitting all energy provided it by the increased temperature. The hole may even begin to glow in different colors as the temperature is increased. Like a burner on your stove, the hole would glow red, then orange, then blue, as the temperature is increased. In time, the hole would continue to glow but the light would be invisible to our eyes. This container is a good model of a perfect blackbody.

It is the analysis of blackbodies that led to one of the most consequential discoveries of the twentieth century. Take a moment to carefully examine Figure 21.2. What relationships exist? What trends can you see? The more time you spend interpreting this figure, the closer you will be to understanding quantum physics!

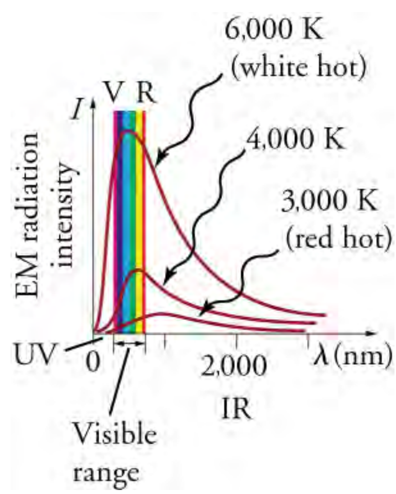


Figure 21.2 Graphs of blackbody radiation (from an ideal radiator) at three different radiator temperatures. The intensity or rate of radiation emission increases dramatically with temperature, and the peak of the spectrum shifts toward the visible and ultraviolet parts of the spectrum. The shape of the spectrum cannot be described with classical physics.

TIPS FOR SUCCESS

When encountering a new graph, it is best to try to interpret the graph before you read about it. Doing this will make the following text more meaningful and will help to remind yourself of some of the key concepts within the section.

Understanding Blackbody Graphs

Figure 21.2 is a plot of radiation intensity against radiated wavelength. In other words, it shows how the intensity of radiated light changes when a blackbody is heated to a particular temperature.

It may help to just follow the bottom-most red line labeled 3,000 K, red hot. The graph shows that when a blackbody acquires a temperature of 3,000 K, it radiates energy across the electromagnetic spectrum. However, the energy is most intensely emitted at a wavelength of approximately 1000 nm. This is in the infrared portion of the electromagnetic spectrum. While a body at this temperature would appear *red-hot* to our eyes, it would truly appear 'infrared-hot' if we were able to see the entire spectrum.

A few other important notes regarding Figure 21.2:

- As temperature increases, the total amount of energy radiated increases. This is shown by examining the area underneath each line.
- Regardless of temperature, all red lines on the graph undergo a consistent pattern. While electromagnetic radiation is emitted throughout the spectrum, the intensity of this radiation peaks at one particular wavelength.
- As the temperature changes, the wavelength of greatest radiation intensity changes. At 4,000 K, the radiation is most intense in the yellow-green portion of the spectrum. At 6,000 K, the blackbody would radiate *white hot,* due to intense radiation throughout the visible portion of the electromagnetic spectrum. Remember that white light is the emission of all visible colors simultaneously.
- As the temperature increases, the frequency of light providing the greatest intensity increases as well. Recall the equation $v = f\lambda$. Because the speed of light is constant, frequency and wavelength are inversely related. This is verified by the leftward movement of the three red lines as temperature is increased.

While in science it is important to categorize observations, theorizing as to why the observations exist is crucial to scientific advancement. Why doesn't a blackbody emit radiation evenly across all wavelengths? Why does the temperature of the body change the peak wavelength that is radiated? Why does an increase in temperature cause the peak wavelength emitted to decrease? It is questions like these that drove significant research at the turn of the twentieth century. And within the context of these questions, Max Planck discovered something of tremendous importance.

Planck's Revolution

The prevailing theory at the time of Max Planck's discovery was that intensity and frequency were related by the equation $I = \frac{2kT}{\lambda^2}$. This equation, derived from classical physics and using wave phenomena, infers that as wavelength increases, the intensity of energy provided will decrease with an inverse-squared relationship. This relationship is graphed in Figure 21.3 and shows a troubling trend. For starters, it should be apparent that the graph from this equation does not match the blackbody graphs found experimentally. Additionally, it shows that for an object of any temperature, there should be an infinite amount of energy quickly emitted in the shortest wavelengths. When theory and experimental results clash, it is important to re-evaluate both models. The disconnect between theory and reality was termed the **ultraviolet catastrophe**.

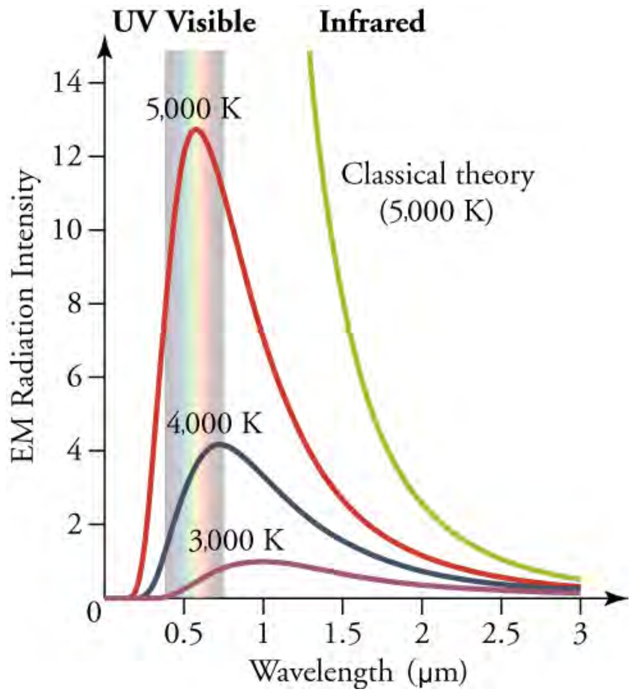


Figure 21.3 The graph above shows the true spectral measurements by a blackbody against those predicted by the classical theory at the time. The discord between the predicted classical theory line and the actual results is known as the ultraviolet catastrophe.

Due to concerns over the ultraviolet catastrophe, Max Planck began to question whether another factor impacted the relationship between intensity and wavelength. This factor, he posited, should affect the probability that short wavelength light would be emitted. Should this factor reduce the probability of short wavelength light, it would cause the radiance curve to not progress infinitely as in the classical theory, but would instead cause the curve to precipitate back downward as is shown in the 5,000 K, 4,000 K, and 3,000 K temperature lines of the graph in Figure 21.3. Planck noted that this factor, whatever it may be, must also be dependent on temperature, as the intensity decreases at lower and lower wavelengths as the temperature increases.

The determination of this *probability factor* was a groundbreaking discovery in physics, yielding insight not just into light but also into energy and matter itself. It would be the basis for Planck's 1918 Nobel Prize in Physics and would result in the transition of physics from classical to modern understanding. In an attempt to determine the cause of the *probability factor,* Max Planck constructed a new theory. This theory, which created the branch of physics called quantum mechanics, speculated that the energy radiated by the blackbody could exist only in specific numerical, or **quantum**, states. This theory is described by the equation $E = nhf$, where n is any nonnegative integer (0, 1, 2, 3, …) and h is Planck's constant, given by $h = 6.626 \times 10^{-34} \text{J} \cdot \text{s}$, and f is frequency.

Through this equation, Planck's probability factor can be more clearly understood. Each frequency of light provides a specific quantized amount of energy. Low frequency light, associated with longer wavelengths would provide a smaller amount of energy, while high frequency light, associated with shorter wavelengths, would provide a larger amount of energy. For specified temperatures with specific total energies, it makes sense that more low frequency light would be radiated than high frequency light. To a degree, the relationship is like pouring coins through a funnel. More of the smaller pennies would be able to pass through the funnel than the larger quarters. In other words, because the value of the coin is somewhat related to the size of the coin, the probability of a quarter passing through the funnel is reduced!

Furthermore, an increase in temperature would signify the presence of higher energy. As a result, the greater amount of total blackbody energy would allow for more of the high frequency, short wavelength, energies to be radiated. This permits the peak of the blackbody curve to drift leftward as the temperature increases, as it does from the 3,000 K to 4,000 K to 5,000 K values. Furthering our coin analogy, consider a wider funnel. This funnel would permit more quarters to pass through and allow for a reduction in concern about the *probability factor*.

In summary, it is the interplay between the predicted classical model and the quantum probability that creates the curve depicted in Figure 21.3. Just as quarters have a higher currency denomination than pennies, higher frequencies come with larger amounts of energy. However, just as the probability of a quarter passing through a fixed diameter funnel is reduced, so is the probability of a high frequency light existing in a fixed temperature object. As is often the case in physics, it is the balancing of multiple incredible ideas that finally allows for better understanding.

Quantization

It may be helpful at this point to further consider the idea of quantum states. Atoms, molecules, and fundamental electron and proton charges are all examples of physical entities that are **quantized**—that is, they appear only in certain discrete values and do not have every conceivable value. On the macroscopic scale, this is not a revolutionary concept. A standing wave on a string allows only particular harmonics described by integers. Going up and down a hill using discrete stair steps causes your potential energy to take on discrete values as you move from step to step. Furthermore, we cannot have a fraction of an atom, or part of an electron's charge, or 14.33 cents. Rather, everything is built of integral multiples of these substructures.

That said, to discover quantum states within a phenomenon that science had always considered continuous would certainly be surprising. When Max Planck was able to use quantization to correctly describe the experimentally known shape of the blackbody spectrum, it was the first indication that energy was quantized on a small scale as well. This discovery earned Planck the Nobel Prize in Physics in 1918 and was such a revolutionary departure from classical physics that Planck himself was reluctant to accept his own idea. The general acceptance of Planck's energy quantization was greatly enhanced by Einstein's explanation of the photoelectric effect (discussed in the next section), which took energy quantization a step further.

Figure 21.4 The German physicist Max Planck had a major influence on the early development of quantum mechanics, being the first to recognize that energy is sometimes quantized. Planck also made important contributions to special relativity and classical physics. (credit: Library of Congress, Prints and Photographs Division, Wikimedia Commons)

WORKED EXAMPLE

How Many Photons per Second Does a Typical Light Bulb Produce?

Assuming that 10 percent of a 100-W light bulb's energy output is in the visible range (typical for incandescent bulbs) with an average wavelength of 580 nm, calculate the number of visible photons emitted per second.

Strategy

The number of visible photons per second is directly related to the amount of energy emitted each second, also known as the bulb's power. By determining the bulb's power, the energy emitted each second can be found. Since the power is given in watts, which is joules per second, the energy will be in joules. By comparing this to the amount of energy associated with each photon, the number of photons emitted each second can be determined.

Solution

The power in visible light production is 10.0 percent of 100 W, or 10.0 J/s. The energy of the average visible photon is found by substituting the given average wavelength into the formula

$$E = nhf = \frac{nhc}{\lambda}.$$

By rearranging the above formula to determine energy per photon, this produces

$$E/n = \frac{(6.63 \times 10^{-34}\,\mathrm{J \cdot s})(3.00 \times 10^{8}\,\mathrm{m/s})}{580 \times 10^{-9}\,\mathrm{m}} = 3.43 \times 10^{-19}\,\mathrm{J/photon}. \tag{21.1}$$

The number of visible photons per second is thus

$$\frac{\text{photons}}{\text{sec}} = \frac{10.0\text{J/s}}{3.43 \times 10^{-19}\text{J/photon}} = 2.92 \times 10^{19}\text{photons/s}.$$

Discussion

This incredible number of photons per second is verification that individual photons are insignificant in ordinary human experience. However, it is also a verification of our everyday experience—on the macroscopic scale, photons are so small that quantization becomes essentially continuous.

WORKED EXAMPLE

How does Photon Energy Change with Various Portions of the EM Spectrum?

Refer to the Graphs of Blackbody Radiation shown in the first figure in this section. Compare the energy necessary to radiate one photon of infrared light and one photon of visible light.

Strategy

To determine the energy radiated, it is necessary to use the equation $E = nhf$. It is also necessary to find a representative frequency for infrared light and visible light.

Solution

According to the first figure in this section, one representative wavelength for infrared light is 2000 nm (2.000×10^{-6} m). The associated frequency of an infrared light is

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}\text{m/s}}{2.000 \times 10^{-6}\text{m}} = 1.50 \times 10^{14}\text{Hz}. \tag{21.2}$$

Using the equation $E = nhf$, the energy associated with one photon of representative infrared light is

$$\frac{E}{n} = h \cdot f = \left(6.63 \times 10^{-34}\text{J} \cdot \text{s}\right)\left(1.50 \times 10^{14}\text{Hz}\right) = 9.95 \times 10^{-20}\frac{\text{J}}{\text{photon}}. \tag{21.3}$$

The same process above can be used to determine the energy associated with one photon of representative visible light. According to the first figure in this section, one representative wavelength for visible light is 500 nm.

$$f = \frac{c}{\lambda} = \frac{3.00 \times 10^{8}\text{m/s}}{5.00 \times 10^{-7}\text{m}} = 6.00 \times 10^{14}\text{Hz}. \tag{21.4}$$

$$\frac{E}{n} = h \cdot f = \left(6.63 \times 10^{-34}\text{J} \cdot \text{s}\right)\left(6.00 \times 10^{14}\text{Hz}\right) = 3.98 \times 10^{-19}\frac{\text{J}}{\text{photon}}. \tag{21.5}$$

Discussion

This example verifies that as the wavelength of light decreases, the quantum energy increases. This explains why a fire burning with a blue flame is considered more dangerous than a fire with a red flame. Each photon of short-wavelength blue light emitted carries a greater amount of energy than a long-wavelength red light. This example also helps explain the differences in the 3,000 K, 4,000 K, and 6,000 K lines shown in the first figure in this section. As the temperature is increased, more energy is available for a greater number of short-wavelength photons to be emitted.

Practice Problems

1. An AM radio station broadcasts at a frequency of 1,530 kHz . What is the energy in Joules of a photon emitted from this station?
   a. 10.1×10^{-26} J
   b. 1.01×10^{-28} J
   c. 1.01×10^{-29} J
   d. 1.01×10^{-27} J
2. A photon travels with energy of 1.0 eV. What type of EM radiation is this photon?
   a. visible radiation

b. microwave radiation
c. infrared radiation
d. ultraviolet radiation

Check Your Understanding

3. Do reflective or absorptive surfaces more closely model a perfect blackbody?
   a. reflective surfaces
   b. absorptive surfaces

4. A black T-shirt is a good model of a blackbody. However, it is not perfect. What prevents a black T-shirt from being considered a perfect blackbody?
   a. The T-shirt reflects some light.
   b. The T-shirt absorbs all incident light.
   c. The T-shirt re-emits all the incident light.
   d. The T-shirt does not reflect light.

5. What is the mathematical relationship linking the energy of a photon to its frequency?
   a. $E = \frac{hf}{n}$
   b. $E = \frac{nh}{f}$
   c. $E = \frac{nf}{h}$
   d. $E = nhf$

6. Why do we not notice quantization of photons in everyday experience?
   a. because the size of each photon is very large
   b. because the mass of each photon is so small
   c. because the energy provided by photons is very large
   d. because the energy provided by photons is very small

7. Two flames are observed on a stove. One is red while the other is blue. Which flame is hotter?
   a. The red flame is hotter because red light has lower frequency.
   b. The red flame is hotter because red light has higher frequency.
   c. The blue flame is hotter because blue light has lower frequency.
   d. The blue flame is hotter because blue light has higher frequency.

8. Your pupils dilate when visible light intensity is reduced. Does wearing sunglasses that lack UV blockers increase or decrease the UV hazard to your eyes? Explain.
   a. Increase, because more high-energy UV photons can enter the eye.
   b. Increase, because less high-energy UV photons can enter the eye.
   c. Decrease, because more high-energy UV photons can enter the eye.
   d. Decrease, because less high-energy UV photons can enter the eye.

9. The temperature of a blackbody radiator is increased. What will happen to the most intense wavelength of light emitted as this increase occurs?
   a. The wavelength of the most intense radiation will vary randomly.
   b. The wavelength of the most intense radiation will increase.
   c. The wavelength of the most intense radiation will remain unchanged.
   d. The wavelength of the most intense radiation will decrease.

21.2 Einstein and the Photoelectric Effect

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Einstein's explanation of the photoelectric effect
- Describe how the photoelectric effect could not be explained by classical physics
- Calculate the energy of a photoelectron under given conditions
- Describe use of the photoelectric effect in biological applications, photoelectric devices and movie soundtracks

Section Key Terms

electric eye photoelectric effect photoelectron photon

The Photoelectric Effect

Teacher Support
[EL]Ask the students what they think the term *photoelectric* means. How does the term relate to its definition?

When light strikes certain materials, it can eject electrons from them. This is called the **photoelectric effect**, meaning that light (*photo*) produces electricity. One common use of the photoelectric effect is in light meters, such as those that adjust the automatic iris in various types of cameras. Another use is in solar cells, as you probably have in your calculator or have seen on a rooftop or a roadside sign. These make use of the photoelectric effect to convert light into electricity for running different devices.

Figure 21.5 The photoelectric effect can be observed by allowing light to fall on the metal plate in this evacuated tube. Electrons ejected by the light are collected on the collector wire and measured as a current. A retarding voltage between the collector wire and plate can then be adjusted so as to determine the energy of the ejected electrons. (credit: P. P. Urone)

Revolutionary Properties of the Photoelectric Effect

When Max Planck theorized that energy was quantized in a blackbody radiator, it is unlikely that he would have recognized just how revolutionary his idea was. Using tools similar to the light meter in Figure 21.5, it would take a scientist of Albert Einstein's stature to fully discover the implications of Max Planck's radical concept.

Through careful observations of the photoelectric effect, Albert Einstein realized that there were several characteristics that could be explained only if *EM radiation is itself quantized*. While these characteristics will be explained a bit later in this section, you can already begin to appreciate why Einstein's idea is very important. It means that the apparently continuous stream of energy in an EM wave is actually not a continuous stream at all. In fact, the EM wave itself is actually composed of tiny quantum packets of energy called **photons**.

In equation form, Einstein found the energy of a photon or **photoelectron** to be

$$E = h\text{f},$$

where E is the energy of a photon of frequency f and h is Planck's constant. A beam from a flashlight, which to this point had been considered a wave, instead could now be viewed as a series of photons, each providing a specific amount of energy see Figure 21.6. Furthermore, the amount of energy within each individual photon is based upon its individual frequency, as dictated by $E = hf$. As a result, the total amount of energy provided by the beam could now be viewed as the sum of all frequency-dependent photon energies added together.

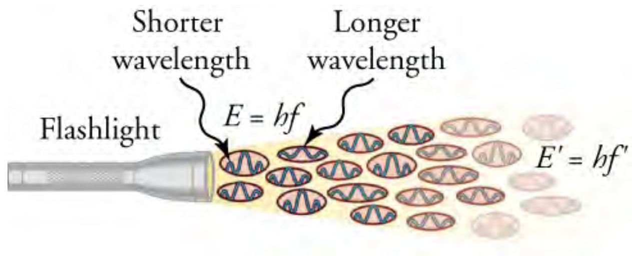


Figure 21.6 An EM wave of frequency *f* is composed of photons, or individual quanta of EM radiation. The energy of each photon is $E = hf$, where *h* is Planck's constant and *f* is the frequency of the EM radiation. Higher intensity means more photons per unit area per second. The flashlight emits large numbers of photons of many different frequencies, hence others have energy $E' = hf'$, and so on.

Just as with Planck's blackbody radiation, Einstein's concept of the photon could take hold in the scientific community only if it could succeed where classical physics failed. The photoelectric effect would be a key to demonstrating Einstein's brilliance.

Consider the following five properties of the photoelectric effect. All of these properties are consistent with the idea that individual photons of EM radiation are absorbed by individual electrons in a material, with the electron gaining the photon's energy. Some of these properties are inconsistent with the idea that EM radiation is a simple wave. For simplicity, let us consider what happens with monochromatic EM radiation in which all photons have the same energy *hf*.

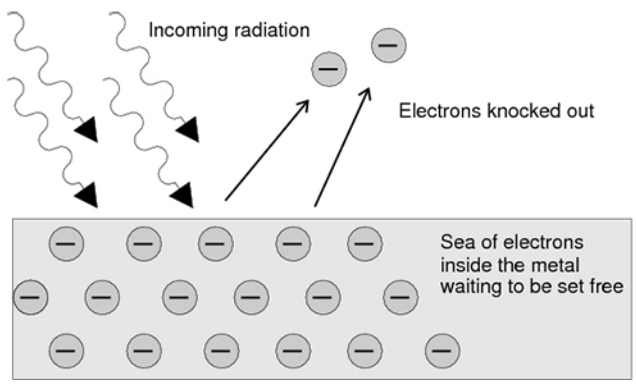


Figure 21.7 Incident radiation strikes a clean metal surface, ejecting multiple electrons from it. The manner in which the frequency and intensity of the incoming radiation affect the ejected electrons strongly suggests that electromagnetic radiation is quantized. This event, called the photoelectric effect, is strong evidence for the existence of photons.

1. If we vary the frequency of the EM radiation falling on a clean metal surface, we find the following: For a given material, there is a threshold frequency f_0 for the EM radiation below which no electrons are ejected, regardless of intensity. Using the photon model, the explanation for this is clear. Individual photons interact with individual electrons. Thus if the energy of an individual photon is too low to break an electron away, no electrons will be ejected. However, if EM radiation were a simple wave, sufficient energy could be obtained simply by increasing the intensity.
2. *Once EM radiation falls on a material, electrons are ejected without delay*. As soon as an individual photon of sufficiently high frequency is absorbed by an individual electron, the electron is ejected. If the EM radiation were a simple wave, several minutes would be required for sufficient energy to be deposited at the metal surface in order to eject an electron.
3. The number of electrons ejected per unit time is proportional to the intensity of the EM radiation and to no other characteristic. High-intensity EM radiation consists of large numbers of photons per unit area, with all photons having the same characteristic energy, *hf*. The increased number of photons per unit area results in an increased number of electrons per unit area ejected.
4. If we vary the intensity of the EM radiation and measure the energy of ejected electrons, we find the following: *The maximum kinetic energy of ejected electrons is independent of the intensity of the EM radiation*. Instead, as noted in point 3 above, increased intensity results in more electrons of the same energy being ejected. If EM radiation were a simple wave, a higher intensity could transfer more energy, and higher-energy electrons would be ejected.
5. The kinetic energy KE of an ejected electron equals the photon energy minus the binding energy BE of the electron in the

specific material. An individual photon can give all of its energy to an electron. The photon's energy is partly used to break the electron away from the material. The remainder goes into the ejected electron's kinetic energy. In equation form, this is given by

$$KE_e = hf - BE, \tag{21.6}$$

where KE_e is the maximum kinetic energy of the ejected electron, hf is the photon's energy, and BE is the binding energy of the electron to the particular material. This equation explains the properties of the photoelectric effect quantitatively and demonstrates that BE is the minimum amount of energy necessary to eject an electron. If the energy supplied is less than BE, the electron cannot be ejected. The binding energy can also be written as $BE = hf_0$, where f_0 is the threshold frequency for the particular material. Figure 21.8 shows a graph of maximum KE_e versus the frequency of incident EM radiation falling on a particular material.

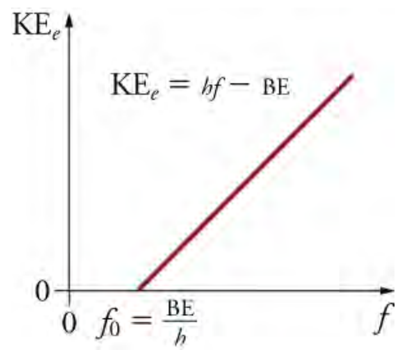


Figure 21.8 A graph of the kinetic energy of an ejected electron, KE$_e$, versus the frequency of EM radiation impinging on a certain material. There is a threshold frequency below which no electrons are ejected, because the individual photon interacting with an individual electron has insufficient energy to break it away. Above the threshold energy, KE$_e$ increases linearly with f, consistent with KE$_e$ = hf – BE. The slope of this line is h, so the data can be used to determine Planck's constant experimentally.

TIPS FOR SUCCESS

The following five pieces of information can be difficult to follow without some organization. It may be useful to create a table of expected results of each of the five properties, with one column showing the classical wave model result and one column showing the modern photon model result.
The table may look something like Table 21.1

	Classical Wave Model	Modern Photon Model
Threshold Frequency		
Electron Ejection Delay		
Intensity of EM Radiation		
Speed of Ejected Electrons		
Relationship between Kinetic Energy and Binding Energy		

Table 21.1 Table of Expected Results

Virtual Physics

Photoelectric Effect

Click to view content (http://www.openstax.org/l/28photoelectric)

In this demonstration, see how light knocks electrons off a metal target, and recreate the experiment that spawned the field of quantum mechanics.

GRASP CHECK

In the circuit provided, what are the three ways to increase the current?

a. decrease the intensity, decrease the frequency, alter the target
b. decrease the intensity, decrease the frequency, don't alter the target
c. increase the intensity, increase the frequency, alter the target
d. increase the intensity, increase the frequency, alter the target

WORKED EXAMPLE

Photon Energy and the Photoelectric Effect: A Violet Light

(a) What is the energy in joules and electron volts of a photon of 420-nm violet light? (b) What is the maximum kinetic energy of electrons ejected from calcium by 420 nm violet light, given that the binding energy of electrons for calcium metal is 2.71 eV?

Strategy

To solve part (a), note that the energy of a photon is given by $E = hf$. For part (b), once the energy of the photon is calculated, it is a straightforward application of $KE_e = hf - BE$ to find the ejected electron's maximum kinetic energy, since BE is given.

Solution for (a)

Photon energy is given by

$$E = hf.$$

Since we are given the wavelength rather than the frequency, we solve the familiar relationship $c = f\lambda$ for the frequency, yielding

$$f = \frac{c}{\lambda} \tag{21.7}$$

Combining these two equations gives the useful relationship

$$E = \frac{hc}{\lambda}. \tag{21.8}$$

Now substituting known values yields

$$E = \frac{\left(6.63 \times 10^{-34}\text{J} \cdot \text{s}\right)\left(3.00 \times 10^{8} m/s\right)}{4.20 \times 10^{-7} m} = 4.74 \times 10^{-19}\text{J}. \tag{21.9}$$

Converting to eV, the energy of the photon is

$$E = (4.74 \times 10^{-19}\text{J} \cdot \text{s})\frac{1 \text{ eV}}{1.60 \times 10^{-19}\text{J}} = 2.96 \text{ eV}. \tag{21.10}$$

Solution for (b)

Finding the kinetic energy of the ejected electron is now a simple application of the equation $KE_e = hf - BE$. Substituting the photon energy and binding energy yields

$$KE_e = hf - BE = 2.96 \text{ eV} - 2.71 \text{ eV} = 0.25 \text{ eV}. \tag{21.11}$$

Discussion

The energy of this 420 nm photon of violet light is a tiny fraction of a joule, and so it is no wonder that a single photon would be difficult for us to sense directly—humans are more attuned to energies on the order of joules. But looking at the energy in electron volts, we can see that this photon has enough energy to affect atoms and molecules. A DNA molecule can be broken with about 1 eV of energy, for example, and typical atomic and molecular energies are on the order of eV, so that the photon in this example could have biological effects, such as sunburn. The ejected electron has rather low energy, and it would not travel far, except in a vacuum. The electron would be stopped by a retarding potential of only 0.26 eV, a slightly larger KE than calculated above. In fact, if the photon wavelength were longer and its energy less than 2.71 eV, then the formula would give a negative kinetic energy, an impossibility. This simply means that the 420 nm photons with their 2.96 eV energy are not much above the frequency threshold. You can see for yourself that the threshold wavelength is 458 nm (blue light). This means that if calcium metal were used in a light meter, the meter would be insensitive to wavelengths longer than those of blue light. Such a light meter would be completely insensitive to red light, for example.

Practice Problems

10. What is the longest-wavelength EM radiation that can eject a photoelectron from silver, given that the bonding energy is 4.73 eV ? Is this radiation in the visible range?

a. 2.63×10^{-7} m; No, the radiation is in microwave region.
b. 2.63×10^{-7} m; No, the radiation is in visible region.
c. 2.63×10^{-7} m; No, the radiation is in infrared region.
d. 2.63×10^{-7} m; No, the radiation is in ultraviolet region.

11. What is the maximum kinetic energy in eV of electrons ejected from sodium metal by 450-nm EM radiation, given that the binding energy is 2.28 eV?

a. 0.48 V
b. 0.82 eV
c. 1.21 eV
d. 0.48 eV

Technological Applications of the Photoelectric Effect

While Einstein's understanding of the photoelectric effect was a transformative discovery in the early 1900s, its presence is ubiquitous today. If you have watched streetlights turn on automatically in response to the setting sun, stopped elevator doors from closing simply by putting your hands between them, or turned on a water faucet by sliding your hands near it, you are familiar with the **electric eye**, a name given to a group of devices that use the photoelectric effect for detection.

All these devices rely on photoconductive cells. These cells are activated when light is absorbed by a semi-conductive material, knocking off a free electron. When this happens, an electron void is left behind, which attracts a nearby electron. The movement of this electron, and the resultant chain of electron movements, produces a current. If electron ejection continues, further holes are created, thereby increasing the electrical conductivity of the cell. This current can turn switches on and off and activate various familiar mechanisms.

One such mechanism takes place where you may not expect it. Next time you are at the movie theater, pay close attention to the sound coming out of the speakers. This sound is actually created using the photoelectric effect! The audiotape in the projector booth is a transparent piece of film of varying width. This film is fed between a photocell and a bright light produced by an exciter lamp. As the transparent portion of the film varies in width, the amount of light that strikes the photocell varies as well. As a result, the current in the photoconductive circuit changes with the width of the filmstrip. This changing current is converted to a changing frequency, which creates the soundtrack commonly heard in the theater.

WORK IN PHYSICS

Solar Energy Physicist

According to the U.S. Department of Energy, Earth receives enough sunlight each hour to power the entire globe for a year. While converting all of this energy is impossible, the job of the solar energy physicist is to explore and improve upon solar energy conversion technologies so that we may harness more of this abundant resource.

The field of solar energy is not a new one. For over half a century, satellites and spacecraft have utilized photovoltaic cells to create current and power their operations. As time has gone on, scientists have worked to adapt this process so that it may be used in homes, businesses, and full-scale power stations using solar cells like the one shown in Figure 21.9.

Figure 21.9 A solar cell is an example of a photovoltaic cell. As light strikes the cell, the cell absorbs the energy of the photons. If this energy exceeds the binding energy of the electrons, then electrons will be forced to move in the cell, thereby producing a current. This current may be used for a variety of purposes. (credit: U.S. Department of Energy)

Solar energy is converted to electrical energy in one of two manners: direct transfer through photovoltaic cells or thermal conversion through the use of a CSP, concentrating solar power, system. Unlike electric eyes, which trip a mechanism when current is lost, photovoltaic cells utilize semiconductors to directly transfer the electrons released through the photoelectric effect into a directed current. The energy from this current can then be converted for storage, or immediately used in an electric process. A CSP system is an indirect method of energy conversion. In this process, light from the Sun is channeled using parabolic mirrors. The light from these mirrors strikes a thermally conductive material, which then heats a pool of water. This water, in turn, is converted to steam, which turns a turbine and creates electricity. While indirect, this method has long been the traditional means of large-scale power generation.

There are, of course, limitations to the efficacy of solar power. Cloud cover, nightfall, and incident angle strike at high altitudes are all factors that directly influence the amount of light energy available. Additionally, the creation of photovoltaic cells requires rare-earth minerals that can be difficult to obtain. However, the major role of a solar energy physicist is to find ways to improve the efficiency of the solar energy conversion process. Currently, this is done by experimenting with new semi conductive materials, by refining current energy transfer methods, and by determining new ways of incorporating solar structures into the current power grid.

Additionally, many solar physicists are looking into ways to allow for increased solar use in impoverished, more remote locations. Because solar energy conversion does not require a connection to a large-scale power grid, research into thinner, more mobile materials will permit remote cultures to use solar cells to convert sunlight collected during the day into stored energy that can then be used at night.

Regardless of the application, solar energy physicists are an important part of the future in responsible energy growth. While a doctoral degree is often necessary for advanced research applications, a bachelor's or master's degree in a related science or engineering field is typically enough to gain access into the industry. Computer skills are very important for energy modeling, including knowledge of CAD software for design purposes. In addition, the ability to collaborate and communicate with others is critical to becoming a solar energy physicist.

GRASP CHECK

What role does the photoelectric effect play in the research of a solar energy physicist?

a. The understanding of photoelectric effect allows the physicist to understand the generation of light energy when using photovoltaic cells.
b. The understanding of photoelectric effect allows the physicist to understand the generation of electrical energy when using photovoltaic cells.
c. The understanding of photoelectric effect allows the physicist to understand the generation of electromagnetic energy when using photovoltaic cells.
d. The understanding of photoelectric effect allows the physicist to understand the generation of magnetic energy when using photovoltaic cells.

Check Your Understanding

12. How did Einstein's model of photons change the view of a beam of energy leaving a flashlight?
   a. A beam of light energy is now considered a continual stream of wave energy, not photons.
   b. A beam of light energy is now considered a collection of photons, each carrying its own individual energy.

13. True or false—Visible light is the only type of electromagnetic radiation that can cause the photoelectric effect.
   a. false
   b. true

14. Is the photoelectric effect a direct consequence of the wave character of EM radiation or the particle character of EM radiation?
   a. The photoelectric effect is a direct consequence of the particle nature of EM radiation.
   b. The photoelectric effect is a direct consequence of the wave nature of EM radiation.
   c. The photoelectric effect is a direct consequence of both the wave and particle nature of EM radiation.
   d. The photoelectric effect is a direct consequence of neither the wave nor the particle nature of EM radiation.

15. Which aspects of the photoelectric effect can only be explained using photons?
   a. aspects 1, 2, and 3
   b. aspects 1, 2, and 4
   c. aspects 1, 2, 4 and 5
   d. aspects 1, 2, 3, 4 and 5

16. In a photovoltaic cell, what energy transformation takes place?
   a. Solar energy transforms into electric energy.
   b. Solar energy transforms into mechanical energy.
   c. Solar energy transforms into thermal energy.
   d. In a photovoltaic cell, thermal energy transforms into electric energy.

17. True or false—A current is created in a photoconductive cell, even if only one electron is expelled from a photon strike.
   a. false
   b. true

18. What is a photon and how is it different from other fundamental particles?
   a. A photon is a quantum packet of energy; it has infinite mass.
   b. A photon is a quantum packet of energy; it is massless.
   c. A photon is a fundamental particle of an atom; it has infinite mass.
   d. A photon is a fundamental particle of an atom; it is massless.

21.3 The Dual Nature of Light

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the Compton effect
- Calculate the momentum of a photon
- Explain how photon momentum is used in solar sails
- Explain the particle-wave duality of light

Section Key Terms

Compton effect	particle-wave duality	photon momentum

Photon Momentum

Do photons abide by the fundamental properties of physics? Can packets of electromagnetic energy possibly follow the same rules as a ping-pong ball or an electron? Although strange to consider, the answer to both questions is yes.

Despite the odd nature of photons, scientists prior to Einstein had long suspected that the fundamental particle of electromagnetic radiation shared properties with our more macroscopic particles. This is no clearer than when considering the photoelectric effect, where photons knock electrons out of a substance. While it is strange to think of a massless particle exhibiting momentum, it is now a well-established fact within the scientific community. Figure 21.10 shows macroscopic evidence of **photon momentum**.

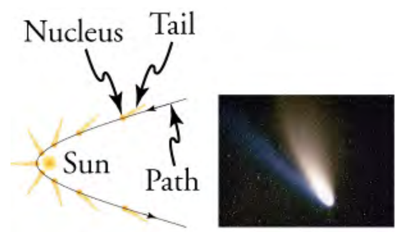


Figure 21.10 The tails of the Hale-Bopp comet point away from the Sun, evidence that light has momentum. Dust emanating from the body of the comet forms this tail. Particles of dust are pushed away from the Sun by light reflecting from them. The blue, ionized gas tail is also produced by photons interacting with atoms in the comet material. (credit: Geoff Chester, U.S. Navy, via Wikimedia Commons)

Figure 21.10 shows a comet with two prominent tails. Comet tails are composed of gases and dust evaporated from the body of the comet and ionized gas. What most people do not know about the tails is that they always point *away* from the Sun rather than trailing behind the comet. This can be seen in the diagram.

Why would this be the case? The evidence indicates that the dust particles of the comet are forced away from the Sun when photons strike them. Evidently, photons carry momentum in the direction of their motion away from the Sun, and some of this momentum is transferred to dust particles in collisions. The blue tail is caused by the solar wind, a stream of plasma consisting primarily of protons and electrons evaporating from the corona of the Sun.

Momentum, The Compton Effect, and Solar Sails

Momentum is conserved in quantum mechanics, just as it is in relativity and classical physics. Some of the earliest direct experimental evidence of this came from the scattering of X-ray photons by electrons in substances, a phenomenon discovered by American physicist Arthur H. Compton (1892–1962). Around 1923, Compton observed that X-rays reflecting from materials had decreased energy and correctly interpreted this as being due to the scattering of the X-ray photons by electrons. This phenomenon could be handled as a collision between two particles—a photon and an electron at rest in the material. After careful observation, it was found that both energy and momentum were conserved in the collision. See Figure 21.11. For the discovery of this conserved scattering, now known as the **Compton effect**, Arthur Compton was awarded the Nobel Prize in 1929.

Shortly after the discovery of Compton scattering, the value of the photon momentum, $\mathbf{p} = \frac{h}{\lambda}$,

was determined by Louis de Broglie. In this equation, called the de Broglie relation, h represents Planck's constant and λ is the photon wavelength.

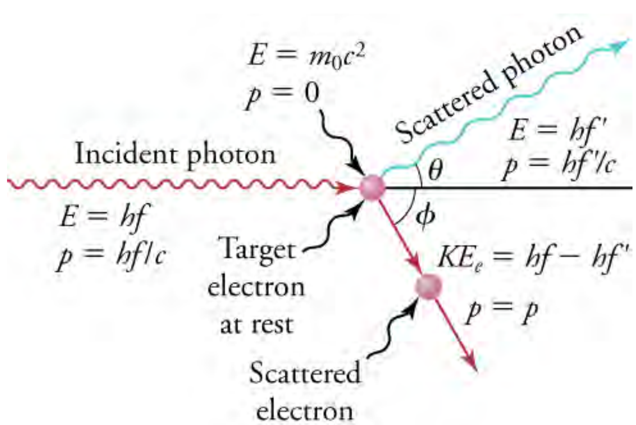


Figure 21.11 The Compton effect is the name given to the scattering of a photon by an electron. Energy and momentum are conserved, resulting in a reduction of both for the scattered photon.

We can see that photon momentum is small, since $\mathbf{p} = h/\lambda$. and h is very small. It is for this reason that we do not ordinarily observe photon momentum. Our mirrors do not recoil when light reflects from them, except perhaps in cartoons. Compton saw the effects of photon momentum because he was observing X-rays, which have a small wavelength and a relatively large momentum, interacting with the lightest of particles, the electron.

WORKED EXAMPLE

Electron and Photon Momentum Compared

(a) Calculate the momentum of a visible photon that has a wavelength of 500 nm. (b) Find the velocity of an electron having the same momentum. (c) What is the energy of the electron, and how does it compare with the energy of the photon?

Strategy

Finding the photon momentum is a straightforward application of its definition: $\mathbf{p} = h/\lambda$. If we find the photon momentum is small, we can assume that an electron with the same momentum will be nonrelativistic, making it easy to find its velocity and kinetic energy from the classical formulas.

Solution for (a)

Photon momentum is given by the de Broglie relation.

$$\mathbf{p} = \frac{h}{\lambda} \tag{21.12}$$

Entering the given photon wavelength yields

$$\mathbf{p} = \frac{6.63 \times 10^{-34} J \cdot s}{5.00 \times 10^{-7} \mathrm{m}} = 1.33 \times 10^{-27} \mathrm{kg} \cdot \mathrm{m/s}. \tag{21.13}$$

Solution for (b)

Since this momentum is indeed small, we will use the classical expression $p = mv$ to find the velocity of an electron with this momentum. Solving for v and using the known value for the mass of an electron gives

$$v = \frac{\mathrm{p}}{m} = \frac{1.33 \times 10^{-27} \mathrm{kg} \cdot \mathrm{m/s}}{9.11 \times 10^{-31} \mathrm{kg}} = 1{,}459.9 \text{ m/s} \approx 1{,}460 \text{ m/s}. \tag{21.14}$$

Solution for (c)

The electron has kinetic energy, which is classically given by

$$KE_e = \frac{1}{2} mv^2. \tag{21.15}$$

Thus,

$$KE_e = \frac{1}{2}(9.11 \times 10^{-31} \mathrm{kg})(1{,}456 \mathrm{m/s})^2 = 9.64 \times 10^{-25} \mathrm{J}. \tag{21.16}$$

Converting this to eV by multiplying by $\frac{(1 \text{ eV})}{(1.602 \times 10^{-19} \mathrm{J})}$ yields

$$KE_e = 6.02 \times 10^{-6} \mathrm{eV}. \tag{21.17}$$

The photon energy E is

$$E = \frac{hc}{\lambda} = \frac{\left(6.63 \times 10^{-34} \mathrm{J} \cdot \mathrm{s}\right)\left(3.00 \times 10^{8} \text{ m/s}\right)}{5.00 \times 10^{-7} \mathrm{m}} = 3.98 \times 10^{-19} \text{ J} = 2.48 \text{ eV}, \tag{21.18}$$

which is about five orders of magnitude greater.

Discussion

Even in huge numbers, the total momentum that photons carry is small. An electron that carries the same momentum as a 500-nm photon will have a 1,460 m/s velocity, which is clearly nonrelativistic. This is borne out by the experimental observation that it takes far less energy to give an electron the same momentum as a photon. That said, for high-energy photons interacting with small masses, photon momentum may be significant. Even on a large scale, photon momentum can have an effect if there are enough of them and if there is nothing to prevent the slow recoil of matter. Comet tails are one example, but there are also proposals to build space sails that use huge low-mass mirrors (made of aluminized Mylar) to reflect sunlight. In the vacuum of space, the mirrors would gradually recoil and could actually accelerate spacecraft within the solar system. See the following figure.

TIPS FOR SUCCESS

When determining energies in particle physics, it is more sensible to use the unit eV instead of Joules. Using eV will help you to recognize differences in magnitude more easily and will make calculations simpler. Also, eV is used by scientists to describe the binding energy of particles and their rest mass, so using eV will eliminate the need to convert energy quantities. Finally, eV is a convenient unit when linking electromagnetic forces to particle physics, as one eV is the amount energy given to an electron when placed in a field of 1-V potential difference.

Practice Problems

19. Find the momentum of a 4.00-cm wavelength microwave photon.

a. 0.83×10^{-32} kg · m/s
b. 1.66×10^{-34} kg · m/s
c. 0.83×10^{-34} kg · m/s
d. 1.66×10^{-32} kg · m/s

20. Calculate the wavelength of a photon that has the same momentum of a proton moving at 1.00 percent of the speed of light.

a. 2.43×10^{-10} m
b. 2.43×10^{-12} m
c. 1.32×10^{-15} m
d. 1.32×10^{-13} m

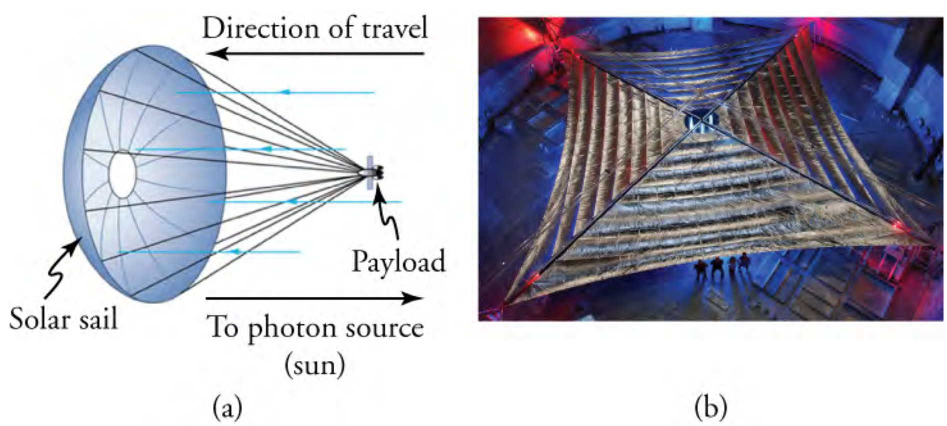


Figure 21.12 (a) Space sails have been proposed that use the momentum of sunlight reflecting from gigantic low-mass sails to propel spacecraft about the solar system. A Russian test model of this (the Cosmos 1) was launched in 2005, but did not make it into orbit due to a rocket failure. (b) A U.S. version of this, labeled LightSail-1, is scheduled for trial launches in 2016. It will have a 40 m2 sail. (credit: Kim Newton/NASA)

LINKS TO PHYSICS

LightSail-1 Project

"Provide ships or sails adapted to the heavenly breezes, and there will be some who will brave even that void."

— *Johannes Kepler (in a letter to Galileo Galilei in 1608)*

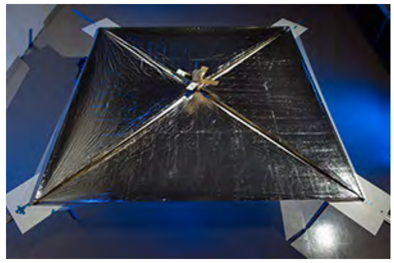


Figure 21.13 NASA's NanoSail-D, a precursor to LightSail-1, with its sails deployed. The Planetary Society will be launching LightSail-1 in early 2016. (credit: NASA/MSFC/D, Wikimedia Commons)

Traversing the Solar System using nothing but the Sun's power has long been a fantasy of scientists and science fiction writers alike. Though physicists like Compton, Einstein, and Planck all provided evidence of light's propulsive capacity, it is only recently that the technology has become available to truly put these visions into motion. In 2016, by sending a lightweight satellite into space, the LightSail-1 project is designed to do just that.

A citizen-funded project headed by the Planetary Society, the 5.45-million-dollar LightSail-1 project is set to launch two crafts into orbit around the Earth. Each craft is equipped with a 32-square-meter solar sail prepared to unfurl once a rocket has launched it to an appropriate altitude. The sails are made of large mirrors, each a quarter of the thickness of a trash bag, which will receive an impulse from the Sun's reflecting photons. Each time the Sun's photon strikes the craft's reflective surface and bounces off, it will provide a momentum to the sail much greater than if the photon were simply absorbed.

Attached to three tiny satellites called CubeSats, whose combined volume is no larger than a loaf of bread, the received momentum from the Sun's photons should be enough to record a substantial increase in orbital speed. The intent of the LightSail-1 mission is to prove that the technology behind photon momentum travel is sound and can be done cheaply. A test flight in May 2015 showed that the craft's Mylar sails could unfurl on command. With another successful result in 2016, the Planetary Society will be planning future versions of the craft with the hopes of eventually achieving interplanetary satellite travel. Though a few centuries premature, Kepler's fantastic vision may not be that far away.

If eventually set into interplanetary launch, what will be the effect of continual photon bombardment on the motion of a craft similar to LightSail-1?

a. It will result in continual acceleration of the craft.
b. It will first accelerate and then decelerate the craft.
c. It will first decelerate and then accelerate the craft.
d. It will result in the craft moving at constant velocity.

Particle-Wave Duality

We have long known that EM radiation is like a wave, capable of interference and diffraction. We now see that light can also be modeled as particles—massless photons of discrete energy and momentum. We call this twofold nature the **particle-wave duality**, meaning that EM radiation has properties of both particles and waves. This may seem contradictory, since we ordinarily deal with large objects that never act like both waves and particles. An ocean wave, for example, looks nothing like a grain of sand. However, this so-called duality is simply a term for properties of the photon analogous to phenomena we can observe directly, on a macroscopic scale. See Figure 21.14. If this term seems strange, it is because we do not ordinarily observe details on the quantum level directly, and our observations yield either particle-like *or* wave-like properties, but never both simultaneously.

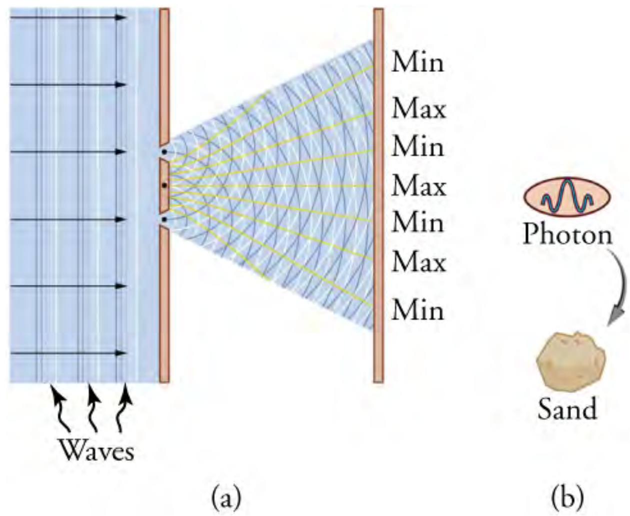


Figure 21.14 (a) The interference pattern for light through a double slit is a wave property understood by analogy to water waves. (b) The properties of photons having quantized energy and momentum and acting as a concentrated unit are understood by analogy to macroscopic particles.

Since we have a particle-wave duality for photons, and since we have seen connections between photons and matter in that both have momentum, it is reasonable to ask whether there is a particle-wave duality for matter as well. If the EM radiation we once thought to be a pure wave has particle properties, is it possible that matter has wave properties? The answer, strangely, is yes. The consequences of this are tremendous, as particle-wave duality has been a constant source of scientific wonder during the twentieth and twenty-first centuries.

Check Your Understanding

21. What fundamental physics properties were found to be conserved in Compton scattering?
  a. energy and wavelength
  b. energy and momentum
  c. mass and energy
  d. energy and angle

22. Why do classical or relativistic momentum equations not work in explaining the conservation of momentum that occurs in Compton scattering?
  a. because neither classical nor relativistic momentum equations utilize mass as a variable in their equations
  b. because relativistic momentum equations utilize mass as a variable in their formulas but classical momentum equations do not
  c. because classical momentum equations utilize mass as a variable in their formulas but relativistic momentum equations do not
  d. because both classical and relativistic momentum equations utilize mass as a variable in their formulas

23. If solar sails were constructed with more massive materials, how would this influence their effectiveness?
  a. The effect of the momentum would increase due to the decreased inertia of the sails.
  b. The effect of the momentum would reduce due to the decreased inertia of the sails.
  c. The effect of the momentum would increase due to the increased inertia of the sails.
  d. The effect of the momentum would be reduced due to the increased inertia of the sails.

24. True or false—It is possible to propel a solar sail craft using just particles within the solar wind.
  a. true
  b. false

25. True or false—Photon momentum more directly supports the wave model of light.
  a. false
  b. true

26. True or false—wave-particle duality exists for objects on the macroscopic scale.
   a. false
   b. true

27. What type of electromagnetic radiation was used in Compton scattering?
   a. visible light
   b. ultraviolet radiation
   c. radio waves
   d. X-rays

KEY TERMS

blackbody object that absorbs all radiated energy that strikes it and also emits energy across all wavelengths of the electromagnetic spectrum

Compton effect phenomenon whereby X-rays scattered from materials have decreased energy

electric eye group of devices that use the photoelectric effect for detection

particle-wave duality property of behaving like either a particle or a wave; the term for the phenomenon that all particles have wave-like characteristics and waves have particle-like characteristics

photoelectric effect phenomenon whereby some materials eject electrons when exposed to light

photoelectron electron that has been ejected from a material by a photon of light

photon a quantum, or particle, of electromagnetic radiation

photon momentum amount of momentum of a photon, calculated by $\mathbf{p} = \frac{h}{\lambda}$

quantized the fact that certain physical entities exist only with particular discrete values and not every conceivable value

quantum discrete packet or bundle of a physical entity such as energy

ultraviolet catastrophe misconception that blackbodies would radiate high frequency energy at a much higher rate than energy radiated at lower frequencies

SECTION SUMMARY

21.1 Planck and Quantum Nature of Light

- A blackbody will radiate energy across all wavelengths of the electromagnetic spectrum.
- Radiation of a blackbody will peak at a particular wavelength, dependent on the temperature of the blackbody.
- Analysis of blackbody radiation led to the field of quantum mechanics, which states that radiated energy can only exist in discrete quantum states.

21.2 Einstein and the Photoelectric Effect

- The photoelectric effect is the process in which EM radiation ejects electrons from a material.
- Einstein proposed photons to be quanta of EM radiation having energy $E = hf$, where f is the frequency of the radiation.
- All EM radiation is composed of photons. As Einstein explained, all characteristics of the photoelectric effect are due to the interaction of individual photons with individual electrons.
- The maximum kinetic energy KE_e of ejected electrons (photoelectrons) is given by $KE_e = hf - BE$, where hf is the photon energy and BE is the binding energy (or work function) of the electron in the particular material.

21.3 The Dual Nature of Light

- Compton scattering provided evidence that photon-electron interactions abide by the principles of conservation of momentum and conservation of energy.
- The momentum of individual photons, quantified by $\mathbf{p} = \frac{h}{\lambda}$, can be used to explain observations of comets and may lead to future space technologies.
- Electromagnetic waves and matter have both wave-like and particle-like properties. This phenomenon is defined as particle-wave duality.

KEY EQUATIONS

21.1 Planck and Quantum Nature of Light

quantum energy	$E = nhf$

21.2 Einstein and the Photoelectric Effect

energy of a photon	$E = hf$
maximum kinetic energy of a photoelectron	$KE_e = hf - BE$
binding energy of an electron	$BE = hf_O$

21.3 The Dual Nature of Light

momentum of a photon (deBroglie relation)	$\mathbf{p} = \frac{h}{\lambda}$

CHAPTER REVIEW

Concept Items

21.1 Planck and Quantum Nature of Light

1. What aspect of the blackbody spectrum forced Planck to propose quantization of energy levels in atoms and molecules?
   a. Radiation occurs at a particular frequency that does not change with the energy supplied.
   b. Certain radiation occurs at a particular frequency that changes with the energy supplied.
   c. Maximum radiation would occur at a particular frequency that does not change with the energy supplied.
   d. Maximum radiation would occur at a particular frequency that changes with the energy supplied.

2. Two lasers shine red light at 650 nm. One laser is twice as bright as the other. Explain this difference using photons and photon energy.
   a. The brighter laser emits twice the number of photons and more energy per photon.
   b. The brighter laser emits twice the number of photons and less energy per photon.
   c. Both lasers emit equal numbers of photons and equivalent amounts of energy per photon.
   d. The brighter laser emits twice the number of photons but both lasers emit equivalent amounts of energy per photon.

3. Consider four stars in the night sky: red, yellow, orange, and blue. The photons of which star will carry the greatest amount of energy?
   a. blue
   b. orange
   c. red
   d. yellow

4. A lightbulb is wired to a variable resistor. What will happen to the color spectrum emitted by the bulb as the resistance of the circuit is increased?
   a. The bulb will emit greener light.
   b. The bulb will emit bluer light.
   c. The bulb will emit more ultraviolet light.
   d. The bulb will emit redder light.

21.2 Einstein and the Photoelectric Effect

5. Light is projected onto a semi-conductive surface. However, no electrons are ejected. What will happen when the light intensity is increased?
   a. An increase in light intensity decreases the number of photons. However, no electrons are ejected.
   b. Increase in light intensity increases the number of photons, so electrons with higher kinetic energy are ejected.
   c. An increase in light intensity increases the number of photons, so electrons will be ejected.
   d. An increase in light intensity increases the number of photons. However, no electrons are ejected.

6. True or false—The concept of a work function (or binding energy) is permissible under the classical wave model.
   a. false
   b. true

7. Can a single microwave photon cause cell damage?
   a. No, there is not enough energy associated with a single microwave photon to result in cell damage.
   b. No, there is zero energy associated with a single microwave photon, so it does not result in cell damage.
   c. Yes, a single microwave photon causes cell damage because it does not have high energy.
   d. Yes, a single microwave photon causes cell damage because it has enough energy.

21.3 The Dual Nature of Light

8. Why don't we feel the momentum of sunlight when we are on the beach?
   a. The momentum of a singular photon is incredibly small.
   b. The momentum is not felt because very few photons strike us at any time, and not all have momentum.
   c. The momentum of a singular photon is large, but very few photons strike us at any time.
   d. A large number of photons strike us at any time, and so their combined momentum is incredibly large.

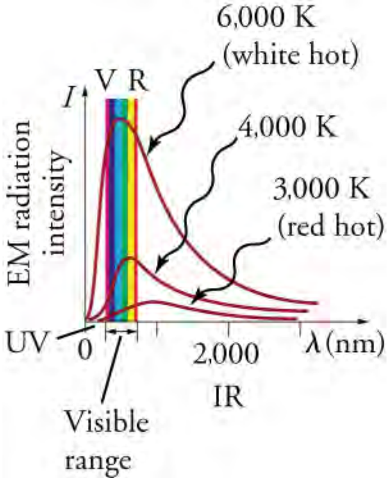
9. If a beam of helium atoms is projected through two slits and onto a screen, will an interference pattern emerge?
   a. No, an interference pattern will not emerge because helium atoms will strike a variety of locations on the screen.
   b. No, an interference pattern will not emerge because helium atoms will strike at certain locations on the screen.
   c. Yes, an interference pattern will emerge because helium atoms will strike a variety of locations on the screen.
   d. Yes, an interference pattern will emerge because helium atoms will strike at certain locations on the screen.

Critical Thinking Items

21.1 Planck and Quantum Nature of Light

10. Explain why the frequency of a blackbody does not double when the temperature is doubled.
  a. Frequency is inversely proportional to temperature.
  b. Frequency is directly proportional to temperature.
  c. Frequency is directly proportional to the square of temperature.
  d. Frequency is directly proportional to the fourth power of temperature.

11. Why does the intensity shown in the blackbody radiation graph decrease after its peak frequency is achieved?



  a. Because after reaching the peak frequency, the photons created at a particular frequency are too many for energy intensity to continue to decrease.
  b. Because after reaching the peak frequency, the photons created at a particular frequency are too few for energy intensity to continue to decrease.
  c. Because after reaching the peak frequency, the photons created at a particular frequency are too many for energy intensity to continue to increase.
  d. Because after reaching the peak frequency, the photons created at a particular frequency are too few for energy intensity to continue to increase.

12. Shortly after the introduction of photography, it was found that photographic emulsions were more sensitive to blue and violet light than they were to red light. Explain why this was the case.
  a. Blue-violet light contains greater amount of energy than red light.
  b. Blue-violet light contains lower amount of energy than red light.
  c. Both blue-violet light and red light have the same frequency but contain different amounts of energy.
  d. Blue-violet light frequency is lower than the frequency of red light.

13. Why is it assumed that a perfect absorber of light (like a blackbody) must also be a perfect emitter of light?
  a. To achieve electrostatic equilibrium with its surroundings
  b. To achieve thermal equilibrium with its surroundings
  c. To achieve mechanical equilibrium with its surroundings
  d. To achieve chemical equilibrium with its surroundings

21.2 Einstein and the Photoelectric Effect

14. Light is projected onto a semi-conductive surface. If the intensity is held constant but the frequency of light is increased, what will happen?
  a. As frequency is increased, electrons will stop being ejected from the surface.
  b. As frequency is increased, electrons will begin to be ejected from the surface.
  c. As frequency is increased, it will have no effect on the electrons being ejected as the intensity is the same.
  d. As frequency is increased, the rate at which the electrons are being ejected will increase.

15. Why is it important to consider what material to use when designing a light meter? Consider the worked example from Section 21-2 for assistance.
  a. A light meter should contain material that responds only to high frequency light.
  b. A light meter should contain material that responds to low frequency light.
  c. A light meter should contain material that has high binding energy.
  d. A light meter should contain a material that does not show any photoelectric effect.

16. Why does overexposure to UV light often result in sunburn when overexposure to visible light does not? This is why you can get burnt even on a cloudy day.
  a. UV light carries less energy than visible light and can penetrate our body.
  b. UV light carries more energy than visible light, so it cannot break bonds at the cellular level.
  c. UV light carries more energy than visible light and can break bonds at the cellular level.
  d. UV light carries less energy than visible light and cannot penetrate the human body.

17. If you pick up and shake a piece of metal that has electrons in it free to move as a current, no electrons fall out. Yet if you heat the metal, electrons can be boiled off. Explain both of these facts as they relate to the amount and distribution of energy involved with shaking the object as compared with heating it.

a. Thermal energy is added to the metal at a much higher rate than energy added due to shaking.
b. Thermal energy is added to the metal at a much lower rate than energy added due to shaking.
c. If the thermal energy added is below the binding energy of the electrons, they may be *boiled off.*
d. If the mechanical energy added is below the binding energy of the electrons, they may be *boiled off.*

21.3 The Dual Nature of Light

18. In many macroscopic collisions, a significant amount of kinetic energy is converted to thermal energy. Explain why this is not a concern for Compton scattering.

a. Because, photons and electrons do not exist on the molecular level, all energy of motion is considered kinetic energy.
b. Because, photons exist on the molecular level while electrons do not exist on the molecular level, all energy of motion is considered kinetic energy.
c. Because, electrons exist on the molecular level while photons do not exist on the molecular level, all energy of motion is considered kinetic energy.
d. Because, photons and electrons exist on the molecular level, all energy of motion is considered kinetic energy.

19. In what region of the electromagnetic spectrum will photons be most effective in accelerating a solar sail?

a. ultraviolet rays
b. infrared rays
c. X-rays
d. gamma rays

20. True or false—Electron microscopes can resolve images that are smaller than the images resolved by light microscopes.

a. false
b. true

21. How would observations of Compton scattering change if ultraviolet light were used in place of X-rays?

a. Ultraviolet light carries less energy than X-rays. As a result, Compton scattering would be easier to detect.
b. Ultraviolet light carries less energy than X-rays. As a result, Compton scattering would be more difficult to detect.
c. Ultraviolet light carries more energy than X-rays. As a result, Compton scattering would be easier to detect.
d. Ultraviolet light has higher energy than X-rays. As a result, Compton scattering would be more difficult to detect.

Problems

21.1 Planck and Quantum Nature of Light

22. How many X-ray photons per second are created by an X-ray tube that produces a flux of X-rays having a power of 1.00 W? Assume the average energy per photon is 75.0 keV.

a. 8.33×10^{15} photons
b. 9.1×10^{7} photons
c. 9.1×10^{8} photons
d. 8.33×10^{13} photons

23. What is the frequency of a photon produced in a CRT using a 25.0-kV accelerating potential? This is similar to the layout as in older color television sets.

a. 6.04×10^{-48} Hz
b. 2.77×10^{-48} Hz
c. 3.02×10^{18} Hz
d. 6.04×10^{18} Hz

21.2 Einstein and the Photoelectric Effect

24. What is the binding energy in eV of electrons in magnesium, if the longest-wavelength photon that can eject electrons is 337 nm?

a. 7.44×10^{-19} J
b. 7.44×10^{-49} J
c. 5.90×10^{-17} J
d. 5.90×10^{-19} J

25. Photoelectrons from a material with a binding energy of 2.71 eV are ejected by 420-nm photons. Once ejected, how long does it take these electrons to travel 2.50 cm to a detection device?

a. 8.5×10^{-6} s
b. 3.5×10^{-7} s
c. 43.5×10^{-9} s
d. 8.5×10^{-8} s

21.3 The Dual Nature of Light

26. What is the momentum of a 0.0100-nm-wavelength photon that could detect details of an atom?

a. 6.626×10^{-27} kg · m/s
b. 6.626×10^{-32} kg · m/s
c. 6.626×10^{-34} kg · m/s
d. 6.626×10^{-23} kg · m/s

27. The momentum of light is exactly reversed when reflected straight back from a mirror, assuming negligible recoil of the mirror. Thus the change in momentum is twice the initial photon momentum. Suppose light of intensity 1.00 kW/m2 reflectsfrom a mirror of area 2.00 m2 each second. Using the most general form of Newton's second law, what is the force on the mirror?

a. 1.33×10^{-5} N
b. 1.33×10^{-6} N
c. 1.33×10^{-7} N
d. 1.33×10^{-8} N

Performance Task

21.3 The Dual Nature of Light

28. Our scientific understanding of light has changed over time. There is evidence to support the wave model of light, just as there is evidence to support the particle model of light.

1. Construct a demonstration that supports the wave model of light. Note—One possible method is to use a piece of aluminum foil, razor blade, and laser to demonstrate wave interference. Can you arrange these materials to create an effective demonstration? In writing, explain how evidence from your demonstration supports the wave model of light.
2. Construct a demonstration that supports the particle model of light. Note—One possible method is to use a negatively charged electroscope, zinc plate, and three light sources of different frequencies. A red laser, a desk lamp, and ultraviolet lamp are typically used. Can you arrange these materials to demonstrate the photoelectric effect? In writing, explain how evidence from your demonstration supports the particle model of light.

TEST PREP

Multiple Choice

21.1 Planck and Quantum Nature of Light

29. A perfect blackbody is a perfect absorber of energy transferred by what method?

a. conduction
b. convection
c. induction
d. radiation

30. Which of the following is a physical entity that is quantized?

a. electric charge of an ion
b. frequency of a sound
c. speed of a car

31. Find the energy in joules of photons of radio waves that leave an FM station that has a 90.0-MHz broadcast frequency.

a. 1.8×10^{-25} J
b. 1.11×10^{-25} J
c. 7.1×10^{-43} J
d. 5.96×10^{-26} J

32. Which region of the electromagnetic spectrum will provide photons of the least energy?

a. infrared light
b. radio waves
c. ultraviolet light
d. X-rays

33. A hot, black coffee mug is sitting on a kitchen table in a dark room. Because it cannot be seen, one assumes that it is not emitting energy in the form of light. Explain the fallacy in this logic.

a. Not all heat is in the form of light energy.
b. Not all light energy falls in the visible portion of the electromagnetic spectrum.
c. All heat is in the form of light energy.
d. All light energy falls in the visible portion of the electromagnetic spectrum.

34. Given two stars of equivalent size, which will have a greater temperature: a red dwarf or a yellow dwarf? Explain. Note—Our sun is considered a yellow dwarf.

a. a yellow dwarf, because yellow light has lower frequency
b. a red dwarf, because red light has lower frequency
c. a red dwarf, because red light has higher frequency
d. a yellow dwarf, because yellow light has higher frequency

21.2 Einstein and the Photoelectric Effect

35. What is a quantum of light called?

a. electron
b. neutron
c. photon
d. proton

36. Which of the following observations from the photoelectric effect is not a violation of classical physics?

a. Electrons are ejected immediately after impact from light.
b. Light can eject electrons from a semi-conductive

material.

c. Light intensity does not influence the kinetic energy of ejected electrons.

d. No electrons are emitted if the light frequency is too low.

37. If $5\ \text{eV}$ of energy is supplied to an electron with a binding energy of $2.3\ \text{eV}$, with what kinetic energy will the electron be launched?

a. $2.3\ \text{eV}$

b. $7.3\ \text{eV}$

c. $11.5\ \text{eV}$

d. $2.7\ \text{eV}$

38. Which of the following terms translates to *light-producing voltage*?

a. photoelectric

b. quantum mechanics

c. photoconductive

d. photovoltaic

39. Why is high frequency EM radiation considered more dangerous than long wavelength EM radiation?

a. Long wavelength EM radiation photons carry less energy and therefore have greater ability to disrupt materials through the photoelectric effect.

b. Long wavelength EM radiation photons carry more energy and therefore have greater ability to disrupt materials through the photoelectric effect.

c. High frequency EM radiation photons carry less energy and therefore have lower ability to disrupt materials through the photoelectric effect.

d. High frequency EM radiation photons carry more energy and therefore have greater ability to disrupt materials through the photoelectric effect.

40. Why are UV, X-rays, and gamma rays considered ionizing radiation?

a. UV, X-rays, and gamma rays are capable of ejecting photons from a surface.

b. UV, X-rays, and gamma rays are capable of ejecting neutrons from a surface.

c. UV, X-rays, and gamma rays are capable of ejecting protons from a surface.

d. UV, X-rays, and gamma rays are capable of ejecting electrons from a surface.

21.3 The Dual Nature of Light

41. What two particles interact in Compton scattering?

a. photon and electron

b. proton and electron

c. neutron and electron

d. proton and neutron

42. What is the momentum of a 500-nm photon?

a. $8.35 \times 10^{-26}\ \text{kg} \cdot \text{m/s}$

b. $3.31 \times 10^{-40}\ \text{kg} \cdot \text{m/s}$

c. $7.55 \times 10^{26}\ \text{kg} \cdot \text{m/s}$

d. $1.33 \times 10^{-27}\ \text{kg} \cdot \text{m/s}$

43. The conservation of what fundamental physics principle is behind the technology of solar sails?

a. charge

b. mass

c. momentum

d. angular momentum

44. Terms like frequency, amplitude, and period are tied to what component of wave-particle duality?

a. neither the particle nor the wave model of light

b. both the particle and wave models of light

c. the particle model of light

d. the wave model of light

45. Why was it beneficial for Compton to scatter electrons using X-rays and not another region of light like microwaves?

a. because X-rays are more penetrating than microwaves

b. because X-rays have lower frequency than microwaves

c. because microwaves have shorter wavelengths than X-rays

d. because X-rays have shorter wavelength than microwaves

Short Answer

21.1 Planck and Quantum Nature of Light

46. Scientists once assumed that all frequencies of light were emitted with equal probability. Explain what the blackbody radiation curve would look like if this were the case.

a. The blackbody radiation curve would look like a circular path.

b. The blackbody radiation curve would look like an elliptical path.

c. The blackbody radiation curve would look like a vertical line.

d. The blackbody radiation curve would look like a horizontal line.

47. Because there are more gradations to high frequency radiation than low frequency radiation, scientists also thought it possible that a curve titled the *ultraviolet catastrophe* would occur. Explain what the blackbody radiation curve would look like if this were the case.

a. The curve would steadily increase in intensity with increasing frequency.
b. The curve would steadily decrease in intensity with increasing frequency.
c. The curve would be much steeper than in the blackbody radiation graph.
d. The curve would be much flatter than in the blackbody radiation graph.

48. Energy provided by a light exists in the following quantities: 150 J, 225 J, 300 J. Define one possible quantum of energy and provide an energy state that cannot exist with this quantum.
a. 65 J; 450 J cannot exist
b. 70 J; 450 J cannot exist
c. 75 J; 375 J cannot exist
d. 75 J; 100 J cannot exist

49. Why is Planck's recognition of quantum particles considered the dividing line between classical and modern physics?
a. Planck recognized that energy is quantized, which was in sync with the classical physics concepts but not in agreement with modern physics concepts.
b. Planck recognized that energy is quantized, which was in sync with modern physics concepts but not in agreement with classical physics concepts.
c. Prior to Planck's hypothesis, all the classical physics calculations were valid for subatomic particles, but quantum physics calculations were not valid.
d. Prior to Planck's hypothesis, all the classical physics calculations were not valid for macroscopic particles, but quantum physics calculations were valid.

50. How many 500-mm microwave photons are needed to supply the 8 kJ of energy necessary to heat a cup of water by 10 degrees Celsius?
a. 8.05×10^{28} photons
b. 8.05×10^{26} photons
c. 2.01×10^{26} photons
d. 2.01×10^{28} photons

51. What is the efficiency of a 100-W, 550-nm lightbulb if a photometer finds that 1×10^{20} photons are emitted each second?
a. 101 percent
b. 72 percent
c. 18 percent
d. 36 percent

52. Rank the following regions of the electromagnetic spectrum by the amount of energy provided per photon: gamma, infrared, microwave, ultraviolet, radio, visible, X-ray.
a. radio, microwave, infrared, visible, ultraviolet, X-ray, gamma
b. radio, infrared, microwave, ultraviolet, visible, X-ray, gamma
c. radio, visible, microwave, infrared, ultraviolet, X-ray, gamma
d. radio, microwave, infrared, visible, ultraviolet, gamma, X-ray

53. Why are photons of gamma rays and X-rays able to penetrate objects more successfully than ultraviolet radiation?
a. Photons of gamma rays and X-rays carry with them less energy.
b. Photons of gamma rays and X-rays have longer wavelengths.
c. Photons of gamma rays and X-rays have lower frequencies.
d. Photons of gamma rays and X-rays carry with them more energy.

21.2 Einstein and the Photoelectric Effect

54. According to wave theory, what is necessary to eject electrons from a surface?
a. Enough energy to overcome the binding energy of the electrons at the surface
b. A frequency that is higher than that of the electrons at the surface
c. Energy that is lower than the binding energy of the electrons at the surface
d. A very small number of photons

55. What is the wavelength of EM radiation that ejects 2.00-eV electrons from calcium metal, given that the binding energy is 2.71 eV?
a. 16.1×10^{5} m
b. 6.21×10^{-5} m
c. 9.94×10^{-26} m
d. 2.63×10^{-7} m

56. Find the wavelength of photons that eject 0.100-eV electrons from potassium, given that the binding energy is $2.24\,\text{eV}$.
a. 6.22×10^{-7} m
b. 5.92×10^{-5} m
c. 1.24×10^{-5} m
d. 5.31×10^{-7} m

57. How do solar cells utilize the photoelectric effect?
a. A solar cell converts all photons that it absorbs to electrical energy using the photoelectric effect.
b. A solar cell converts all electrons that it absorbs to electrical energy using the photoelectric effect.
c. A solar cell absorbs the photons with energy less

than the energy gap of the material of the solar cell and converts it to electrical energy using the photoelectric effect.

d. A solar cell absorbs the photons with energy greater than the energy gap of the material of the solar cell and converts it to electrical energy using the photoelectric effect.

58. Explain the advantages of the photoelectric effect to other forms of energy transformation.

a. The photoelectric effect is able to work on the Sun's natural energy.
b. The photoelectric effect is able to work on energy generated by burning fossil fuels.
c. The photoelectric effect can convert heat energy into electrical energy.
d. The photoelectric effect can convert electrical energy into light energy.

21.3 The Dual Nature of Light

59. Upon collision, what happens to the frequency of a photon?

a. The frequency of the photon will drop to zero.
b. The frequency of the photon will remain the same.
c. The frequency of the photon will increase.
d. The frequency of the photon will decrease.

60. How does the momentum of a photon compare to the momentum of an electron of identical energy?

a. Momentum of the photon is greater than the momentum of an electron.
b. Momentum of the photon is less than the momentum of an electron.
c. Momentum of the photon is equal to the momentum of an electron.
d. Momentum of the photon is zero due to zero rest mass but the momentum of an electron is finite.

61. A 500-nm photon strikes an electron and loses 20 percent of its energy. What is the new momentum of the photon?

a. 4.24×10^{-27} kg · m/s
b. 3.18×10^{-27} kg · m/s
c. 2.12×10^{-27} kg · m/s
d. 1.06×10^{-27} kg · m/s

62. A 500-nm photon strikes an electron and loses 20 percent of its energy. What is the speed of the recoiling electron?

a. 7.18×10^5 m/s
b. 6.18×10^5 m/s
c. 5.18×10^5 m/s
d. 4.18×10^5 m/s

63. When a photon strikes a solar sail, what is the direction of impulse on the photon?

a. parallel to the sail
b. perpendicular to the sail
c. tangential to the sail
d. opposite to the sail

64. What is a fundamental difference between solar sails and sails that are used on sailboats?

a. Solar sails rely on disorganized strikes from light particles, while sailboats rely on disorganized strikes from air particles.
b. Solar sails rely on disorganized strikes from air particles, while sailboats rely on disorganized strikes from light particles.
c. Solar sails rely on organized strikes from air particles, while sailboats rely on organized strikes from light particles.
d. Solar sails rely on organized strikes from light particles, while sailboats rely on organized strikes from air particles.

65. The wavelength of a particle is called the de Broglie wavelength, and it can be found with the equation $p = \frac{h}{\lambda}$.
Yes or no—Can the wavelength of an electron match that of a proton?

a. Yes, a slow-moving electron can achieve the same momentum as a slow-moving proton.
b. No, a fast-moving electron cannot achieve the same momentum, and hence the same wavelength, as a proton.
c. No, an electron can achieve the same momentum, and hence not the same wavelength, as a proton.
d. Yes, a fast-moving electron can achieve the same momentum, and hence have the same wavelength, as a slow-moving proton.

66. Large objects can move with great momentum. Why then is it difficult to see their wave-like nature?

a. Their wavelength is equal to the object's size.
b. Their wavelength is very small compared to the object's size.
c. Their wavelength is very large compared to the object's size.
d. Their frequency is very small compared to the object's size.

Extended Response

21.1 Planck and Quantum Nature of Light

67. Some television tubes are CRTs. They use an approximately 30-kV accelerating potential to send electrons to the screen, where the electrons stimulate phosphors to emit the light that forms the pictures we watch. Would you expect X-rays also to be created? Explain.
   a. No, because the full spectrum of EM radiation is not emitted at any temperature.
   b. No, because the full spectrum of EM radiation is not emitted at certain temperatures.
   c. Yes, because the full spectrum of EM radiation is emitted at any temperature.
   d. Yes, because the full spectrum of EM radiation is emitted at certain temperatures.

68. If Planck's constant were large, say 10^{34} times greater than it is, we would observe macroscopic entities to be quantized. Describe the motion of a child's swing under such circumstances.
   a. The child would not be able to swing with particular energies.
   b. The child could be released from any height.
   c. The child would be able to swing with constant velocity.
   d. The child could be released only from particular heights.

69. What is the accelerating voltage of an X-ray tube that produces X-rays with the shortest wavelength of 0.0103 nm?
   a. 1.21×10^{10} V
   b. 2.4×10^{5} V
   c. 3.0×10^{-33} V
   d. 1.21×10^{5} V

70. Patients in a doctor's office are rightly concerned about receiving a chest X-ray. Yet visible light is also a form of electromagnetic radiation and they show little concern about sitting under the bright lights of the waiting room. Explain this discrepancy.
   a. X-ray photons carry considerably more energy so they can harm the patients.
   b. X-ray photons carry considerably less energy so they can harm the patients.
   c. X-ray photons have considerably longer wavelengths so they cannot harm the patients.
   d. X-ray photons have considerably lower frequencies so they can harm the patients.

21.2 Einstein and the Photoelectric Effect

71. When increasing the intensity of light shining on a metallic surface, it is possible to increase the current created on that surface. Classical theorists would argue that this is evidence that intensity causes charge to move with a greater kinetic energy. Argue this logic from the perspective of a modern physicist.
   a. The increased intensity increases the number of ejected electrons. The increased current is due to the increase in the number of electrons.
   b. The increased intensity decreases the number of ejected electrons. The increased current is due to the decrease in the number of electrons ejected.
   c. The increased intensity does not alter the number of electrons ejected. The increased current is due to the increase in the kinetic energy of electrons.
   d. The increased intensity alters the number of electrons ejected, but an increase in the current is due to an increase in the kinetic energy of electrons.

72. What impact does the quantum nature of electromagnetic radiation have on the understanding of speed at the particle scale?
   a. Speed must also be quantized at the particle scale.
   b. Speed will not be quantized at the particle scale.
   c. Speed must be zero at the particle scale.
   d. Speed will be infinite at the particle scale.

73. A 500 nm photon of light strikes a semi-conductive surface with a binding energy of 2 eV. With what velocity will an electron be emitted from the semi-conductive surface?
   a. 8.38×10^{5} m/s
   b. 9.33×10^{5} m/s
   c. 3×10^{8} m/s
   d. 4.11×10^{5} m/s

74. True or false—Treating food with ionizing radiation helps keep it from spoiling.
   a. true
   b. false

21.3 The Dual Nature of Light

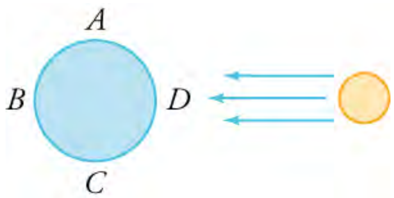
75. When testing atomic bombs, scientists at Los Alamos recognized that huge releases of energy resulted in problems with power and communications systems in the area surrounding the blast site. Explain the possible tie to Compton scattering.
   a. The release of light energy caused large-scale emission of electrons.
   b. The release of light energy caused large-scale emission of protons.
   c. The release of light energy caused large-scale emission of neutrons.
   d. The release of light energy caused large-scale

emission of photons.

76. Sunlight above the Earth's atmosphere has an intensity of 1.30 kW/m2 . If this is reflected straight back from a mirror that has only a small recoil, the light's momentum is exactly reversed, giving the mirror twice the incident momentum. If the mirror were attached to a solar sail craft, how fast would the craft be moving after 24 hr? Note—The average mass per square meter of the craft is 0.100 kg.

a. 8.67×10^{-5} m/s^2
b. 8.67×10^{-6} m/s^2
c. 94.2 m/s
d. 7.49 m/s

77. Consider the counter-clockwise motion of LightSail-1 around Earth. When will the satellite move the fastest?



a. point A
b. point B
c. point C
d. point D

78. What will happen to the interference pattern created by electrons when their velocities are increased?

a. There will be more zones of constructive interference and fewer zones of destructive interference.
b. There will be more zones of destructive interference and fewer zones of constructive interference.
c. There will be more zones of constructive and destructive interference.
d. There will be fewer zones of constructive and destructive interference.

CHAPTER 22
The Atom

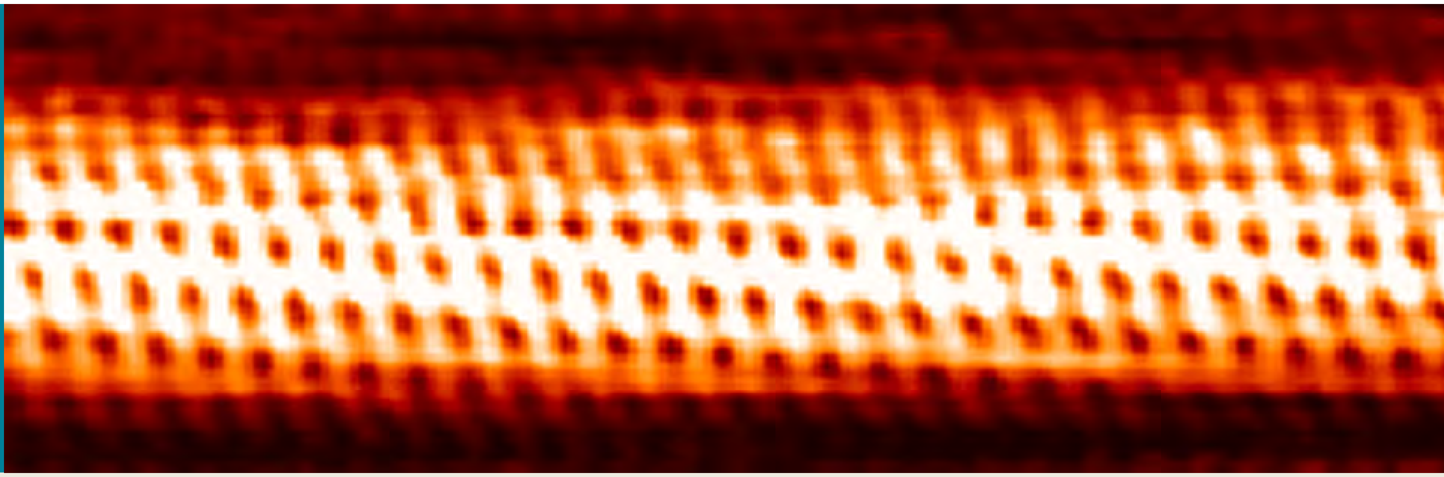


Figure 22.1 Individual carbon atoms are visible in this image of a carbon nanotube made by a scanning tunneling electron microscope. (credit: Taner Yildirim, National Institute of Standards and Technology, Wikimedia Commons)

Chapter Outline

INTRODUCTION From childhood on, we learn that atoms are a substructure of all things around us, from the air we breathe to the autumn leaves that blanket a forest trail. Invisible to the eye, the atoms have properties that are used to explain many phenomena—a theme found throughout this text. In this chapter, we discuss the discovery of atoms and their own substructures. We will then learn about the forces that keep them together and the tremendous energy they release when we break them apart. Finally, we will see how the knowledge and manipulation of atoms allows us to better understand geology, biology, and the world around us.

22.1 The Structure of the Atom

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe Rutherford's experiment and his model of the atom
- Describe emission and absorption spectra of atoms
- Describe the Bohr model of the atom
- Calculate the energy of electrons when they change energy levels
- Calculate the frequency and wavelength of emitted photons when electrons change energy levels
- Describe the quantum model of the atom

Section Key Terms

energy-level diagram	excited state	Fraunhofer lines
ground state	Heisenberg Uncertainty Principle	hydrogen-like atoms
planetary model of the atom	Rutherford scattering	Rydberg constant

How do we know that atoms are really there if we cannot see them with our own eyes? While often taken for granted, our knowledge of the existence and structure of atoms is the result of centuries of contemplation and experimentation. The earliest known speculation on the atom dates back to the fifth century B.C., when Greek philosophers Leucippus and Democritus contemplated whether a substance could be divided without limit into ever smaller pieces. Since then, scientists such as John Dalton (1766–1844), Amadeo Avogadro (1776–1856), and Dmitri Mendeleev (1834–1907) helped to discover the properties of that fundamental structure of matter. While much could be written about any number of important scientific philosophers, this section will focus on the role played by Ernest Rutherford (1871–1937). Though his understanding of our most elemental matter is rooted in the success of countless prior investigations, his surprising discovery about the interior of the atom is most fundamental in explaining so many well-known phenomena.

Rutherford's Experiment

In the early 1900's, the *plum pudding* model was the accepted model of the atom. Proposed in 1904 by J. J. Thomson, the model suggested that the atom was a spherical ball of positive charge, with negatively charged electrons scattered evenly throughout. In that model, the positive charges made up the pudding, while the electrons acted as isolated plums. During its short life, the model could be used to explain why most particles were neutral, although with an unbalanced number of plums, electrically charged atoms could exist.

When Ernest Rutherford began his gold foil experiment in 1909, it is unlikely that anyone would have expected that the plum pudding model would be challenged. However, using a radioactive source, a thin sheet of gold foil, and a phosphorescent screen, Rutherford would uncover something so great that he would later call it "the most incredible event that has ever happened to me in my life"[James, L. K. (1993). *Nobel Laureates in Chemistry, 1901–1992*. Washington, DC: American Chemical Society.]

The experiment that Rutherford designed is shown in Figure 22.2. As you can see in, a radioactive source was placed in a lead container with a hole in one side to produce a beam of positively charged helium particles, called *alpha particles*. Then, a thin gold foil sheet was placed in the beam. When the high-energy alpha particles passed through the gold foil, they were scattered. The scattering was observed from the bright spots they produced when they struck the phosphor screen.

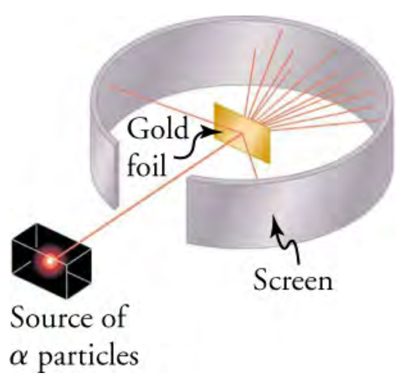


Figure 22.2 Rutherford's experiment gave direct evidence for the size and mass of the nucleus by scattering alpha particles from a thin gold foil. The scattering of particles suggests that the gold nuclei are very small and contain nearly all of the gold atom's mass. Particularly significant in showing the size of the nucleus are alpha particles that scatter to very large angles, much like a soccer ball bouncing off a goalie's head.

The expectation of the plum pudding model was that the high-energy alpha particles would be scattered only slightly by the presence of the gold sheet. Because the energy of the alpha particles was much higher than those typically associated with atoms, the alpha particles should have passed through the thin foil much like a supersonic bowling ball would crash through a few dozen rows of bowling pins. Any deflection was expected to be minor, and due primarily to the electrostatic Coulomb force between the alpha particles and the foil's interior electric charges.

However, the true result was nothing of the sort. While the majority of alpha particles passed through the foil unobstructed, Rutherford and his collaborators Hans Geiger and Ernest Marsden found that alpha particles occasionally were scattered to large angles, and some even came back in the direction from which they came! The result, called **Rutherford scattering**, implied that the gold nuclei were actually very small when compared with the size of the gold atom. As shown in Figure 22.3, the dense nucleus is surrounded by mostly empty space of the atom, an idea verified by the fact that only 1 in 8,000 particles was scattered backward.

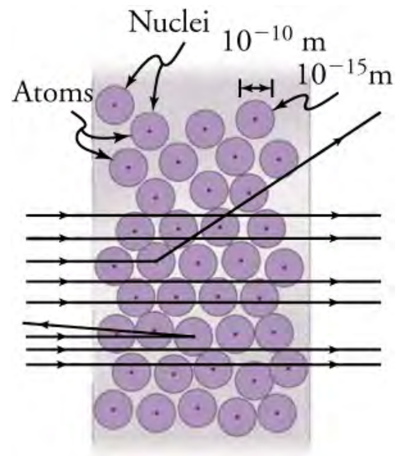


Figure 22.3 An expanded view of the atoms in the gold foil in Rutherford's experiment. Circles represent the atoms that are about 10^{-10} m in diameter, while the dots represent the nuclei that are about 10^{-15} m in diameter. To be visible, the dots are much larger than scale—if the nuclei were actually the size of the dots, each atom would have a diameter of about five meters! Most alpha particles crash through but are relatively unaffected because of their high energy and the electron's small mass. Some, however, strike a nucleus and are scattered straight back. A detailed analysis of their interaction gives the size and mass of the nucleus.

Although the results of the experiment were published by his colleagues in 1909, it took Rutherford two years to convince himself of their meaning. Rutherford later wrote: "It was almost as incredible as if you fired a 15-inch shell at a piece of tissue paper and it came back and hit you. On consideration, I realized that this scattering backwards ... [meant] ... the greatest part of the mass of the atom was concentrated in a tiny nucleus." In 1911, Rutherford published his analysis together with a proposed model of the atom, which was in part based on Geiger's work from the previous year. As a result of the paper, the size of the nucleus was determined to be about 10^{-15} m, or 100,000 times smaller than the atom. That implies a huge density, on the order of 10^{15} g/cm^3, much greater than any macroscopic matter.

Based on the size and mass of the nucleus revealed by his experiment, as well as the mass of electrons, Rutherford proposed the **planetary model of the atom**. The planetary model of the atom pictures low-mass electrons orbiting a large-mass nucleus. The sizes of the electron orbits are large compared with the size of the nucleus, and most of the atom is a vacuum. The model is analogous to how low-mass planets in our solar system orbit the large-mass Sun. In the atom, the attractive Coulomb force is analogous to gravitation in the planetary system (see Figure 22.4).

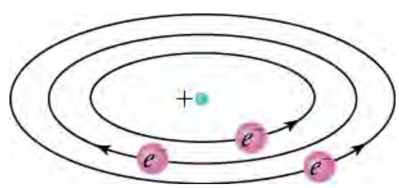


Figure 22.4 Rutherford's planetary model of the atom incorporates the characteristics of the nucleus, electrons, and the size of the atom. The model was the first to recognize the structure of atoms, in which low-mass electrons orbit a very small, massive nucleus in orbits much larger than the nucleus. The atom is mostly empty and is analogous to our planetary system.

Virtual Physics

Rutherford Scattering

Click to view content (https://www.openstax.org/l/28rutherford)

How did Rutherford figure out the structure of the atom without being able to see it? Explore the answer through this simulation of the famous experiment in which he disproved the plum pudding model by observing alpha particles bouncing off atoms and determining that they must have a small core.

TIPS FOR SUCCESS

As you progress through the model of the atom, consider the effect that experimentation has on the scientific process. Ask yourself the following: What would our model of the atom be without Rutherford's gold foil experiment? What further understanding of the atom would not have been gained? How would that affect our current technologies? Though often confusing, experiments taking place today to further understand composition of the atom could perhaps have a similar effect.

Absorption and Emission Spectra

In 1900, Max Planck recognized that all energy radiated from a source is emitted by atoms in quantum states. How would that radical idea relate to the interior of an atom? The answer was first found by investigating the spectrum of light or emission spectrum produced when a gas is highly energized.

Figure 22.5 shows how to isolate the emission spectrum of one such gas. The gas is placed in the discharge tube at the left, where it is energized to the point at which it begins to radiate energy or emit light. The radiated light is channeled by a thin slit and then passed through a diffraction grating, which will separate the light into its constituent wavelengths. The separated light will then strike the photographic film on the right.

The line spectrum shown in part (b) of Figure 22.5 is the output shown on the film for excited iron. Note that this spectrum is not continuous but discrete. In other words, only particular wavelengths are emitted by the iron source. Why would that be the case?

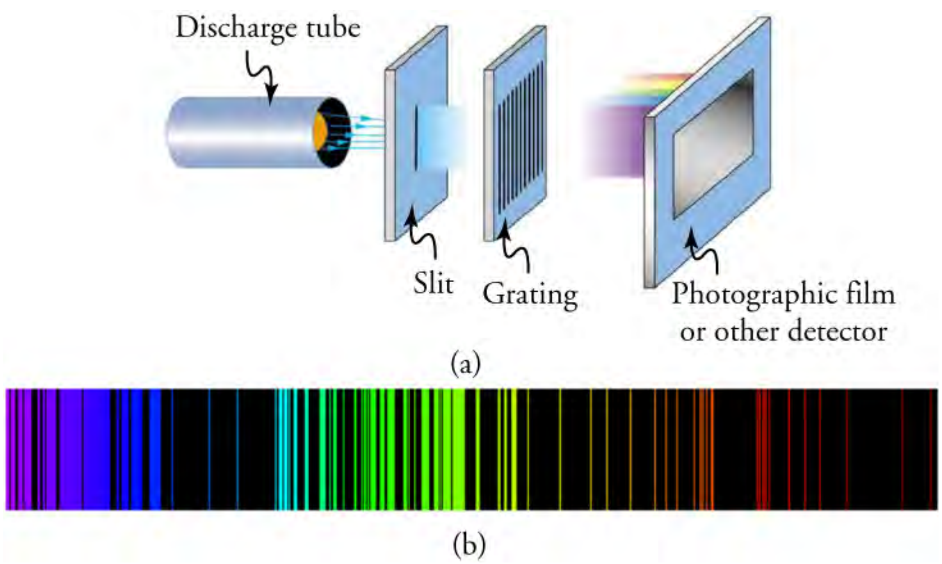


Figure 22.5 Part (a) shows, from left to right, a discharge tube, slit, and diffraction grating producing a line spectrum. Part (b) shows the emission spectrum for iron. The discrete lines imply quantized energy states for the atoms that produce them. The line spectrum for each element is unique, providing a powerful and much-used analytical tool, and many line spectra were well known for many years before they could be explained with physics. (credit:(b) Yttrium91, Wikimedia Commons)

The spectrum of light created by excited iron shows a variety of discrete wavelengths emitted within the visible spectrum. Each element, when excited to the appropriate degree, will create a discrete emission spectrum as in part (b) of Figure 22.5. However, the wavelengths emitted will vary from element to element. The emission spectrum for iron was chosen for Figure 22.5 solely because a substantial portion of its emission spectrum is within the visible spectrum. Figure 22.6 shows the emission spectrum for hydrogen. Note that, while discrete, a large portion of hydrogen emission takes place in the ultraviolet and infrared regions.

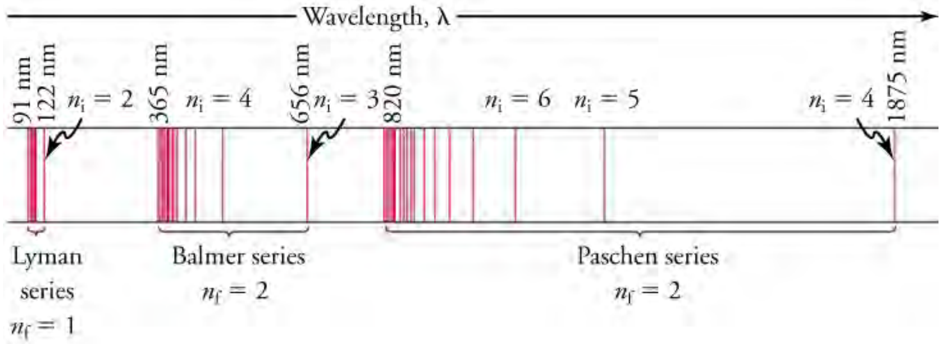


Figure 22.6 A schematic of the hydrogen spectrum shows several series named for those who contributed most to their determination. Part of the Balmer series is in the visible spectrum, while the Lyman series is entirely in the ultraviolet, and the Paschen series and others are in the infrared. Values of n_f and n_i are shown for some of the lines. Their importance will be described shortly.

Just as an emission spectrum shows all discrete wavelengths emitted by a gas, an absorption spectrum will show all light that is absorbed by a gas. Black lines exist where the wavelengths are absorbed, with the remainder of the spectrum lit by light is free to pass through. What relationship do you think exists between the black lines of a gas's absorption spectrum and the colored lines of its emission spectrum? Figure 22.7 shows the absorption spectrum of the Sun. The black lines are called **Fraunhofer lines**, and they correspond to the wavelengths absorbed by gases in the Sun's exterior.

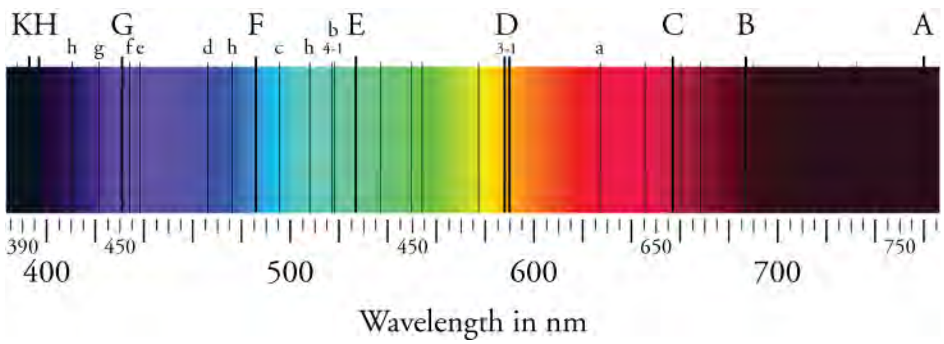


Figure 22.7 The absorption spectrum of the Sun. The black lines appear at wavelengths absorbed by the Sun's gas exterior. The energetic photons emitted from the Sun's interior are absorbed by gas in its exterior and reemitted in directions away from the observer. That results in dark lines within the absorption spectrum. The lines are called Fraunhofer lines, in honor of the German physicist who discovered them. Lines similar to those are used to determine the chemical composition of stars well outside our solar system.

Bohr's Explanation of the Hydrogen Spectrum

To tie the unique signatures of emission spectra to the composition of the atom itself would require clever thinking. Niels Bohr (1885–1962), a Danish physicist, did just that, by making immediate use of Rutherford's planetary model of the atom. Bohr, shown in Figure 22.8, became convinced of its validity and spent part of 1912 at Rutherford's laboratory. In 1913, after returning to Copenhagen, he began publishing his theory of the simplest atom, hydrogen, based on Rutherford's planetary model.

Figure 22.8 Niels Bohr, Danish physicist, used the planetary model of the atom to explain the atomic spectrum and size of the hydrogen

atom. His many contributions to the development of atomic physics and quantum mechanics, his personal influence on many students and colleagues, and his personal integrity, especially in the face of Nazi oppression, earned him a prominent place in history. (credit: Unknown Author, Wikimedia Commons)

Bohr was able to derive the formula for the hydrogen spectrum using basic physics, the planetary model of the atom, and some very important new conjectures. His first conjecture was that only certain orbits are allowed: In other words, in an atom, the orbits of electrons are quantized. Each quantized orbit has a different distinct energy, and electrons can move to a higher orbit by absorbing energy or drop to a lower orbit by emitting energy. Because of the quantized orbits, the amount of energy emitted or absorbed must also be quantized, producing the discrete spectra seen in Figure 22.5 and Figure 22.7. In equation form, the amount of energy absorbed or emitted can be found as

$$\Delta E = E_i - E_f, \tag{22.1}$$

where E_i refers to the energy of the initial quantized orbit, and E_f refers to the energy of the final orbits. Furthermore, the wavelength emitted can be found using the equation

$$hf = E_i - E_f, \tag{22.2}$$

and relating the wavelength to the frequency found using the equation $v = f\lambda$, where v corresponds to the speed of light.

It makes sense that energy is involved in changing orbits. For example, a burst of energy is required for a satellite to climb to a higher orbit. What is not expected is that atomic orbits should be quantized. Quantization is not observed for satellites or planets, which can have any orbit, given the proper energy (see Figure 22.9).

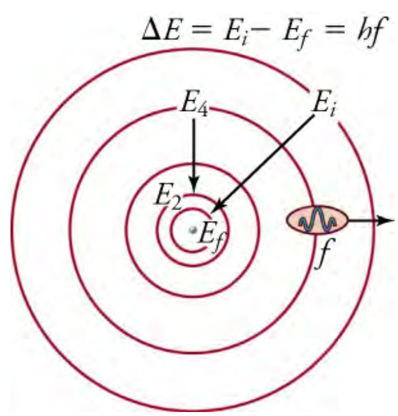


Figure 22.9 The planetary model of the atom, as modified by Bohr, has the orbits of the electrons quantized. Only certain orbits are allowed, explaining why atomic spectra are discrete or quantized. The energy carried away from an atom by a photon comes from the electron dropping from one allowed orbit to another and is thus quantized. The same is true for atomic absorption of photons.

Figure 22.10 shows an **energy-level diagram**, a convenient way to display energy states. Each of the horizontal lines corresponds to the energy of an electron in a different orbital. Energy is plotted vertically with the lowest or **ground state** at the bottom and with **excited states** above. The vertical arrow downwards shows energy being emitted out of the atom due to an electron dropping from one excited state to another. That would correspond to a line shown on the atom's emission spectrum. The Lyman series shown in Figure 22.6 results from electrons dropping to the ground state, while the Balmer and Paschen series result to electrons dropping to the $n = 2$ and $n = 3$ states, respectively.

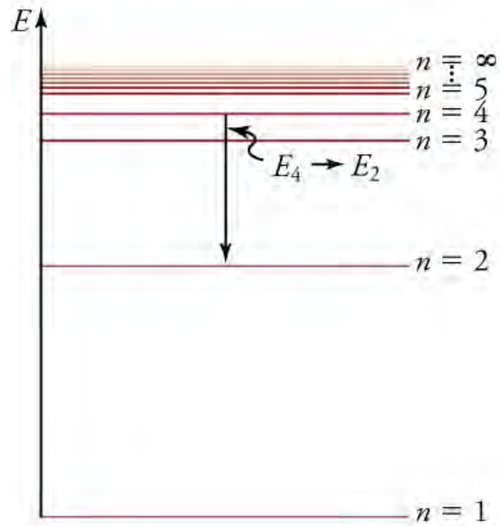


Figure 22.10 An energy-level diagram plots energy vertically and is useful in visualizing the energy states of a system and the transitions between them. This diagram is for the hydrogen-atom electrons, showing a transition between two orbits having energies E_4 and E_2 . The energy transition results in a Balmer series line in an emission spectrum.

Energy and Wavelength of Emitted Hydrogen Spectra

The energy associated with a particular orbital of a hydrogen atom can be found using the equation

$$E_n = -\frac{13.6\text{ eV}}{n^2}(n = 1, 2, 3, \ldots), \tag{22.3}$$

where n corresponds to the orbital value from the atom's nucleus. The negative value in the equation is based upon a baseline energy of zero when the electron is infinitely far from the atom. As a result, the negative value shows that energy is necessary to free the electron from its orbital state. The minimum energy to free the electron is also referred to as its *binding energy*. The equation is only valid for atoms with single electrons in their orbital shells (like hydrogen). For ionized atoms similar to hydrogen, the following formula may be used.

$$E_n = \frac{Z^2}{n^2}E_o\ (n = 1, 2, 3, \ldots) \tag{22.4}$$

Please note that E_o corresponds to −13.6 eV, as mentioned earlier. Additionally, Z refers to the atomic number of the element studied. The atomic number is the number of protons in the nucleus—it is different for each element. The above equation is derived from some basic physics principles, namely conservation of energy, conservation of angular momentum, Coulomb's law, and centripetal force. There are three derivations that result in the orbital energy equations, and they are shown below. While you can use the energy equations without understanding the derivations, they will help to remind you of just how valuable those fundamental concepts are.

Derivation 1 (Finding the Radius of an Orbital)

One primary difference between the planetary model of the solar system and the planetary model of the atom is the cause of the circular motion. While gravitation causes the motion of orbiting planets around an interior star, the Coulomb force is responsible for the circular shape of the electron's orbit. The magnitude of the centripetal force is $\frac{m_e v^2}{r_n}$, while the magnitude of the Coulomb force is $\frac{k(Zq_e)(q_e)}{{r_e}^2}$. The assumption here is that the nucleus is more massive than the stationary electron, and the electron orbits about it. That is consistent with the planetary model of the atom. Equating the Coulomb force and the centripetal force,

$$\frac{m_e v^2}{r_n} = \frac{k(Zq_e)(q_e)}{{r_e}^2}, \tag{22.5}$$

which yields

$$r_n = \frac{k\left(Zq_e{}^2\right)}{mv^2}. \tag{22.6}$$

Derivation 2 (Finding the Velocity of the Orbiting Electron)

Bohr was clever enough to find a way to calculate the electron orbital energies in hydrogen. That was an important first step that has been improved upon, but it is well worth repeating here, because it does correctly describe many characteristics of hydrogen. Assuming circular orbits, Bohr proposed that the angular momentum L of an electron in its orbit is also quantized, that is, it has only specific, discrete values. The value for L is given by the formula

$$L = m_e v r_n = n\frac{h}{2\pi}(n = 1, 2, 3, \ldots), \tag{22.7}$$

where L is the angular momentum, m_e is the electron's mass, r_n is the radius of the n th orbit, and h is Planck's constant. Note that angular momentum is $L = I\omega$. For a small object at a radius r, $I = mr^2$, and $\omega = \frac{v}{r}$, so that $L = I\omega = \left(mr^2\right)\left(\frac{v}{r}\right) = mvr$. Quantization says that the value of mvr can only be equal to $h/2$, $2h/2$, $3h/2$, etc. At the time, Bohr himself did not know why angular momentum should be quantized, but by using that assumption, he was able to calculate the energies in the hydrogen spectrum, something no one else had done at the time.

Derivation 3 (Finding the Energy of the Orbiting Electron)

To get the electron orbital energies, we start by noting that the electron energy is the sum of its kinetic and potential energy.

$$E_n = KE + PE \tag{22.8}$$

Kinetic energy is the familiar $KE = \frac{1}{2}mv^2$, assuming the electron is not moving at a relativistic speed. Potential energy for the electron is electrical, or $PE = q_e V$, where V is the potential due to the nucleus, which looks like a point charge. The nucleus has a positive charge Zq_e ; thus, $V = \frac{kZq_e}{r_n}$, recalling an earlier equation for the potential due to a point charge from the chapter on Electricity and Magnetism. Since the electron's charge is negative, we see that $PE = \frac{-kZq_e{}^2}{r_n}$. Substituting the expressions for KE and PE,

$$E_n = \frac{1}{2}m_e v^2 - \frac{kZq_e{}^2}{r_n}. \tag{22.9}$$

Now we solve for r_n and v using the equation for angular momentum $L = m_e v r_n = n\frac{h}{2\pi}(n = 1, 2, 3, \ldots)$, giving

$$v = n\frac{h}{2\pi m_e r_n}(n = 1, 2, 3, \ldots) \tag{22.10}$$

and

$$r_n = n\frac{h}{2\pi m_e v}(n = 1, 2, 3, \ldots). \tag{22.11}$$

Substituting the expression for r_n and v into the above expressions for energy (KE and PE), and performing algebraic manipulation, yields

$$E_n = -\frac{Z^2}{n^2}E_o(n = 1, 2, 3, \ldots) \tag{22.12}$$

for the orbital energies of **hydrogen-like atoms**. Here, E_o is the ground-state energy ($n = 1$) for hydrogen ($Z = 1$) and is given by

$$E_o = \frac{2\pi^2 q_e{}^4 m_e k^2}{h^2} = 13.6\text{ eV}. \tag{22.13}$$

Thus, for hydrogen,

$$E_n = -\frac{13.6\text{ eV}}{n^2}(n = 1, 2, 3, \ldots). \tag{22.14}$$

The relationship between orbital energies and orbital states for the hydrogen atom can be seen in Figure 22.11.

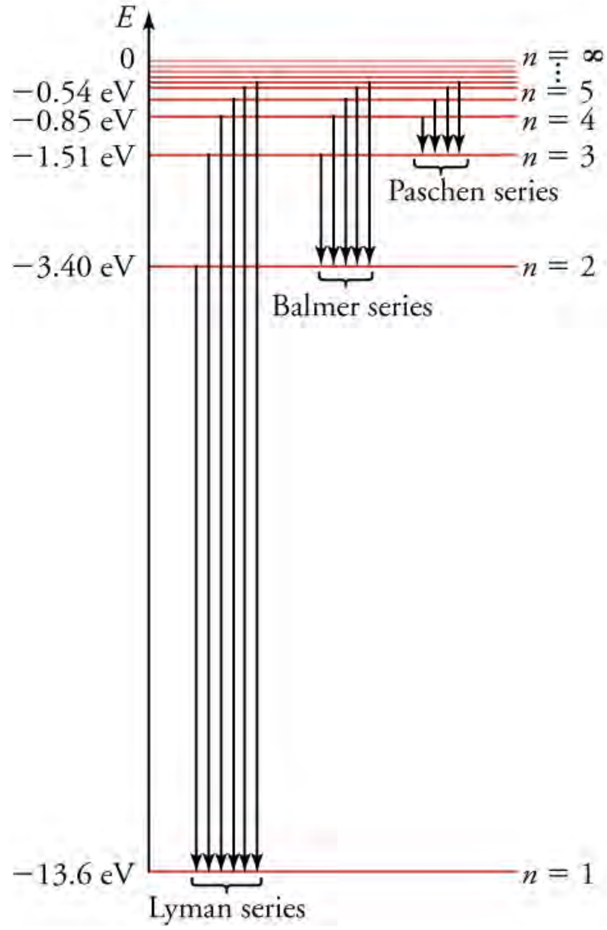


Figure 22.11 Energy-level diagram for hydrogen showing the Lyman, Balmer, and Paschen series of transitions. The orbital energies are calculated using the above equation, first derived by Bohr.

WORKED EXAMPLE

A hydrogen atom is struck by a photon. How much energy must be absorbed from the photon to raise the electron of the hydrogen atom from its ground state to its second orbital?

Strategy

The hydrogen atom has an atomic number of $Z = 1$. Raising the electron from the ground state to its second orbital will increase its orbital level from $n = 1$ to $n = 2$. The energy determined will be measured in electron-volts.

Solution

The amount of energy necessary to cause the change in electron state is the difference between the final and initial energies of the electron. The final energy state of the electron can be found using

$$E_n = \frac{Z^2}{n^2} E_o \; (n = 1, 2, 3, \ldots) . \tag{22.15}$$

Knowing the n and Z values for the hydrogen atom, and knowing that $E_o = -13.6$ eV, the result is

$$E_f = \frac{1^2}{2^2}(-13.6 \text{ eV}) = -3.4 \text{ eV}. \tag{22.16}$$

The original amount of energy associated with the electron is equivalent to the ground state orbital, or

$$E_O = \frac{1^2}{1^2}(-13.6 \text{ eV}) = -13.6 \text{ eV}. \tag{22.17}$$

The amount of energy necessary to change the orbital state of the electron can be found by determining the electron's change in energy.

$$\Delta E = E_f - E_o = (-3.4 \text{ eV}) - (-13.6 \text{ eV}) = +10.2 \text{ eV} \tag{22.18}$$

Discussion

The energy required to change the orbital state of the electron is positive. That means that for the electron to move to a state with greater energy, energy must be added to the atom. Should the electron drop back down to its original energy state, a change of −10.2 eV would take place, and 10.2 eV of energy would be emitted from the atom. Just as only quantum amounts of energy may be absorbed by the atom, only quantum amounts of energy can be emitted from the atom. That helps to explain many of the quantum light effects that you have learned about previously.

WORKED EXAMPLE

Characteristic X-Ray Energy

Calculate the approximate energy of an X-ray emitted for an $n = 2$ to $n = 1$ transition in a tungsten anode in an X-ray tube.

Strategy

How do we calculate energies in a multiple-electron atom? In the case of characteristic X-rays, the following approximate calculation is reasonable. Characteristic X-rays are produced when an inner-shell vacancy is filled. Inner-shell electrons are nearer the nucleus than others in an atom and thus feel little net effect from the others. That is similar to what happens inside a charged conductor, where its excess charge is distributed over the surface so that it produces no electric field inside. It is reasonable to assume the inner-shell electrons have hydrogen-like energies, as given by

$$E_n = \frac{Z^2}{n^2} E_o \ (n = 1, 2, 3, \ldots) \tag{22.19}$$

For tungsten, $Z = 74$, so that the effective charge is 73.

Solution

The amount of energy given off as an X-ray is found using

$$\Delta E = hf = E_i - E_f, \tag{22.20}$$

where

$$E_f = -\frac{Z^2}{1^2} E_o = -\frac{73^2}{1}(13.6 \text{ eV}) = -72.5 \text{ keV} \tag{22.21}$$

and

$$E_i = -\frac{Z^2}{2^2} E_o = -\frac{73^2}{4}(13.6 \text{ eV}) = -18.1 \text{ keV}. \tag{22.22}$$

Thus,

$$\Delta E = E_i - E_f = (-18.1 \text{ keV}) - (-72.5 \text{ keV}) = 54.4 \text{ keV}. \tag{22.23}$$

Discussion

This large photon energy is typical of characteristic X-rays from heavy elements. It is large compared with other atomic emissions because it is produced when an inner-shell vacancy is filled, and inner-shell electrons are tightly bound. Characteristic X-ray energies become progressively larger for heavier elements because their energy increases approximately as Z^2. Significant accelerating voltage is needed to create such inner-shell vacancies, because other shells are filled and you cannot simply bump one electron to a higher filled shell. You must remove it from the atom completely. In the case of tungsten, at least 72.5 kV is needed. Tungsten is a common anode material in X-ray tubes; so much of the energy of the impinging electrons is absorbed, raising its temperature, that a high-melting-point material like tungsten is required.

The wavelength of light emitted by an atom can also be determined through basic derivations. Let us consider the energy of a photon emitted from a hydrogen atom in a downward transition, given by the equation

$$\Delta E = hf = E_i - E_f \tag{22.24}$$

Substituting $E_n = \left(\frac{-13.6\text{ eV}}{n^2}\right)$, we get

$$hf = (13.6\text{ eV})\left(\frac{1}{{n_f}^2} - \frac{1}{{n_i}^2}\right). \tag{22.25}$$

Dividing both sides of the equation by hc gives us an expression for $\frac{1}{\lambda}$,

$$\frac{hf}{hc} = \frac{f}{c} = \frac{1}{\lambda} = \frac{13.6\text{ eV}}{hc}\left(\frac{1}{{n_f}^2} - \frac{1}{{n_i}^2}\right). \tag{22.26}$$

It can be shown that

$$\left(\frac{13.6\text{ eV}}{hc}\right) = \frac{(13.6\text{ eV})(1.602 \times 10^{-19}\text{J/eV})}{(6.602 \times 10^{-34}\text{J}\cdot\text{s})(2.998 \times 10^{8}\text{m/s})} = 1.097 \times 10^{7}\text{m}^{-1} = R, \tag{22.27}$$

where R is the **Rydberg constant**.

Simplified, the formula for determining emitted wavelength can now be written as

$$\frac{1}{\lambda} = R\left(\frac{1}{{n_f}^2} - \frac{1}{{n_i}^2}\right). \tag{22.28}$$

WORKED EXAMPLE

What wavelength of light is emitted by an electron dropping from the third orbital to the ground state of a hydrogen atom?

Strategy

The ground state of a hydrogen atom is considered the first orbital of the atom. As a result, $n_f = 1$ and $n_i = 3$. The Rydberg constant has already been determined and will be constant regardless of atom chosen.

Solution

$$\frac{1}{\lambda} = R\left(\frac{1}{{n_f}^2} - \frac{1}{{n_i}^2}\right) \tag{22.29}$$

For the equation above, calculate wavelength based on the known energy states.

$$\frac{1}{\lambda} = 1.097 \times 10^{7}\left(\frac{1}{1^2} - \frac{1}{3^2}\right) = 9.751 \times 10^{6}\text{m}^{-1} \tag{22.30}$$

Rearranging the equation for wavelength yields

$$\lambda = 1.026 \times 10^{-7}\text{m} = 102.6\text{ nm}. \tag{22.31}$$

Discussion

This wavelength corresponds to light in the ultraviolet spectrum. As a result, we would not be able to see the photon of light emitted when an electron drops from its third to first energy state. However, it is worth noting that by supplying light of wavelength precisely 102.6 nm, we can cause the electron in hydrogen to move from its first to its third orbital state.

Limits of Bohr's Theory and the Quantum Model of the Atom

There are limits to Bohr's theory. It does not account for the interaction of bound electrons, so it cannot be fully applied to multielectron atoms, even one as simple as the two-electron helium atom. Bohr's model is what we call *semiclassical*. The orbits are quantized (nonclassical) but are assumed to be simple circular paths (classical). As quantum mechanics was developed, it became clear that there are no well-defined orbits; rather, there are clouds of probability. Additionally, Bohr's theory did not explain that some spectral lines are doublets or split into two when examined closely. While we shall examine a few of those aspects of quantum mechanics in more detail, it should be kept in mind that Bohr did not fail. Rather, he made very important steps along the path to greater knowledge and laid the foundation for all of atomic physics that has since evolved.

DeBroglie's Waves

Following Bohr's initial work on the hydrogen atom, a decade was to pass before Louis de Broglie proposed that matter has wave properties. The wave-like properties of matter were subsequently confirmed by observations of electron interference when scattered from crystals. Electrons can exist only in locations where they interfere constructively. How does that affect electrons in atomic orbits? When an electron is bound to an atom, its wavelength must fit into a small space, something like a standing wave on a string (see Figure 22.12). Orbits in which an electron can constructively interfere with itself are allowed. All orbits in which constructive interference cannot occur are not able to exist. Thus, only certain orbits are allowed. The wave nature of an electron, according to de Broglie, is why the orbits are quantized!

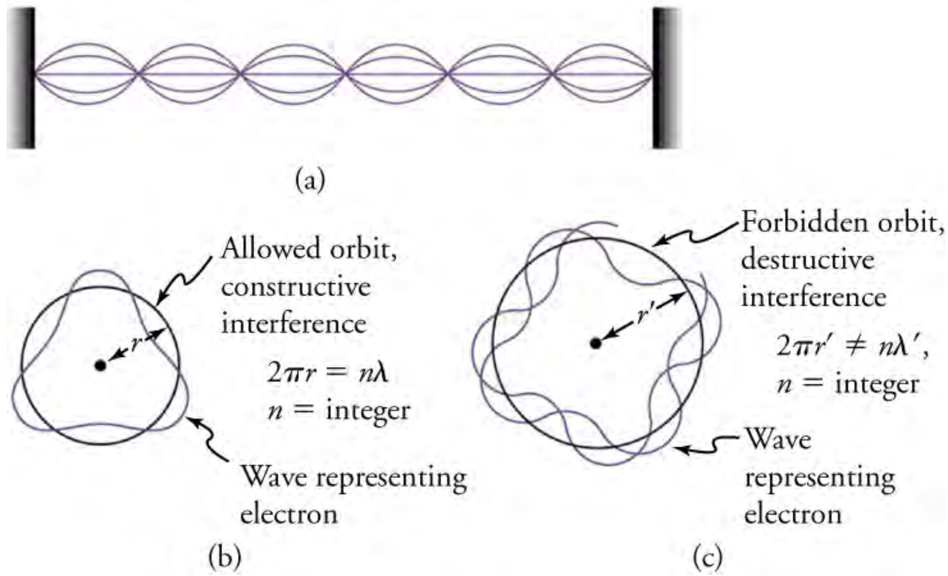


Figure 22.12 (a) Standing waves on a string have a wavelength related to the length of the string, allowing them to interfere constructively. (b) If we imagine the string formed into a closed circle, we get a rough idea of how electrons in circular orbits can interfere constructively. (c) If the wavelength does not fit into the circumference, the electron interferes destructively; it cannot exist in such an orbit.

For a circular orbit, constructive interference occurs when the electron's wavelength fits neatly into the circumference, so that wave crests always align with crests and wave troughs align with troughs, as shown in Figure 22.12(b). More precisely, when an integral multiple of the electron's wavelength equals the circumference of the orbit, constructive interference is obtained. In equation form, the condition for constructive interference and an allowed electron orbit is

$$n\lambda_n = 2\pi r_n (n = 1, 2, 3, ...), \tag{22.32}$$

where λ_n is the electron's wavelength and r_n is the radius of that circular orbit. Figure 22.13 shows the third and fourth orbitals of a hydrogen atom.

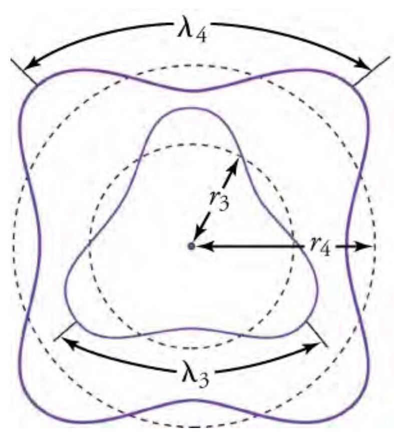


Figure 22.13 The third and fourth allowed circular orbits have three and four wavelengths, respectively, in their circumferences.

Heisenberg Uncertainty

How does determining the location of an electron change its trajectory? The answer is fundamentally important—measurement affects the system being observed. It is impossible to measure a physical quantity exactly, and greater precision in measuring one quantity produces less precision in measuring a related quantity. It was Werner Heisenberg who first stated that limit to knowledge in 1929 as a result of his work on quantum mechanics and the wave characteristics of all particles (see Figure 22.14).

Figure 22.14 Werner Heisenberg was the physicist who developed the first version of true quantum mechanics. Not only did his work give a description of nature on the very small scale, it also changed our view of the availability of knowledge. Although he is universally recognized for the importance of his work by receiving the Nobel Prize in 1932, for example, Heisenberg remained in Germany during World War II and headed the German effort to build a nuclear bomb, permanently alienating himself from most of the scientific community. (credit: Unknown Author, Wikimedia Commons)

For example, you can measure the position of a moving electron by scattering light or other electrons from it. However, by doing so, you are giving the electron energy, and therefore imparting momentum to it. As a result, the momentum of the electron is affected and cannot be determined precisely. This change in momentum could be anywhere from close to zero up to the relative momentum of the electron ($p \approx h/\lambda$). Note that, in this case, the particle is an electron, but the principle applies to any particle.

Viewing the electron through the model of wave-particle duality, Heisenberg recognized that, because a wave is not located at one fixed point in space, there is an uncertainty associated with any electron's position. That uncertainty in position, Δx , is approximately equal to the wavelength of the particle. That is, $\Delta x \approx \lambda$. There is an interesting trade-off between position and momentum. The uncertainty in an electron's position can be reduced by using a shorter-wavelength electron, since $\Delta x \approx \lambda$. But shortening the wavelength increases the uncertainty in momentum, since $\Delta p \approx h/\lambda$. Conversely, the uncertainty in momentum can be reduced by using a longer-wavelength electron, but that increases the uncertainty in position. Mathematically, you can express the trade-off by multiplying the uncertainties. The wavelength cancels, leaving

$$\Delta x \Delta p \approx h.$$

Therefore, if one uncertainty is reduced, the other must increase so that their product is $\approx h$. With the use of advanced mathematics, Heisenberg showed that the best that can be done in a simultaneous measurement of position and momentum is

$$\Delta x \Delta p \geq \frac{h}{4\pi}. \tag{22.33}$$

That relationship is known as the **Heisenberg uncertainty principle**.

The Quantum Model of the Atom

Because of the wave characteristic of matter, the idea of well-defined orbits gives way to a model in which there is a cloud of probability, consistent with Heisenberg's uncertainty principle. Figure 22.15 shows how the principle applies to the ground state of hydrogen. If you try to follow the electron in some well-defined orbit using a probe that has a wavelength small enough to measure position accurately, you will instead knock the electron out of its orbit. Each measurement of the electron's position will find it to be in a definite location somewhere near the nucleus. Repeated measurements reveal a cloud of probability like that in the figure, with each speck the location determined by a single measurement. There is not a well-defined, circular-orbit type of distribution. Nature again proves to be different on a small scale than on a macroscopic scale.

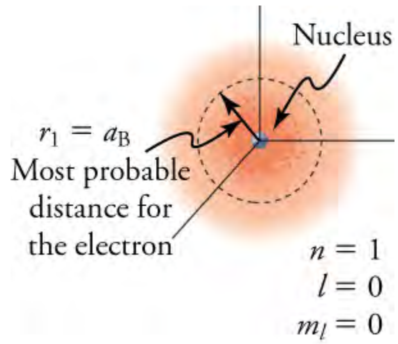


Figure 22.15 The ground state of a hydrogen atom has a probability cloud describing the position of its electron. The probability of finding the electron is proportional to the darkness of the cloud. The electron can be closer or farther than the Bohr radius, but it is very unlikely to be a great distance from the nucleus.

Virtual Physics

Models of the Hydrogen Atom

Click to view content (https://www.openstax.org/l/28atom_model)

How did scientists figure out the structure of atoms without looking at them? Try out different models by shooting light at the atom. Use this simulation to see how the prediction of the model matches the experimental results.

Check Your Understanding

1. Alpha particles are positively charged. What influence did their charge have on the gold foil experiment?
   a. The positively charged alpha particles were attracted by the attractive electrostatic force from the positive nuclei of the gold atoms.
   b. The positively charged alpha particles were scattered by the attractive electrostatic force from the positive nuclei of the gold atoms.
   c. The positively charged alpha particles were scattered by the repulsive electrostatic force from the positive nuclei of the gold atoms.
   d. The positively charged alpha particles were attracted by the repulsive electrostatic force from the positive nuclei of the gold atoms.

22.2 Nuclear Forces and Radioactivity

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe the structure and forces present within the nucleus
- Explain the three types of radiation
- Write nuclear equations associated with the various types of radioactive decay

Section Key Terms

alpha decay	atomic number	beta decay	gamma decay	Geiger tube
isotope	mass number	nucleons	radioactive	radioactive decay

radioactivity	scintillator	strong nuclear force	transmutation

There is an ongoing quest to find the substructures of matter. At one time, it was thought that atoms would be the ultimate substructure. However, just when the first direct evidence of atoms was obtained, it became clear that they have a substructure and a tiny nucleus. The nucleus itself has spectacular characteristics. For example, certain nuclei are unstable, and their decay emits radiations with energies millions of times greater than atomic energies. Some of the mysteries of nature, such as why the core of Earth remains molten and how the Sun produces its energy, are explained by nuclear phenomena. The exploration of radioactivity and the nucleus has revealed new fundamental particles, forces, and conservation laws. That exploration has evolved into a search for further underlying structures, such as quarks. In this section, we will explore the fundamentals of the nucleus and nuclear radioactivity.

The Structure of the Nucleus

At this point, you are likely familiar with the neutron and proton, the two fundamental particles that make up the nucleus of an atom. Those two particles, collectively called **nucleons**, make up the small interior portion of the atom. Both particles have nearly the same mass, although the neutron is about two parts in 1,000 more massive. The mass of a proton is equivalent to 1,836 electrons, while the mass of a neutron is equivalent to that of 1,839 electrons. That said, each of the particles is significantly more massive than the electron.

When describing the mass of objects on the scale of nucleons and atoms, it is most reasonable to measure their mass in terms of atoms. The atomic mass unit (u) was originally defined so that a neutral carbon atom would have a mass of exactly 12 u. Given that protons and neutrons are approximately the same mass, that there are six protons and six neutrons in a carbon atom, and that the mass of an electron is minuscule in comparison, measuring this way allows for both protons and neutrons to have masses close to 1 u. Table 22.1 shows the mass of protons, neutrons, and electrons on the new scale.

TIPS FOR SUCCESS

For most conceptual situations, the difference in mass between the proton and neutron is insubstantial. In fact, for calculations that require fewer than four significant digits, both the proton and neutron masses may be considered equivalent to one atomic mass unit. However, when determining the amount of energy released in a nuclear reaction, as in Equation 22.40, the difference in mass cannot be ignored.

Another other useful mass unit on the atomic scale is the MeV/c^2 . While rarely used in most contexts, it is convenient when one uses the equation $E = mc^2$, as will be addressed later in this text.

	Proton Mass	Neutron Mass	Electron Mass
Kilograms (kg)	1.673×10^{-27}	1.675×10^{-27}	9.109×10^{-31}
Atomic mass units (u)	1.007	1.009	5.486×10^{-4}

Table 22.1 Atomic Masses for Multiple Units

To more completely characterize nuclei, let us also consider two other important quantities: the atomic number and the mass number. The **atomic number**, Z, represents the number of protons within a nucleus. That value determines the elemental quality of each atom. Every carbon atom, for instance, has a Z value of 6, whereas every oxygen atom has a Z value of 8. For clarification, only oxygen atoms may have a Z value of 8. If the Z value is not 8, the atom cannot be oxygen.

The **mass number**, A, represents the total number of protons and neutrons, or nucleons, within an atom. For an ordinary carbon atom the mass number would be 12, as there are typically six neutrons accompanying the six protons within the atom. In the case of carbon, the mass would be exactly 12 u. For oxygen, with a mass number of 16, the atomic mass is 15.994915 u. Of course, the difference is minor and can be ignored for most scenarios. Again, because the mass of an electron is so small compared to the nucleons, the mass number and the atomic mass can be essentially equivalent. Figure 22.16 shows an example of Lithium-7, which has an atomic number of 3 and a mass number of 7.

How does the mass number help to differentiate one atom from another? If each atom of carbon has an atomic number of 6, then what is the value of including the mass number at all? The intent of the mass number is to differentiate between various isotopes of an atom. The term **isotope** refers to the variation of atoms based upon the number of neutrons within their nucleus. While it is most common for there to be six neutrons accompanying the six protons within a carbon atom, it is possible to find carbon atoms with seven neutrons or eight neutrons. Those carbon atoms are respectively referred to as carbon-13 and carbon-14 atoms, with their mass numbers being their primary distinction. The isotope distinction is an important one to make, as the number of neutrons within an atom can affect a number of its properties, not the least of which is nuclear stability.

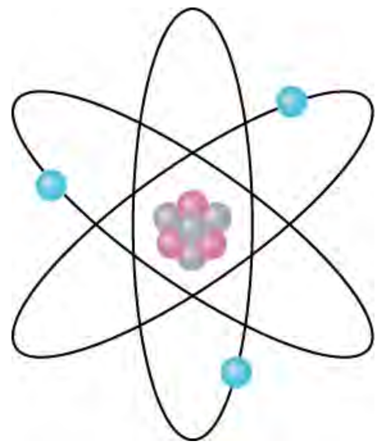


Figure 22.16 Lithium-7 has three protons and four neutrons within its nucleus. As a result, its mass number is 7, while its atomic number is 3. The actual mass of the atom is 7.016 u. Lithium 7 is an isotope of lithium.

To more easily identify various atoms, their atomic number and mass number are typically written in a form of representation called the nuclide. The nuclide form appears as follows: ${}^{Z}_{A}X_{N}$, where *X* is the atomic symbol and *N* represents the number of neutrons.

Let us look at a few examples of nuclides expressed in the ${}^{Z}_{A}X_{N}$ notation. The nucleus of the simplest atom, hydrogen, is a single proton, or ${}^{1}_{1}\text{H}$ (the zero for no neutrons is often omitted). To check the symbol, refer to the periodic table—you see that the atomic number *Z* of hydrogen is 1. Since you are given that there are no neutrons, the mass number *A* is also 1. There is a scarce form of hydrogen found in nature called *deuterium*; its nucleus has one proton and one neutron and, hence, twice the mass of common hydrogen. The symbol for deuterium is, thus, ${}^{2}_{1}\text{H}_2$. An even rarer—and radioactive—form of hydrogen is called *tritium*, since it has a single proton and two neutrons, and it is written ${}^{3}_{1}\text{H}_2$. The three varieties of hydrogen have nearly identical chemistries, but the nuclei differ greatly in mass, stability, and other characteristics. Again, the different nuclei are referred to as isotopes of the same element.

There is some redundancy in the symbols *A*, *X*, *Z*, and *N*. If the element *X* is known, then *Z* can be found in a periodic table. If both *A* and *X* are known, then *N* can also be determined by first finding *Z*; then, $N = A - Z$. Thus the simpler notation for nuclides is

$$^{A}X, \tag{22.34}$$

which is sufficient and is most commonly used. For example, in this simpler notation, the three isotopes of hydrogen are ${}^{1}\text{H}$, ${}^{2}\text{H}$, and ${}^{3}\text{H}$. For ${}^{238}\text{U}$, should we need to know, we can determine that $Z = 92$ for uranium from the periodic table, and thus, $N = 238 - 92 = 146$.

Radioactivity and Nuclear Forces

In 1896, the French physicist Antoine Henri Becquerel (1852–1908) noticed something strange. When a uranium-rich mineral called pitchblende was placed on a completely opaque envelope containing a photographic plate, it darkened spots on the photographic plate.. Becquerel reasoned that the pitchblende must emit invisible rays capable of penetrating the opaque material. Stranger still was that no light was shining on the pitchblende, which means that the pitchblende was emitting the invisible rays continuously without having any energy input! There is an apparent violation of the law of conservation of energy, one that scientists can now explain using Einstein's famous equation $E = mc^2$. It was soon evident that Becquerel's rays originate in the nuclei of the atoms and have other unique characteristics.

To this point, most reactions you have studied have been chemical reactions, which are reactions involving the electrons surrounding the atoms. However, two types of experimental evidence implied that Becquerel's rays did not originate with electrons, but instead within the nucleus of an atom.

First, the radiation is found to be only associated with certain elements, such as uranium. Whether uranium was in the form of an element or compound was irrelevant to its radiation. In addition, the presence of radiation does not vary with temperature, pressure, or ionization state of the uranium atom. Since all of those factors affect electrons in an atom, the radiation cannot come from electron transitions, as atomic spectra do.

The huge energy emitted during each event is the second piece of evidence that the radiation cannot be atomic. Nuclear radiation has energies on the order of 10^6 eV per event, which is much greater than typical atomic energies that are a few eV, such as those observed in spectra and chemical reactions, and more than ten times as high as the most energetic X-rays.

But why would reactions within the nucleus take place? And what would cause an apparently stable structure to begin emitting energy? Was there something special about Becquerel's uranium-rich pitchblende? To answer those questions, it is necessary to look into the structure of the nucleus. Though it is perhaps surprising, you will find that many of the same principles that we observe on a macroscopic level still apply to the nucleus.

Nuclear Stability

A variety of experiments indicate that a nucleus behaves something like a tightly packed ball of nucleons, as illustrated in Figure 22.17. Those nucleons have large kinetic energies and, thus, move rapidly in very close contact. Nucleons can be separated by a large force, such as in a collision with another nucleus, but strongly resist being pushed closer together. The most compelling evidence that nucleons are closely packed in a nucleus is that the radius of a nucleus, r, is found to be approximately

$$r = r_o A^{1/3}, \tag{22.35}$$

where $r_o = 1.2$ femtometer (fm) and A is the mass number of the nucleus.

Note that $r^3 \propto A$. Since many nuclei are spherical, and the volume of a sphere is $V = \left(\frac{4}{3}\right)\pi r^3$, we see that $V \propto A$—that is, the volume of a nucleus is proportional to the number of nucleons in it. That is what you expect if you pack nucleons so close that there is no empty space between them.

Figure 22.17 Nucleons are held together by nuclear forces and resist both being pulled apart and pushed inside one another. The volume of the nucleus is the sum of the volumes of the nucleons in it, here shown in different colors to represent protons and neutrons.

So what forces hold a nucleus together? After all, the nucleus is very small and its protons, being positive, should exert tremendous repulsive forces on one another. Considering that, it seems that the nucleus would be forced apart, not together!

The answer is that a previously unknown force holds the nucleus together and makes it into a tightly packed ball of nucleons. This force is known as the **strong nuclear force**. The strong force has such a short range that it quickly fall to zero over a distance of only 10^{-15} meters. However, like glue, it is very strong when the nucleons get close to one another.

The balancing of the electromagnetic force with the nuclear forces is what allows the nucleus to maintain its spherical shape. If, for any reason, the electromagnetic force should overcome the nuclear force, components of the nucleus would be projected outward, creating the very radiation that Becquerel discovered!

Understanding why the nucleus would break apart can be partially explained using Table 22.2. The balance between the strong nuclear force and the electromagnetic force is a tenuous one. Recall that the attractive strong nuclear force exists between any two nucleons and acts over a very short range while the weaker repulsive electromagnetic force only acts between protons, although over a larger range. Considering the interactions, an imperfect balance between neutrons and protons can result in a nuclear reaction, with the result of regaining equilibrium.

	Range of Force	Direction	Nucleon Interaction	Magnitude of Force
Electromagnetic Force	Long range, though decreasing by $1/r^2$	Repulsive	Proton –proton repulsion	Relatively small
Strong Nuclear Force	Very short range, essentially zero at 1 femtometer	Attractive	Attraction between any two nucleons	100 times greater than the electromagnetic force

Table 22.2 Comparing the Electromagnetic and Strong Forces

The radiation discovered by Becquerel was due to the large number of protons present in his uranium-rich pitchblende. In short, the large number of protons caused the electromagnetic force to be greater than the strong nuclear force. To regain stability, the nucleus needed to undergo a nuclear reaction called **alpha (α) decay**.

The Three Types of Radiation

Radioactivity refers to the act of emitting particles or energy from the nucleus. When the uranium nucleus emits energetic nucleons in Becquerel's experiment, the radioactive process causes the nucleus to alter in structure. The alteration is called **radioactive decay**. Any substance that undergoes radioactive decay is said to be **radioactive**. That those terms share a root with the term *radiation* should not be too surprising, as they all relate to the transmission of energy.

Alpha Decay

Alpha decay refers to the type of decay that takes place when too many protons exist in the nucleus. It is the most common type of decay and causes the nucleus to regain equilibrium between its two competing internal forces. During alpha decay, the nucleus ejects two protons and two neutrons, allowing the strong nuclear force to regain balance with the repulsive electromagnetic force. The nuclear equation for an alpha decay process can be shown as follows.

$$ {}^{A}_{Z}X_N \rightarrow {}^{A-4}_{Z-2}Y_N + {}^{4}_{2}\text{He} \tag{22.36}$$

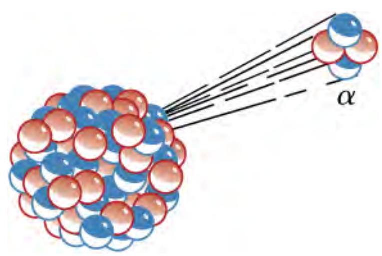


Figure 22.18 A nucleus undergoes alpha decay. The alpha particle can be seen as made up of two neutrons and two protons, which constitute a helium-4 atom.

Three things to note as a result of the above equation:

1. By ejecting an alpha particle, the original nuclide decreases in atomic number. That means that Becquerel's uranium nucleus, upon decaying, is actually transformed into thorium, two atomic numbers lower on the periodic table! The process of changing elemental composition is called **transmutation**.
2. Note that the two protons and two neutrons ejected from the nucleus combine to form a helium nucleus. Shortly after decay, the ejected helium ion typically acquires two electrons to become a stable helium atom.
3. Finally, it is important to see that, despite the elemental change, physical conservation still takes place. The mass number of the new element and the alpha particle together equal the mass number of the original element. Also, the net charge of all particles involved remains the same before and after the transmutation.

Beta Decay

Like alpha decay, **beta (β) decay** also takes place when there is an imbalance between neutrons and protons within the nucleus. For beta decay, however, a neutron is transformed into a proton and electron or vice versa. The transformation allows for the total mass number of the atom to remain the same, although the atomic number will increase by one (or decrease by one). Once again, the transformation of the neutron allows for a rebalancing of the strong nuclear and electromagnetic forces. The nuclear equation for a beta decay process is shown below.

$${}^{A}_{Z}X_{N} \rightarrow {}^{A}_{Z+1}Y_{N-1} + e + v$$

The symbol v in the equation above stands for a high-energy particle called the neutrino. A nucleus may also emit a positron, and in that case Z decreases and N increases. It is beyond the scope of this section and will be discussed in further detail in the chapter on particles. It is worth noting, however, that the mass number and charge in all beta-decay reactions are conserved.

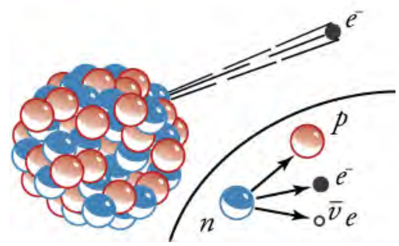


Figure 22.19 A nucleus undergoes beta decay. The neutron splits into a proton, electron, and neutrino. This particular decay is called β^- decay.

Gamma Decay

Gamma decay is a unique form of radiation that does not involve balancing forces within the nucleus. Gamma decay occurs when a nucleus drops from an excited state to the ground state. Recall that such a change in energy state will release energy from the nucleus in the form of a photon. The energy associated with the photon emitted is so great that its wavelength is shorter than that of an X-ray. Its nuclear equation is as follows.

$${}^{A}_{Z}X_{N} \rightarrow X_{N} + \gamma \tag{22.37}$$

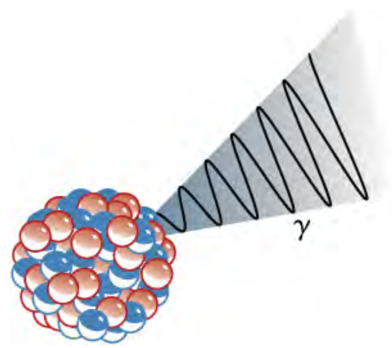


Figure 22.20 A nucleus undergoes gamma decay. The nucleus drops in energy state, releasing a gamma ray.

WORKED EXAMPLE

Creating a Decay Equation

Write the complete decay equation in ${}^{A}_{Z}X_{N}$ notation for beta decay producing ${}^{137}\text{Ba}$. Refer to the periodic table for values of Z.

Strategy

Beta decay results in an increase in atomic number. As a result, the original (or parent) nucleus, must have an atomic number of one fewer proton.

Solution

The equation for beta decay is as follows

$${}^{A}_{Z}X_{N} \rightarrow {}_{Z+1}{}^{A}Y_{N-1} + e + v. \tag{22.38}$$

Considering that barium is the product (or daughter) nucleus and has an atomic number of 56, the original nucleus must be of an atomic number of 55. That corresponds to cesium, or Cs.

$${}^{137}_{55}\text{Cs}_{N} \rightarrow {}^{137}_{56}\text{Ba}_{N-1} + e + v \tag{22.39}$$

The number of neutrons in the parent cesium and daughter barium can be determined by subtracting the atomic number from the mass number (137 – 55 for cesium, 137 – 56 for barium). Substitute those values for the *N* and *N – 1* subscripts in the above equation.

$$^{137}_{55}\text{Cs}_{82} \rightarrow \,^{137}_{56}\text{Ba}_{81} + e + v \quad (22.40)$$

Discussion

The terms *parent* and *daughter* nucleus refer to the reactants and products of a nuclear reaction. The terminology is not just used in this example, but in all nuclear reaction examples. The cesium-137 nuclear reaction poses a significant health risk, as its chemistry is similar to that of potassium and sodium, and so it can easily be concentrated in your cells if ingested.

WORKED EXAMPLE

Alpha Decay Energy Found from Nuclear Masses

Find the energy emitted in the α decay of ^{239}Pu.

Strategy

Nuclear reaction energy, such as released in α decay, can be found using the equation $E = mc^2$. We must first find Δm, the difference in mass between the parent nucleus and the products of the decay.

The mass of pertinent particles is as follows

^{239}Pu: 239.052157 u

^{235}U: 235.043924 u

^{4}He: 4.002602 u.

Solution

The decay equation for ^{239}Pu is

$$^{239}\text{Pu} \rightarrow \,^{235}\text{U} + \,^{4}\text{He}. \quad (22.41)$$

Determine the amount of mass lost between the parent and daughter nuclei.

$$\begin{aligned} \Delta m &= m\left(^{239}\text{P u}\right) - \left(m\left(^{239}\text{U}\right) \ + \ m\left(^{4}\text{He}\right)\right) \\ \Delta m &= 239.052157 \text{ u} - (235.043924 \text{ u} + 4.002602 \text{ u}) \\ \Delta m &= 0.0005631 \text{ u} \end{aligned} \quad (22.42)$$

Now we can find *E* by entering Δm into the equation.

$$E = (\Delta m)\, c^2 = (0.005631 \text{ u})\, c^2 \quad (22.43)$$

And knowing that $1 \text{ u} = 931.5 \text{ meV}/c^2$, we can find that

$$E = (0.005631)\left(931.5 \text{ MeV}/c^2\right)\left(c^2\right) = 5.25 \text{ MeV}. \quad (22.44)$$

Discussion

The energy released in this α decay is in the MeV range, about 10^6 times as great as typical chemical reaction energies, consistent with previous discussions. Most of the energy becomes kinetic energy of the α particle (or ^{4}He nucleus), which moves away at high speed.

The energy carried away by the recoil of the ^{235}U nucleus is much smaller, in order to conserve momentum. The ^{235}U nucleus can be left in an excited state to later emit photons (γ rays). The decay is spontaneous and releases energy, because the products have less mass than the parent nucleus.

Properties of Radiation

The charges of the three radiated particles differ. Alpha particles, with two protons, carry a net charge of +2. Beta particles, with one electron, carry a net charge of –1. Meanwhile, gamma rays are solely photons, or light, and carry no charge. The difference in charge plays an important role in how the three radiations affect surrounding substances.

Alpha particles, being highly charged, will quickly interact with ions in the air and electrons within metals. As a result, they have a short range and short penetrating distance in most materials. Beta particles, being slightly less charged, have a larger range and larger penetrating distance. Gamma rays, on the other hand, have little electric interaction with particles and travel much farther. Two diagrams below show the importance of difference in penetration. Table 22.3 shows the distance of radiation penetration, and Figure 22.21 shows the influence various factors have on radiation penetration distance.

Type of Radiation	Range
α particles	A sheet of paper, a few cm of air, fractions of a millimeter of tissue
β particles	A thin aluminum plate, tens of cm of tissue
γ rays	Several cm of lead, meters of concrete

Table 22.3 Comparing Ranges of Radioactive Decay

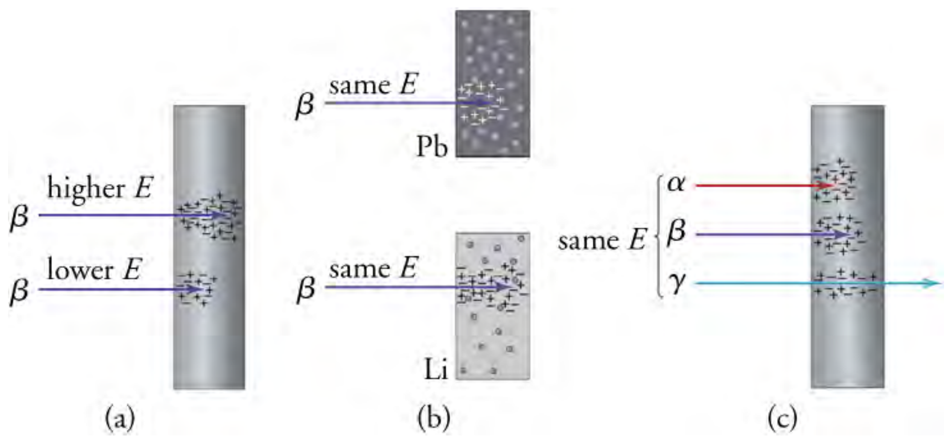


Figure 22.21 The penetration or range of radiation depends on its energy, the material it encounters, and the type of radiation. (a) Greater energy means greater range. (b) Radiation has a smaller range in materials with high electron density. (c) Alphas have the smallest range, betas have a greater range, and gammas have the greatest range.

LINKS TO PHYSICS

Radiation Detectors

The first direct detection of radiation was Becquerel's darkened photographic plate. Photographic film is still the most common detector of ionizing radiation, being used routinely in medical and dental X-rays. Nuclear radiation can also be captured on film, as seen in Figure 22.22. The mechanism for film exposure by radiation is similar to that by photons. A quantum of energy from a radioactive particle interacts with the emulsion and alters it chemically, thus exposing the film. Provided the radiation has more than the few eV of energy needed to induce the chemical change, the chemical alteration will occur. The amount of film darkening is related to the type of radiation and amount of exposure. The process is not 100 percent efficient, since not all incident radiation interacts and not all interactions produce the chemical change.

Figure 22.22 Film badges contain film similar to that used in this dental X-ray film. It is sandwiched between various absorbers to determine the penetrating ability of the radiation as well as the amount. Film badges are worn to determine radiation exposure. (credit: Werneuchen, Wikimedia Commons)

Another very common radiation detector is the **Geiger tube**. The clicking and buzzing sound we hear in dramatizations and documentaries, as well as in our own physics labs, is usually an audio output of events detected by a Geiger counter. These relatively inexpensive radiation detectors are based on the simple and sturdy Geiger tube, shown schematically in Figure 22.23. A conducting cylinder with a wire along its axis is filled with an insulating gas so that a voltage applied between the cylinder and wire produces almost no current. Ionizing radiation passing through the tube produces free ion pairs that are attracted to the wire and cylinder, forming a current that is detected as a count. Not every particle is detected, since some radiation can pass through without producing enough ionization. However, Geiger counters are very useful in producing a prompt output that reveals the existence and relative intensity of ionizing radiation.

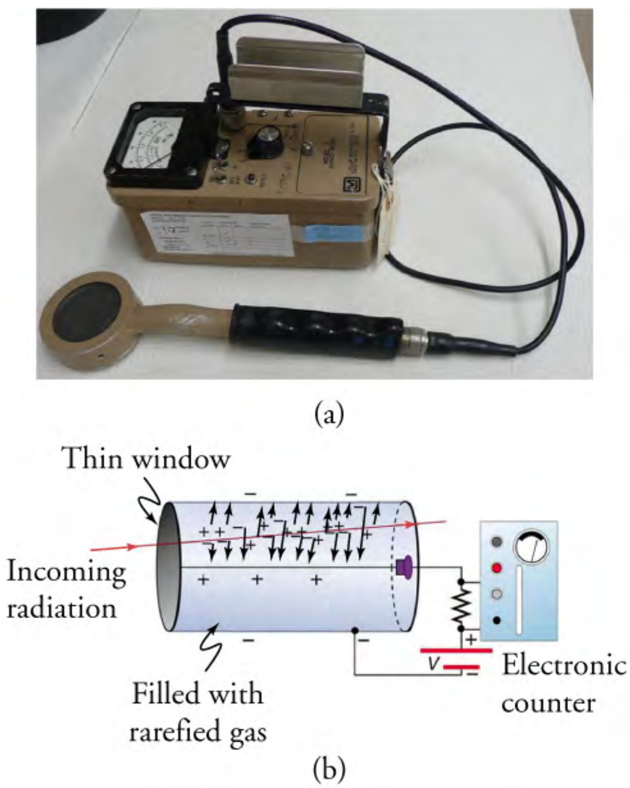


Figure 22.23 (a) Geiger counters such as this one are used for prompt monitoring of radiation levels, generally giving only relative intensity and not identifying the type or energy of the radiation. (credit: Tim Vickers, Wikimedia Commons) (b) Voltage applied between the cylinder and wire in a Geiger tube affects ions and electrons produced by radiation passing through the gas-filled cylinder. Ions move toward the cylinder and electrons toward the wire. The resulting current is detected and registered as a count.

Another radiation detection method records light produced when radiation interacts with materials. The energy of the radiation is sufficient to excite atoms in a material that may fluoresce, such as the phosphor used by Rutherford's group. Materials called **scintillators** use a more complex process to convert radiation energy into light. Scintillators may be liquid or solid, and they can be very efficient. Their light output can provide information about the energy, charge, and type of radiation. Scintillator light flashes are very brief in duration, allowing the detection of a huge number of particles in short periods of time. Scintillation detectors are used in a variety of research and diagnostic applications. Among those are the detection of the radiation from distant galaxies using satellite-mounted equipment and the detection of exotic particles in accelerator laboratories.

Virtual Physics

Beta Decay

Click to view content (https://www.openstax.org/l/21betadecayvid)

Watch beta decay occur for a collection of nuclei or for an individual nucleus. With this applet, individuals or groups of students can compare half-lives!

Check Your Understanding

2. What leads scientists to infer that the nuclear strong force exists?

a. A strong force must hold all the electrons outside the nucleus of an atom.
b. A strong force must counteract the highly attractive Coulomb force in the nucleus.
c. A strong force must hold all the neutrons together inside the nucleus.
d. A strong force must counteract the highly repulsive Coulomb force between protons in the nucleus.

22.3 Half Life and Radiometric Dating

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain radioactive half-life and its role in radiometric dating
- Calculate radioactive half-life and solve problems associated with radiometric dating

Section Key Terms

activity	becquerel	carbon-14 dating
decay constant	half-life	radioactive dating

Half-Life and the Rate of Radioactive Decay

Unstable nuclei decay. However, some nuclides decay faster than others. For example, radium and polonium, discovered by Marie and Pierre Curie, decay faster than uranium. That means they have shorter lifetimes, producing a greater rate of decay. Here we will explore half-life and activity, the quantitative terms for lifetime and rate of decay.

Why do we use the term like *half-life* rather than *lifetime*? The answer can be found by examining Figure 22.24, which shows how the number of radioactive nuclei in a sample decreases with time. The time in which half of the original number of nuclei decay is defined as the **half-life**, $t_{1/2}$. After one half-life passes, half of the remaining nuclei will decay in the next half-life. Then, half of that amount in turn decays in the following half-life. Therefore, the number of radioactive nuclei decreases from N to $N/2$ in one half-life, to $N/4$ in the next, to $N/8$ in the next, and so on. Nuclear decay is an example of a purely statistical process.

TIPS FOR SUCCESS

A more precise definition of half-life is that each nucleus has a 50 percent chance of surviving for a time equal to one half-life. If an individual nucleus survives through that time, it still has a 50 percent chance of surviving through another half-life. Even if it happens to survive hundreds of half-lives, it still has a 50 percent chance of surviving through one more. Therefore, the decay of a nucleus is like random coin flipping. The chance of heads is 50 percent, no matter what has happened before. The probability concept aligns with the traditional definition of half-life. Provided the number of nuclei is reasonably large, half of the original nuclei should decay during one half-life period.

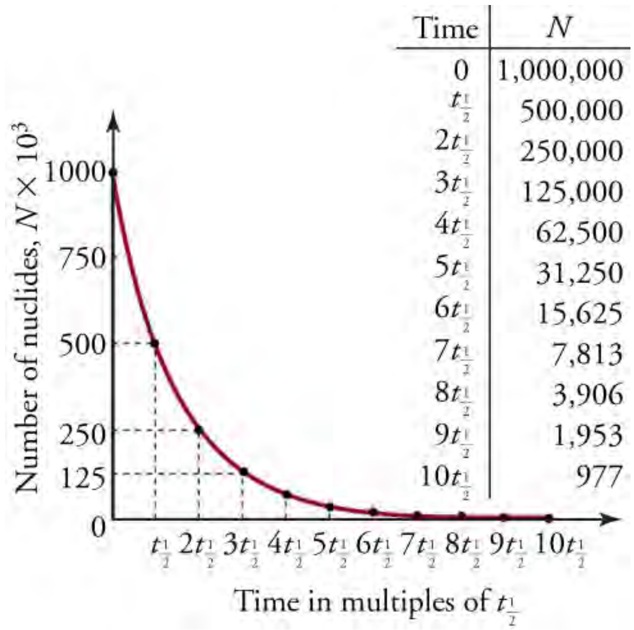


Figure 22.24 Radioactive decay reduces the number of radioactive nuclei over time. In one half-life ($t_{1/2}$), the number decreases to half of its original value. Half of what remains decays in the next half-life, and half of that in the next, and so on. This is exponential decay, as seen in the graph of the number of nuclei present as a function of time.

The following equation gives the quantitative relationship between the original number of nuclei present at time zero (N_O) and the number (N) at a later time t

$$N = N_O e^{-\lambda t}, \tag{22.45}$$

where $e = 2.71828...$ is the base of the natural logarithm, and λ is the **decay constant** for the nuclide. The shorter the half-life, the larger is the value of λ, and the faster the exponential $e^{-\lambda t}$ decreases with time. The decay constant can be found with the equation

$$\lambda = \frac{\ln(2)}{t_{1/2}} \approx \frac{0.693}{t_{1/2}}. \tag{22.46}$$

Activity, the Rate of Decay

What do we mean when we say a source is highly radioactive? Generally, it means the number of decays per unit time is very high. We define **activity** R to be the rate of decay expressed in decays per unit time. In equation form, this is

$$R = \frac{\Delta N}{\Delta t}, \tag{22.47}$$

where ΔN is the number of decays that occur in time Δt .

Activity can also be determined through the equation

$$R = \lambda N, \tag{22.48}$$

which shows that as the amount of radiative material (N) decreases, the rate of decay decreases as well.

The SI unit for activity is one decay per second and it is given the name **becquerel** (Bq) in honor of the discoverer of radioactivity. That is,

$1 \text{ Bq} = 1 \text{ decay/second}.$

Activity R is often expressed in other units, such as decays per minute or decays per year. One of the most common units for activity is the curie (Ci), defined to be the activity of 1 g of ^{226}Ra, in honor of Marie Curie's work with radium. The definition of the curie is

$$1 \text{ Ci} = 3.70 \times 10^{10} \text{ Bq}, \tag{22.49}$$

or 3.70×10^{10} decays per second.

Radiometric Dating

Radioactive dating or radiometric dating is a clever use of naturally occurring radioactivity. Its most familiar application is **carbon-14 dating**. Carbon-14 is an isotope of carbon that is produced when solar neutrinos strike ^{14}N particles within the atmosphere. Radioactive carbon has the same chemistry as stable carbon, and so it mixes into the biosphere, where it is consumed and becomes part of every living organism. Carbon-14 has an abundance of 1.3 parts per trillion of normal carbon, so if you know the number of carbon nuclei in an object (perhaps determined by mass and Avogadro's number), you can multiply that number by 1.3×10^{-12} to find the number of ^{14}C nuclei within the object. Over time, carbon-14 will naturally decay back to ^{14}N with a half-life of 5,730 years (note that this is an example of beta decay). When an organism dies, carbon exchange with the environment ceases, and ^{14}C is not replenished. By comparing the abundance of ^{14}C in an artifact, such as mummy wrappings, with the normal abundance in living tissue, it is possible to determine the artifact's age (or time since death). Carbon-14 dating can be used for biological tissues as old as 50 or 60 thousand years, but is most accurate for younger samples, since the abundance of ^{14}C nuclei in them is greater.

One of the most famous cases of carbon-14 dating involves the Shroud of Turin, a long piece of fabric purported to be the burial shroud of Jesus (see Figure 22.25). This relic was first displayed in Turin in 1354 and was denounced as a fraud at that time by a French bishop. Its remarkable negative imprint of an apparently crucified body resembles the then-accepted image of Jesus. As a result, the relic has been remained controversial throughout the centuries. Carbon-14 dating was not performed on the shroud until 1988, when the process had been refined to the point where only a small amount of material needed to be destroyed. Samples were tested at three independent laboratories, each being given four pieces of cloth, with only one unidentified piece from the shroud, to avoid prejudice. All three laboratories found samples of the shroud contain 92 percent of the ^{14}C found in living tissues, allowing the shroud to be dated (see Equation 22.57).

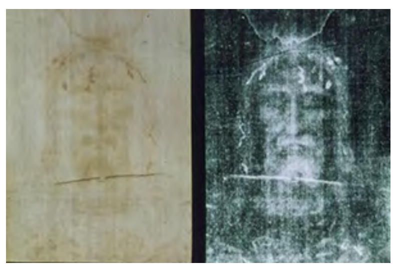


Figure 22.25 Part of the Shroud of Turin, which shows a remarkable negative imprint likeness of Jesus complete with evidence of crucifixion wounds. The shroud first surfaced in the 14th century and was only recently carbon-14 dated. It has not been determined how the image was placed on the material. (credit: Butko, Wikimedia Commons)

WORKED EXAMPLE

Carbon-11 Decay

Carbon-11 has a half-life of 20.334 min. (a) What is the decay constant for carbon-11?
If 1 kg of carbon-11 sample exists at the beginning of an hour, (b) how much material will remain at the end of the hour and (c) what will be the decay activity at that time?

Strategy

Since N_O refers to the amount of carbon-11 at the start, then after one half-life, the amount of carbon-11 remaining will be $N_O/2$. The decay constant is equivalent to the probability that a nucleus will decay each second. As a result, the half-life will need to be converted to seconds.

Solution

(a)

$$N = N_O e^{-\lambda t} \tag{22.50}$$

Since half of the carbon-11 remains after one half-life, $N/N_O = 0.5$.

$$0.5 = e^{-\lambda t} \tag{22.51}$$

Take the natural logarithm of each side to isolate the decay constant.

$$\ln(0.5) = -\lambda t \tag{22.52}$$

Convert the 20.334 min to seconds.

$$\begin{aligned}-0.693 &= (-\lambda)(20.334\ \text{min})\left(\tfrac{60\ \text{s}}{1\ \text{min}}\right)\\ -0.693 &= (-\lambda)(1{,}220.04\ \text{s})\\ \tfrac{-0.693}{1{,}220.04\ \text{s}} &= -\lambda\\ \lambda &= 5.68\ \times\ 10^{-4}\text{s}^{-1}\end{aligned} \tag{22.53}$$

(b) The amount of material after one hour can be found by using the equation

$$N = N_O e^{-\lambda t}, \tag{22.54}$$

with t converted into seconds and N_O written as 1,000 g

$$\begin{aligned}N &= (1{,}000\ \ \text{g})e - (0.000568)(60.60)\\ N &= 129.4\ g\end{aligned} \tag{22.55}$$

(c) The decay activity after one hour can be found by using the equation

$$R = \lambda N \tag{22.56}$$

for the mass value after one hour.

$$R = \lambda N = \left(0.000568\ \frac{\text{decays}}{\text{second}}\right)(129.4\ \text{grams}) = 0.0735\ \text{Bq} \tag{22.57}$$

Discussion

(a) The decay constant shows that 0.0568 percent of the nuclei in a carbon-11 sample will decay each second. Another way of considering the decay constant is that a given carbon-11 nuclei has a 0.0568 percent probability of decaying each second. The decay of carbon-11 allows it to be used in positron emission topography (PET) scans; however, its 20.334 min half-life does pose challenges for its administration.

(b) One hour is nearly three full half-lives of the carbon-11 nucleus. As a result, one would expect the amount of sample remaining to be approximately one eighth of the original amount. The 129.4 g remaining is just a bit larger than one-eighth, which is sensible given a half-life of just over 20 min.

(c) Label analysis shows that the unit of Becquerel is sensible, as there are 0.0735 g of carbon-11 decaying each second. That is smaller amount than at the beginning of the hour, when $R = \left(0.000568\ \frac{\text{decay}}{\text{s}}\right)(1{,}000\ \text{g}) = 0.568$ g of carbon-11 were decaying each second.

WORKED EXAMPLE

How Old is the Shroud of Turin?

Calculate the age of the Shroud of Turin given that the amount of ^{14}C found in it is 92 percent of that in living tissue.

Strategy

Because 92 percent of the ^{14}C remains, $N/N_O = 0.92$. Therefore, the equation $N = N_O e^{-\lambda t}$ can be used to find λt. We also know that the half-life of ^{14}C is 5,730 years, and so once λt is known, we can find λ and then find t as requested. Here, we assume that the decrease in ^{14}C is solely due to nuclear decay.

Solution

Solving the equation $N = N_O e^{-\lambda t}$ for N/N_O gives

$$\frac{N}{N_O} = e^{-\lambda t}. \tag{22.58}$$

Thus,

$$0.92 = e^{-\lambda t}. \tag{22.59}$$

Taking the natural logarithm of both sides of the equation yields

$$\ln\ 0.92 = -\lambda t \tag{22.60}$$

so that

$$-0.0834 = -\lambda t. \tag{22.61}$$

Rearranging to isolate *t* gives

$$t = \frac{0.0834}{\lambda}. \tag{22.62}$$

Now, the equation $\lambda = \frac{0.693}{t_{1/2}}$ can be used to find λ for ${}^{14}\text{C}$. Solving for λ and substituting the known half-life gives

$$\lambda = \frac{0.693}{t_{1/2}} = \frac{0.693}{5{,}730\ \text{years}} = 1.21 \times 10^{-4}\text{y}^{-1}. \tag{22.63}$$

We enter that value into the previous equation to find *t*.

$$t = \frac{0.0834}{1.21 \times 10^{-4}} = 690\ \text{years}. \tag{22.64}$$

Discussion

This dates the material in the shroud to 1988–690 = 1300. Our calculation is only accurate to two digits, so that the year is rounded to 1300. The values obtained at the three independent laboratories gave a weighted average date of 1320 ± 60. That uncertainty is typical of carbon-14 dating and is due to the small amount of 14 C in living tissues, the amount of material available, and experimental uncertainties (reduced by having three independent measurements). That said, is it notable that the carbon-14 date is consistent with the first record of the shroud's existence and certainly inconsistent with the period in which Jesus lived.

There are other noncarbon forms of radioactive dating. Rocks, for example, can sometimes be dated based on the decay of ${}^{238}\text{U}$. The decay series for ${}^{238}\text{U}$ ends with ${}^{206}\text{Pb}$, so the ratio of those nuclides in a rock can be used an indication of how long it has been since the rock solidified. Knowledge of the ${}^{238}\text{U}$ half-life has shown, for example, that the oldest rocks on Earth solidified about 3.5×10^{9} years ago.

Virtual Physics

Radioactive Dating Game

[Click to view content (https://www.openstax.org/l/02radioactive_dating_game)](https://www.openstax.org/l/02radioactive_dating_game)

Learn about different types of radiometric dating, such as carbon dating. Understand how decay and half-life work to enable radiometric dating to work. Play a game that tests your ability to match the percentage of the dating element that remains to the age of the object.

22.4 Nuclear Fission and Fusion

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Explain nuclear fission
- Explain nuclear fusion
- Describe how the processes of fission and fusion work in nuclear weapons and in generating nuclear power

Section Key Terms

chain reaction	critical mass	liquid drop model
nuclear fission	nuclear fusion	proton-proton cycle

The previous section dealt with naturally occurring nuclear decay. Without human intervention, some nuclei will change composition in order to achieve a stable equilibrium. This section delves into a less-natural process. Knowing that energy can be emitted in various forms of nuclear change, is it possible to create a nuclear reaction through our own intervention? The answer to this question is yes. Through two distinct methods, humankind has discovered multiple ways of manipulating the atom to release its internal energy.

Nuclear Fission

In simplest terms, **nuclear fission** is the splitting of an atomic bond. Given that it requires great energy separate two nucleons, it may come as a surprise to learn that splitting a nucleus can *release* vast potential energy. And although it is true that huge amounts of energy can be released, considerable effort is needed to do so in practice.

An unstable atom will naturally decay, but it may take millions of years to do so. As a result, a physical catalyst is necessary to produce useful energy through nuclear fission. The catalyst typically occurs in the form of a free neutron, projected directly at the nucleus of a high-mass atom.

As shown in Figure 22.26, a neutron strike can cause the nucleus to elongate, much like a drop of liquid water. This is why the model is known as the **liquid drop model**. As the nucleus elongates, nucleons are no longer so tightly packed, and the repulsive electromagnetic force can overcome the short-range strong nuclear force. The imbalance of forces can result in the two ends of the drop flying apart, with some of the nuclear binding energy released to the surroundings.

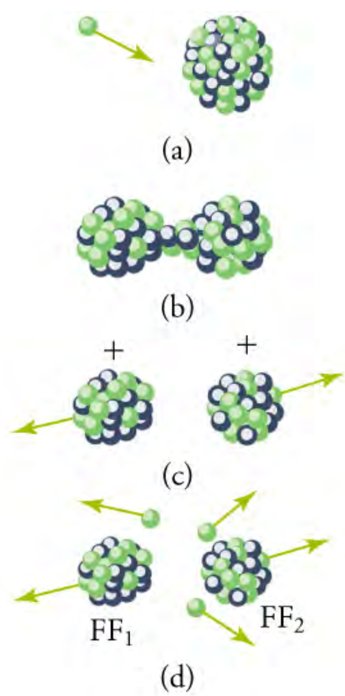


Figure 22.26 Neutron-induced fission is shown. First, energy is put into a large nucleus when it absorbs a neutron. Acting like a struck liquid drop, the nucleus deforms and begins to narrow in the middle. Since fewer nucleons are in contact, the repulsive Coulomb force is able to break the nucleus into two parts with some neutrons also flying away.

As you can imagine, the consequences of the nuclei splitting are substantial. When a nucleus is split, it is not only energy that is released, but a small number of neutrons as well. Those neutrons have the potential to cause further fission in other nuclei, especially if they are directed back toward the other nuclei by a dense shield or neutron reflector (see part (d) of Figure 22.26).

However, not every neutron produced by fission induces further fission. Some neutrons escape the fissionable material, while others interact with a nucleus without making it split. We can enhance the number of fissions produced by neutrons by having a large amount of fissionable material as well as a neutron reflector. The minimum amount necessary for self-sustained fission of a given nuclide is called its **critical mass**. Some nuclides, such as ^{239}Pu, produce more neutrons per fission than others, such as ^{235}U. Additionally, some nuclides are easier to make fission than others. In particular, ^{235}U and ^{239}Pu are easier to fission than the much more abundant ^{238}U. Both factors affect critical mass, which is smallest for ^{239}Pu. The self-sustained fission of nuclei is commonly referred to as a **chain reaction**, as shown in Figure 22.27.

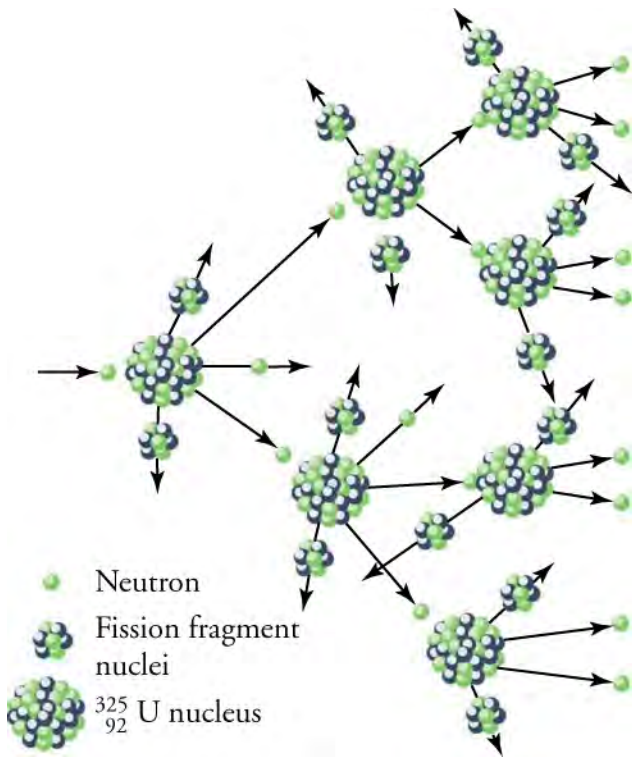


Figure 22.27 A chain reaction can produce self-sustained fission if each fission produces enough neutrons to induce at least one more fission. This depends on several factors, including how many neutrons are produced in an average fission and how easy it is to make a particular type of nuclide fission.

A chain reaction can have runaway results. If each atomic split results in two nuclei producing a new fission, the number of nuclear reactions will increase exponentially. One fission will produce two atoms, the next round of fission will create four atoms, the third round eight atoms, and so on. Of course, each time fission occurs, more energy will be emitted, further increasing the power of the atomic reaction. And that is just if two neutrons create fission reactions each round. Perhaps you can now see why so many people consider atomic energy to be an exciting energy source!

To make a self-sustained nuclear fission reactor with ^{235}U, it is necessary to slow down the neutrons. Water is very effective at this, since neutrons collide with protons in water molecules and lose energy. Figure 22.28 shows a schematic of a reactor design called the *pressurized water reactor*.

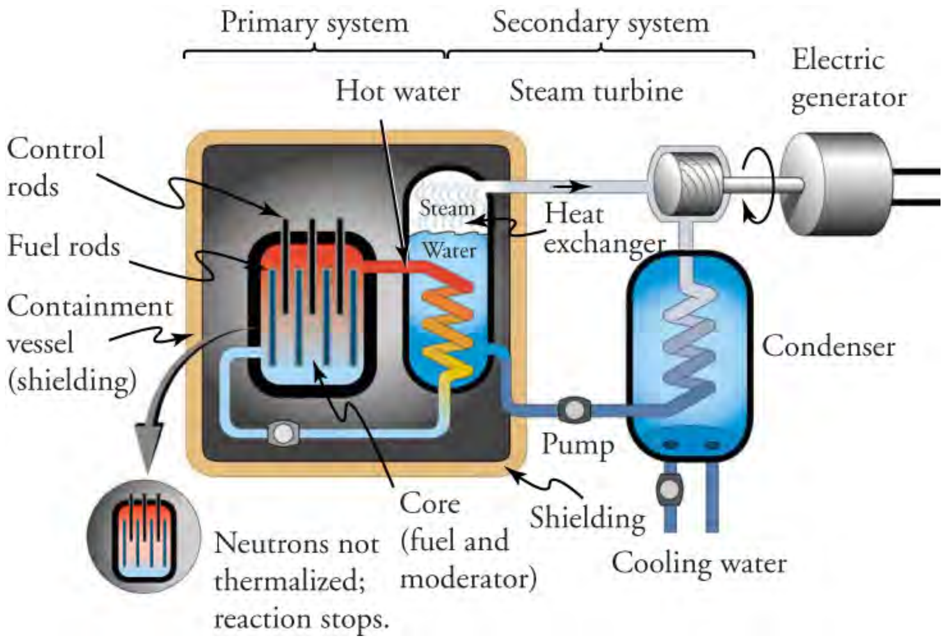


Figure 22.28 A pressurized water reactor is cleverly designed to control the fission of large amounts of ^{235}U, while using the heat produced in the fission reaction to create steam for generating electrical energy. Control rods adjust neutron flux so that it is self-sustaining. In case the reactor overheats and boils the water away, the chain reaction terminates, because water is needed to slow down the neutrons. This inherent safety feature can be overwhelmed in extreme circumstances.

Control rods containing nuclides that very strongly absorb neutrons are used to adjust neutron flux. To produce large amounts of power, reactors contain hundreds to thousands of critical masses, and the chain reaction easily becomes self-sustaining. Neutron flux must be carefully regulated to avoid an out-of-control exponential increase in the rate of fission.

Control rods help prevent overheating, perhaps even a meltdown or explosive disassembly. The water that is used to slow down neutrons, necessary to get them to induce fission in ^{235}U, and achieve criticality, provides a negative feedback for temperature increase. In case the reactor overheats and boils the water to steam or is breached, the absence of water kills the chain reaction. Considerable heat, however, can still be generated by the reactor's radioactive fission products. Other safety features, thus, need to be incorporated in the event of a loss of coolant accident, including auxiliary cooling water and pumps.

Energies in Nuclear Fission

The following are two interesting facts to consider:

- The average fission reaction produces 200 MeV of energy.
- If you were to measure the mass of the products of a nuclear reaction, you would find that their mass was slightly less than the mass of the original nucleus.

How are those things possible? Doesn't the fission reaction's production of energy violate the conservation of energy? Furthermore, doesn't the loss in mass in the reaction violate the conservation of mass? Those are important questions, and they can both be answered with one of the most famous equations in scientific history.

$$E = mc^2 \tag{22.65}$$

Recall that, according to Einstein's theory, energy and mass are essentially the same thing. In the case of fission, the mass of the products is less than that of the reactants because the missing mass appears in the form of the energy released in the reaction, with a constant value of c^2 Joules of energy converted for each kilogram of material. The value of c^2 is substantial—from Einstein's equation, the amount of energy in just 1 gram of mass would be enough to support the average U.S. citizen for more than 270 years! The example below will show you how a mass-energy transformation of this type takes place.

WORKED EXAMPLE

Calculating Energy from a Kilogram of Fissionable Fuel

Calculate the amount of energy produced by the fission of 1.00 kg of ^{235}U, given the average fission reaction of

$$^{235}\text{U produces 200 MeV.} \tag{22.66}$$

Strategy

The total energy produced is the number of ^{235}U atoms times the given energy per ^{235}U fission. We should therefore find the number of ^{235}U atoms in 1.00 kg.

Solution

The number of ^{235}U atoms in 1.00 kg is Avogadro's number times the number of moles. One mole of ^{235}U has a mass of 235.04 g; thus, there are $(1{,}000 \text{ g})/(235.04.00 \text{ g/mol}) = 4.25 \text{ mol}$. The number of ^{235}U atoms is therefore

$$(4.25 \text{ mol})\left(6.02 \times 10^{23}\text{U/mol}\right) = 2.56 \times 10^{24}\text{atomsof235U}.$$

So the total energy released is

$$E = \left(2.56 \times 10^{24}\right)\left(\frac{200 \text{ MeV}}{^{235}\text{U}}\right)\left(\frac{1.60 \times 10^{-13}\text{J}}{\text{MeV}}\right) = 8.21 \times 10^{13}\text{J}. \tag{22.67}$$

Discussion

The result is another impressively large amount of energy, equivalent to about 14,000 barrels of crude oil or 600,000 gallons of gasoline. But, it is only one fourth the energy produced by the fusion of a kilogram of a mixture of deuterium and tritium. Even though each fission reaction yields about ten times the energy of a fusion reaction, the energy per kilogram of fission fuel is less, because there are far fewer moles per kilogram of the heavy nuclides. Fission fuel is also much scarcer than fusion fuel, and less than 1 percent of uranium (the 235 U) is readily usable.

Virtual Physics

Nuclear Fission

Click to view content (https://www.openstax.org/l/16fission)

Start a chain reaction, or introduce nonradioactive isotopes to prevent one. Use the applet to control energy production in a nuclear reactor!

Nuclear Fusion

Nuclear fusion is defined as the combining, or fusing, of two nuclei and, the combining of nuclei also results in an emission of energy. For many, the concept is counterintuitive. After all, if energy is released when a nucleus is split, how can it also be released when nucleons are combined together? The difference between fission and fusion, which results from the size of the nuclei involved, will be addressed next.

Remember that the structure of a nucleus is based on the interplay of the compressive nuclear strong force and the repulsive electromagnetic force. For nuclei that are less massive than iron, the nuclear force is actually stronger than that of the Coulomb force. As a result, when a low-mass nucleus absorbs nucleons, the added neutrons and protons bind the nucleus more tightly. The increased nuclear strong force does work on the nucleus, and energy is released.

Once the size of the created nucleus exceeds that of iron, the short-ranging nuclear force does not have the ability to bind a nucleus more tightly, and the emission of energy ceases. In fact, for fusion to occur for elements of greater mass than iron, energy must be added to the system! Figure 22.29 shows an energy-mass curve commonly used to describe nuclear reactions. Notice the location of iron (Fe) on the graph. All low-mass nuclei to the left of iron release energy through fusion, while all high-mass particles to the right of iron produce energy through fission.

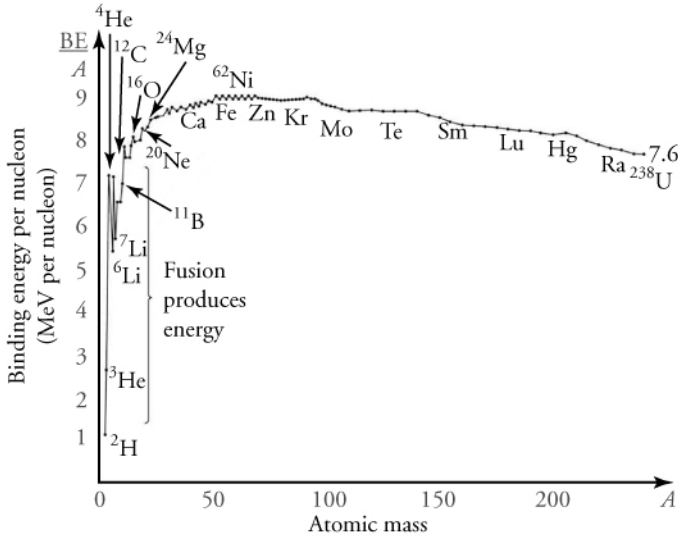


Figure 22.29 Fusion of light nuclei to form medium-mass nuclei converts mass to energy, because binding energy per nucleon (BE/A) is greater for the product nuclei. The larger BE/A is, the less mass per nucleon, and so mass is converted to energy and released in such fusion reactions.

TIPS FOR SUCCESS

Just as it is not possible for the elements to the left of iron in the figure to naturally fission, it is not possible for elements to the right of iron to naturally undergo fusion, as that process would require the addition of energy to occur. Furthermore, notice that elements commonly discussed in fission and fusion are elements that can provide the greatest change in binding energy, such as uranium and hydrogen.

Iron's location on the energy-mass curve is important, and explains a number of its characteristics, including its role as an elemental endpoint in fusion reactions in stars.

The major obstruction to fusion is the Coulomb repulsion force between nuclei. Since the attractive nuclear force that can fuse nuclei together is short ranged, the repulsion of like positive charges must be overcome in order to get nuclei close enough to induce fusion. Figure 22.30 shows an approximate graph of the potential energy between two nuclei as a function of the distance between their centers. The graph resembles a hill with a well in its center. A ball rolled to the left must have enough kinetic energy to get over the hump before it falls into the deeper well with a net gain in energy. So it is with fusion. If the nuclei are given enough kinetic energy to overcome the electric potential energy due to repulsion, then they can combine, release energy, and fall into a deep well. One way to accomplish that end is to heat fusion fuel to high temperatures so that the kinetic energy of thermal motion is sufficient to get the nuclei together.

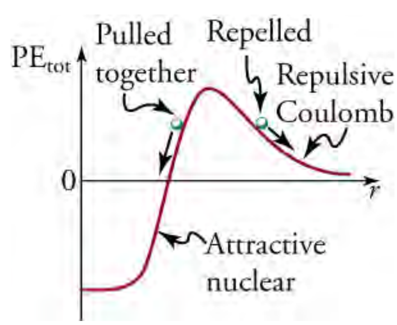


Figure 22.30 Potential energy between two light nuclei graphed as a function of distance between them. If the nuclei have enough kinetic energy to get over the Coulomb repulsion hump, they combine, release energy, and drop into a deep attractive well.

You might think that, in our Sun, nuclei are constantly coming into contact and fusing. However, this is only partially true. Only at the Sun's core are the particles close enough and the temperature high enough for fusion to occur!

In the series of reactions below, the Sun produces energy by fusing protons, or hydrogen nuclei ($^{1}\mathrm{H}$, by far the Sun's most abundant nuclide) into helium nuclei ^4He . The principal sequence of fusion reactions forms what is called the **proton-proton cycle**

$$\begin{array}{ll} {}^1\text{H} + {}^1\text{H} \rightarrow {}^2\text{H} + e^+ + v_e & (0.42 \text{ MeV}) \\ {}^1\text{H} + {}^2\text{H} \rightarrow {}^3\text{He} + \gamma & (5.49 \text{ MeV}) \\ {}^3\text{He} + {}^3\text{He} \rightarrow {}^4\text{He} + {}^1\text{H} + {}^1\text{H} & (12.86 \text{ MeV}), \end{array} \tag{22.68}$$

where e^+ stands for a positron and v_e is an electron neutrino. The energy in parentheses is *released* by the reaction. Note that the first two reactions must occur twice for the third to be possible, so the cycle consumes six protons (^1H) but gives back two. Furthermore, the two positrons produced will find two electrons and annihilate to form four more γ rays, for a total of six. The overall cycle is thus

$$2e - +4{}^1\text{H} \rightarrow 4 \text{ He} + 2v_e + 6\gamma \quad (26.7 \text{ MeV}),$$

where the 26.7 MeV includes the annihilation energy of the positrons and electrons and is distributed among all the reaction products. The solar interior is dense, and the reactions occur deep in the Sun where temperatures are highest. It takes about 32,000 years for the energy to diffuse to the surface and radiate away. However, the neutrinos can carry their energy out of the Sun in less than two seconds, because they interact so weakly with other matter. Negative feedback in the Sun acts as a thermostat to regulate the overall energy output. For instance, if the interior of the Sun becomes hotter than normal, the reaction rate increases, producing energy that expands the interior. The expansion cools it and lowers the reaction rate. Conversely, if the interior becomes too cool, it contracts, increasing the temperature and therefore the reaction rate (see Figure 22.31). Stars like the Sun are stable for billions of years, until a significant fraction of their hydrogen has been depleted.

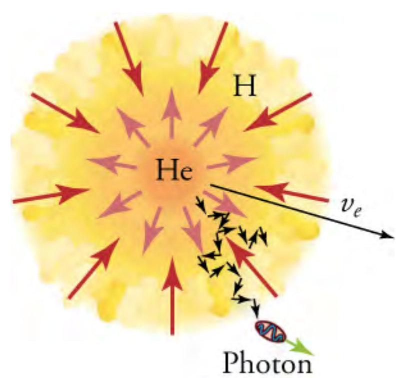


Figure 22.31 Nuclear fusion in the Sun converts hydrogen nuclei into helium; fusion occurs primarily at the boundary of the helium core, where the temperature is highest and sufficient hydrogen remains. Energy released diffuses slowly to the surface, with the exception of neutrinos, which escape immediately. Energy production remains stable because of negative-feedback effects.

Nuclear Weapons and Nuclear Power

The world was in political turmoil when fission was discovered in 1938. Compounding the troubles, the possibility of a self-sustained chain reaction was immediately recognized by leading scientists the world over. The enormous energy known to be in nuclei, but considered inaccessible, now seemed to be available on a large scale.

Within months after the announcement of the discovery of fission, Adolf Hitler banned the export of uranium from newly occupied Czechoslovakia. It seemed that the possible military value of uranium had been recognized in Nazi Germany, and that a serious effort to build a nuclear bomb had begun.

Alarmed scientists, many of whom fled Nazi Germany, decided to take action. None was more famous or revered than Einstein. It was felt that his help was needed to get the American government to make a serious effort at constructing nuclear weapons as a matter of survival. Leo Szilard, a Hungarian physicist who had emigrated to America, took a draft of a letter to Einstein, who, although a pacifist, signed the final version. The letter was for President Franklin Roosevelt, warning of the German potential to build extremely powerful bombs of a new type. It was sent in August of 1939, just before the German invasion of Poland that marked the start of World War II.

It was not until December 6, 1941, the day before the Japanese attack on Pearl Harbor, that the United States made a massive commitment to building a nuclear bomb. The top secret Manhattan Project was a crash program aimed at beating the Germans.

It was carried out in remote locations, such as Los Alamos, New Mexico, whenever possible, and eventually came to cost billions of dollars and employ the efforts of more than 100,000 people. J. Robert Oppenheimer (1904–1967), a talented physicist, was chosen to head the project. The first major step was made by Enrico Fermi and his group in December 1942, when they completed the first self-sustaining nuclear reactor. This first *atomic pile*, built in a squash court at the University of Chicago, proved that a fission chain reaction was possible.

Plutonium was recognized as easier to fission with neutrons and, hence, a superior fission material very early in the Manhattan Project. Plutonium availability was uncertain, and so a uranium bomb was developed simultaneously. Figure 22.32 shows a gun-type bomb, which takes two subcritical uranium masses and shoots them together. To get an appreciable yield, the critical mass must be held together by the explosive charges inside the cannon barrel for a few microseconds. Since the buildup of the uranium chain reaction is relatively slow, the device to bring the critical mass together can be relatively simple. Owing to the fact that the rate of spontaneous fission is low, a neutron source is at the center the assembled critical mass.

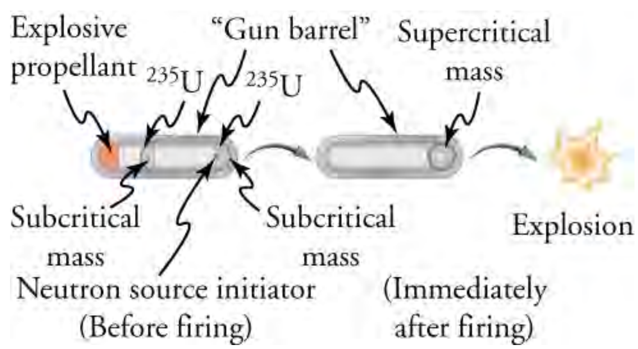


Figure 22.32 A gun-type fission bomb for ^{235}U utilizes two subcritical masses forced together by explosive charges inside a cannon barrel. The energy yield depends on the amount of uranium and the time it can be held together before it disassembles itself.

Plutonium's special properties necessitated a more sophisticated critical mass assembly, shown schematically in Figure 22.33. A spherical mass of plutonium is surrounded by shaped charges (high explosives that focus their blast) that implode the plutonium, crushing it into a smaller volume to form a critical mass. The implosion technique is faster and more effective, because it compresses three-dimensionally rather than one-dimensionally as in the gun-type bomb. Again, a neutron source is included to initiate the chain reaction.

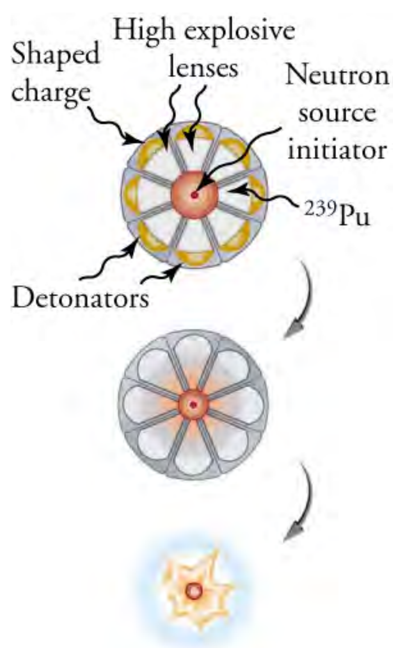


Figure 22.33 An implosion created by high explosives compresses a sphere of ^{239}Pu into a critical mass. The superior fissionability of plutonium has made it the preferred bomb material.

Owing to its complexity, the plutonium bomb needed to be tested before there could be any attempt to use it. On July 16, 1945, the test named Trinity was conducted in the isolated Alamogordo Desert in New Mexico, about 200 miles south of Los Alamos (see Figure 22.34). A new age had begun. The yield of the Trinity device was about 10 kilotons (kT), the equivalent of 5,000 of the largest conventional bombs.

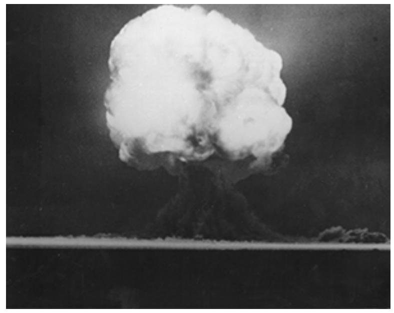


Figure 22.34 Trinity test (1945), the first nuclear bomb (credit: U.S. Department of Energy)

Although Germany surrendered on May 7, 1945, Japan had been steadfastly refusing to surrender for many months, resulting large numbers of civilian and military casualties. Invasion plans by the Allies estimated a million casualties of their own and untold losses of Japanese lives. The bomb was viewed as a way to end the war. The first bomb used was a gun-type uranium bomb dropped on Hiroshima on August 6 by the United States. Its yield of about 15 kT destroyed the city and killed an estimated 80,000 people, with 100,000 more being seriously injured. The second bomb was an implosion-type plutonium bomb dropped on Nagasaki only three days later. Its 20-kT yield killed at least 50,000 people, something less than Hiroshima because of the hilly terrain and the fact that it was a few kilometers off target. The Japanese were told that one bomb a week would be dropped until they surrendered unconditionally, which they did on August 14. In actuality, the United States had only enough plutonium for one more bomb, as yet unassembled.

Knowing that fusion produces several times more energy per kilogram of fuel than fission, some scientists pursued the idea of constructing a fusion bomb. The first such bomb was detonated by the United States several years after the first fission bombs, on October 31, 1952, at Eniwetok Atoll in the Pacific Ocean. It had a yield of 10 megatons (MT), about 670 times that of the fission bomb that destroyed Hiroshima. The Soviet Union followed with a fusion device of its own in August 1953, and a weapons race, beyond the aim of this text to discuss, continued until the end of the Cold War.

Figure 22.35 shows a simple diagram of how a thermonuclear bomb is constructed. A fission bomb is exploded next to fusion fuel in the solid form of lithium deuteride. Before the shock wave blows it apart, γ rays heat and compress the fuel, and neutrons create tritium through the reaction $n + ^6\text{Li} \rightarrow ^3\text{H} + ^4\text{He}$. Additional fusion and fission fuels are enclosed in a dense shell of ^{238}U. At the same time that the uranium shell reflects the neutrons back into the fuel to enhance its fusion, the fast-moving neutrons cause the plentiful and inexpensive ^{238}U to fission, part of what allows thermonuclear bombs to be so large.

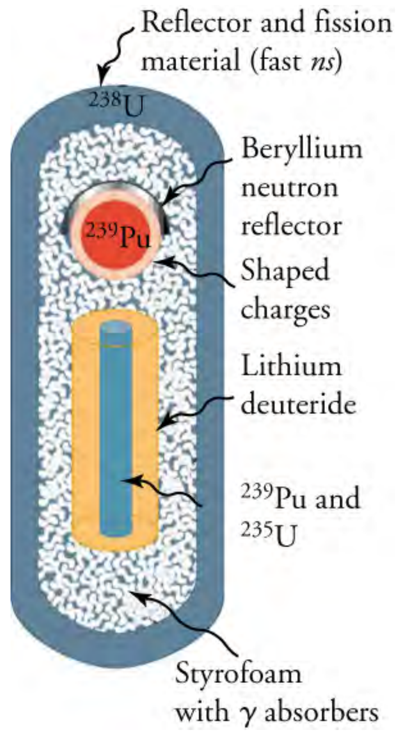


Figure 22.35 This schematic of a fusion bomb (H-bomb) gives some idea of how the ^{239}Pu fission trigger is used to ignite fusion fuel. Neutrons and γ rays transmit energy to the fusion fuel, create tritium from deuterium, and heat and compress the fusion fuel. The outer shell of ^{238}U serves to reflect some neutrons back into the fuel, causing more fusion, and it boosts the energy output by fissioning itself when neutron energies become high enough.

Of course, not all applications of nuclear physics are as destructive as the weapons described above. Hundreds of nuclear fission power plants around the world attest to the fact that controlled fission is both practical and economical. Given growing concerns over global warming, nuclear power is often seen as a viable alternative to energy derived from fossil fuels.

BOUNDLESS PHYSICS

Fusion Reactors

For decades, fusion reactors have been deemed the *energy of the future*. A safer, cleaner, and more abundant potential source of energy than its fission counterpart, images of the fusion reactor have been conjured up each time the need for a renewable, environmentally friendly resource is discussed. Now, after more than half a century of speculating, some scientists believe that fusion reactors are nearly here.

In creating energy by combining atomic nuclei, the fusion reaction holds many advantages over fission. First, fusion reactions are more efficient, releasing 3 to 4 times more energy than fission per gram of fuel. Furthermore, unlike fission reactions that require *heavy* elements like uranium that are difficult to obtain, fusion requires *light* elements that are abundant in nature. The greatest advantage of the fusion reaction, however, is in its ability to be controlled. While traditional nuclear reactors create worries about meltdowns and radioactive waste, neither is a substantial concern with the fusion reaction. Consider that fusion reactions require a large amount of energy to overcome the repulsive Coulomb force and that the byproducts of a fusion reaction are largely limited to helium nuclei.

In order for fusion to occur, hydrogen isotopes of deuterium and tritium must be acquired. While deuterium can easily be gathered from ocean water, tritium is slightly more difficult to come by, though it can be manufactured from Earth's abundant lithium. Once acquired, the hydrogen isotopes are injected into an empty vessel and subjected to temperature and pressure great enough to mimic the conditions at the core of our Sun. Using carefully controlled high-frequency radio waves, the hydrogen isotopes are broken into plasma and further controlled through an electromagnetic field. As the electromagnetic field continues to exert pressure on the hydrogen plasma, enough energy is supplied to cause the hydrogen plasma to fuse into helium.

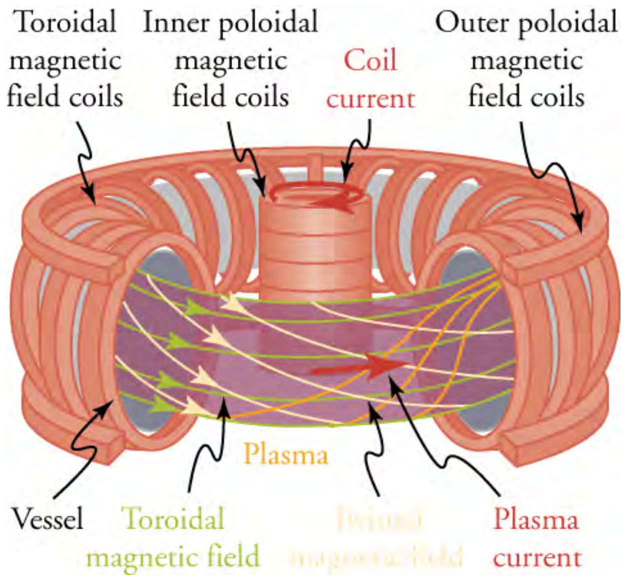


Figure 22.36 Tokamak confinement of nuclear fusion plasma. The magnetic field lines are used to confine the high-temperature plasma (purple). Research is currently being done to increase the efficiency of the tokamak confinement model.

Once the plasma fuses, high-velocity neutrons are ejected from the newly formed helium atoms. Those high velocity neutrons, carrying the excess energy stored within bonds of the original hydrogen, are able to travel unaffected by the applied magnetic field. In doing so, they strike a barrier around the nuclear reactor, transforming their excess energy to heat. The heat is then harvested to make steam that drives turbines. Hydrogen's tremendous power is now usable!

The historical concern with nuclear fusion reactors is that the energy required to control the electromagnetic field is greater than the energy harvested from the hydrogen atoms. However, recent research by both Lockheed Martin engineers and scientists at the Lawrence Livermore National Laboratory has yielded exciting theoretical improvements in efficiency. At the time of this writing, a test facility called ITER (International Thermonuclear Experimental Reactor) is being constructed in southern France. A joint venture of the European Union, the United States, Japan, Russia, China, South Korea, and India, ITER is designed for further study into the future of nuclear fusion energy production.

22.5 Medical Applications of Radioactivity: Diagnostic Imaging and Radiation

Section Learning Objectives

By the end of this section, you will be able to do the following:

- Describe how nuclear imaging works (e.g., radioisotope imaging, PET)
- Describe the ionizing effects of radiation and how they can be used for medical treatment

Section Key Terms

Anger camera	rad	radiopharmaceutical	therapeutic ratio
relative biological effectiveness (RBE)	roentgen equivalent man (rem)	tagged	

Medical Applications of Nuclear Physics

Applications of nuclear physics have become an integral part of modern life. From the bone scan that detects one cancer to the radioiodine treatment that cures another, nuclear radiation has diagnostic and therapeutic effects on medicine.

Medical Imaging

A host of medical imaging techniques employ nuclear radiation. What makes nuclear radiation so useful? First, γ radiation can easily penetrate tissue; hence, it is a useful probe to monitor conditions inside the body. Second, nuclear radiation depends on the nuclide and not on the chemical compound it is in, so that a radioactive nuclide can be put into a compound designed for specific purposes. When that is done, the compound is said to be **tagged**. A tagged compound used for medical purposes is called a **radiopharmaceutical**. Radiation detectors external to the body can determine the location and concentration of a radiopharmaceutical to yield medically useful information. For example, certain drugs are concentrated in inflamed regions of the body, and their locations can aid diagnosis and treatment as seen in Figure 22.37. Another application utilizes a radiopharmaceutical that the body sends to bone cells, particularly those that are most active, to detect cancerous tumors or healing points. Images can then be produced of such bone scans. Clever use of radioisotopes determines the functioning of body organs, such as blood flow, heart muscle activity, and iodine uptake in the thyroid gland. For instance, a radioactive form of iodine can be used to monitor the thyroid, a radioactive thallium salt can be used to follow the blood stream, and radioactive gallium can be used for cancer imaging.

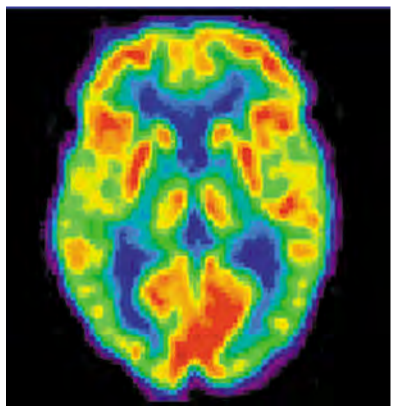


Figure 22.37 A radiopharmaceutical was used to produce this brain image of a patient with Alzheimer's disease. Certain features are computer enhanced. (credit: National Institutes of Health)

Once a radioactive compound has been ingested, a device like that shown in Figure 22.38 is used to monitor nuclear activity. The device, called an **Anger camera** or gamma camera uses a piece of lead with holes bored through it. The gamma rays are redirected through the collimator to narrow their beam, and are then interpreted using a device called a scintillator. The computer analysis of detector signals produces an image. One of the disadvantages of this detection method is that there is no depth information (i.e., it provides a two-dimensional view of the tumor as opposed to a three-dimensional view), because radiation from any location under that detector produces a signal.

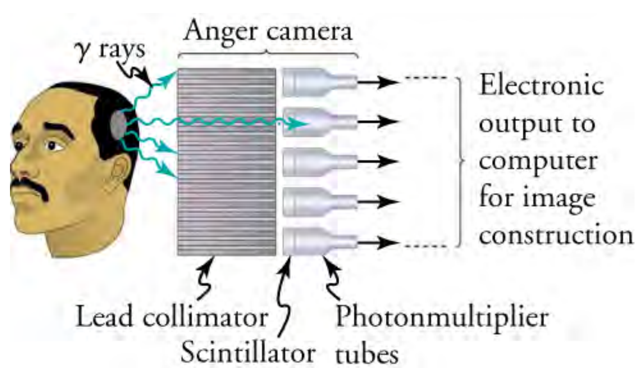


Figure 22.38 An Anger or gamma camera consists of a lead collimator and an array of detectors. Gamma rays produce light flashes in the scintillators. The light output is converted to an electrical signal by the photomultipliers. A computer constructs an image from the detector output.

Single-photon-emission computer tomography (SPECT) used in conjunction with a CT scanner improves on the process carried out by the gamma camera. Figure 22.39 shows a patient in a circular array of SPECT detectors that may be stationary or rotated, with detector output used by a computer to construct a detailed image. The spatial resolution of this technique is poor, but the three-dimensional image created results in a marked improvement in contrast.

Figure 22.39 SPECT uses a rotating camera to form an image of the concentration of a radiopharmaceutical compound. (credit: Woldo, Wikimedia Commons)

Positron emission tomography (or PET) scans utilize images produced by β^+ emitters. When the emitted positron β^+ encounters an electron, mutual annihilation occurs, producing two γ rays. Those γ rays have identical 0.511 MeV energies (the energy comes from the destruction of an electron or positron mass) and they move directly away from each other, allowing detectors to determine their point of origin accurately (as shown in Figure 22.40). It requires detectors on opposite sides to simultaneously (i.e., at the same time) detect photons of 0.511 MeV energy and utilizes computer imaging techniques similar to those in SPECT and CT scans. PET is used extensively for diagnosing brain disorders. It can note decreased metabolism in certain regions that accompany Alzheimer's disease. PET can also locate regions in the brain that become active when a person carries out specific activities, such as speaking, closing his or her eyes, and so on.

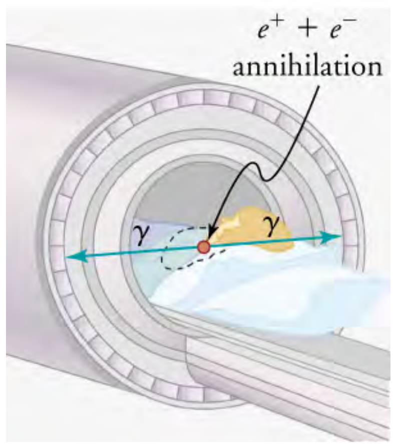


Figure 22.40 A PET system takes advantage of the two identical γ -ray photons produced by positron-electron annihilation. The γ rays are emitted in opposite directions, so that the line along which each pair is emitted is determined. Various events detected by several pairs of detectors are then analyzed by the computer to form an accurate image.

Ionizing Radiation on the Body

We hear many seemingly contradictory things about the biological effects of ionizing radiation. It can cause cancer, burns, and hair loss, and yet it is used to treat and even cure cancer. How do we understand such effects? Once again, there is an underlying simplicity in nature, even in complicated biological organisms. All the effects of ionizing radiation on biological tissue can be understood by knowing that ionizing radiation affects molecules within cells, particularly DNA molecules. Let us take a brief look at molecules within cells and how cells operate. Cells have long, double-helical DNA molecules containing chemical patterns called genetic codes that govern the function and processes undertaken by the cells. Damage to DNA consists of breaks in chemical bonds or other changes in the structural features of the DNA chain, leading to changes in the genetic code. In human cells, we can have as many as a million individual instances of damage to DNA per cell per day. The repair ability of DNA is vital for maintaining the integrity of the genetic code and for the normal functioning of the entire organism. A cell with a damaged ability to repair DNA, which could have been induced by ionizing radiation, can do one of the following:

- The cell can go into an irreversible state of dormancy, known as senescence.
- The cell can commit suicide, known as programmed cell death.
- The cell can go into unregulated cell division, leading to tumors and cancers.

Since ionizing radiation damages the DNA, ionizing radiation has its greatest effect on cells that rapidly reproduce, including most types of cancer. Thus, cancer cells are more sensitive to radiation than normal cells and can be killed by it easily. Cancer is characterized by a malfunction of cell reproduction, and can also be caused by ionizing radiation. There is no contradiction to say that ionizing radiation can be both a cure and a cause.

Radiotherapy

Radiotherapy is effective against cancer because cancer cells reproduce rapidly and, consequently, are more sensitive to radiation. The central problem in radiotherapy is to make the dose for cancer cells as high as possible while limiting the dose for normal cells. The ratio of abnormal cells killed to normal cells killed is called the **therapeutic ratio**, and all radiotherapy techniques are designed to enhance that ratio. Radiation can be concentrated in cancerous tissue by a number of techniques. One of the most prevalent techniques for well-defined tumors is a geometric technique shown in Figure 22.41. A narrow beam of radiation is passed through the patient from a variety of directions with a common crossing point in the tumor. The technique concentrates the dose in the tumor while spreading it out over a large volume of normal tissue.

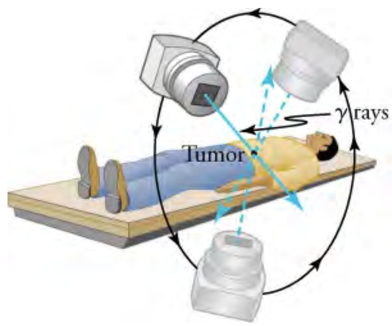


Figure 22.41 The ^{60}Co source of γ -radiation is rotated around the patient so that the common crossing point is in the tumor, concentrating the dose there. This geometric technique works for well-defined tumors.

Another use of radiation therapy is through radiopharmaceuticals. Cleverly, radiopharmaceuticals are used in cancer therapy by tagging antibodies with radioisotopes. Those antibodies are extracted from the patient, cultured, loaded with a radioisotope, and then returned to the patient. The antibodies are then concentrated almost entirely in the tissue they developed to fight, thus localizing the radiation in abnormal tissue. This method is used with radioactive iodine to fight thyroid cancer. While the therapeutic ratio can be quite high for such short-range radiation, there can be a significant dose for organs that eliminate radiopharmaceuticals from the body, such as the liver, kidneys, and bladder. As with most radiotherapy, the technique is limited by the tolerable amount of damage to the normal tissue.

Radiation Dosage

To quantitatively discuss the biological effects of ionizing radiation, we need a radiation dose unit that is directly related to those effects. To do define such a unit, it is important to consider both the biological organism and the radiation itself. Knowing that the amount of ionization is proportional to the amount of deposited energy, we define a radiation dose unit called the **rad**. It 1/100 of a joule of ionizing energy deposited per kilogram of tissue, which is

$$1 \text{ rad} = 0.01 \text{ J/kg}. \tag{22.69}$$

For example, if a 50.0-kg person is exposed to ionizing radiation over her entire body and she absorbs 1.00 J, then her whole-body radiation dose is

$$(1.00 \text{ J})/(50.0 \text{ kg}) = 0.0200 \text{ J/kg} = 2.00 \text{ rad}. \tag{22.70}$$

If the same 1.00 J of ionizing energy were absorbed in her 2.00-kg forearm alone, then the dose to the forearm would be

$$(1.00 \text{ J})/(2.00 \text{ kg}) = 0.500 \text{ J/kg} = 50.0 \text{ rad}, \tag{22.71}$$

and the unaffected tissue would have a zero rad dose. When calculating radiation doses, you divide the energy absorbed by the mass of affected tissue. You must specify the affected region, such as the whole body or forearm in addition to giving the numerical dose in rads. Although the energy per kilogram in 1 rad is small, it can still have significant effects. Since only a few eV cause ionization, just 0.01 J of ionizing energy can create a huge number of ion pairs and have an effect at the cellular level.

The effects of ionizing radiation may be directly proportional to the dose in rads, but they also depend on the type of radiation and the type of tissue. That is, for a given dose in rads, the effects depend on whether the radiation is α, β, γ, X-ray, or some other type of ionizing radiation. The **relative biological effectiveness** (RBE) relates to the amount of biological damage that can occur from a given type of radiation and is given in Table 22.4 for several types of ionizing radiation.

Type and energy of radiation	RBE
X-rays	1
γ rays	1
β rays greater than 32 keV	1
β rays less than 32 keV	1.7
Neutrons, thermal to slow (< 20 keV)	2–5
Neutrons, fast (1–10 MeV)	10 (body), 32 (eyes)
Protons (1–10 MeV)	10 (body), 32 (eyes)
α rays from radioactive decay	10–20
Heavy ions from accelerators	10–20

Table 22.4 Relative Biological Effectiveness

TIPS FOR SUCCESS

The RBEs given in Table 22.4 are approximate, but they yield certain valuable insights.

- The eyes are more sensitive to radiation, because the cells of the lens do not repair themselves.
- Though both are neutral and have large ranges, neutrons cause more damage than γ rays because neutrons often cause secondary radiation when they are captured.
- Short-range particles such as α rays have a severely damaging effect to internal anatomy, as their damage is concentrated and more difficult for the biological organism to repair. However, the skin can usually block alpha particles from entering the body.

Can you think of any other insights from the table?

A final dose unit more closely related to the effect of radiation on biological tissue is called the **roentgen equivalent man**, or rem. A combination of all factors mentioned previously, the roentgen equivalent man is defined to be the dose in rads multiplied by the relative biological effectiveness.

$$\text{rem} = \text{rad} \times \text{RBE} \tag{22.72}$$

The large-scale effects of radiation on humans can be divided into two categories: immediate effects and long-term effects. Table 22.5 gives the immediate effects of whole-body exposures received in less than one day. If the radiation exposure is spread out over more time, greater doses are needed to cause the effects listed. Any dose less than 10 rem is called a low dose, a dose 10 to 100 rem is called a moderate dose, and anything greater than 100 rem is called a high dose.

Dose (rem)	Effect
0–10	No observable effect
10–100	Slight to moderate decrease in white blood cell counts

Table 22.5 Immediate Effects of Radiation (Adults, Whole Body, Single Exposure)

Dose (rem)	Effect
50	Temporary sterility
100–200	Significant reduction in blood cell counts, brief nausea, and vomiting; rarely fatal
200–500	Nausea, vomiting, hair loss, severe blood damage, hemorrhage, fatalities
450	LD50/32; lethal to 50% of the population within 32 days after exposure if untreated
500–2,000	Worst effects due to malfunction of small intestine and blood systems; limited survival
> 2,000	Fatal within hours due to collapse of central nervous system

Table 22.5 Immediate Effects of Radiation (Adults, Whole Body, Single Exposure)

WORK IN PHYSICS

Health Physicist

Are you interested in learning more about radiation? Are you curious about studying radiation dosage levels and ensuring the safety of the environment and people that are most closely affected by it? If so, you may be interested in becoming a health physicist.

The field of health physics draws from a variety of science disciplines with the central aim of mitigating radiation concerns. Those that work as health physicists have a diverse array of potential jobs available to them, including those in research, industry, education, environmental protection, and governmental regulation. Furthermore, while the term *health physicist* may lead many to think of the medical field, there are plenty of applications within the military, industrial, and energy fields as well.

As a researcher, a health physicist can further environmental studies on the effects of radiation, design instruments for more accurate measurements, and assist in establishing valuable radiation standards. Within the energy field, a health physicsist often acts as a manager, closely tied to all operations at all levels, from procuring appropriate equipment to monitoring health data. Within industry, the health physicist acts as a consultant, assisting industry management in important decisions, designing facilities, and choosing appropriate detection tools. The health physicist possesses a unique knowledge base that allows him or her to operate in a wide variety of interesting disciplines!

To become a health physicist, it is necessary to have a background in the physical sciences. Understanding the fields of biology, physiology, biochemistry, and genetics are all important as well. The ability to analyze and solve new problems is critical, and a natural aptitude for science and mathematics will assist in the continued necessary training. There are two possible certifications for health physicists: from the American Board of Health Physicists (ABHP) and the National Registry of Radiation Protection Technologists (NRRPT).

KEY TERMS

activity rate of decay for radioactive nuclides

alpha decay type of radioactive decay in which an atomic nucleus emits an alpha particle

anger camera common medical imaging device that uses a scintillator connected to a series of photomultipliers

atomic number number of protons in a nucleus

becquerel SI unit for rate of decay of a radioactive material

beta decay type of radioactive decay in which an atomic nucleus emits a beta particle

carbon-14 dating radioactive dating technique based on the radioactivity of carbon-14

chain reaction self-sustaining sequence of events, exemplified by the self-sustaining nature of a fission reaction at critical mass

critical mass minimum amount necessary for self-sustained fission of a given nuclide

decay constant quantity that is inversely proportional to the half-life and that is used in the equation for number of nuclei as a function of time

energy-level diagram a diagram used to analyze the energy levels of electrons in the orbits of an atom

excited state any state beyond the $n = 1$ orbital in which the electron stores energy

Fraunhofer lines black lines shown on an absorption spectrum that show the wavelengths absorbed by a gas

gamma decay type of radioactive decay in which an atomic nucleus emits a gamma ray

Geiger tube very common radiation detector that usually gives an audio output

ground state the n=1 orbital of an electron

half-life time in which there is a 50 percent chance that a nucleus will decay

Heisenberg uncertainty principle fundamental limit to the precision with which pairs of quantities such as momentum and position can be measured

hydrogen-like atom any atom with only a single electron

isotope nuclei having the same Z and different N's

liquid drop model model of the atomic nucleus (useful only to understand some of its features) in which nucleons in a nucleus act like atoms in a drop

mass number number of nucleons in a nucleus

nuclear fission reaction in which a nucleus splits

nuclear fusion reaction in which two nuclei are combined, or fused, to form a larger nucleus

nucleons particles found inside nuclei

planetary model of the atom model of the atom that shows electrons orbiting like planets about a Sun-like nucleus

proton-proton cycle combined reactions $^1\text{H} + ^1\text{H} \rightarrow ^2\text{H} + e^- + v_e$, $^1\text{H} + ^2\text{H} \rightarrow ^3\text{He} + \gamma$ and $^3\text{He} + ^3\text{He} \rightarrow ^4\text{He} + ^1\text{H} + ^1\text{H}$ that begins with hydrogen and ends with helium

rad amount of ionizing energy deposited per kilogram of tissue

radioactive substance or object that emits nuclear radiation

radioactive dating application of radioactive decay in which the age of a material is determined by the amount of radioactivity of a particular type that occurs

radioactive decay process by which an atomic nucleus of an unstable atom loses mass and energy by emitting ionizing particles

radioactivity emission of rays from the nuclei of atoms

radiopharmaceutical compound used for medical imaging

relative biological effectiveness (RBE) number that expresses the relative amount of damage that a fixed amount of ionizing radiation of a given type can inflict on biological tissues

roentgen equivalent man (rem) dose unit more closely related to effects in biological tissue

Rutherford scattering scattering of alpha particles by gold nuclei in the gold foil experiment

Rydberg constant a physical constant related to atomic spectra, with an established value of $1.097 \times 10^7 \text{m}^{-1}$

scintillator radiation detection method that records light produced when radiation interacts with materials

strong nuclear force attractive force that holds nucleons together within the nucleus

tagged having a radioactive substance attached (to a chemical compound)

therapeutic ratio the ratio of abnormal cells killed to normal cells killed

transmutation process of changing elemental composition

SECTION SUMMARY

22.1 The Structure of the Atom

- Rutherford's gold foil experiment provided evidence that the atom is composed of a small, dense nucleus with electrons occupying the mostly empty space around it.
- Analysis of emission spectra shows that energy is emitted from energized gas in discrete quantities.
- The Bohr model of the atom describes electrons existing in discrete orbits, with discrete energies emitted and absorbed as the electrons decrease and increase in orbital energy.
- The energy emitted or absorbed by an electron as it changes energy state can be determined with the equation $\Delta E = E_i - E_f$, where

$E_n = \frac{Z^2}{n^2} E_o \ (n = 1, 2, 3, ...)$.

- The wavelength of energy absorbed or emitted by an electron as it changes energy state can be determined by the equation $\frac{1}{\lambda} = R\left(\frac{1}{{n_f}^2} - \frac{1}{{n_i}^2}\right)$, where $R = 1.097 \times 10^7 \mathrm{m}^{-1}$.
- Described as an electron cloud, the quantum model of the atom is the result of de Broglie waves and Heisenberg's uncertainty principle.

22.2 Nuclear Forces and Radioactivity

- The structure of the nucleus is defined by its two nucleons, the neutron and proton.
- Atomic numbers and mass numbers are used to differentiate between various atoms and isotopes. Those numbers can be combined into an easily recognizable form called a *nuclide*.
- The size and stability of the nucleus is based upon two forces: the electromagnetic force and strong nuclear force.
- Radioactive decay is the alteration of the nucleus through the emission of particles or energy.
- Alpha decay occurs when too many protons exist in the nucleus. It results in the ejection of an alpha particle, as described in the equation ${}^{A}_{Z}X_N \rightarrow {}^{A-4}_{Z-2}Y_N + {}^{4}_{2}\mathrm{He}$.
- Beta decay occurs when too many neutrons (or protons) exist in the nucleus. It results in the transmutation of a neutron into a proton, electron, and neutrino. The decay is expressed through the equation ${}^{A}_{Z}X_N \rightarrow {}_{Z+1}{}^{A}Y_{N-1} + e + v$. (Beta decay may also transform a proton into a neutron.)
- Gamma decay occurs when a nucleus in an excited state move to a more stable state, resulting in the release of a photon. Gamma decay is represented with the equation ${}^{A}_{Z}X_N \rightarrow X_N + \gamma$.
- The penetration distance of radiation depends on its energy, charge, and type of material it encounters.

22.3 Half Life and Radiometric Dating

- Radioactive half-life is the time it takes a sample of nuclei to decay to half of its original amount.
- The rate of radioactive decay is defined as the sample's activity, represented by the equation $R = \frac{\Delta N}{\Delta t}$.
- Knowing the half-life of a radioactive isotope allows for the process of radioactive dating to determine the age of a material.
- If the half-life of a material is known, the age of the material can be found using the equation $N = N_O e^{-\lambda t}$.
- The age of organic material can be determined using the decay of the carbon-14 isotope, while the age of rocks can be determined using the decay of uranium-238.

22.4 Nuclear Fission and Fusion

- Nuclear fission is the splitting of an atomic bond, releasing a large amount of potential energy previously holding the atom together. The amount of energy released can be determined through the equation $E = mc^2$.
- Nuclear fusion is the combining, or fusing together, of two nuclei. Energy is also released in nuclear fusion as the combined nuclei are closer together, resulting in a decreased strong nuclear force.
- Fission was used in two nuclear weapons at the conclusion of World War II: the gun-type uranium bomb and the implosion-type plutonium bomb.
- While fission has been used in both nuclear weapons and nuclear reactors, fusion is capable of releasing more energy per reaction. As a result, fusion is a well-researched, if not yet well-controlled, energy source.

22.5 Medical Applications of Radioactivity: Diagnostic Imaging and Radiation

- Medical imaging occurs when a radiopharmaceutical placed in the body provides information to an array of radiation detectors outside the body.
- Devices utilizing medical imaging include the Anger camera, SPECT detector, and PET scan.
- Ionizing radiation can both cure and cause cancer through the manipulation of DNA molecules.
- Radiation dosage and its effect on the body can be measured using the quantities radiation dose unit (rad), relative biological effectiveness (RBE), and the roentgen equivalent man (rem).

KEY EQUATIONS

22.1 The Structure of the Atom

energy of hydrogen electron in an orbital	$E_n = -\frac{13.6 \text{ eV}}{n^2} (n = 1, 2, 3, ...)$
energy of any hydrogen-like electron in orbital	$E_n = \frac{Z^2}{n^2} E_o \ (n = 1, 2, 3, ...)$

wavelength of light emitted by an electron changing states	$\frac{1}{\lambda} = R\left(\frac{1}{{n_f}^2} - \frac{1}{{n_i}^2}\right)$
wavelength of an orbital	$n\lambda_n = 2\pi r_n \; (n = 1, 2, 3, \ldots)$
heisenberg's uncertainty principle	$\Delta x \Delta p \geq \frac{h}{4\pi}$

22.2 Nuclear Forces and Radioactivity

alpha decay equation	${}^{A}_{Z}X_N \rightarrow {}^{A-4}_{Z-2}Y_N + {}^{4}_{2}\text{He}$
beta decay equation	${}^{A}_{Z}X_N \rightarrow {}_{Z+1}{}^{A}Y_{N-1} + e + v$
gamma decay equation	${}^{A}_{Z}X_N \rightarrow X_N + \gamma$

22.3 Half Life and Radiometric Dating

radioactive half-life	$N = N_O e^{-\lambda t}$

22.4 Nuclear Fission and Fusion

energy–mass conversion	$E = mc^2$
proton-proton chain	$2\text{e}- \; + \; 4^1\text{H} \; \rightarrow \; 4 \text{ He} \; + \; 2v_e \; + \; 6\gamma$

22.5 Medical Applications of Radioactivity: Diagnostic Imaging and Radiation

roentgen equivalent man	$\text{rem} = \text{rad} \; \times \; \text{RBE}$

CHAPTER REVIEW

Concept Items

22.1 The Structure of the Atom

1. A star emits light from its core. One observer views the emission unobstructed while a second observer views the emission while obstructed by a cloud of hydrogen gas. Describe the difference between their observations.
   a. Intensity of the light in the spectrum will increase.
   b. Intensity of the light in the spectrum will decrease.
   c. Frequencies will be absorbed from the spectrum.
   d. Frequencies will be added to the spectrum.

2. How does the orbital energy of a hydrogen-like atom change as it increases in atomic number?
   a. The orbital energy will increase.
   b. The orbital energy will decrease.
   c. The orbital energy will remain constant.
   d. The orbital energy will be halved.

22.4 Nuclear Fission and Fusion

3. Aside from energy yield, why are nuclear fusion reactors more desirable than nuclear fission reactors?
   a. Nuclear fusion reactors have a low installation cost.
   b. Radioactive waste is greater for a fusion reactor.
   c. Nuclear fusion reactors are easy to design and build.
   d. A fusion reactor produces less radioactive waste.

Critical Thinking Items

22.1 The Structure of the Atom

4. How would the gold foil experiment have changed if electrons were used in place of alpha particles, assuming that the electrons hit the gold foil with the same force as the alpha particles?
   a. Being less massive, the electrons might have been scattered to a greater degree than the alpha particles.
   b. Being less massive, the electrons might have been scattered to a lesser degree than the alpha particles.
   c. Being more massive, the electrons would have been scattered to a greater degree than the alpha particles.
   d. Being more massive, the electrons would have been scattered to a lesser degree than the alpha particles.

5. Why does the emission spectrum of an isolated gas differ from the emission spectrum created by a white light?
   a. White light and an emission spectrum are different varieties of continuous distribution of frequencies.
   b. White light and an emission spectrum are different series of discrete frequencies.

c. White light is a continuous distribution of frequencies, and an emission spectrum is a series of discrete frequencies.
d. White light is a series of discrete frequencies, and an emission spectrum is a continuous distribution of frequencies.

6. Why would it most likely be difficult to observe quantized orbital states for satellites orbiting the earth?
a. On a macroscopic level, the orbital states do exist for satellites orbiting Earth but are too closely spaced for us to see.
b. On a macroscopic level, the orbital states do not exist for satellites orbiting Earth.
c. On a macroscopic level, we cannot control the amount of energy that we give to an artificial satellite and thus control its orbital altitude.
d. On a macroscopic level, we cannot control the amount of energy that we give to an artificial satellite but we can control its orbital altitude.

7. Do standing waves explain why electron orbitals are quantized?
a. no
b. yes

8. Some terms referring to the observation of light include *emission spectrum* and *absorption spectrum*. Based on these definitions, what would a *reflection spectrum* describe?
a. The reflection spectrum would describe when incident waves are selectively reflected by a substance.
b. The reflection spectrum would describe when incident waves are completely reflected by a substance.
c. The reflection spectrum would describe when incident waves are not absorbed by a substance.
d. The reflection spectrum would describe when incident waves are completely absorbed by a substance.

22.2 Nuclear Forces and Radioactivity

9. Explain why an alpha particle can have a greater range in air than a beta particle in lead.
a. While the alpha particle has a lesser charge than a beta particle, the electron density in lead is much less than that in air.
b. While the alpha particle has a greater charge than a beta particle, the electron density in lead is much lower than that in air.
c. While the alpha particle has a lesser charge than a beta particle, the electron density in lead is much greater than that in air.
d. While the alpha particle has a greater charge than a beta particle, the electron density in lead is much higher than that in air.

10. What influence does the strong nuclear force have on the electrons in an atom?
a. It attracts them toward the nucleus.
b. It repels them away from the nucleus.
c. The strong force makes electrons revolve around the nucleus.
d. It does not have any influence.

22.3 Half Life and Radiometric Dating

11. Provide an example of something that decreases in a manner similar to radioactive decay.
a. The potential energy of an object falling under the influence of gravity
b. The kinetic energy of a ball that is dropped from a building to the ground
c. Theh charge transfer from an ebonite rod to fur
d. The heat transfer from a hot to a cold object

12. A sample of radioactive material has a decay constant of 0.05 s^{-1}. Why is it wrong to presume that the sample will take just 20 seconds to fully decay?
a. The decay constant varies with the mass of the sample.
b. The decay constant results vary with the amount of the sample.
c. The decay constant represents a percentage of the sample that cannot decay.
d. The decay constant represents only the fraction of a sample that decays in a unit of time, not the decay of the entire sample.

22.4 Nuclear Fission and Fusion

13. What is the atomic number of the most strongly bound nuclide?
a. 25
b. 26
c. 27
d. 28

14. Why are large electromagnets necessary in nuclear fusion reactors?
a. Electromagnets are used to slow down the movement of charge hydrogen plasma.
b. Electromagnets are used to decrease the temperature of hydrogen plasma.
c. Electromagnets are used to confine the hydrogen plasma.
d. Electromagnets are used to stabilize the temperature of the hydrogen plasma.

22.5 Medical Applications of Radioactivity: Diagnostic Imaging and Radiation

15. Why are different radiopharmaceuticals used to image different parts of the body?
  a. The different radiopharmaceuticals travel through different blood vessels.
  b. The different radiopharmaceuticals travel to different parts of the body.
  c. The different radiopharmaceuticals are used to treat different diseases of the body.
  d. The different radiopharmaceuticals produce different amounts of ionizing radiation.

16. Why do people think carefully about whether to receive a diagnostic test such as a CT scan?
  a. The radiation from a CT scan is capable of creating cancerous cells.
  b. The radiation from a CT scan is capable of destroying cancerous cells.
  c. The radiation from a CT scan is capable of creating diabetic cells.
  d. The radiation from a CT scan is capable of destroying diabetic cells.

17. Sometimes it is necessary to take a PET scan very soon after ingesting a radiopharmaceutical. Why is that the case?
  a. The radiopharmaceutical may have a short half-life.
  b. The radiopharmaceutical may have a long half-life.
  c. The radiopharmaceutical quickly passes through the digestive system.
  d. The radiopharmaceutical can become lodged in the digestive system.

Performance Task

22.5 Medical Applications of Radioactivity: Diagnostic Imaging and Radiation

18. On the Environmental Protection Agency's website, a helpful tool exists to allow you to determine your average annual radiation dose. Use the tool to determine whether the radiation level you have been exposed to is dangerous and to compare your radiation dosage to other radiative events.

  1. Visit the webpage (http://www.openstax.org/l/28calculate) and answer the series of questions provided to determine the average annual radiation dosage that you receive.
  2. Table 22.5 shows the immediate effects of a radiation dosage. Using the table, explain what you would experience if your yearly dosage of radiation was received all over the course of one day. Also, determine whether your dosage is considered a low, moderate, or high.
  3. Using the information input into the webpage, what percentage of your dosage comes from natural sources? The average percentage of radiation from natural sources for an individual is around 85 percent.
  4. Research radiation dosages for evacuees from events like the Chernobyl and Fukushima meltdowns. How does your annual radiation exposure rate compare to the net dosage for evacuees of each event. Use numbers to support your answer.
  5. The U.S. Department of Labor limits the amount of radiation that a given worker may receive in a 12 month period.
     a. Research the present maximum value and compare your annual exposure rate to that of a radiation worker. Use numbers to support your answer.
     b. What types of work are likely to cause an increase in the radiation exposure of a particular worker?

Provide one question based upon the information gathered on the EPA website.

TEST PREP

Multiple Choice

22.1 The Structure of the Atom

19. If electrons are negatively charged and the nucleus is positively charged, why do they not attract and collide with each other?
  a. The pull from the nucleus provides a centrifugal force, which is not strong enough to draw the electrons into the nucleus.
  b. The pull from the nucleus provides a centripetal force, which is not strong enough to draw the electrons into the nucleus.

c. The pull from the nucleus provides a helical motion.
d. The pull from the nucleus provides a cycloid motion.

22.4 Nuclear Fission and Fusion

20. If a nucleus elongates due to a neutron strike, which of the following forces will decrease?
a. Nuclear force between neutrons only
b. Coulomb force between protons only
c. Strong nuclear force between all nucleons and Coulomb force between protons, but the strong force will decrease more
d. Strong nuclear force between neutrons and Coulomb force between protons, but Coulomb force will decrease more

Short Answer

22.1 The Structure of the Atom

21. Why do Bohr's calculations for electron energies not work for all atoms?
a. In atoms with more than one electron is an atomic shell, the electrons will interact. That requires a more complex formula than Bohr's calculations accounted for.
b. In atoms with 10 or more electrons in an atomic shell, the electrons will interact. That requires a more complex formula than Bohr's calculations accounted for.
c. In atoms with more than one electron in an atomic shell, the electrons will not interact. That requires a more complex formula than Bohr's calculations accounted for.
d. In atoms with 10 or more electrons in an atomic shell, the electrons will not interact. That requires a more complex formula than Bohr's calculations accounted for.

22.2 Nuclear Forces and Radioactivity

22. Does transmutation occur within chemical reactions?
a. no
b. yes

22.3 Half Life and Radiometric Dating

23. How does the radioactive activity of a sample change with time?
a. The radioactive activity decreases exponentially.
b. The radioactive activity undergoes linear decay.
c. The radioactive activity undergoes logarithmic decay.
d. The radioactive activity will not change with time.

22.4 Nuclear Fission and Fusion

24. Why does fission of heavy nuclei result in the release of neutrons?
a. Heavy nuclei require more neutrons to achieve stability.
b. Heavy nuclei require more neutrons to balance charge.
c. Light nuclei require more neutrons to achieve stability.
d. Light nuclei require more neutrons to balance charge.

22.5 Medical Applications of Radioactivity: Diagnostic Imaging and Radiation

25. Why is radioactive iodine used to monitor the thyroid?
a. Radioactive iodine can be used by the thyroid while absorbing information about the thyroid.
b. Radioactive iodine can be used by the thyroid while emitting information about the thyroid.
c. Radioactive iodine can be secreted by the thyroid while absorbing information about the thyroid.
d. Radioactive iodine can be secreted by the thyroid while emitting information about the thyroid.

Extended Response

22.1 The Structure of the Atom

26. Compare the standing wavelength of an $n = 2$ orbital to the standing wavelength of an $n = 4$ orbital.
a. The standing wavelength of an $n = 2$ orbital is greater than the standing wavelength of an $n = 4$ orbital.
b. The standing wavelength of an $n = 2$ orbital is less than the standing wavelength of an $n = 4$ orbital.
c. There is no relation between the standing wavelength of an $n = 2$ orbital and the standing wavelength of an $n = 4$ orbital.
d. The standing wavelength of an $n = 2$ orbital is the same as the standing wavelength of an$n = 4$ orbital.

27. Describe the shape of the electron cloud, based on total energy levels, for an atom with electrons in multiple orbital states.
a. There are multiple regions of high electron

probability of various shapes surrounding the nucleus.
b. There is a single solid spherical region of high electron probability surrounding the nucleus.
c. There are multiple concentric shells of high electron probability surrounding the nucleus.
d. There is a single spherical shell of high electron probability surrounding the nucleus.

22.2 Nuclear Forces and Radioactivity

28. How did Becquerel's observations of pitchblende imply the existence of radioactivity?
  a. A chemical reaction occurred on the photographic plate without any external source of energy.
  b. Bright spots appeared on the photographic plate due to an external source of energy.
  c. Energy from the Sun was absorbed by the pitchblende and reflected onto the photographic plate.
  d. Dark spots appeared on the photographic plate due to an external source of energy.

22.4 Nuclear Fission and Fusion

29. Describe the potential energy of two nuclei as they approach each other.
  a. The potential energy will decrease as the nuclei are brought together and then rapidly increase once a minimum is reached.
  b. The potential energy will decrease as the nuclei are brought together.
  c. The potential energy will increase as the nuclei are brought together.
  d. The potential energy will increase as the nuclei are brought together and then rapidly decrease once a maximum is reached.

22.5 Medical Applications of Radioactivity: Diagnostic Imaging and Radiation

30. Why do X-rays and gamma rays have equivalent RBE values if they provide different amounts of energy to the body?
  a. The penetration distance, which depends on energy, is short for both X-rays and gamma rays.
  b. The penetration distance, which depends on energy, is long for both X-rays and gamma rays.
  c. The penetration distance, as determined by their high mass, is different for both X-rays and gamma rays.
  d. The penetration distance, as determined by their low mass, is the same for both X-rays and gamma rays.

CHAPTER 23
Particle Physics

Figure 23.1 Part of the Large Hadron Collider (LHC) at CERN, on the border of Switzerland and France. The LHC is a particle accelerator, designed to study fundamental particles. (credit: Image Editor, Flickr)

Chapter Outline

INTRODUCTION Following ideas remarkably similar to those of the ancient Greeks, we continue to look for smaller and smaller structures in nature, hoping ultimately to find and understand the most fundamental building blocks that exist. Atomic physics deals with the smallest units of elements and compounds. In its study, we have found a relatively small number of atoms with systematic properties, and these properties have explained a tremendous range of phenomena. Nuclear physics is concerned with the nuclei of atoms and their substructures. Here, a smaller number of components—the proton and neutron—make up all nuclei. Exploring the systematic behavior of their interactions has revealed even more about matter, forces, and energy. **Particle physics** deals with the substructures of atoms and nuclei and is particularly aimed at finding those truly fundamental particles that have no further substructure. Just as in atomic and nuclear physics, we have found a complex array of particles and properties with systematic characteristics analogous to the periodic table and the chart of nuclides. An underlying structure is apparent, and there is some reason to think that we *are* finding particles that have no substructure. Of course, we have been in similar situations before. For example, atoms were once thought to be the ultimate substructures. It is possible that we could continue to find deeper and deeper structures without ever discovering the ultimate substructure—in science there is never complete certainty. See Figure 23.2.

The properties of matter are based on substructures called molecules and atoms. Each atom has the substructure of a nucleus surrounded by electrons, and their interactions explain atomic properties. Protons and neutrons—and the interactions between them—explain the stability and abundance of elements and form the substructure of nuclei. Protons and neutrons are not fundamental—they are composed of quarks. Like electrons and a few other particles, quarks may be the fundamental building blocks of all matter, lacking any further substructure. But the story is not complete because quarks and electrons may have substructures smaller than details that are presently observable.

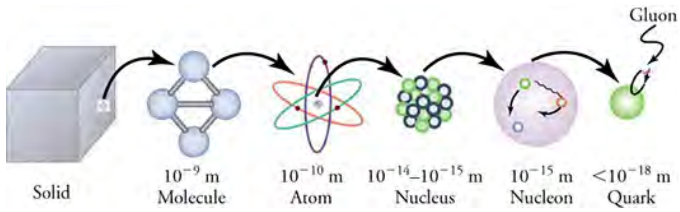


Figure 23.2 A solid, a molecule, an atom, a nucleus, a nucleon (a particle that makes up the nucleus—either a proton or a neutron), and a quark.

This chapter covers the basics of particle physics as we know it today. An amazing convergence of topics is evolving in particle physics. We find that some particles are intimately related to forces and that nature on the smallest scale may have its greatest influence on the large scale character of the universe. It is an adventure exceeding the best science fiction because it is not only fantastic but also real.

23.1 The Four Fundamental Forces

Section Learning Objectives

By the end of the section, you will be able to do the following:

- Define, describe, and differentiate the four fundamental forces
- Describe the carrier particles and explain how their exchange transmits force
- Explain how particle accelerators work to gather evidence about particle physics

Section Key Terms

carrier particle	colliding beam	cyclotron	Feynman diagram	graviton
particle physics	pion	quantum electrodynamics	synchrotron	W^- boson
W^+ boson	weak nuclear force	Z^0 boson		

Despite the apparent complexity within the universe, there remain just four basic forces. These forces are responsible for all interactions known to science: from the very small to the very large to those that we experience in our day-to-day lives. These forces describe the movement of galaxies, the chemical reactions in our laboratories, the structure within atomic nuclei, and the cause of radioactive decay. They describe the true cause behind familiar terms like friction and the normal force. These four basic forces are known as fundamental because they alone are responsible for all observations of forces in nature. The four fundamental forces are gravity, electromagnetism, **weak nuclear force**, and strong nuclear force.

Understanding the Four Forces

The gravitational force is most familiar to us because it describes so many of our common observations. It explains why a dropped ball falls to the ground and why our planet orbits the Sun. It gives us the property of weight and determines much about the motion of objects in our daily lives. Because gravitational force acts between all objects of mass and has the ability to act over large distances, the gravitational force can be used to explain much of what we observe and can even describe the motion of objects on astronomical scales! That said, gravity is incredibly weak compared to the other fundamental forces and is the weakest of all of the fundamental forces. Consider this: The entire mass of Earth is needed to hold an iron nail to the ground. Yet with a simple magnet, the force of gravity can be overcome, allowing the nail to accelerate upward through space.

The electromagnetic force is responsible for both electrostatic interactions and the magnetic force seen between bar magnets. When focusing on the electrostatic relationship between two charged particles, the electromagnetic force is known as the coulomb force. The electromagnetic force is an important force in the chemical and biological sciences, as it is responsible for molecular connections like ionic bonding and hydrogen bonding. Additionally, the electromagnetic force is behind the common physics forces of friction and the normal force. Like the gravitational force, the electromagnetic force is an inverse square law. However, the electromagnetic force does not exist between any two objects of mass, only those that are charged.

When considering the structure of an atom, the electromagnetic force is somewhat apparent. After all, the electrons are held in place by an attractive force from the nucleus. But what causes the nucleus to remain intact? After all, if all protons are positive, it makes sense that the coulomb force between the protons would repel the nucleus apart immediately. Scientists theorized that another force must exist within the nucleus to keep it together. They further theorized that this nuclear force must be significantly stronger than gravity, which has been observed and measured for centuries, and also stronger than the electromagnetic force, which would cause the protons to want to accelerate away from each other.

The strong nuclear force is an attractive force that exists between all nucleons. This force, which acts equally between proton-proton connections, proton-neutron connections, and neutron-neutron connections, is the strongest of all forces at short ranges. However, at a distance of 10^{-13} cm, or the diameter of a single proton, the force dissipates to zero. If the nucleus is large (it has many nucleons), then the distance between each nucleon could be much larger than the diameter of a single proton.

The weak nuclear force is responsible for beta decay, as seen in the equation ${}^{A}_{Z}X_N \rightarrow {}^{A}_{Z+1}Y_{N-1} + e + v$. Recall that beta decay is when a beta particle is ejected from an atom. In order to accelerate away from the nucleus, the particle must be acted on by a force. Enrico Fermi was the first to envision this type of force. While this force is appropriately labeled, it remains stronger than the gravitational force. However, its range is even smaller than that of the strong force, as can be seen in Table 23.1. The weak nuclear force is more important than it may appear at this time, as will be addressed when we discuss quarks.

Force	Approximate Relative Strength[1]	Range
Gravity	10^{-38}	∞
Weak	10^{-13}	$< 10^{-18}$ m
Electromagnetic	10^{-2}	∞
Strong	1	$< 10^{-15}$ m

[1]Relative strength is based on the strong force felt by a proton–proton pair.

Table 23.1 Relative strength and range of the four fundamental forces

Transmitting the Four Fundamental Forces

Just as it troubled Einstein prior to formulating the gravitational field theory, the concept of forces acting over a distance had greatly troubled particle physicists. That is, how does one proton *know* that another exists? Furthermore, what causes one proton to make a second proton repel? Or, for that matter, what is it about a proton that causes a neutron to attract? These mysterious interactions were first considered by Hideki Yukawa in 1935 and laid the foundation for much of what we now understand about particle physics.

Hideki Yukawa's focus was on the strong nuclear force and, in particular, its incredibly short range. His idea was a blend of particles, relativity, and quantum mechanics that was applicable to all four forces. Yukawa proposed that the nuclear force is actually transmitted by the exchange of particles, called **carrier particles**, and that what we commonly refer to as the force's field consists of these carrier particles. Specifically for the strong nuclear force, Yukawa proposed that a previously unknown particle, called a **pion**, is exchanged between nucleons, transmitting the force between them. Figure 23.3 illustrates how a pion would carry a force between a proton and a neutron.

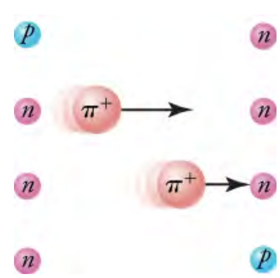


Figure 23.3 The strong nuclear force is transmitted between a proton and neutron by the creation and exchange of a pion. The pion, created through a temporary violation of conservation of mass-energy, travels from the proton to the neutron and is recaptured. It is not

directly observable and is called a virtual particle. Note that the proton and neutron change identity in the process. The range of the force is limited by the fact that the pion can exist for only the short time allowed by the Heisenberg uncertainty principle. Yukawa used the finite range of the strong nuclear force to estimate the mass of the pion; the shorter the range, the larger the mass of the carrier particle.

In Yukawa's strong force, the carrier particle is assumed to be transmitted at the speed of light and is continually transferred between the two nucleons shown. The particle that Yukawa predicted was finally discovered within cosmic rays in 1947. Its name, the pion, stands for pi meson, where meson means *medium mass*; it's a medium mass because it is smaller than a nucleon but larger than an electron. Yukawa launched the field that is now called quantum chromodynamics, and the carrier particles are now called gluons due to their strong binding power. The reason for the change in the particle name will be explained when quarks are discussed later in this section.

As you may assume, the strong force is not the only force with a carrier particle. Nuclear decay from the weak force also requires a particle transfer. In the weak force are the following three: the weak negative carrier, W^-; the weak positive carrier, W^+; and the zero charge carrier, Z^0. As we will see, Fermi inferred that these particles must carry mass, as the total mass of the products of nuclear decay is slightly larger than the total mass of all reactants after nuclear decay.

The carrier particle for the electromagnetic force is, not surprisingly, the photon. After all, just as a lightbulb can emit photons from a charged tungsten filament, the photon can be used to transfer information from one electrically charged particle to another. Finally, the **graviton** is the proposed carrier particle for gravity. While it has not yet been found, scientists are currently looking for evidence of its existence (see Boundless Physics: Searching for the Graviton).

So how does a carrier particle transmit a fundamental force? Figure 23.4 shows a virtual photon transmitted from one positively charged particle to another. The transmitted photon is referred to as a virtual particle because it cannot be directly observed while transmitting the force. Figure 23.5 shows a way of graphing the exchange of a virtual photon between the two positively charged particles. This graph of time versus position is called a **Feynman diagram**, after the brilliant American physicist Richard Feynman (1918–1988), who developed it.

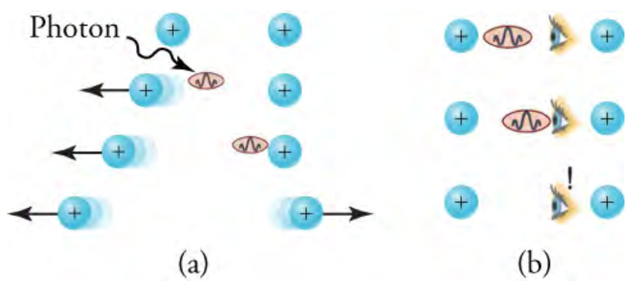


Figure 23.4 The image in part (a) shows the exchange of a virtual photon transmitting the electromagnetic force between charges, just as virtual pion exchange carries the strong nuclear force between nucleons. The image in part (b) shows that the photon cannot be directly observed in its passage because this would disrupt it and alter the force. In this case, the photon does not reach the other charge.

The Feynman diagram should be read from the bottom up to show the movement of particles over time. In it, you can see that the left proton is propelled leftward from the photon emission, while the right proton feels an impulse to the right when the photon is received. In addition to the Feynman diagram, Richard Feynman was one of the theorists who developed the field of **quantum electrodynamics** (QED), which further describes electromagnetic interactions on the submicroscopic scale. For this work, he shared the 1965 Nobel Prize with Julian Schwinger and S.I. Tomonaga. A Feynman diagram explaining the strong force interaction hypothesized by Yukawa can be seen in Figure 23.6. Here, you can see the change in particle type due to the exchange of the pi meson.

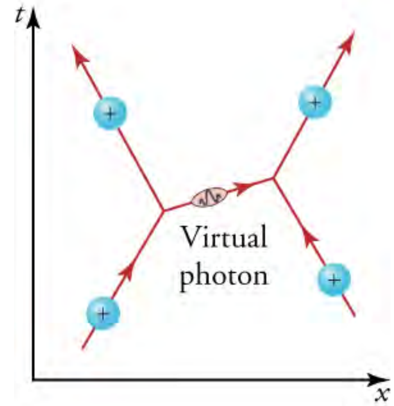


Figure 23.5 The Feynman diagram for the exchange of a virtual photon between two positively charged particles illustrates how electromagnetic force is transmitted on a quantum mechanical scale. Time is graphed vertically, while the distance is graphed horizontally. The two positively charged particles are seen to repel each other by the photon exchange.

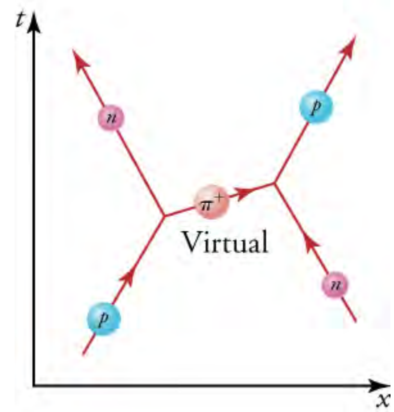


Figure 23.6 The image shows a Feynman diagram for the exchange of a π+ (pion) between a proton and a neutron, carrying the strong nuclear force between them. This diagram represents the situation shown more pictorially in Figure 23.3.

The relative masses of the listed carrier particles describe something valuable about the four fundamental forces, as can be seen in Table 23.2. **W bosons** (consisting of W^- and W^+ bosons) and **Z bosons** (Z^0 bosons), carriers of the weak nuclear force, are nearly 1,000 times more massive than pions, carriers of the strong nuclear force. Simultaneously, the distance that the weak nuclear force can be transmitted is approximately $\frac{1}{1,000}$ times the strong force transmission distance. Unlike carrier particles, which have a limited range, the photon is a massless particle that has no limit to the transmission distance of the electromagnetic force. This relationship leads scientists to understand that the yet-unfound graviton is likely massless as well.

Force	Carrier Particle	Range	Relative Strength[1]
Gravity	Graviton (theorized)	∞	10^{-38}
Weak	W and Z bosons	∞	10^{-2}
Electromagnetic	Photon	$< 10^{-18}$ m	10^{-13}
Strong	Pi mesons or pions (now known as gluons)	$< 10^{-15}$ m	1

[1]Relative strength is based on the strong force felt by a proton-proton pair.

Table 23.2 Carrier particles and their relative masses compared to pions for the four fundamental forces

BOUNDLESS PHYSICS

Searching for the Graviton

From Newton's Universal Law of Gravitation to Einstein's field equations, gravitation has held the focus of scientists for centuries. Given the discovery of carrier particles during the twentieth century, the importance of understanding gravitation has yet again gained the interest of prominent physicists everywhere.

With carrier particles discovered for three of the four fundamental forces, it is sensible to scientists that a similar particle, titled the **graviton**, must exist for the gravitational force. While evidence of this particle is yet to be uncovered, scientists are working diligently to discover its existence.

So what do scientists think about the unfound particle? For starters, the graviton (like the photon) should be a massless particle traveling at the speed of light. This is assumed because, like the electromagnetic force, gravity is an inverse square law, $F \approx \frac{1}{r^2}$. Scientists also theorize that the graviton is an electrically neutral particle, as an empty space within the influence of gravity is chargeless.

However, because gravity is such a weak force, searching for the graviton has resulted in some unique methods. LIGO, the Laser Interferometer Gravitational-Wave Observatory, is one tool currently being utilized (see Figure 23.7). While searching for a gravitational wave to find a carrier particle may seem counterintuitive, it is similar to the approach taken by Planck and Einstein to learn more about the photon. According to wave-particle duality, if a gravitational wave can be found, the graviton should be present along with it. Predicted by Einstein's theory of general relativity, scientists have been monitoring binary star systems for evidence of these gravitational waves.

Figure 23.7 In searching for gravitational waves, scientists are using the Laser Interferometer Gravitational-Wave Observatory (LIGO). Here we see the control room of LIGO in Hanford, Washington.

Particle accelerators like the Large Hadron Collider (LHC) are being used to search for the graviton through high-energy collisions. While scientists at the LHC speculate that the particle may not exist long enough to be seen, evidence of its prior existence, like footprints in the sand, can be found through gaps in projected energy and momentum.

Some scientists are even searching the remnants of the Big Bang in an attempt to find the graviton. By observing the cosmic background radiation, they are looking for anomalies in gravitational waves that would provide information about the gravity particles that existed at the start of our universe.

Regardless of the method used, scientists should know the graviton once they find it. A massless, chargeless particle with a spin of 2 and traveling at the speed of light—there is no other particle like it. Should it be found, its discovery would surely be considered by future generations to be on par with those of Newton and Einstein.

GRASP CHECK

Why are binary star systems used by LIGO to find gravitational waves?

a. Binary star systems have high temperature.
b. Binary star systems have low density.
c. Binary star systems contain a large amount of mass, but because they are orbiting each other, the gravitational field between the two is much less.
d. Binary star systems contain a large amount of mass. As a result, the gravitational field between the two is great.

Accelerators Create Matter From Energy

Before looking at all the particles that make up our universe, let us first examine some of the machines that create them. The fundamental process in creating unknown particles is to accelerate known particles, such as protons or electrons, and direct a beam of them toward a target. Collisions with target nuclei provide a wealth of information, such as information obtained by Rutherford in the gold foil experiment. If the energy of the incoming particles is large enough, new matter can even be created in the collision. The more energy input or ΔE, the more matter m can be created, according to mass energy equivalence $m = \Delta E/c^2$. Limitations are placed on what can occur by known conservation laws, such as conservation of mass-energy, momentum, and charge. Even more interesting are the unknown limitations provided by nature. While some expected reactions do occur, others do not, and still other unexpected reactions may appear. New laws are revealed, and the vast majority of what we know about particle physics has come from accelerator laboratories. It is the particle physicist's favorite indoor sport.

Our earliest model of a particle accelerator comes from the Van de Graaff generator. The relatively simple device, which you have likely seen in physics demonstrations, can be manipulated to produce potentials as great as 50 million volts. While these machines do not have energies large enough to produce new particles, analysis of their accelerated ions was instrumental in exploring several aspects of the nucleus.

Another equally famous early accelerator is the **cyclotron**, invented in 1930 by the American physicist, E.O. Lawrence (1901–1958). Figure 23.8 is a visual representation with more detail. Cyclotrons use fixed-frequency alternating electric fields to accelerate particles. The particles spiral outward in a magnetic field, making increasingly larger radius orbits during acceleration. This clever arrangement allows the successive addition of electric potential energy with each loop. As a result, greater particle energies are possible than in a Van de Graaff generator.

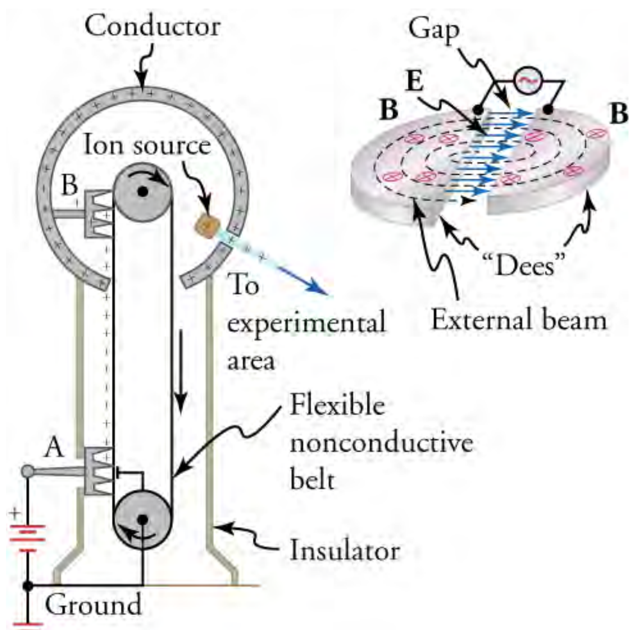


Figure 23.8 On the left is an artist's rendition of the popular physics demonstration tool, the Van de Graaff generator. A battery (A) supplies excess positive charge to a pointed conductor, the points of which spray the charge onto a moving insulating belt near the bottom. The pointed conductor (B) on top in the large sphere picks up the charge. (The induced electric field at the points is so large that it removes the charge from the belt.) This can be done because the charge does not remain inside the conducting sphere but moves to its outer surface. An ion source inside the sphere produces positive ions, which are accelerated away from the positive sphere to high velocities. On the right is a cyclotron. Cyclotrons use a magnetic field to cause particles to move in circular orbits. As the particles pass between the plates of the Dees, the voltage across the gap is oscillated to accelerate them twice in each orbit.

A **synchrotron** is a modification of the cyclotron in which particles continually travel in a fixed-radius orbit, increasing speed each time. Accelerating voltages are synchronized with the particles to accelerate them, hence the name. Additionally, magnetic field strength is increased to keep the orbital radius constant as energy increases. A ring of magnets and accelerating tubes, as shown in Figure 23.9, are the major components of synchrotrons. High-energy particles require strong magnetic fields to steer them, so superconducting magnets are commonly employed. Still limited by achievable magnetic field strengths, synchrotrons need to be very large at very high energies since the radius of a high-energy particle's orbit is very large.

To further probe the nucleus, physicists need accelerators of greater energy and detectors of shorter wavelength. To do so requires not only greater funding but greater ingenuity as well. **Colliding beams** used at both the Fermi National Accelerator Laboratory (Fermilab; see Figure 23.11) near Chicago and the LHC in Switzerland are designed to reduce energy loss in particle collisions. Typical stationary particle detectors lose a large amount of energy to the recoiling target struck by the accelerating particle. By providing head-on collisions between particles moving in opposite directions, colliding beams make it possible to create particles with momenta and kinetic energies near zero. This allows for particles of greater energy and mass to be created. Figure 23.10 is a schematic representation of this effect. In addition to circular accelerators, linear accelerators can be used to reduce energy radiation losses. The Stanford Linear Accelerator Center (now called the SLAC National Accelerator Laboratory) in California is home to the largest such accelerator in the world.

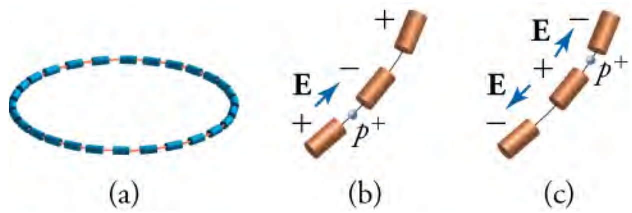


Figure 23.9 (a) A synchrotron has a ring of magnets and accelerating tubes. The frequency of the accelerating voltages is increased to cause the beam particles to travel the same distance in a shorter time. The magnetic field should also be increased to keep each beam burst traveling in a fixed-radius path. Limits on magnetic field strength require these machines to be very large in order to accelerate particles to very high energies. (b) A positively charged particle is shown in the gap between accelerating tubes. (c) While the particle passes through the tube, the potentials are reversed so that there is another acceleration at the next gap. The frequency of the reversals needs to be varied as the particle is accelerated to achieve successive accelerations in each gap.

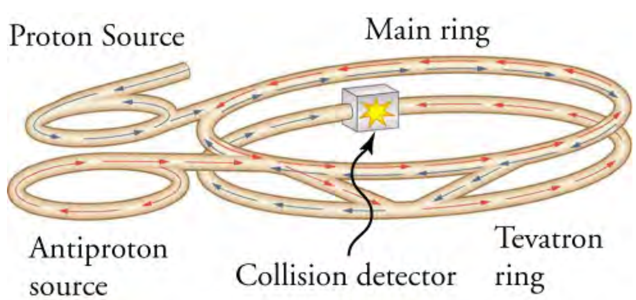


Figure 23.10 This schematic shows the two rings of Fermilab's accelerator and the scheme for colliding protons and antiprotons (not to scale).

Figure 23.11 The Fermi National Accelerator Laboratory, near Batavia, Illinois, was a subatomic particle collider that accelerated protons and antiprotons to attain energies up to 1 Tev (a trillion electronvolts). The circular ponds near the rings were built to dissipate waste heat. This accelerator was shut down in September 2011. (credit: Fermilab, Reidar Hahn)

Check Your Understanding

1. Which of the four forces is responsible for radioactive decay?
    a. the electromagnetic force

b. the gravitational force
c. the strong nuclear force
d. the weak nuclear force

2. What force or forces exist between an electron and a proton?
   a. the strong nuclear force, the electromagnetic force, and gravity
   b. the weak nuclear force, the strong nuclear force, and gravity
   c. the weak nuclear force, the strong nuclear force, and the electromagnetic force
   d. the weak nuclear force, the electromagnetic force, and gravity

3. What is the proposed carrier particle for the gravitational force?
   a. boson
   b. graviton
   c. gluon
   d. photon

4. What is the relationship between the mass and range of a carrier particle?
   a. Range of a carrier particle is inversely proportional to its mass.
   b. Range of a carrier particle is inversely proportional to square of its mass.
   c. Range of a carrier particle is directly proportional to its mass.
   d. Range of a carrier particle is directly proportional to square of its mass.

5. What type of particle accelerator uses fixed-frequency oscillating electric fields to accelerate particles?
   a. cyclotron
   b. synchrotron
   c. betatron
   d. Van de Graaff accelerator

6. How does the expanding radius of the cyclotron provide evidence of particle acceleration?
   a. A constant magnetic force is exerted on particles at all radii. As the radius increases, the velocity of the particle must increase to maintain this constant force.
   b. A constant centripetal force is exerted on particles at all radii. As the radius increases, the velocity of the particle must decrease to maintain this constant force.
   c. A constant magnetic force is exerted on particles at all radii. As the radius increases, the velocity of the particle must decrease to maintain this constant force.
   d. A constant centripetal force is exerted on particles at all radii. As the radius increases, the velocity of the particle must increase to maintain this constant force.

7. Which of the four forces is responsible for the structure of galaxies?
   a. electromagnetic force
   b. gravity
   c. strong nuclear force
   d. weak nuclear force

23.2 Quarks

Section Learning Objectives

By the end of the section, you will be able to do the following:

- Describe quarks and their relationship to other particles
- Distinguish hadrons from leptons
- Distinguish matter from antimatter
- Describe the standard model of the atom
- Define a Higgs boson and its importance to particle physics

Section Key Terms

annihilation	antimatter	baryon	bottom quark	charmed quark
color	down quark	flavor	gluon	hadron
Higgs boson	Higgs field	lepton	meson	pair production
positron	quantum chromodynamics	quark	Standard Model	strange quark
top quark	up quark			

Quarks

“The first principles of the universe are atoms and empty space. Everything else is merely thought to exist…”

“… Further, the atoms are unlimited in size and number, and they are borne along with the whole universe in a vortex, and thereby generate all composite things—fire, water, air, earth. For even these are conglomerations of given atoms. And it because of their solidity that these atoms are impassive and unalterable.”

—Diogenes Laertius (summarizing the views of Democritus, circa 460–370 B.C.)

The search for fundamental particles is nothing new. Atomists of the Greek and Indian empires, like Democritus of fifth century B.C., openly wondered about the most finite components of our universe. Though dormant for centuries, curiosity about the atomic nature of matter was reinvigorated by Rutherford’s gold foil experiment and the discovery of the nucleus. By the early 1930s, scientists believed they had fully determined the tiniest constituents of matter—in the form of the proton, neutron, and electron.

This would be only partially true. At present, scientists know that there are hundreds of particles not unlike our electron and nucleons, all making up what some have termed the *particle zoo*. While we are confident that the electron remains fundamental, it is surrounded by a plethora of similar sounding terms, like leptons, hadrons, baryons, and mesons. Even though not every particle is considered fundamental, they all play a vital role in understanding the intricate structure of our universe.

A fundamental particle is defined as a particle with no substructure and no finite size. According to the **Standard Model**, there are three types of fundamental particles: leptons, quarks, and carrier particles. As you may recall, carrier particles are responsible for transmitting fundamental forces between their interacting masses. **Leptons** are a group of six particles not bound by the strong nuclear force, of which the electron is one. As for quarks, they are the fundamental building blocks of a group of particles called **hadrons**, a group that includes both the proton and the neutron.

Now for a brief history of **quarks**. Quarks were first proposed independently by American physicists Murray Gell-Mann and George Zweig in 1963. Originally, three quark types—or **flavors**—were proposed with the names **up** (u), **down** (d), and **strange** (s).

At first, physicists expected that, with sufficient energy, we should be able to free quarks and observe them directly. However, this has not proved possible, as the current understanding is that the force holding quarks together is incredibly great and, much like a spring, increases in magnitude as the quarks are separated. As a result, when large energies are put into collisions, other particles are created—but no quarks emerge. With that in mind, there is compelling evidence for the existence of quarks. By 1967, experiments at the SLAC National Accelerator Laboratory scattering 20-GeV electrons from protons produced results like Rutherford had obtained for the nucleus nearly 60 years earlier. The SLAC scattering experiments showed unambiguously that there were three point-like (meaning they had sizes considerably smaller than the probe’s wavelength) charges inside the proton as seen in Figure 23.12. This evidence made all but the most skeptical admit that there was validity to the quark substructure of hadrons.

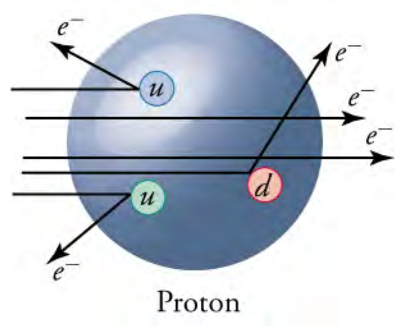


Figure 23.12 Scattering of high-energy electrons from protons at facilities like SLAC produces evidence of three point-like charges consistent with proposed quark properties. This experiment is analogous to Rutherford's discovery of the small size of the nucleus by scattering α particles. High-energy electrons are used so that the probe wavelength is small enough to see details smaller than the proton.

The inclusion of the strange quark with Zweig and Gell-Mann's model concerned physicists. While the up and down quarks demonstrated fairly clear symmetry and were present in common fundamental particles like protons and neutrons, the strange quark did not have a counterpart of its own. This thought, coupled with the four known leptons at the time, caused scientists to predict that a fourth quark, yet to be found, also existed.

In 1974, two groups of physicists independently discovered a particle with this new quark, labeled **charmed**. This completed the second *exotic* quark pair, strange (s) and charmed (c). A final pair of quarks was proposed when a third pair of leptons was discovered in 1975. The existence of the **bottom** (b) quark and the **top** (t) quark was verified through experimentation in 1976 and 1995, respectively. While it may seem odd that so much time would elapse between the original quark discovery in 1967 and the verification of the top quark in 1995, keep in mind that each quark discovered had a progressively larger mass. As a result, each new quark has required more energy to discover.

TIPS FOR SUCCESS

Note that a very important tenet of science occurred throughout the period of quark discovery. The charmed, bottom, and top quarks were all speculated on, and then were discovered some time later. Each of their discoveries helped to verify and strengthen the quark model. This process of speculation and verification continues to take place today and is part of what drives physicists to search for evidence of the graviton and Grand Unified Theory.

One of the most confounding traits of quarks is their electric charge. Long assumed to be discrete, and specifically a multiple of the elementary charge of the electron, the electric charge of an individual quark is fractional and thus seems to violate a presumed tenet of particle physics. The fractional charge of quarks, which are $\pm\left(\frac{2}{3}\right)q_e$ and $\pm\left(\frac{1}{3}\right)q_e$, are the only structures found in nature with a nonintegral number of charge q. However, note that despite this odd construction, the fractional value of the quark does not violate the quantum nature of the charge. After all, free quarks cannot be found in nature, and all quarks are bound into arrangements in which an integer number of charge is constructed. Table 23.3 shows the six known quarks, in addition to their antiquark components, as will be discussed later in this section.

Flavor	Symbol	Antiparticle	Charge[1][2]
Up	u	$\overline{u}$	$\pm\frac{2}{3}q_e$
Down	d	$\overline{d}$	$\mp\frac{1}{3}q_e$
Strange	s	$\overline{s}$	$\mp\frac{1}{3}q_e$
Charmed	c	$\overline{c}$	$\pm\frac{2}{3}q_e$

[1]The lower of the ± symbols are the values for antiquarks.
[2]There are further qualities that differentiate between quarks. However, they are beyond the discussion in this text.

Table 23.3 Quarks and Antiquarks

Flavor	Symbol	Antiparticle	Charge[1][2]
Bottom	b	$\overline{b}$	$\mp\frac{1}{3}q_e$
Top	t	$\overline{t}$	$\pm\frac{2}{3}q_e$

[1]The lower of the $\pm$ symbols are the values for antiquarks.
[2]There are further qualities that differentiate between quarks. However, they are beyond the discussion in this text.

Table 23.3 Quarks and Antiquarks

While the term *flavor* is used to differentiate between types of quarks, the concept of **color** is more analogous to the electric charge in that it is primarily responsible for the force interactions between quarks. Note—Take a moment to think about the electrostatic force. It is the electric charge that causes attraction and repulsion. It is the same case here but with a *color* charge. The three colors available to a quark are red, green, and blue, with antiquarks having colors of anti-red (or cyan), anti-green (or magenta), and anti-blue (or yellow).

Why use colors when discussing quarks? After all, the quarks are not actually colored with visible light. The reason colors are used is because the properties of a quark are analogous to the three primary and secondary colors mentioned above. Just as different colors of light can be combined to create white, different *colors* of quark may be combined to construct a particle like a proton or neutron. In fact, for each hadron, the quarks must combine such that their color sums to white! Recall that two up quarks and one down quark construct a proton, as seen in Figure 23.12. The sum of the three quarks' colors—red, green, and blue—yields the color white. This theory of color interaction within particles is called **quantum chromodynamics**, or QCD. As part of QCD, the strong nuclear force can be explained using color. In fact, some scientists refer to the color force, not the strong force, as one of the four fundamental forces. Figure 23.13 is a Feynman diagram showing the interaction between two quarks by using the transmission of a colored **gluon**. Note that the gluon is also considered the charge carrier for the strong nuclear force.

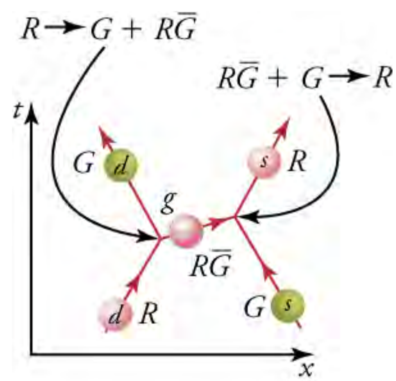


Figure 23.13 The exchange of gluons between quarks carries the strong force and may change the color of the interacting quarks. While the colors of the individual quarks change, their flavors do not.

Note that quark flavor may have any color. For instance, in Figure 23.13, the down quark has a red color and a green color. In other words, colors are not specific to a particle quark flavor.

Hadrons and Leptons

Particles can be revealingly grouped according to what forces they feel between them. All particles (even those that are massless) are affected by gravity since gravity affects the space and time in which particles exist. All charged particles are affected by the electromagnetic force, as are neutral particles that have an internal distribution of charge (such as the neutron with its magnetic moment). Special names are given to particles that feel the strong and weak nuclear forces. Hadrons are particles that feel the strong nuclear force, whereas leptons are particles that do not. All particles feel the weak nuclear force. This means that hadrons are distinguished by being able to feel both the strong and weak nuclear forces. Leptons and hadrons are distinguished in other ways as well. Leptons are fundamental particles that have no measurable size, while hadrons are composed of quarks and have a diameter on the order of 10 to 15 m. Six particles, including the electron and neutrino, make up the list of known leptons. There are hundreds of complex particles in the hadron class, a few of which (including the proton and neutron) are listed in Table 23.4.

Category	Particle Name	Symbol	Antiparticle	Rest Mass $\left(\mathrm{MeV/c^2}\right)$	Mean Lifetime (s)
Leptons	Electron	e^-	e^+	0.511	Stable
	Neutrino (e)	ν_e	$\overline{\nu}_e$	$0\ (7.0\,\mathrm{eV})$ [1]	Stable
	Muon	μ^-	μ^+	105.7	2.20×10^{-6}
	Neutrino (μ)	ν_μ	$\overline{\nu}_\mu$	$0\ (<0.27)$ [1]	Stable
	Tau	τ^-	τ^+	1,777	2.91×10^{-6}
	Neutrino (τ)	ν_τ	$\overline{\nu}_\tau$	$0\ (<31)$ [1]	Stable
Hadrons – Mesons[2]	Pion	π^+	π^-	139.6	2.60×10^{-8}
		π^0	Self	135.0	8.40×10^{-17}
	Kaon	K^+	K^-	493.7	1.24×10^{-8}
		K^0	K^0	497.6	0.90×10^{-10}
	Eta	η^0	Self	547.9	2.53×10^{-19}
Hadrons – Baryons[3]	Proton	p	$\overline{p}$	938.3	Stable
	Neutron	n	$\overline{n}$	939.6	882
	Lambda	Λ^0	$\overline{\Lambda}^0$	1,115.7	2.63×10^{-10}
	Omega	Ω^-	Ω^+	1,672.5	0.82×10^{-10}

[1]Neutrino masses may be zero. Experimental upper limits are given in parentheses.
[2]Many other mesons known
[3]Many other baryons known

Table 23.4 List of Leptons and Hadrons.

There are many more leptons, mesons, and baryons yet to be discovered and measured. The purpose of trying to uncover the smallest indivisible things in existence is to explain the world around us through forces and the interactions between particles, galaxies and objects. This is why a handful of scientists devote their life's work to smashing together small particles.

What internal structure makes a proton so different from an electron? The proton, like all hadrons, is made up of quarks. A few examples of hadron quark composition can be seen in Figure 23.14. As shown, each hadron is constructed of multiple quarks. As mentioned previously, the fractional quark charge in all four hadrons sums to the particle's integral value. Also, notice that the color composition for each of the four particles adds to white. Each of the particles shown is constructed of up, down, and their antiquarks. This is not surprising, as the quarks strange, charmed, top, and bottom are found in only our most exotic particles.

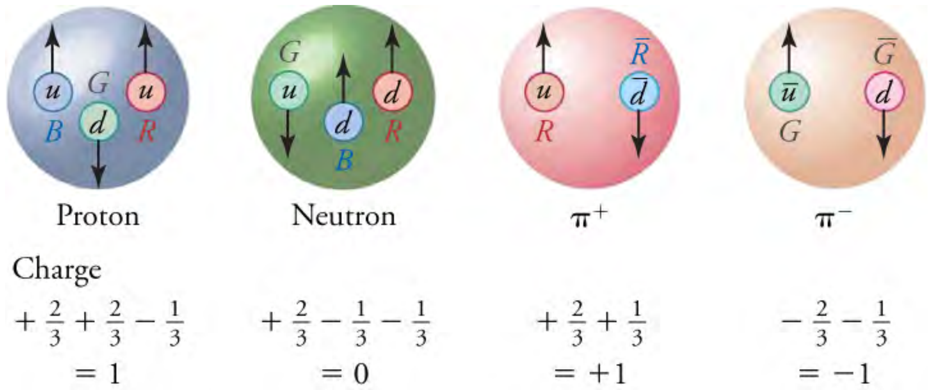


Figure 23.14 All baryons, such as the proton and neutron shown here, are composed of three quarks. All mesons, such as the pions shown here, are composed of a quark–antiquark pair. Arrows represent the spins of the quarks. The colors are such that they need to add to white for any possible combination of quarks.

You may have noticed that while the proton and neutron in Figure 23.14 are composed of three quarks, both pions are comprised of only two quarks. This refers to a final delineation in particle structure. Particles with three quarks are called **baryons**. These are heavy particles that can decay into another baryon. Particles with only two quarks—a-quark–anti-quark pair—are called **mesons**. These are particles of moderate mass that cannot decay into the more massive baryons.

Before continuing, take a moment to view Figure 23.15. In this figure, you can see the strong force reimagined as a color force. The particles interacting in this figure are the proton and neutron, just as they were in Figure 23.6. This reenvisioning of the strong force as an interaction between colored quarks is the critical concept behind quantum chromodynamics.

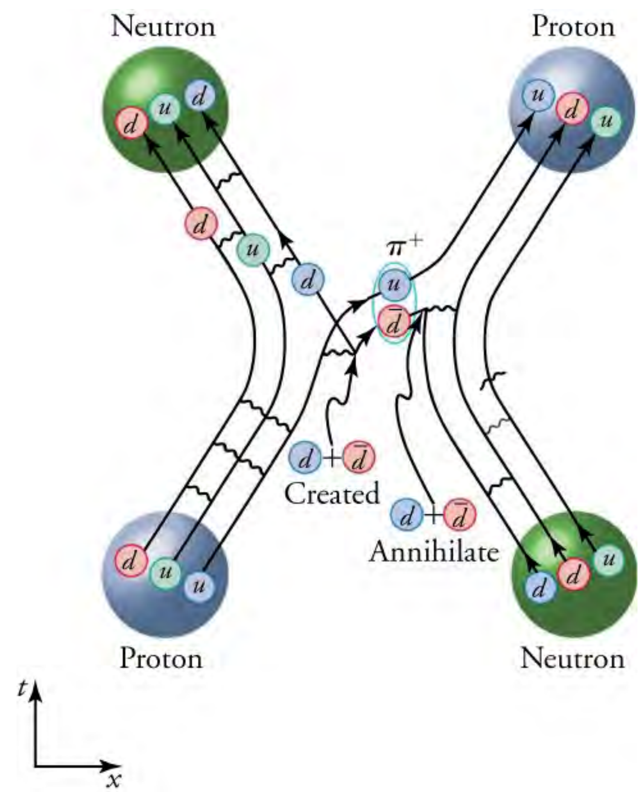


Figure 23.15 This Feynman diagram shows the interaction between a proton and a neutron, corresponding to the interaction shown in Figure 23.6. This diagram, however, shows the quark and gluon details of the strong nuclear force interaction.

Matter and Antimatter

Antimatter was first discovered in the form of the **positron**, the positively charged electron. In 1932, American physicist Carl Anderson discovered the positron in cosmic ray studies. Through a cloud chamber modified to curve the trajectories of cosmic rays, Anderson noticed that the curves of some particles followed that of a negative charge, while others curved like a positive charge. However, the positive curve showed not the mass of a proton but the mass of an electron. This outcome is shown in Figure 23.16 and suggests the existence of a positively charged version of the electron, created by the destruction of solar photons.

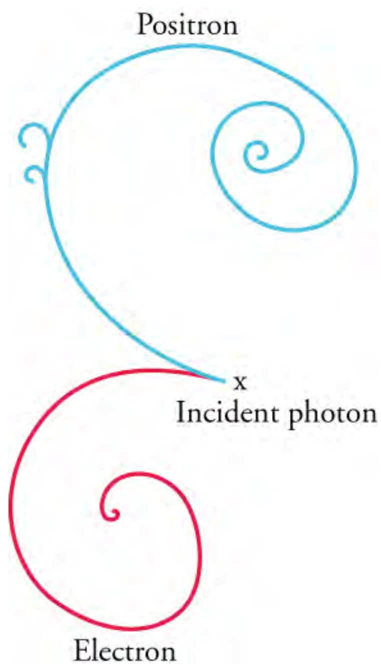


Figure 23.16 The image above is from the Fermilab *15 foot bubble chamber* and shows the production of an electron and positron (or antielectron) from an incident photon. This event is titled **pair production** and provides evidence of antimatter, as the two repel each other.

Antimatter is considered the opposite of matter. For most antiparticles, this means that they share the same properties as their original particles with the exception of their charge. This is why the positron can be considered a positive electron while the antiproton is considered a negative proton. The idea of an opposite charge for neutral particles (like the neutron) can be confusing, but it makes sense when considered from the quark perspective. Just as the neutron is composed of one up quark and two down quarks (of charge $+\frac{2}{3}$ and $-\frac{1}{3}$, respectively), the antineutron is composed of one anti–up quark and two anti–down quarks (of charge $-\frac{2}{3}$ and $+\frac{1}{3}$, respectively). While the overall charge of the neutron remains the same, its constituent particles do not!

A word about antiparticles: Like regular particles, antiparticles could function just fine on their own. In fact, a universe made up of antimatter may operate just as our own matter-based universe does. However, we do not know fully whether this is the case. The reason for this is **annihilation**. Annihilation is the process of destruction that occurs when a particle and its antiparticle interact. As soon as two particles (like a positron and an electron) coincide, they convert their masses to energy through the equation $E = mc^2$. This mass-to-energy conversion, which typically results in photon release, happens instantaneously and makes it very difficult for scientists to study antimatter. That said, scientists have had success creating antimatter through high-energy particle collisions. Both antineutrons and antiprotons were created through accelerator experiments in 1956, and an anti–hydrogen atom was even created at CERN in 1995! As referenced in , the annihilation of antiparticles is currently used in medical studies to determine the location of radioisotopes.

Completing the Standard Model of the Atom

The Standard Model of the atom refers to the current scientific view of the fundamental components and interacting forces of matter. The Standard Model (Figure 23.17) shows the six quarks that bind to form all hadrons, the six lepton particles already considered fundamental, the four carrier particles (or gauge bosons) that transmit forces between the leptons and quarks, and the recently added **Higgs boson** (which will be discussed shortly). This totals 17 fundamental particles, combinations of which are responsible for all known matter in our entire universe! When adding the antiquarks and antileptons, 31 components make up the Standard Model.

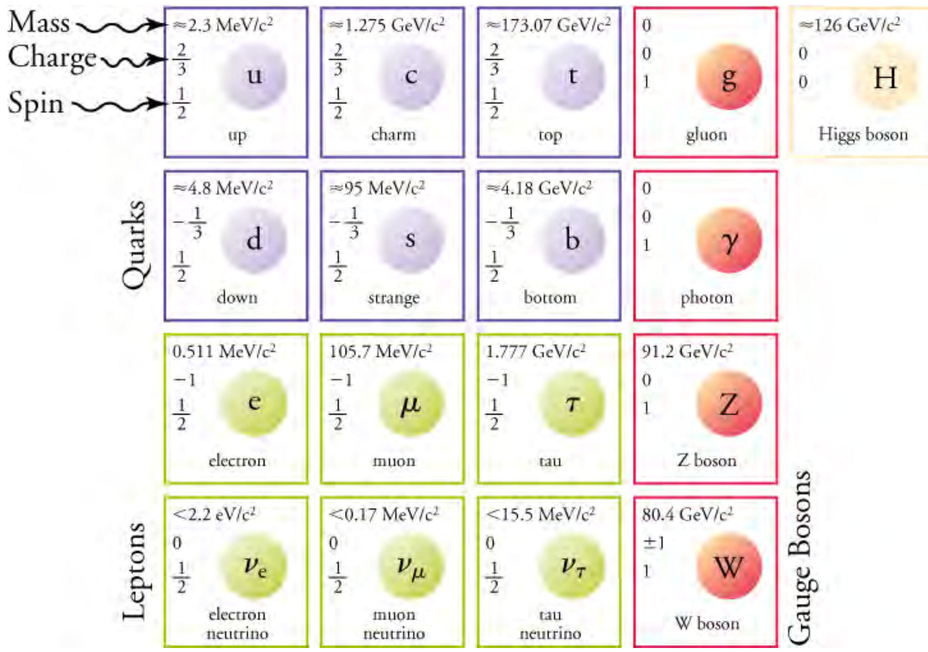


Figure 23.17 The Standard Model of elementary particles shows an organized view of all fundamental particles, as currently known: six quarks, six leptons, and four gauge bosons (or carrier particles). The Higgs boson, first observed in 2012, is a new addition to the Standard Model.

Figure 23.17 shows all particles within the Standard Model of the atom. Not only does this chart divide all known particles by color-coded group, but it also provides information on particle stability. Note that the color-coding system in this chart is separate from the red, green, and blue color labeling system of quarks. The first three columns represent the three *families* of matter. The first column, considered Family 1, represents particles that make up normal matter, constructing the protons, neutrons, and electrons that make up the common world. Family 2, represented from the charm quark to the muon neutrino, is comprised of particles that are more massive. The leptons in this group are less stable and more likely to decay. Family 3, represented by the third column, are more massive still and decay more quickly. The order of these families also conveniently represents the order in which these particles were discovered.

TIPS FOR SUCCESS

Look for trends that exist within the Standard Model. Compare the charge of each particle. Compare the spin. How does mass relate to the model structure? Recognizing each of these trends and asking questions will yield more insight into the organization of particles and the forces that dictate particle relationships. Our understanding of the Standard Model is still young, and the questions you may have in analyzing the Standard Model may be some of the same questions that particle physicists are searching for answers to today!

The Standard Model also summarizes the fundamental forces that exist as particles interact. A closer look at the Standard Model, as shown in Figure 23.18, reveals that the arrangement of carrier particles describes these interactions.

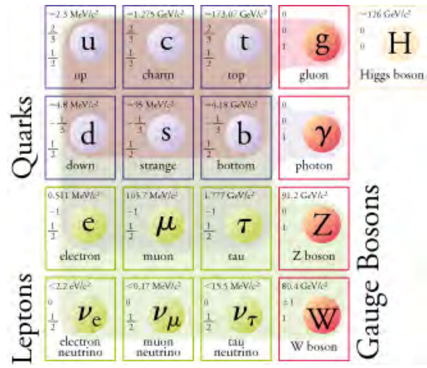


Figure 23.18 The revised Standard Model shows the interaction between gauge bosons and other fundamental particles. These interactions are responsible for the fundamental forces, three of which are described through the chart's shaded areas.

Each of the shaded areas represents a fundamental force and its constituent particles. The red shaded area shows all particles involved in the strong nuclear force, which we now know is due to quantum chromodynamics. The blue shaded area corresponds to the electromagnetic force, while the green shaded area corresponds to the weak nuclear force, which affects all quarks and leptons. The electromagnetic force and weak nuclear force are considered united by the electroweak force within the Standard Model. Also, because definitive evidence of the graviton is yet to be found, it is not included in the Standard Model.

The Higgs Boson

One interesting feature of the Standard Model shown in Figure 23.18 is that, while the gluon and photon have no mass, the Z and W bosons are very massive. What supplies these quickly moving particles with mass and not the gluons and photons? Furthermore, what causes some quarks to have more mass than others?

In the 1960s, British physicist Peter Higgs and others speculated that the W and Z bosons were actually just as massless as the gluon and photon. However, as the W and Z bosons traveled from one particle to another, they were slowed down by the presence of a **Higgs field**, much like a fish swimming through water. The thinking was that the existence of the Higgs field would slow down the bosons, causing them to decrease in energy and thereby transfer this energy to mass. Under this theory, all particles pass through the Higgs field, which exists throughout the universe. The gluon and photon travel through this field as well but are able to do so unaffected.

The presence of a force from the Higgs field suggests the existence of its own carrier particle, the Higgs boson. This theorized boson interacts with all particles but gluons and photons, transferring force from the Higgs field. Particles with large mass (like the top quark) are more likely to receive force from the Higgs boson.

While it is difficult to examine a field, it is somewhat simpler to find evidence of its carrier. On July 4, 2012, two groups of scientists at the LHC independently confirmed the existence of a Higgs-like particle. By examining trillions of proton–proton collisions at energies of 7 to 8 TeV, LHC scientists were able to determine the constituent particles that created the protons. In this data, scientists found a particle with similar mass, spin, parity, and interactions with other particles that matched the Higgs boson predicted decades prior. On March 13, 2013, the existence of the Higgs boson was tentatively confirmed by CERN. Peter Higgs and Francois Englert received the Nobel Prize in 2013 for the "theoretical discovery of a mechanism that contributes to our understanding of the origin and mass of subatomic particles."

WORK IN PHYSICS

Particle Physicist

If you have an innate desire to unravel life's great mysteries and further understand the nature of the physical world, a career in particle physics may be for you!

Particle physicists have played a critical role in much of society's technological progress. From lasers to computers, televisions to space missions, splitting the atom to understanding the DNA molecule to MRIs and PET scans, much of our modern society is based on the work done by particle physicists.

While many particle physicists focus on specialized tasks in the fields of astronomy and medicine, the main goal of particle physics is to further scientists' understanding of the Standard Model. This may mean work in government, industry, or academics. Within the government, jobs in particle physics can be found within the National Institute for Standards and Technology, Department of Energy, NASA, and Department of Defense. Both the electronics and computer industries rely on the expertise of particle physicists. College teaching and research positions can also be potential career opportunities for particle physicists, though they often require some postgraduate work as a prerequisite. In addition, many particle physicists are employed to work on high-energy colliders. Domestic collider labs include the Brookhaven National Laboratory in New York, the Fermi National Accelerator Laboratory near Chicago, and the SLAC National Accelerator Laboratory operated by Stanford University. For those who like to travel, work at international collider labs can be found at the CERN facility in Switzerland in addition to institutes like the Budker Institute of Nuclear Physics in Russia, DESY in Germany, and KEK in Japan.

Shirley Jackson became the first African American woman to earn a Ph.D. from MIT back in 1973, and she went on to lead a highly successful career in the field of particle physics. Like Dr. Jackson, successful students of particle physics grow up with a strong curiosity in the world around them and a drive to continually learn more. If you are interested in exploring a career in particle physics, work to achieve good grades and SAT scores, and find time to read popular books on physics topics that interest you. While some math may be challenging, recognize that this is only a tool of physics and should not be considered prohibitive to the field. High-level work in particle physics often requires a Ph.D.; however, it is possible to find work with a master's degree. Additionally, jobs in industry and teaching can be achieved with solely an undergraduate degree.

GRASP CHECK

What is the primary goal of all work in particle physics?

a. The primary goal is to further our understanding of the Standard Model.
b. The primary goal is to further our understanding of Rutherford's model.
c. The primary goal is to further our understanding of Bohr's model.
d. The primary goal is to further our understanding of Thomson's model.

Check Your Understanding

8. In what particle were quarks originally discovered?

a. the electron
b. the neutron
c. the proton
d. the photon

9. Why was the existence of the charm quark speculated, even though no direct evidence of it existed?

a. The existence of the charm quark was symmetrical with up and down quarks. Additionally, there were two known leptons at the time and only two quarks.
b. The strange particle lacked the symmetry that existed with the up and down quarks. Additionally, there were four known leptons at the time and only three quarks.
c. The bottom particle lacked the symmetry that existed with the up and down quarks. Additionally, there were two known leptons at the time and only two quarks.
d. The existence of charm quarks was symmetrical with up and down quarks. Additionally, there were four known leptons at the time and only three quarks.

10. What type of particle is the electron?

a. The electron is a lepton.
b. The electron is a hadron.
c. The electron is a baryon.
d. The electron is an antibaryon.

11. How do the number of fundamental particles differ between hadrons and leptons?

a. Hadrons are constructed of at least three fundamental quark particles, while leptons are fundamental particles.
b. Hadrons are constructed of at least three fundamental quark particles, while leptons are constructed of two fundamental particles.
c. Hadrons are constructed of at least two fundamental quark particles, while leptons are constructed of three

fundamental particles.
d. Hadrons are constructed of at least two fundamental quark particles, while leptons are fundamental particles.

12. Does antimatter exist?
a. no
b. yes

13. How does the deconstruction of a photon into an electron and a positron uphold the principles of mass and charge conservation?
a. The sum of the masses of an electron and a positron is equal to the mass of the photon before pair production. The sum of the charges on an electron and a positron is equal to the zero charge of the photon.
b. The sum of the masses of an electron and a positron is equal to the mass of the photon before pair production. The sum of the same charges on an electron and a positron is equal to the charge on a photon.
c. During the particle production the total energy of the photon is converted to the mass of an electron and a positron. The sum of the opposite charges on the electron and positron is equal to the zero charge of the photon.
d. During particle production, the total energy of the photon is converted to the mass of an electron and a positron. The sum of the same charges on an electron and a positron is equal to the charge on a photon.

14. How many fundamental particles exist in the Standard Model, including the Higgs boson and the graviton (not yet observed)?
a. 12
b. 15
c. 13
d. 19

15. Why do gluons interact only with particles in the first two rows of the Standard Model?
a. The leptons in the third and fourth rows do not have mass, but the gluons can interact between the quarks through gravity only.
b. The leptons in the third and fourth rows do not have color, but the gluons can interact between quarks through color interactions only.
c. The leptons in the third and fourth rows do not have spin, but the gluons can interact between quarks through spin interactions only.
d. The leptons in the third and fourth rows do not have charge, but the gluons can interact between quarks through charge interactions only.

16. What fundamental property is provided by particle interaction with the Higgs boson?
a. charge
b. mass
c. spin
d. color

17. Considering the Higgs field, what differentiates more massive particles from less massive particles?
a. More massive particles interact more with the Higgs field than the less massive particles.
b. More massive particles interact less with the Higgs field than the less massive particles.

18. What particles were launched into the proton during the original discovery of the quark?
a. bosons
b. electrons
c. neutrons
d. photons

23.3 The Unification of Forces

Section Learning Objectives

By the end of the section, you will be able to do the following:

- Define a grand unified theory and its importance
- Explain the evolution of the four fundamental forces from the Big Bang onward
- Explain how grand unification theories can be tested

Section Key Terms

Big Bang	Electroweak Epoch	electroweak theory	Grand Unification Epoch	Grand Unified Theory
Inflationary Epoch	Planck Epoch	Quark Era	superforce	Theory of Everything

Understanding the Grand Unified Theory

Present quests to show that the four basic forces are different manifestations of a single unified force that follow a long tradition. In the nineteenth century, the distinct electric and magnetic forces were shown to be intimately connected and are now collectively called the electromagnetic force. More recently, the weak nuclear force was united with the electromagnetic force. As shown in Figure 23.19, carrier particles transmit three of the four fundamental forces in very similar ways. With these considerations in mind, it is natural to suggest that a theory may be constructed in which the strong nuclear, weak nuclear, and electromagnetic forces are all unified. The search for a correct theory linking the forces, called the **Grand Unified Theory (GUT)**, is explored in this section.

In the 1960s, the **electroweak theory** was developed by Steven Weinberg, Sheldon Glashow, and Abdus Salam. This theory proposed that the electromagnetic and weak nuclear forces are identical at sufficiently high energies. At lower energies, like those in our present-day universe, the two forces remain united but manifest themselves in different ways. One of the main consequences of the electroweak theory was the prediction of three short-range carrier particles, now known as the W^+, W^-, and Z^0 bosons. Not only were three particles predicted, but the mass of each W^+ and W^- boson was predicted to be 81 GeV/c^2, and that of the Z^0 boson was predicted to be 90 GeV/c^2. In 1983, these carrier particles were observed at CERN with the predicted characteristics, including masses having those predicted values as given in .

How can forces be unified? They are definitely distinct under most circumstances. For example, they are carried by different particles and have greatly different strengths. But experiments show that at extremely short distances and at extremely high energies, the strengths of the forces begin to become more similar, as seen in Figure 23.20.

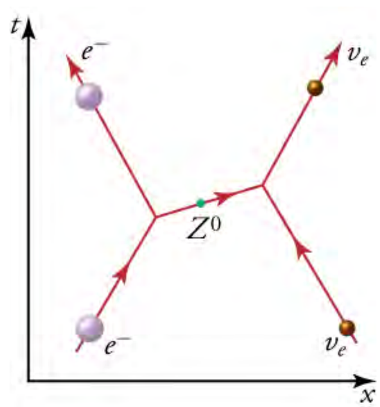


Figure 23.19 The exchange of a virtual Z^0 particle (boson) carries the weak nuclear force between an electron and a neutrino in this Feynman diagram. This diagram is similar to the diagrams in Figure 23.6 and for the electromagnetic and strong nuclear forces.

As discussed earlier, the short ranges and large masses of the weak carrier bosons require correspondingly high energies to create them. Thus, the energy scale on the horizontal axis of Figure 23.20 also corresponds to shorter and shorter distances (going from left to right), with 100 GeV corresponding to approximately 10^{-18} m, for example. At that distance, the strengths of the electromagnetic and weak nuclear forces are the same. To test this, energies of about 100 GeV are put into the system. When this occurs, the W^+, W^-, and Z^0 carrier particles are created and released. At those and higher energies, the masses of the carrier particles become less and less relevant, and the Z^0 boson in particular resembles the massless, chargeless photon. As further energy is added, the W^+, W^-, and Z^0 particles are further transformed into massless carrier particles even more similar to photons and gluons.

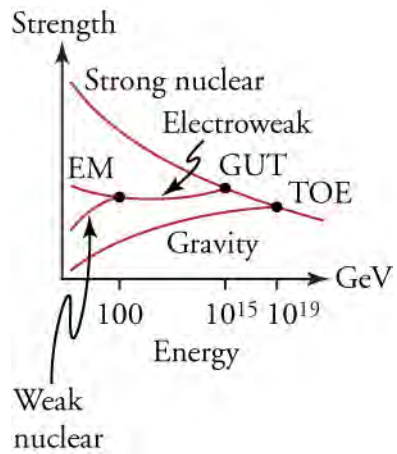


Figure 23.20 The relative strengths of the four basic forces vary with distance, and, hence, energy is needed to probe small distances. At ordinary energies (a few eV or less), the forces differ greatly. However, at energies available in accelerators, the weak nuclear and electromagnetic (EM) forces become unified. Unfortunately, the energies at which the strong nuclear and electroweak forces become the same are unreachable in any conceivable accelerator. The universe may provide a laboratory, and nature may show effects at ordinary energies that give us clues about the validity of this graph.

The extremely short distances and high energies at which the electroweak force becomes identical with the strong nuclear force are not reachable with any conceivable human-built accelerator. At energies of about 10^{14} GeV (16,000 J per particle), distances of about 10 to 30 m can be probed. Such energies are needed to test the theory directly, but these are about 10^{10} times higher than the maximum energy associated with the LHC, and the distances are about 10 to 12 smaller than any structure we have direct knowledge of. This would be the realm of various GUTs, of which there are many, since there is no constraining evidence at these energies and distances. Past experience has shown that anytime you probe so many orders of magnitude further, you find the unexpected.

While direct evidence of a GUT is not presently possible, that does not rule out the ability to assess a GUT through an indirect process. Current GUTs require various other events as a consequence of their theory. Some GUTs require the existence of magnetic monopoles, very massive individual north- and south-pole particles, which have not yet been proven to exist, while others require the use of extra dimensions. However, not all theories result in the same consequences. For example, disproving the existence of magnetic monopoles will not disprove all GUTs. Much of the science we accept in our everyday lives is based on different models, each with their own strengths and limitations. Although a particular model may have drawbacks, that does not necessarily mean that it should be discounted completely.

One consequence of GUTs that can theoretically be assessed is proton decay. Multiple current GUTs hypothesize that the stable proton should actually decay at a lifetime of 10^{31} years. While this time is incredibly large (keep in mind that the age of the universe is less than 14 billion years), scientists at the Super-Kamiokande in Japan have used a 50,000-ton tank of water to search for its existence. The decay of a single proton in the Super-Kamiokande tank would be observed by a detector, thereby providing support for the predicting GUT model. However, as of 2014, 17 years into the experiment, decay is yet to be found. This time span equates to a minimum limit on proton life of 5.9×10^{33} years. While this result certainly does not support many grand unifying theories, an acceptable model may still exist.

TIPS FOR SUCCESS

The Super-Kamiokande experiment is a clever use of proportional reasoning. Because it is not feasible to test for 10^{31} years in order for a single proton to decay, scientists chose instead to manipulate the proton–time ratio. If one proton decays in 10^{31}

years, then in one year 10^{-31} protons will decay. With this in mind, if scientists wanted to test the proton decay theory in one year, they would need 10^{31} protons. While this is also unfeasible, the use of a 50,000-ton tank of water helps to bring both the wait time and proton number to within reason.

The Standard Model and the Big Bang

Nature is full of examples where the macroscopic and microscopic worlds intertwine. Newton realized that the nature of gravity on Earth that pulls an apple to the ground could explain the motion of the moon and planets so much farther away. Decays of tiny nuclei explain the hot interior of the Earth. Fusion of nuclei likewise explains the energy of stars. Today, the patterns in particle physics seem to be explaining the evolution and character of the universe. And the nature of the universe has implications for unexplored regions of particle physics.

In 1929, Edwin Hubble observed that all but the closest galaxies surrounding our own had a red shift in their hydrogen spectra that was proportional to their distance from us. Applying the Doppler Effect, Hubble recognized that this meant that all galaxies were receding from our own, with those farther away receding even faster. Knowing that our place in the universe was no more unique than any other, the implication was clear: The space within the universe itself was expanding. Just like pen marks on an expanding balloon, everything in the universe was accelerating away from everything else.

Figure 23.21 shows how the recession of galaxies looks like the remnants of a gigantic explosion, the famous **Big Bang**. Extrapolating backward in time, the Big Bang would have occurred between 13 and 15 billion years ago, when all matter would have been at a single point. From this, questions instantly arise. What caused the explosion? What happened before the Big Bang? Was there a before, or did time start then? For our purposes, the biggest question relating to the Big Bang is this: How does the Big Bang relate to the unification of the fundamental forces?

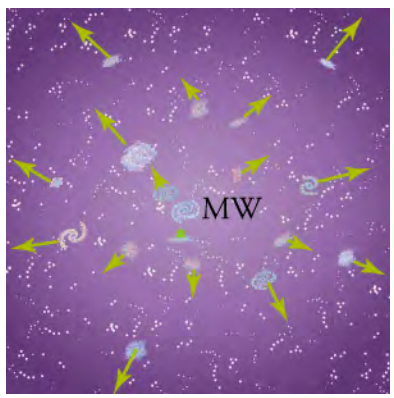


Figure 23.21 Galaxies are flying apart from one another, with the more distant ones moving faster, as if a primordial explosion expelled the matter from which they formed. The most distant known galaxies move nearly at the speed of light relative to us.

To fully understand the conditions of the very early universe, recognize that as the universe contracts to the size of the Big Bang, changes will occur. The density and temperature of the universe will increase dramatically. As particles become closer together, they will become too close to exist as we know them. The high energies will create other, more unusual particles to exist in greater abundance. Knowing this, let's move forward from the start of the universe, beginning with the Big Bang, as illustrated in Figure 23.22.

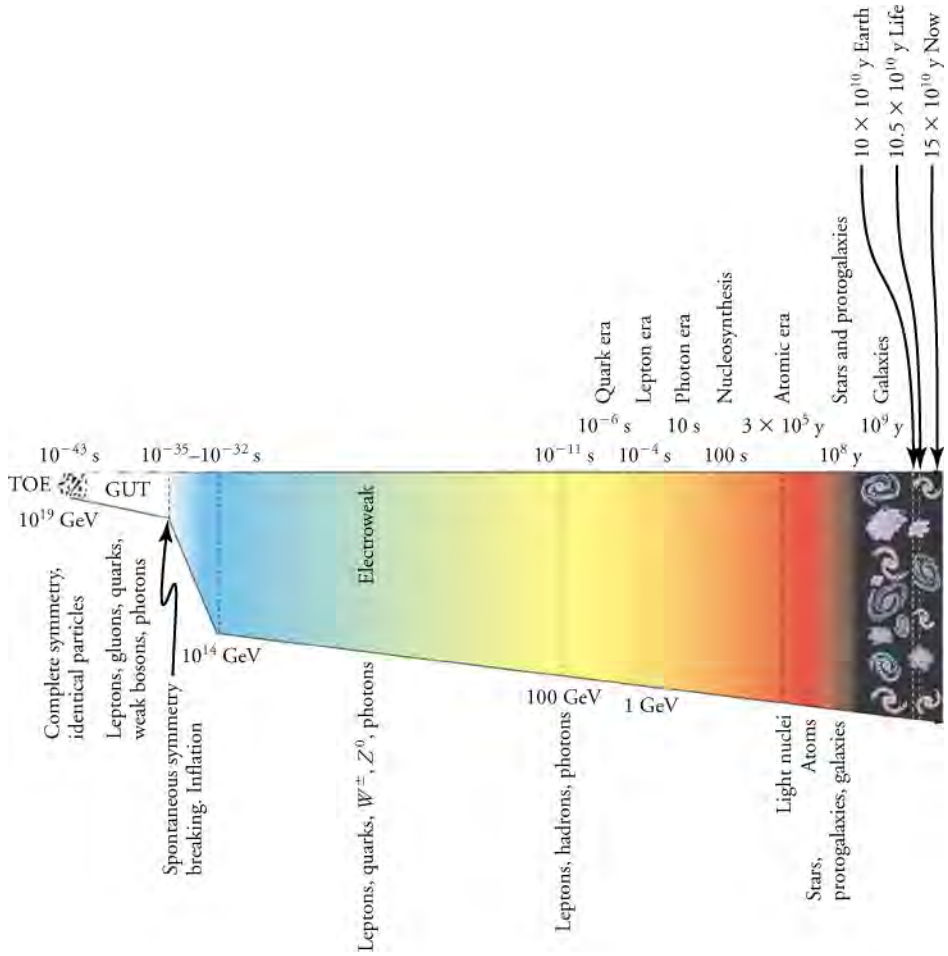


Figure 23.22 The evolution of the universe from the Big Bang onward (from left to right) is intimately tied to the laws of physics, especially those of particle physics at the earliest stages. Theories of the unification of forces at high energies may be verified by their shaping of the universe and its evolution.

The **Planck Epoch** $\left(0 \rightarrow 10^{-43}\text{ s}\right)$ —Though scientists are unable to model the conditions of the Planck Epoch in the laboratory, speculation is that at this time compressed energy was great enough to reach the immense 10^{19} GeV necessary to unify gravity with all other forces. As a result, modern cosmology suggests that all four forces would have existed as one force, a hypothetical **superforce** as suggested by the **Theory of Everything**.

The **Grand Unification Epoch** $\left(10^{-43} \rightarrow 10^{-36}\text{ s}\right)$ —As the universe expands, the temperatures necessary to maintain the superforce decrease. As a result, gravity separates, leaving the electroweak and strong nuclear forces together. At this time, the electromagnetic, weak, and strong forces are identical, matching the conditions requested in the Grand Unification Theory.

The **Inflationary Epoch** $\left(10^{-36} \rightarrow 10^{-32}\text{ s}\right)$ —The separation of the strong nuclear force from the electroweak force during this time is thought to have been responsible for the massive inflation of the universe. Corresponding to the steep diagonal line on the left side of Figure 23.22, the universe may have expanded by a factor of 10^{50} or more in size. In fact, the expansion was so great during this time that it actually occurred faster than the speed of light! Unfortunately, there is little hope that we may be able to test the inflationary scenario directly since it occurs at energies near 10^{14} GeV, vastly greater than the limits of modern accelerators.

The **Electroweak Epoch** $\left(10^{-32} \rightarrow 10^{-11}\text{ s}\right)$ —Now separated from both gravity and the strong nuclear force, the electroweak force exists as a singular force during this time period. As stated earlier, scientists are able to create the energies at this stage in the universe's expansion, needing only 100 GeV, as shown in Figure 23.20. W and Z bosons, as well as the Higgs boson, are released during this time.

The **Quark Era** $\left(10^{-11} \rightarrow 10^{-6}\text{ s}\right)$ —During the Quark Era, the universe has expanded and temperatures have decreased to the point at which all four fundamental forces have separated. Additionally, quarks began to take form as energies decreased.

As the universe expanded, further eras took place, allowing for the existence of hadrons, leptons, and photons, the fundamental particles of the standard model. Eventually, in nucleosynthesis, nuclei would be able to form, and the basic building blocks of atomic matter could take place. Using particle accelerators, we are very much working backwards in an attempt to understand the universe. It is encouraging to see that the macroscopic conditions of the Big Bang align nicely with our submicroscopic particle theory.

Check Your Understanding

19. Is there one grand unified theory or multiple grand unifying theories?
  a. one grand unifying theory
  b. multiple grand unifying theories

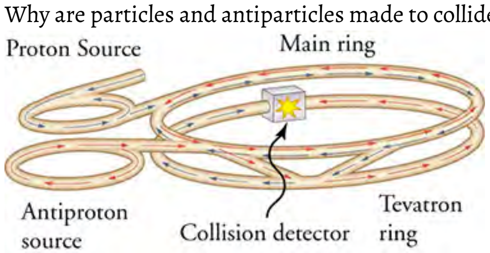
20. In what manner is $E = mc^2$ considered a precursor to the Grand Unified Theory?
  a. The grand unified theory seeks relate the electroweak and strong nuclear forces to one another just as $E = mc^2$ related energy and mass.
  b. The grand unified theory seeks to relate the electroweak force and mass to one another just as $E = mc^2$ related energy and mass.
  c. The grand unified theory seeks to relate the mass and strong nuclear forces to one another just as $E = mc^2$ related energy and mass.
  d. The grand unified theory seeks to relate gravity and strong nuclear force to one another, just as $E = mc^2$ related energy and mass.

21. List the following eras in order of occurrence from the Big Bang: Electroweak Epoch, Grand Unification Epoch, Inflationary Epoch, Planck Epoch, Quark Era.
  a. Quark Era, Grand Unification Epoch, Inflationary Epoch, Electroweak Epoch, Planck Epoch
  b. Planck Epoch, Inflationary Epoch, Grand Unification Epoch, Electroweak Epoch, Quark Era
  c. Planck Epoch, Electroweak Epoch, Grand Unification Epoch, Inflationary Epoch, Quark Era
  d. Planck Epoch, Grand Unification Epoch, Inflationary Epoch, Electroweak Epoch, Quark Era

22. How did the temperature of the universe change as it expanded?
  a. The temperature of the universe increased.
  b. The temperature of the universe decreased.
  c. The temperature of the universe first decreased and then increased.
  d. The temperature of the universe first increased and then decreased.

23. Under current conditions, is it possible for scientists to use particle accelerators to verify the Grand Unified Theory?
  a. No, there is not enough energy.
  b. Yes, there is enough energy.

24. Why are particles and antiparticles made to collide as shown in this image?



  a. Particles and antiparticles have the same mass.
  b. Particles and antiparticles have different mass.
  c. Particles and antiparticles have the same charge.
  d. Particles and antiparticles have opposite charges.

25. The existence of what particles were predicted as a consequence of the electroweak theory?
  a. fermions
  b. Higgs bosons

c. leptons
d. W^+, W^-, and Z° bosons

KEY TERMS

W^+ boson positive carrier particle of the weak nuclear force

W^- boson negative carrier particle of the weak nuclear force

Z^0 boson neutral carrier particle of the weak nuclear force

annihilation the process of destruction that occurs when a particle and antiparticle interact

antimatter matter constructed of antiparticles; antimatter shares most of the same properties of regular matter, with charge being the only difference between many particles and their antiparticle analogues

baryon hadrons that always decay to another baryon

Big Bang a gigantic explosion that threw out matter a few billion years ago

bottom quark a quark flavor

carrier particle a virtual particle exchanged in the transmission of a fundamental force

charmed quark a quark flavor, which is the counterpart of the strange quark

colliding beam head-on collisions between particles moving in opposite directions

color a property of quarks the relates to their interactions through the strong force

cyclotron accelerator that uses fixed-frequency alternating electric fields and fixed magnets to accelerate particles in a circular spiral path

down quark the second lightest of all quarks

Electroweak Epoch the stage before 10^{-11} back to 10^{-34} seconds after the Big Bang

electroweak theory theory showing connections between EM and weak forces

Feynman diagram a graph of time versus position that describes the exchange of virtual particles between subatomic particles

flavor quark type

gluons exchange particles of the nuclear strong force

Grand Unification Epoch the time period from 10^{-43} to 10^{-34} seconds after the Big Bang, when Grand Unification Theory, in which all forces except gravity are identical, governed the universe

Grand Unified Theory theory that shows unification of the strong and electroweak forces

graviton hypothesized particle exchanged between two particles of mass, transmitting the gravitational force between them

hadron particles composed of quarks that feel the strong and weak nuclear force

Higgs boson a massive particle that provides mass to the weak bosons and provides validity to the theory that carrier particles are identical under certain circumstances

Higgs field the field through which all fundamental particles travel that provides them varying mass through the transport of the Higgs boson

Inflationary Epoch the rapid expansion of the universe by an incredible factor of 10^{-50} for the brief time from 10^{-35} to about 10^{-32} seconds

lepton fundamental particles that do not feel the nuclear strong force

meson hadrons that can decay to leptons and leave no hadrons

pair production the creation of a particle and antiparticle, commonly an electron and positron, due to the annihilation of a photon

particle physics the study of and the quest for those truly fundamental particles having no substructure

pion particle exchanged between nucleons, transmitting the strong nuclear force between them

Planck Epoch the earliest era of the universe, before 10^{-43} seconds after the Big Bang

positron a particle of antimatter that has the properties of a positively charged electron

quantum chromodynamics the theory of color interaction between quarks that leads to understanding of the nuclear strong force

quantum electrodynamics the theory of electromagnetism on the particle scale

quark an elementary particle and fundamental constituent of matter that is a substructure of hadrons

Quark Era the time period from 10^{-11} to 10^{-6} seconds at which all four fundamental forces are separated and quarks begin to exit

Standard Model an organization of fundamental particles and forces that is a result of quantum chromodynamics and electroweak theory

strange quark the third lightest of all quarks

superforce the unification of all four fundamental forces into one force

synchrotron a version of a cyclotron in which the frequency of the alternating voltage and the magnetic field strength are increased as the beam particles are accelerated

Theory of Everything the theory that shows unification of all four fundamental forces

top quark a quark flavor

up quark the lightest of all quarks

weak nuclear force fundamental force responsible for particle decay

SECTION SUMMARY

23.1 The Four Fundamental Forces

- The four fundamental forces are gravity, the electromagnetic force, the weak nuclear force, and the strong nuclear force.
- A variety of particle accelerators have been used to explore the nature of subatomic particles and to test predictions of particle theories.

23.2 Quarks

- There are three types of fundamental particles—leptons, quarks, and carrier particles.
- Quarks come in six flavors and three colors and occur only in combinations that produce white.
- Hadrons are thought to be composed of quarks, with baryons having three quarks and mesons having a quark and an antiquark.
- Known particles can be divided into three major groups—leptons, hadrons, and carrier particles (gauge bosons).
- All particles of matter have an antimatter counterpart that has the opposite charge and certain other quantum numbers. These matter–antimatter pairs are otherwise very similar but will annihilate when brought together.
- The strong force is carried by eight proposed particles called gluons, which are intimately connected to a quantum number called color—their governing theory is thus called quantum chromodynamics (QCD). Taken together, QCD and the electroweak theory are widely accepted as the Standard Model of particle physics.

23.3 The Unification of Forces

- Attempts to show unification of the four forces are called Grand Unified Theories (GUTs) and have been partially successful, with connections proven between EM and weak forces in electroweak theory.
- Unification of the strong force is expected at such high energies that it cannot be directly tested, but it may have observable consequences in the as-yet-unobserved decay of the proton. Although unification of forces is generally anticipated, much remains to be done to prove its validity.

CHAPTER REVIEW

Concept Items

23.1 The Four Fundamental Forces

1. What forces does the inverse square law describe?
   a. the electromagnetic and weak nuclear force
   b. the electromagnetic force and strong nuclear force
   c. the electromagnetic force and gravity
   d. the strong nuclear force and gravity

2. Do the carrier particles explain the loss of mass in nuclear decay?
   a. no
   b. yes

3. What happens to the rate of voltage oscillation within a synchrotron each time the particle completes a loop?
   a. The rate of voltage oscillation increases as the particle travels faster and faster on each loop.
   b. The rate of voltage oscillation decreases as the particle travels faster and faster on each loop.
   c. The rate of voltage oscillation remains the same each time the particle completes a loop.
   d. The rate of voltage oscillation first increases and then remains constant each time the particle completes a loop.

4. Which of the four forces is responsible for ionic bonding?
   a. electromagnetic force
   b. gravity
   c. strong force
   d. weak nuclear force

5. What type of particle accelerator uses oscillating electric fields to accelerate particles around a fixed radius track?
   a. LINAC
   b. synchrotron
   c. SLAC
   d. Van de Graaff accelerator

23.2 Quarks

6. How does the charge of an individual quark determine hadron structure?
   a. Since the hadron must have an integral value, the individual quarks must be combined such that the average of their charges results in the value of a quark.
   b. Since the hadron must have an integral value, the individual atoms must be combined such that the sum of their charges is less than zero.
   c. The individual quarks must be combined such that the product of their charges is equal to the total charge of the hadron structure.
   d. Since the hadron must have an integral value of charge, the individual quarks must be combined such that the sum of their charges results in an

integral value.

7. Why do leptons not feel the strong nuclear force?
   a. Gluons are the carriers of the strong nuclear force that interacts between quarks through color interactions, but leptons are constructed of quarks that do not have gluons.
   b. Gluons are the carriers of the strong nuclear force that interacts between quarks through mass interactions, but leptons are not constructed of quarks and are not massive.
   c. Gluons are the carriers of the strong nuclear force that interacts between quarks through mass interactions, but leptons are constructed of the quarks that are not massive.
   d. Gluons are the carriers of the strong nuclear force that interacts between quarks through color interactions, but leptons are not constructed of quarks, nor do they have color constituents.

8. What property commonly distinguishes antimatter from its matter analogue?
   a. mass
   b. charge
   c. energy
   d. speed

9. Can the Standard Model change as new information is gathered?
   a. yes
   b. no

10. What is the relationship between the Higgs field and the Higgs boson?
   a. The Higgs boson is the carrier that transfers force for the Higgs field.
   b. The Higgs field is the time duration over which the Higgs particles transfer force to the other particles.
   c. The Higgs field is the magnitude of momentum transferred by the Higgs particles to the other particles.
   d. The Higgs field is the magnitude of torque transfers by the Higgs particles on the other particles.

11. What were the original three flavors of quarks discovered?
   a. up, down, and charm
   b. up, down, and bottom
   c. up, down, and strange
   d. up, down, and top

12. Protons are more massive than electrons. The three quarks in the proton account for only a small amount of this mass difference. What accounts for the remaining excess mass in protons compared to electrons?
   a. The highly energetic gluons connecting the quarks account for the remaining excess mass in protons compared to electrons.
   b. The highly energetic photons connecting the quarks account for the remaining excess mass in protons compared to electrons.
   c. The antiparallel orientation of the quarks present in a proton accounts for the remaining excess mass in protons compared to electrons.
   d. The parallel orientation of the quarks present in a proton accounts for the remaining excess mass in protons compared to electrons.

23.3 The Unification of Forces

13. Why is the unification of fundamental forces important?
   a. The unification of forces will help us understand fundamental structures of the universe.
   b. The unification of forces will help in the proof of the graviton.
   c. The unification of forces will help in achieving a speed greater than the speed of light.
   d. The unification of forces will help in studying antimatter particles.

14. Why are scientists unable to model the conditions of the universe at time periods shortly after the Big Bang?
   a. The amount of energy necessary to replicate the Planck Epoch is too high.
   b. The amount of energy necessary to replicate the Planck Epoch is too low.
   c. The volume of setup necessary to replicate the Planck Epoch is too high.
   d. The volume of setup necessary to replicate the Planck Epoch is too low.

15. What role does proton decay have in the search for GUTs?
   a. Proton decay is a premise of a number of GUTs.
   b. Proton decay negates the validity of a number of GUTs.

16. What is the name for the theory of unification of all four fundamental forces?
   a. the theory of everything
   b. the theory of energy-to-mass conversion
   c. the theory of relativity
   d. the theory of the Big Bang

17. Is it easier for scientists to find evidence for the Grand Unified Theory or the Theory of Everything? Explain.
   a. Theory of Everything, because it requires 10^{19} GeV of energy
   b. Theory of Everything, because it requires 10^{14} GeV of energy
   c. Grand Unified Theory, because it requires

10^{19} GeV of energy

d. Grand Unified Theory, because it requires 10^{14} GeV of energy

Critical Thinking Items

23.1 The Four Fundamental Forces

18. The gravitational force is considered a very weak force. Yet, it is strong enough to hold Earth in orbit around the Sun. Explain this apparent disparity.

a. At the level of the Earth-to-Sun distance, gravity is the strongest acting force because neither the strong nor the weak nuclear force exists at this distance.

b. At the level of the Earth-to-Sun distance, gravity is the strongest acting force because both the strong and the weak nuclear force is minimal at this distance.

19. True or False—Given that their carrier particles are massless, some may argue that the electromagnetic and gravitational forces should maintain the same value at all distances from their source. However, both forces decrease with distance at a rate of $\frac{1}{r}$

a. false

b. true

20. Why is a stationary target considered inefficient in a particle accelerator?

a. The stationary target recoils upon particle strike, thereby transferring much of the particle's energy into its motion. As a result, a greater amount of energy goes into breaking the particle into its constituent components.

b. The stationary target contains zero kinetic energy, so it requires more energy to break the particle into its constituent components.

c. The stationary target contains zero potential energy, so it requires more energy to break the particle into its constituent components.

d. The stationary target recoils upon particle strike, transferring much of the particle's energy into its motion. As a result, a lesser amount of energy goes into breaking the particle into its constituent components.

21. Compare the total strong nuclear force in a lithium atom to the total strong nuclear force in a lithium ion (Li^{+1}).

a. The total strong nuclear force in a lithium atom is thrice the total strong nuclear force in a lithium ion.

b. The total strong nuclear force in a lithium atom is twice the total strong nuclear force in a lithium ion.

c. The total strong nuclear force in a lithium atom is the same as the total strong nuclear force in a lithium ion.

d. The total strong nuclear force in a lithium atom is half the total strong nuclear force in a lithium ion.

23.2 Quarks

22. Explain why it is not possible to find a particle composed of just two quarks.

a. A particle composed of two quarks will have an integral charge and a white color. Hence, it cannot exist.

b. A particle composed of two quarks will have an integral charge and a color that is not white. Hence, it cannot exist.

c. A particle composed of two quarks will have a fractional charge and a white color. Hence, it cannot exist.

d. A particle composed of two quarks will have a fractional charge and a color that is not white. Hence, it cannot exist.

23. Why are mesons considered unstable?

a. Mesons are composites of two antiparticles that quickly annihilate each other.

b. Mesons are composites of two particles that quickly annihilate each other.

c. Mesons are composites of a particle and antiparticle that quickly annihilate each other.

d. Mesons are composites of two particles and one antiparticle that quickly annihilate each other.

24. Does antimatter have a negative mass?

a. No, antimatter does not have a negative mass.

b. Yes, antimatter does have a negative mass.

25. What similarities exist between the Standard Model and the periodic table of elements?

a. During their invention, both the Standard Model and the periodic table organized material by mass.

b. At the times of their invention, both the Standard Model and the periodic table organized material by charge.

c. At the times of their invention, both the Standard Model and the periodic table organized material by interaction with other available particles.

d. At the times of their invention, both the Standard Model and the periodic table organized material by size.

26. How were particle collisions used to provide evidence of the Higgs boson?

a. Because some particles do not contain the Higgs boson, the collisions of such particles will cause their destruction.
b. Because only the charged particles contain the Higgs boson, the collisions of such particles will cause their destruction and will expel the Higgs boson.
c. Because all particles with mass contain the Higgs boson, the collisions of such particles will cause their destruction and will absorb the Higgs boson.
d. Because all particles with mass contain the Higgs boson, the collisions of such particles will cause their destruction and will expel the Higgs boson.

27. Explain how the combination of a quark and antiquark can result in the creation of a hadron.
a. The combination of a quark and antiquark can result in a particle with an integer charge and color of white, therefore satisfying the properties for a hadron.
b. The combination of a quark and antiquark must result in a particle with a negative charge and color of white, therefore satisfying the properties for a hadron.
c. The combination of a quark and antiquark can result in a particle with an integer charge and color that is not white, therefore satisfying the properties for a hadron.
d. The combination of a quark and antiquark can result in particle with a fractional charge and color that is not white, therefore satisfying the properties for a hadron.

23.3 The Unification of Forces

28. Why does the strength of the strong force diminish under high-energy conditions?
a. Under high-energy conditions, particles interacting under the strong force will be compressed closer together. As a result, the force between them will decrease.
b. Under high-energy conditions, particles interacting under the strong force will start oscillating. As a result, the force between them will increase.
c. Under high-energy conditions, particles interacting under the strong force will have high velocity. As a result, the force between them will decrease.
d. Under high-energy conditions, particles interacting under the strong force will start moving randomly. As a result, the force between them will decrease.

29. If some unknown cause of the red shift, such as light becoming *tired* from traveling long distances through empty space, is discovered, what effect would there be on cosmology?
a. The effect would be substantial, as the Big Bang is based on the idea that the red shift is evidence that galaxies are moving toward one another.
b. The effect would be substantial, as the Big Bang is based on the idea that the red shift is evidence that the galaxies are moving away from one another.
c. The effect would be substantial, as the Big Bang is based on the idea that the red shift is evidence that galaxies are neither moving away from nor moving toward one another.
d. The effect would be substantial, as the Big Bang is based on the idea that the red shift is evidence that galaxies are sometimes moving away from and sometimes moving toward one another.

30. How many molecules of water are necessary if scientists wanted to check the 10^{31} -yr estimate of proton decay within the course of one calendar year?
a. 10^{29} molecules
b. 10^{30} molecules
c. 10^{31} molecules
d. 10^{32} molecules

31. As energy of interacting particles increases toward the theory of everything, the gravitational force between them increases. Why does this occur?
a. As energy increases, the masses of the interacting particles will increase.
b. As energy increases, the masses of the interacting particles will decrease.
c. As energy increases, the masses of the interacting particles will remain constant.
d. As energy increases, the masses of the interacting particles starts changing (increasing or decreasing). As a result, the gravitational force between the particles will increase.

Performance Task

23.3 The Unification of Forces

32. Communication is an often overlooked and useful skill for a scientist, especially in a competitive field where financial resources are limited. Scientists are often required to explain their findings or the relevance of their work to agencies within the government in order to maintain funding to continue their research. Let's say you are an ambitious young particle physicist, heading an expensive project, and you need to justify its existence to the appropriate funding agency. Write a brief paper (about one page) explaining why molecular-level structure is important in the functioning of designed materials in a specific industry.
    - First, think of an industry where molecular-level structure is important.
    - Research what materials are used in that industry as well as what are the desired properties of the materials.
    - What molecular-level characteristics lead to what properties?

    One example would be explaining how flexible but durable materials are made up of long-chained molecules and how this is useful for finding more environmentally friendly alternatives to plastics. Another example is explaining why electrically conductive materials are often made of metal and how this is useful for developing better batteries.

TEST PREP

Multiple Choice

23.1 The Four Fundamental Forces

33. Which of the following is not one of the four fundamental forces?
    a. gravity
    b. friction
    c. strong nuclear
    d. electromagnetic

34. What type of carrier particle has not yet been found?
    a. gravitons
    b. W bosons
    c. Z bosons
    d. pions

35. What effect does an increase in electric potential have on the accelerating capacity of a Van de Graaff generator?
    a. It increases accelerating capacity.
    b. It decreases accelerating capacity.
    c. The accelerating capacity of a Van de Graaff generator is constant regardless of electric potential.
    d. Van de Graaff generators do not have the capacity to accelerate particles.

36. What force or forces exist between a proton and a second proton?
    a. The weak electrostatic force and strong magnetic force
    b. The weak electrostatic and strong gravitational force
    c. The weak frictional force and strong gravitational force
    d. The weak nuclear force, the strong nuclear force, and the electromagnetic force

23.2 Quarks

37. To what color must quarks combine for a particle to be constructed?
    a. black
    b. green
    c. red
    d. white

38. What type of hadron is always constructed partially of an antiquark?
    a. baryon
    b. lepton
    c. meson
    d. photno

39. What particle is typically released when two particles annihilate?
    a. graviton
    b. antimatter
    c. pion
    d. photno

40. Which of the following categories is not one of the three main categories of the Standard Model?
    a. gauge bosons
    b. hadrons
    c. leptons
    d. quarks

41. Analysis of what particles began the search for the Higgs boson?
    a. W and Z bosons
    b. up and down quarks
    c. mesons and baryons

d. neutrinos and photons

42. What similarities exist between the discovery of the quark and the discovery of the neutron?

a. Both the quark and the neutron were discovered by launching charged particles through an unknown structure and observing the particle recoil.
b. Both the quark and the neutron were discovered by launching electrically neutral particles through an unknown structure and observing the particle recoil.
c. Both quarks and neutrons were discovered by studying their deflection under an electric field.

23.3 The Unification of Forces

43. Which two forces were first combined, signifying the eventual desire for a Grand Unified Theory?

a. electric force and magnetic forces
b. electric force and weak nuclear force
c. gravitational force and the weak nuclear force
d. electroweak force and strong nuclear force

44. After the Big Bang, what was the first force to separate from the others?

a. electromagnetic force
b. gravity
c. strong nuclear force
d. weak nuclear force

45. What is the name of the device used by scientists to check for proton decay?

a. the cyclotron
b. the Large Hadron Collider
c. the Super-Kamiokande
d. the synchrotron

46. How do Feynman diagrams suggest the Grand Unified Theory?

a. The electromagnetic, weak, and strong nuclear forces all have similar Feynman diagrams.
b. The electromagnetic, weak, and gravitational forces all have similar Feynman diagrams.
c. The electromagnetic, weak, and strong forces all have different Feynman diagrams.

Short Answer

23.1 The Four Fundamental Forces

47. Why do people tend to be more aware of the gravitational and electromagnetic forces than the strong and weak nuclear forces?

a. The gravitational and electromagnetic forces act at short ranges, while strong and weak nuclear forces act at comparatively long range.
b. The strong and weak nuclear forces act at short ranges, while gravitational and electromagnetic forces act at comparatively long range.
c. The strong and weak nuclear forces act between all objects, while gravitational and electromagnetic forces act between smaller objects.
d. The strong and weak nuclear forces exist in outer space, while gravitational and electromagnetic forces exist everywhere.

48. What fundamental force is responsible for the force of friction?

a. the electromagnetic force
b. the strong nuclear force
c. the weak nuclear force

49. How do carrier particles relate to the concept of a force field?

a. Carrier particles carry mass from one location to another within a force field.
b. Carrier particles carry force from one location to another within a force field.
c. Carrier particles carry charge from one location to another within a force field.
d. Carrier particles carry volume from one location to another within a force field.

50. Which carrier particle is transmitted solely between nucleons?

a. graviton
b. photon
c. pion
d. W and Z bosons

51. Two particles of the same mass are traveling at the same speed but in opposite directions when they collide head-on.
What is the final kinetic energy of this two-particle system?

a. infinite
b. the sum of the kinetic energies of the two particles
c. zero
d. the product of the kinetic energies of the two particles

52. Why do colliding beams result in the location of smaller particles?

a. Colliding beams create energy, allowing more energy to be used to separate the colliding particles.
b. Colliding beams lower the energy of the system, so it requires less energy to separate the colliding particles.
c. Colliding beams reduce energy loss, so less energy

is required to separate colliding particles.
d. Colliding beams reduce energy loss, allowing more energy to be used to separate the colliding particles.

23.2 Quarks

53. What two features of quarks determine the structure of a particle?
a. the color and charge of individual quarks
b. the color and size of individual quarks
c. the charge and size of individual quarks
d. the charge and mass of individual quarks

54. What fundamental force does quantum chromodynamics describe?
a. the weak nuclear force
b. the strong nuclear force
c. the electromagnetic force
d. the gravitational force

55. Is it possible for a baryon to be constructed of two quarks and an antiquark?
a. Yes, the color of the three particles would be able to sum to white.
b. No, the color of the three particles would not be able to sum to white.

56. Can baryons be more massive than mesons?
a. no
b. yes

57. If antimatter exists, why is it so difficult to find?
a. There is a smaller amount of antimatter than matter in the universe; antimatter is quickly annihilated by its matter analogue.
b. There is a smaller amount of matter than antimatter in the universe; matter is annihilated by its antimatter analogue.
c. There is a smaller amount of antimatter than matter in universe; antimatter and its matter analogue coexist.
d. There is a smaller amount of matter than antimatter in the universe; matter and its antimatter analogue coexist.

58. Does a neutron have an antimatter counterpart?
a. No, the antineutron does not exist.
b. Yes, the antineutron does exist.

59. How are the four fundamental forces incorporated into the Standard Model of the atom?
a. The four fundamental forces are represented by their carrier particles, the electrons.
b. The four fundamental forces are represented by their carrier particles, the gauge bosons.
c. The four fundamental forces are represented by their carrier particles, the leptons.
d. The four fundamental forces are represented by their carrier particles, the quarks.

60. Which particles in the Standard Model account for the majority of matter with which we are familiar?
a. particles in fourth column of the Standard Model
b. particles in third column of the Standard Model
c. particles in the second column of the Standard Model
d. particles in the first column of the Standard Model

61. How can a particle gain mass by traveling through the Higgs field?
a. The Higgs field slows down passing particles; the decrease in kinetic energy is transferred to the particle's mass.
b. The Higgs field accelerates passing particles; the decrease in kinetic energy is transferred to the particle's mass.
c. The Higgs field slows down passing particles; the increase in kinetic energy is transferred to the particle's mass.
d. The Higgs field accelerates passing particles; the increase in kinetic energy is transferred to the particle's mass.

62. How does mass-energy conservation relate to the Higgs field?
a. The increase in a particle's energy when traveling through the Higgs field is countered by its increase in mass.
b. The decrease in a particle's kinetic energy when traveling through the Higgs field is countered by its increase in mass.
c. The decrease in a particle's energy when traveling through the Higgs field is countered by its decrease in mass.
d. The increase in a particle's energy when traveling through the Higgs field is countered by its decrease in mass.

23.3 The Unification of Forces

63. Why do scientists believe that the strong nuclear force and the electroweak force will combine under high energies?
a. The electroweak force will have greater strength.
b. The strong nuclear force and electroweak force will achieve the same strength.
c. The strong nuclear force will have greater strength.

64. At what energy will the strong nuclear force theoretically unite with the electroweak force?

a. 10^{12} eV
b. 10^{13} eV
c. 10^{14} eV
d. 10^{15} eV

65. While we can demonstrate the unification of certain forces within the laboratory, for how long were the four forces naturally unified within the universe?
a. 10^{-43} seconds
b. 10^{-41} seconds
c. 10^{-39} seconds
d. 10^{-38} seconds

66. How does the search for the Grand Unified Theory help test the standard cosmological model?
a. Scientists are increasing energy in the lab that models the energy in earlier, denser stages of the universe.
b. Scientists are increasing energy in the lab that models the energy in earlier, less dense stages of the universe.
c. Scientists are decreasing energy in the lab that models the energy in earlier, denser stages of the universe.
d. Scientists are decreasing energy in the lab that models the energy in earlier, less dense stages of the universe.

67. Why does finding proof that protons do not decay not disprove all GUTs?
a. Proton decay is not a premise of all GUTs, and current GUTs can be amended in response to new findings.
b. Proton decay is a premise of all GUTs, but current GUTs can be amended in response to new findings.

68. When accelerating elementary particles in a particle accelerator, they quickly achieve a speed approaching the speed of light. However, as time continues, the particles maintain this speed yet continue to increase their kinetic energy. How is this possible?
a. The speed remains the same, but the masses of the particles increase.
b. The speed remains the same, but the masses of the particles decrease.
c. The speed remains the same, and the masses of the particles remain the same.
d. The speed and masses will remain the same, but temperature will increase.

Extended Response

23.1 The Four Fundamental Forces

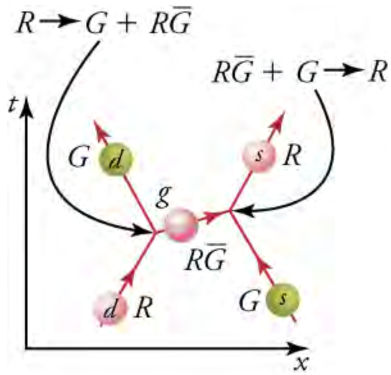
69. If the strong attractive force is the greatest of the four fundamental forces, are all masses fated to combine together at some point in the future? Explain.
a. No, the strong attractive force acts only at incredibly small distances. As a result, only masses close enough to be within its range will combine.
b. No, the strong attractive force acts only at large distances. As a result, only masses far enough apart will combine.
c. Yes, the strong attractive force acts at any distance. As a result, all masses are fated to combine together at some point in the future.
d. Yes, the strong attractive force acts at large distances. As a result, all masses are fated tocombine together at some point in the future.

70. How does the discussion of carrier particles relate to the concept of relativity?
a. Calculations of mass and energy during their transfer are relativistic, because carrier particles travel more slowly than the speed of sound.
b. Calculations of mass and energy during their transfer are relativistic, because carrier particles travel at or near the speed of light.
c. Calculations of mass and energy during their transfer are relativistic, because carrier particles travel at or near the speed of sound.
d. Calculations of mass and energy during their transfer are relativistic, because carrier particles travel faster than the speed of light.

71. Why are synchrotrons constructed to be very large?
a. By using a large radius, high particle velocities can be achieved using a large centripetal force created by large electromagnets.
b. By using a large radius, high particle velocities can be achieved without a large centripetal force created by large electromagnets.
c. By using a large radius, the velocities of particles can be reduced without a large centripetal force created by large electromagnets.
d. By using a large radius, the acceleration of particles can be decreased without a large centripetal force created by large electromagnets.

23.2 Quarks

72. In this image, how does the emission of the gluon cause the down quark to change from a red color to a green color?



a. The emitted red gluon is made up of a green and a red color. As a result, the down quark changes from a red color to a green color.
b. The emitted red gluon is made up of an anti-green and an anti-red color. As a result, the down quark changes from a red color to a green color.
c. The emitted red gluon is made up of a green and an anti-red color. As a result, the down quark changes from a red color to a green color.
d. The emitted red gluon is made up of an anti-green and a red color. As a result, the down quark changes from a red color to a green color.

73. Neutrinos are much more difficult for scientists to find when compared to other hadrons and leptons. Why is this?
a. Neutrinos are hadrons, and they lack charge.
b. Neutrinos are not hadrons, and they lack charge.
c. Neutrinos are hadrons, and they have positive charge.
d. Neutrinos are not hadrons, and they have a positive charge.

74. What happens to the masses of a particle and its antiparticle when the two annihilate at low energies?
a. The masses of the particle and antiparticle are transformed into energy in the form of photons.
b. The masses of the particle and antiparticle are converted into kinetic energy of the particle and antiparticle respectively.
c. The mass of the antiparticle is converted into kinetic energy of the particle.
d. The mass of the particle is converted into radiation energy of the antiparticle.

75. When a star erupts in a supernova explosion, huge numbers of electron neutrinos are formed in nuclear reactions. Such neutrinos from the 1987A supernova in the relatively nearby Magellanic Cloud were observed within hours of the initial brightening, indicating that they traveled to earth at approximately the speed of light. Explain how this data can be used to set an upper limit on the mass of the neutrino.
a. If the velocity of the neutrino is known, then the upper limit on mass of the neutrino can be set.
b. If only the kinetic energy of the neutrino is known, then the upper limit on mass of the neutrino can be set.
c. If either the velocity or the kinetic energy is known, then the upper limit on the mass of the neutrino can be set.
d. If both the kinetic energy and the velocity of the neutrino are known, then the upper limit on the mass of the neutrino can be set.

76. The term *force carrier particle* is shorthand for the scientific term *vector gauge boson*. From that perspective, can the Higgs boson truly be considered a *force carrier particle*?
a. No, the mass quality provided by the Higgs boson is a scalar quantity.
b. Yes, the mass quality provided by the Higgs boson results in a change of particle's direction.

23.3 The Unification of Forces

77. If a Grand Unified Theory is proven and the four forces are unified, it will still be correct to say that the orbit of the Moon is determined by the gravitational force. Explain why.
a. Gravity will not be a property of the unified force.
b. Gravity will be one property of the unified force.
c. Apart from gravity, no other force depends on the mass of the object.
d. Apart from gravity, no other force can make an object move in a fixed orbit.

78. As the universe expanded and temperatures dropped, the strong nuclear force separated from the electroweak force. Is it likely that under cooler conditions, the force of electricity will separate from the force of magnetism?
a. No, the electric force relies on the magnetic force and vice versa.
b. Yes, the electric and magnetic forces can be separated from each other.

79. Two pool balls collide head-on and stop. Their original kinetic energy is converted to heat and sound. Given that this is not possible for particles, what happens to their converted energy?
a. The kinetic energy is converted into relativistic potential energy, governed by the equation $E = \lambda mch$.
b. The kinetic energy is converted into relativistic mass, governed by the equation $E = \lambda m^2 c$.
c. The kinetic energy is converted into relativistic potential energy, governed by the equation $E = \lambda mgh$.

d. Their kinetic energy is converted into relativistic mass, governed by the equation $E = \lambda mc^2$.

APPENDIX A

Reference Tables

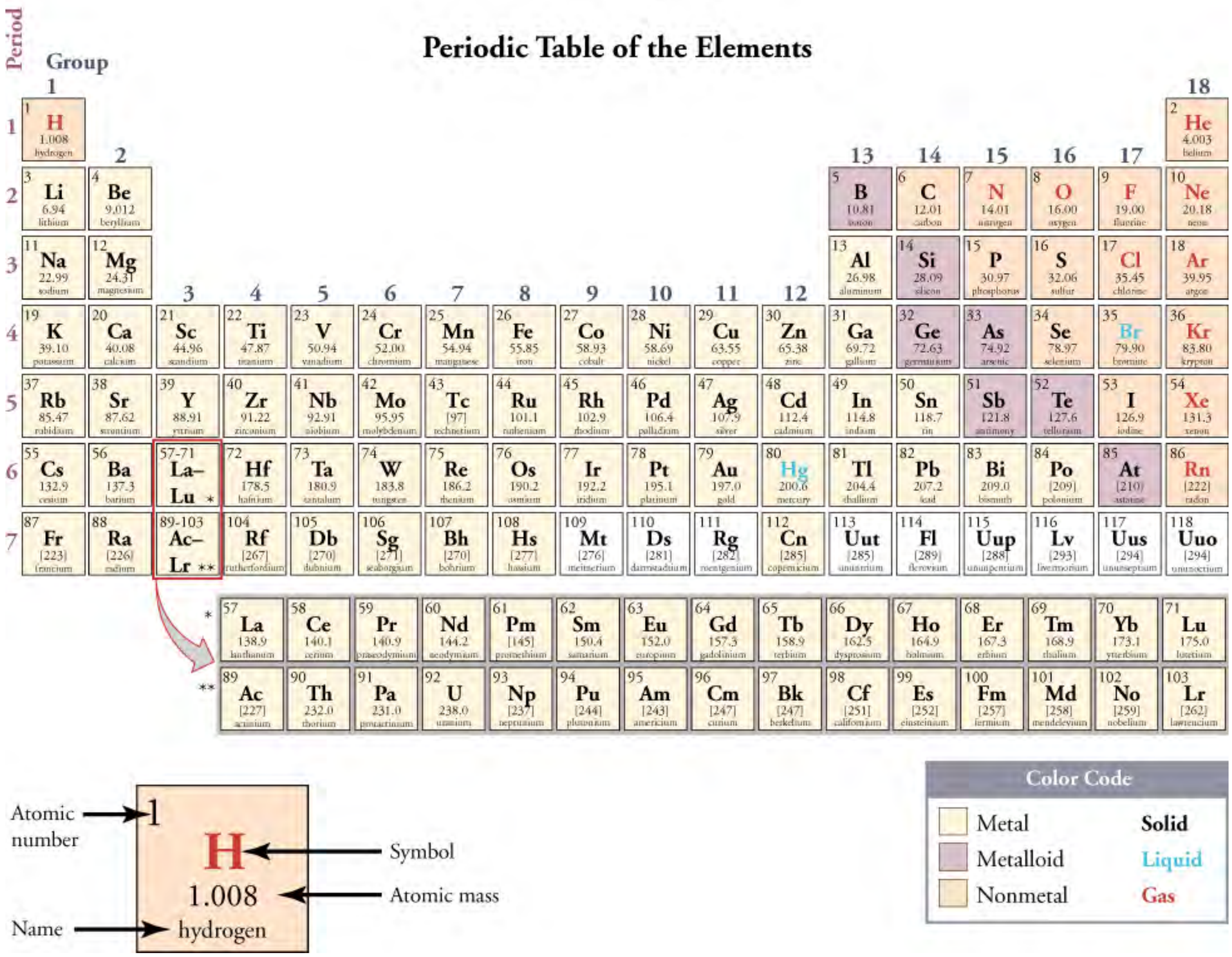


Figure A1 Periodic Table of Elements

Prefix	Symbol	Value	Prefix	Symbol	Value
tera	T	10^{12}	deci	d	10^{-1}
giga	G	10^{9}	centi	c	10^{-2}
mega	M	10^{6}	milli	m	10^{-3}
kilo	k	10^{3}	micro	µ	10^{-6}
hecto	h	10^{2}	nano	n	10^{-9}
deka	da	10^{1}	pico	p	10^{-12}

Table A1 Metric Prefixes for Powers of Ten and Their Symbols

Prefix	Symbol	Value	Prefix	Symbol	Value
___		10^0	femto	f	10^{-15}

Table A1 Metric Prefixes for Powers of Ten and Their Symbols

	Entity	Abbreviation	Name
Fundamental units	Length	m	meter
	Mass	kg	kilogram
	Time	s	second
	Current	A	ampere
Supplementary unit	Angle	rad	radian
Derived units	Force	$N = kg \cdot \frac{m}{s^2}$	newton
	Energy	$J = kg \cdot m^2$	joule
	Power	$W = \frac{J}{s}$	watt
	Pressure	$Pa = \frac{N}{m^2}$	pascal
	Frequency	$Hz = \frac{1}{s}$	hertz
	Electronic potential	$V = \frac{J}{C}$	volt
	Capacitance	$F = \frac{C}{V}$	farad
	Charge	$C = s \cdot A$	coulomb
	Resistance	$\Omega = \frac{V}{A}$	ohm
	Magnetic field	$T = \frac{N}{A \cdot m}$	tesla
	Nuclear decay rate	$Bq = \frac{1}{s}$	becquerel

Table A2 SI Units

Length	1 inch (in.) = 2.54 cm (exactly)
	1 foot (ft) = 0.3048 m
	1 mile (mi) = 1.609 km

Table A3 Selected British Units

Force	1 pound (lb) = 4.448 N
Energy	1 British thermal unit (Btu) = 1.055×10^3 J
Power	1 horsepower (hp) = 746 W
Pressure	1 lb/in^2 = 6.895×10^3 Pa

Table A3 Selected British Units

Length	1 light year (ly) = 9.46×10^{15} m
	1 astronomical unit (au) = 1.50×10^{11} m
	1 nautical mile = 1.852 km
	1 angstrom(Å) = 10^{-10} m
Area	1 acre (ac) = 4.05×10^3 m2
	1 square foot (ft^2) 9.29×10^{-2} m^3
	1 barn (*b*) = 10^{-28} m^2
Volume	1 liter (*L*) = 10^{-3} m^3
	1 U.S. gallon (gal) = 3.785×10^{-3} m^3
Mass	1 solar mass = 1.99×10^{30} kg
	1 metric ton = 10^3 kg
	1 atomic mass unit (*u*) = 1.6605×10^{-27} kg
Time	1 year (*y*) = 3.16×10^7 s
	1 day (*d*) = 86,400 s
Speed	1 mile per hour (mph) = 1.609 km / h
	1 nautical mile per hour (naut) = 1.852 km / h
Angle	1 degree (°) = 1.745x10^{-2} rad
	1 minute of arc (') = 1 / 60 degree
	1 second of arc ('') = 1 / 60 minute of arc
	1 grad = 1.571×10^{-2} rad

Table A4 Other Units

Energy	1 kiloton TNT (kT) = 4.2×10^{12} J
	1 kilowatt hour (kW · *h*) = 3.60 × 106 J
	1 food calorie (kcal) = 4186 J
	1 calorie (cal) = 4.186 J
	1 electron volt (cV) = 1.60×10^{-19} J
Pressure	1 atmosphere (atm) = 1.013×10^5 Pa
	1 millimeter of mercury (mm Hg) = 133.3 Pa
	1 torricelli (torr) = 1 mm Hg = 133.3 Pa
Nuclear decay rate	1 curie (Ci) = 3.70×10^{10} Bq

Table A4 Other Units

Circumference of a circle with radius *r* or diameter *d*	$C = 2\pi = \pi d$
Area of a circle with radius *r* or diameter *d*	$A = \pi r^2 = \pi d^2/4$
Area of a sphere with radius *r*	$A = 4\pi r^2$
Volume of a sphere with radius *r*	$V = (4/3)\left(\pi r^3\right)$

Table A5 Useful formulae

Symbol	Meaning	Best Value	Approximate Value
c	Speed of light in vacuum	$2.99792458 \times 10^8 \text{m/s}$	$3.00 \times 10^8 \text{m/s}$
G	Gravitational constant	$6.67384\,(80) \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2$	$6.67 \times 10^{-11} \text{N} \cdot \text{m}^2/\text{kg}^2$
${}^{N}A$	Avogadro's number	$6.02214129\,(27) \times 10^{23} \text{J/K}$	6.02×10^{23}
k	Boltzmann's constant	$1.3806488\,(13) \times 10^{-23} \text{J/K}$	$1.38 \times 10^{-23} \text{J/K}$
R	Gas constant	$8.3144621\,(75)\ \text{J/mol} \cdot \text{K}$	$8.31\ \text{J/mol} \cdot \text{K} = 1.99\ \text{cal/mol} \cdot \text{K}$ $= 0.0821\ \text{atm} \cdot \text{L/mol} \cdot \text{K}$
σ	Stefan-Boltzmann Constant	$5.670373\,(21) \times 10^{-8} \text{W/m}^2 \cdot \text{K}$	$5.67 \times 10^{-8} \text{W/m}^2 \cdot \text{K}$
k	Coulomb force constant	$8.987551788... \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2$	$8.99 \times 10^9 \text{N} \cdot \text{m}^2/\text{C}^2$

Table A6 Important Constants

Symbol	Meaning	Best Value	Approximate Value
q_e	Charge on electron	$-1.602176565\,(35) \times 10^{-19}\,\mathrm{C}$	$-1.60 \times 10^{-19}\,\mathrm{C}$
ε_0	Permittivity of free space	$8.854187817\ldots \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$	$8.85 \times 10^{-12}\,\mathrm{C^2/N \cdot m^2}$
μ_0	Permeability of free space	$4\pi \times 10^{-7}\,\mathrm{T \cdot m/A}$	$1.26 \times 10^{-6}\,\mathrm{T \cdot m/A}$
h	Planck's constant	$6.62606957\,(29) \times 10^{-34}\,\mathrm{J \cdot s}$	$6.63 \times 10^{-34}\,\mathrm{J \cdot s}$

Table A6 Important Constants

Alpha	Α	α
Beta	Β	β
Gamma	Γ	γ
Delta	Δ	δ
Epsilon	Ε	ε
Zeta	Ζ	ζ
Eta	Η	η
Theta	Θ	θ
Iota	Ι	ι
Kappa	Κ	κ
Lambda	Λ	λ
Mu	Μ	μ
Nu	Ν	ν
Xi	Ξ	ξ
Omicron	Ο	ο
Pi	Π	π
Rho	Ρ	ρ
Sigma	Σ	σ
Tau	Τ	τ

Table A7 The Greek Alphabet

Upsilon	Υ	υ
Phi	Φ	ϕ
Chi	Χ	χ
Psi	Ψ	ψ
Omega	Ω	ω

Table A7 The Greek Alphabet

Sun	mass	1.99×10^{30} kg
	average radius	6.96×10^{8} m
	Earth-sun distance (average)	1.496×10^{11} m
Earth	mass	5.9736×10^{24} kg
	average radius	6.376×10^{6} m
	orbital period	3.16×10^{7} s
Moon	mass	7.35×10^{22} kg
	average radius	1.74×10^{6} s
	orbital period (average)	2.36×10^{6} s
	Earth-moon distance (average)	3.84×10^{8} m

Table A8 Solar System Data

Atomic number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $^{t}1/2$
0	neutron	1	n	1.008 665	β^-	10.37 min
1	Hydrogen	1	^1H	1.007 825	99.985%	
	Deuterium	2	^2H or D	2.014 102	0.015%	
	Tritium	3	^3H or T	3.016 050	β^-	12.33 y
2	Helium	3	^3He	3.016 030	1.38×10^{-4} %	

Table A9 Atomic Masses and Decay

Atomic number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		4	^{4}He	4.002 603	≈100%	
3	Lithium	6	^{6}Li	6.015 121	7.5%	
		7	^{7}Li	7.016 003	92.5%	
4	Beryllium	7	^{7}Be	7.016 928	EC	53.29 d
		9	^{9}Be	9.012 182	100%	
5	Boron	10	^{10}B	10.012 937	19.9%	
		11	^{11}B	11.009 305	80.1%	
6	Carbon	11	^{11}C	11.011 432	EC, β^+	
		12	^{12}C	12.000 000	98.90%	
		13	^{13}C	13.003 355	1.10%	
		14	^{14}C	14.003 241	β^-	5730 y
7	Nitrogen	13	^{12}N	13.005 738	β^+	9.96 min
		14	^{13}N	14.003 074	99.63%	
		15	^{14}N	15.000 108	0.37%	
8	Oxygen	15	^{15}O	15.003 065	EC, β^+	122 s
		16	^{16}O	15.994 915	99.76%	
		18	^{18}O	17.999 160	0.200%	
9	Fluorine	18	^{18}F	18.000 937	EC, β^+	1.83 h
		19	^{19}F	18.998 403	100%	
10	Neon	20	^{20}Ne	19.992 435	90.51%	
		22	^{22}Ne	21.991 383	9.22%	
11	Sodium	22	^{22}Na	21.994 434	β^+	2.602 y
		23	^{23}Na	22.989 767	100%	
		24	^{24}Na	23.990 961	β^-	14.96 h

Table A9 Atomic Masses and Decay

Atomic number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
12	Magnesium	24	^{24}Mg	23.985 042	78.99%	
13	Aluminum	27	^{27}Al	26.981 539	100%	
14	Silicon	28	^{28}Si	27.976 927	92.23%	2.62h
		31	^{31}Si	30.975 362	β^-	
15	Phosphorus	31	^{31}P	30.973 762	100%	
		32	^{32}P	31.973 907	β^-	14.28 d
16	Sulfur	32	^{32}S	31.972 070	95.02%	
		35	^{35}S	34.969 031	β^-	87.4 d
17	Chlorine	35	^{35}Cl	34.968 852	75.77%	
		37	^{37}Cl	36.965 903	24.23%	
18	Argon	40	^{40}Ar	39.962 384	99.60%	
19	Potassium	39	^{39}K	38.963 707	93.26%	
		40	^{40}K	39.963 999	0.0117%, EC, β^-	1.28×10^9 y
20	Calcium	40	^{40}Ca	39.962 591	96.94%	
21	Scandium	45	^{45}Sc	44.955 910	100%	
22	Titanium	48	^{48}Ti	47.947 947	73.8%	
23	Vanadium	51	^{51}V	50.943 962	99.75%	
24	Chromium	52	^{52}Cr	51.940 509	83.79%	
25	Manganese	55	^{55}Mn	54.938 047	100%	
26	Iron	56	^{56}Fe	55.934 939	91.72%	
27	Cobalt	59	^{59}Co	58.933 198	100%	
		60	^{60}Co	59.933 819	β^-	5.271 y
28	Nickel	58	^{58}Ni	57.935 346	68.27%	

Table A9 Atomic Masses and Decay

Atomic number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		60	^{60}Ni	59.930 788	26.10%	
29	Copper	63	^{63}Cu	62.939 598	69.17%	
			^{65}Cu	64.927 793	30.83%	
30	Zinc	64	^{64}Zn	63.929 145	48.6%	
		66	^{66}Zn	65.926 034	27.9%	
31	Gallium	69	^{69}Ga	68.925 580	60.1%	
32	Germanium	72	^{72}Ge	71.922 079	27.4%	
		74	^{74}Ge	73.921 177	36.5%	
33	Arsenic	75	^{75}As	74.921 594	100%	
34	Selenium	80	^{80}Se	79.916 520	49.7%	
35	Bromine	79	^{79}Br	78.918 336	50.69%	
36	Krypton	84	^{84}Kr	83.911 507	57.0%	
37	Rubidium	85	^{85}Rb	84.911 794	72.17%	
38	Strontium	86	^{86}Sr	85.909 267	9.86%	
		88	^{88}Sr	87.905 619	82.58%	
		90	^{90}Sr	89.907 738	β^-	28.8 y
39	Yttrium	89	^{89}Y	88.905 849	100%	
		90	^{90}Y	89.907 152	β^-	64.1 h
40	Zirconium	90	^{90}Zr	89.904 703	51.45%	
41	Niobium	93	^{93}Nb	92.906 377	100%	
42	Molybdenum	98	^{98}Mo	97.905 406	24.13%	
43	Technetium	98	^{98}Tc	97.907 215	β^-	4.2×10^6 y
44	Ruthenium	102	^{102}Ru	101.904 348	31.6%	

Table A9 Atomic Masses and Decay

Atomic number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
45	Rhodium	103	^{103}Rh	102.905 500	100%	
46	Palladium	106	^{106}Pd	105.903 478	27.33%	
47	Silver	107	^{107}Ag	106.905 092	51.84%	
		109	^{109}Ag	108.904 757	48.16%	
48	Cadmium	114	^{114}Cd	113.903 357	28.73%	
49	Indium	115	^{115}In	114.903 880	95.7%, β^-	4.4×10^{14} y
50	Tin	120	^{120}Sn	119.902 200	32.59%	
51	Antimony	121	^{121}Sb	120.903 821	57.3%	
52	Tellurium	130	^{130}Te	129.906 229	33.8%, β^-	2.5×10^{21} y
53	Iodine	127	^{127}I	126.904 473	100%	
		131	^{131}I	130.906 114	β^-	8.040 d
54	Xenon	132	^{132}Xe	131.904 144	26.9%	
		136	^{136}Xe	135.907 214	8.9%	
55	Cesium	133	^{133}Cs	132.905 429	100%	
		134	^{134}Cs	133.906 696	EC, β^-	2.06 y
56	Barium	137	^{137}Ba	136.905 812	11.23%	
		138	^{138}Ba	137.905 232	71.70%	
57	Lanthanum	139	^{139}La	138.906 346	99.91%	
58	Cerium	140	^{140}Ce	139.905 433	88.48%	
59	Praseodymium	141	^{141}Pr	140.907 647	100%	
60	Neodymium	142	^{142}Nd	141.907 719	27.13%	
61	Promethium	145	^{145}Pm	144.912 743	EC, α	17.7 y
62	Samarium	152	^{152}Sm	151.919 729	26.7%	

Table A9 Atomic Masses and Decay

Atomic number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
63	Europium	153	^{153}Eu	152.921 225	52.2%	
64	Gadolinium	158	^{158}Gd	157.924 099	24.84%	
65	Terbium	159	^{159}Tb	158.925 342	100%	
66	Dysprosium	164	^{164}Dy	163.929 171	28.2%	
67	Holmium	165	^{165}Ho	164.930 319	100%	
68	Erbium	166	^{166}Ho	165.930 290	33.6%	
69	Thulium	169	^{169}Tm	168.934 212	100%	
70	Ytterbium	174	^{174}Yb	173.938 859	31.8%	
71	Lutecium	175	^{175}Lu	174.940 770	97.41%	
72	Hafnium	180	^{180}Hf	179.946 545	35.10%	
73	Tantalum	181	^{181}Ta	180.947 992	99.98%	
74	Tungsten	184	^{184}W	183.950 928	30.67%	
75	Rhenium	187	^{187}Re	186.955 744	62.6%, β^-	4.6×10^{10}y
76	Osmium	191	^{191}Os	190.960 920	β^-	15.4 d
		192	^{192}Os	191.961 467	41.0%	
77	Iridium	191	^{191}Ir	190.960 584	37.3%	
		193	^{193}Ir	192.962 917	62.7%	
78	Platinum	195	^{195}Pt	194.964 766	33.8%	
79	Gold	197	^{197}Au	196.966 543	100%	
		198	^{198}Au	197.968 217	β^-	2.696 d
80	Mercury	199	^{199}Hg	198.968 253	16.87%	
		202	^{202}Hg	201.970 617	29.86%	
81	Thallium	205	^{205}Tl	204.974 401	70.48%	

Table A9 Atomic Masses and Decay

Atomic number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
82	Lead	206	^{206}Pb	205.974 440	24.1%	
		207	^{207}Pb	206.975 872	22.1%	
		208	^{208}Pb	207.976 627	52.4%	
		210	^{210}Pb	209.984 163	α, β^-	22.3 y
		211	^{211}Pb	210.988 735	β^-	36.1 min
		212	^{212}Pb	211.991 871	β^-	10.64 h
83	Bismuth	209	^{209}Bi	208.980 374	100%	
		211	^{211}Bi	210.987 255	α, β^-	2.14 min
84	Polonium	210	^{210}Po	209.982 848	α	138.38 d
85	Astatine	218	^{218}At	218.008 684	α, β^-	1.6 s
86	Radon	222	^{222}Rn	222.017 570	α	3.82 d
87	Francium 2	223	^{223}Fr	223.019 733	α, β^-	21.8 min
88	Radium	226	^{226}Ra	226.025 402	α	1.60×10^3 y
89	Actinium	227	^{227}Ac	227.027 750	α, β^-	21.8 y
90	Thorium	228	^{228}Th	228.028 715	α	1.91 y
		232	^{232}Th	232.038 054	100%, α	1.41×10^{10} y
91	Protactinium	231	^{231}Pa	231.035 880	α	3.28×10^4 y
92	Uranium	233	^{233}U	233.039 628	α	1.59×10^3 y
		235	^{235}U	235.043 924	0.720%, α	7.04×10^8 y
		236	^{236}U	236.045 562	α	2.34×10^7 y

Table A9 Atomic Masses and Decay

Atomic number, Z	Name	Atomic Mass Number, A	Symbol	Atomic Mass (u)	Percent Abundance or Decay Mode	Half-life, $t_{1/2}$
		238	^{238}U	238.050 784	99.2745%, α	4.47×10^9 y
		239	^{239}U	239.054 289	β^-	23.5 min
93	Neptunium	239	^{239}Np	239.052 933	β^-	2.355 d
94	Plutonium	239	^{239}Pu	239.052 157	α	2.41×10^4 y
95	Americium	243	^{243}Am	243.061 375	α, fission	7.37×10^3 y
96	Curium	245	^{245}Cm	245.065 483	α	8.50×10^3 y
97	Berkelium	245	^{247}Bk	247.070 300	α	1.38×10^3 y
98	Californium	249	^{249}Cf	249.074 844	α	351 y
99	Einsteinium	254	^{254}Es	254.088 019	α, β^-	276 d
100	Fermium	253	^{253}Fm	253.085 173	EC, α	3.00 d
101	Mendelevium	255	^{255}Md	255.091 081	EC, α	27 min
102	Nobelium	255	^{255}No	255.093 260	EC, α	3.1 min
103	Lawrencium	257	^{257}Lr	257.099 480	EC, α	0.646 s
104	Rutherfordium	261	^{261}Rf	261.108 690	α	1.08 mim
105	Dubnium	262	^{262}Db	262.113 760	α, fission	34 s
106	Seaborgium	263	^{263}Sg	263.11 86	α, fission	0.8 s
107	Bohrium	262	^{262}Bh	262.123 1	α	0.102 s
108	Hassium	264	^{264}Hs	264.128 5	α	0.08 ms
108	Meitnerium	266	^{266}Mt	266.137 8	α	3.4 ms

Table A9 Atomic Masses and Decay

Isotope	$^{t}1/2$	Decay Mode	Energy(MeV)	Percent		T-Ray Energy(MeV)	Percent
^{3}H	12.33 y	β^-	0.0186	100%			
^{14}C	5730 y	β^-	0.156	100%			
^{13}N	9.96 min	β^+	1.20	100%			
^{22}Na	2.602 y	β^+	1.20	90%	γ	1.27	100%
^{32}P	14.28 d	β^-	1.71	100%			
^{35}S	87.4 d	β^-	0.167	100%			
^{36}Ci	3.00 × 105 y	β^-	0.710	100%			
^{40}K	1.28 × 109 y	β^-	1.31	89%			
^{43}K	22.3 h	β^-	0.827	87%	γs	0.373	87%
						0.618	87%
^{45}Ca	165 d	β^-	0.257	100%			
^{51}Cr	27.70 d	EC			γ	0.320	10%
^{52}Mn	5.59d	β^+	3.69	28%	γ s	1.33	28%
						1.43	28%
^{52}Fe	8.27 h	β^+	1.80	43%		0.169	43%
						0.378	43%
^{59}Fe	44.6 d	β^- s	0.273	45%	γ s	1.10	57%
			0.466	55%		1.29	43%
^{60}Co	5.271 y	β^-	0.318	100%	γ s	1.17	100%
						1.33	100%
^{65}Zn	244.1 d	EC			γ	1.12	51%
^{67}Ga	78.3 h	EC			γ s	0.0933	70%
						0.185	35%
						0.300	19%

Table A10 Selected Radioactive Isotopes

Isotope	$^{t}1/2$	Decay Mode	Energy(MeV)	Percent		T-Ray Energy(MeV)	Percent
						others	
^{75}Se	118.5 d	EC			γ s	0.121	20%
						0.136	65%
						0.265	68%
						0.280	20%
						others	
^{86}Rb	18.8 d	β^- s	0.69	9%	γ	1.08	9%
			1.77	91%			
^{85}Sr	64.8 d	EC			γ	0.514 1	100%
^{90}Sr	28.8 y	β^-	0.546	100%			
^{90}Y	64.1 h	β^-	2.28	100%			
^{99m}Tc	6.02 h	IT			γ	0.142	100%
^{113}mIn	99.5 ^{m}in	IT			γ	0.392	100%
^{123}I	13.0 h	EC			γ	0.159	≈100%
^{131}I	8.040 d	β^- s	0.248	7%	γ s	0.364	85%
			0.607	93%		others	
			others				
^{129}Cs	32.3 h	EC			γ s	0.0400	35%
						0.372	32%
						0.411	25%
						others	
^{137}Cs	30.17 y	β^- s	0.511	95%	γ	0.662	95%
			1.17	5%			
^{140}Ba	12.79 d	β^-	1.035	≈100%	γ s	0.030	25%

Table A10 Selected Radioactive Isotopes

Isotope	$^{t}1/2$	Decay Mode	Energy(MeV)	Percent		T-Ray Energy(MeV)	Percent
						0.044	65%
						0.537	24%
						others	
^{198}Au	2.696 d	β^-	1.161	≈100%	γ	0.412	≈100%
^{197}Hg	64.1 h	EC			γ	0.0733	100%
^{210}Po	138.38 d	α	5.41	100%			
^{226}Ra	1.60 × 103 y	αs	4.68	5%	γ	0.186 1	100%
			4.87	95%			
^{235}U	7.038 × 108 y	α	4.68	≈100%	γ s	Numerous	<0.400%
^{238}U	4.468 × 109 y	αs	4.22	23%	γ	0.050	23%
			4.27	77%			
^{237}Np	2.14 × 106 y	αs	numerous		γ s	numerous	<0.250%
			4.96 (max.)				
^{239}Pu	2.41 × 104 y	αs	5.19	11%	γ s	7.5 × 10-5	73%
			5.23	15%		0.013	15%
			5.24	73%		0.052	15%
						others	
^{243}Am	7.37 × 103 y	αs	Max. 5.44		γ s	0.075	
			5.37	88%		others	
			5.32	11%			
			others				

Table A10 Selected Radioactive Isotopes

Symbol	Meaning	Best Value	Approximate Value
m_e	Electron mass	$9.10938291(40) \times 10^{-31}$ kg	9.11×10^{-31} kg

Table A11 Submicroscopic masses

Symbol	Meaning	Best Value	Approximate Value
m_p	Proton mass	$1.672621777(74) \times 10^{-27}$ kg	1.6726×10^{-27} kg
m_n	Neutron mass	$1.674927351(74) \times 10^{-27}$ kg	1.6749×10^{-27} kg
u	Atomic mass unit	$1.660538921(73) \times 10^{-27}$ kg	1.6605×10^{-27} kg

Table A11 Submicroscopic masses

Substance	$p(\mathrm{kg/m^3})$	Substance	$p(\mathrm{kg/m^3})$
Air	1.29	Iron	7.86×10^3
Air (at 20°C and Atmospheric pressure)	1.20	Lead	11.3×10^3
Aluminum	2.70×10^3	Mercury	13.6×10^3
Benzene	0.879×10^3	Nitrogen gas	1.25
Brass	8.4×10^3	Oak	0.710×10^3
Copper	8.92×10^3	Osmium	22.6×10^3
Ethyl alcohol	0.806×10^3	Oxygen gas	1.43
Fresh water	1.00×10^3	Pine	0.373×10^3
Glycerin	1.26×10^3	Platinum	21.4×10^3
Gold	1.93×10^3	Seawater	1.03×10^3
Helium gas	1.79×10^{-1}	Silver	10.5×10^3
Hydrogen gas	8.99×10^{-2}	Tin	7.30×10^3
Ice	0.917×10^3	Uranium	18.7×10^3

Table A12 Densities of common substances (including water at various temperatures)

Substance	Specific Heat (J/kg · °C)	Substance	Specific Heat (J/kg · °C)
Elemental solids		*Other solids*	
Aluminum	900	Brass	380
Beryllium	1830	Glass	837
Cadmium	230	Ice (–5 °C)	2090

Table A13 Specific heats of common substances

Substance	Specific Heat (J/kg · °C)	Substance	Specific Heat (J/kg · °C)
Copper	387	Marble	860
Germanium	322	Wood	1700
Gold	129	*Liquids*	
Iron	448	Alcohol (ethyl)	2400
Lead	128	Mercury	140
Silicon	703	Water (15 °C)	4186
Silver	234	*Gas*	
		Steam (100 °C)	2010
Note: To convert values to units of cal/g · °C, divide by 4186			

Table A13 Specific heats of common substances

Substance	Melting Point (°C)	Latent Heat of Fusion (J/kg)	Boiling Point (°C)	Latent Heat of Vaporization (J/kg)
Helium	−272.2	5.23×10^3	−268.93	2.09×10^4
Oxygen	−218.79	1.38×10^4	−182.97	2.13×10^5
Nitrogen	−209.97	2.55×10^4	−195.81	2.01×10^5
Ethyl Alcohol	−114	1.04×10^5	78	8.54×10^5
Water	0.00	3.33×10^5	100.00	2.26×10^6
Sulfur	119	3.81×10^4	444.60	2.90×10^5
Lead	327.3	3.97×10^5	1750	8.70×10^5
Aluminum	660	3.97×10^5	2516	1.05×10^7
Silver	960.80	8.82×10^4	2162	2.33×10^6
Gold	1063.00	6.44×10^4	2856	1.58×10^6
Copper	1083	1.34×10^5	2562	5.06×10^6

Table A14 Heats of fusion and vaporization for common substances

Materials (Solids)	Average Linear Expansion Coefficient (α)(°C)$^{-1}$	Material (Liquids and Gases)	Average Volume Expansion Coefficient (B)(°C)$^{-1}$
Aluminum	24×10^{-6}	Acetone	1.5×10^{-4}
Brass and Bronze	19×10^{-6}	Alcohol, ethyl	1.12×10^{-4}
Concrete	12×10^{-6}	Benzene	1.24×10^{-4}
Copper	17×10^{-6}	Gasoline	9.6×10^{-4}
Glass (ordinary)	9×10^{-6}	Glycerin	4.85×10^{-4}
Glass (Pyrex)	3.2×10^{-6}	Mercury	1.82×10^{-4}
Invar (Ni-Fe alloy)	1.3×10^{-6}	Turpentine	9.0×10^{-4}
Lead	29×10^{-6}	Air* at 0°C	3.67×10^{-3}
Steel	13×10^{-6}	Helium*	3.665×10^{-3}

* The values given here assume the gases undergo expansion at constant pressure. However, the expansion of gases depends on the pressure applied to the gas. Therefore, gases do not have a specific value for the volume expansion coefficient.

Table A15 Coefficients of thermal expansion for common substances

Medium	v(m/s)	Medium	v(m/s)	Medium	v(m/s)
Gases		Liquids at 25°C		Solids*	
Hydrogen	1286	Glycerol	1904	Pyrex glass	5640
Helium	972	Seawater	1533	Iron	5950
Air	343	Water	1493	Aluminum	5100
Air	331	Mercury	1450	Brass	4700
Oxygen	317	Kerosene	1324	Copper	3560
		Methyl Alcohol	1143	Gold	3240
		Carbon tetrachloride	926	Lucite	2680
				Lead	1322
				Rubber	1600

Table A16 Speed of sound in various substances

Medium	v(m/s)	Medium	v(m/s)	Medium	v(m/s)

*Values given here are for propagation of longitudinal waves in bulk media. However, speeds for longitudinal waves in thin rods are slower, and speeds of transverse waves in bulk are even slower.

Table A16 Speed of sound in various substances

Source of Sound	*B(dB)*
Nearby jet airplane	150
Jackhammer machine gun	130
Siren; rock concert	120
Subway; power lawn mower	100
Busy traffic	80
Vacuum cleaner	70
Normal Conversation	60
Mosquito buzzing	40
whisper	30
Rustling leaves	10
Threshold of hearing	0

Table A17 Conversion of sound intensity to decibel level

Wavelength Range (nm)	Color Description
400-430	Violet
430-485	Blue
485-560	Green
560-590	Yellow
590-625	Orange
625-700	Red

Table A18 Wavelengths of visible light

Substance	Index of Refraction	Substance	Index of Refraction
Solids at 20°C		Liquids at 20°C	
Cubic zirconia	2.15	Benzene	1.501
Diamond (C)	2.419	Carbon disulfide	1.628
Flourite (CaF_2)	1.434	Carbon tetrachloride	1.461
Fused quartz (SiO_2)	1.458	Ethyl alcohol	1.361
Gallium phosphide	3.50	Glycerin	1.473
Glass, crown	1.52	Water	1.333
Glass, flint	1.66		
Ice (H_2O)	1.309	Gases at 0°C, 1 *atm*	
Polystyrene	1.49	Air	1.000 293
Sodium chloride (NaCl)	1.544	Carbon dioxide	1.000 45

Note: These values assume that light has a wavelength of 589 nm in vacuum.

Table A19 Indices of refraction

Hoop or thin cylindrical shell	$I_{CM} = MR^2$
Hollow cylinder	$I_{CM} = \frac{1}{2}M(R_1^2 + R_2^2)$
Solid cylinder or disk	$I_{CM} = \frac{1}{2}MR^2$
Rectangular plane	$I_{CM} = \frac{1}{12}M(a^2 + b^2)$
Long, thin rod with rotation axis through center	$I_{CM} = \frac{1}{12}ML^2$
Long, thin rod with rotation axis through end	$I_{CM} = \frac{1}{3}ML^2$
Solid sphere	$I_{CM} = \frac{2}{5}MR^2$
Thin spherical shell	$I_{CM} = \frac{2}{3}MR^2$

Table A20 Moments of inertia for different shapes

	μ_s	μ_k
Rubber on dry concrete	1.0	0.8

Table A21 Coefficients of friction for common objects on other objects

	μ_s	μ_k
Steel on steel	0.74	0.57
Aluminum on steel	0.61	0.47
Glass on glass	0.94	0.4
Copper on steel	0.53	0.36
Wood on wood	0.25-0.5	0.2
Waxed wood on wet snow	0.14	0.1
Waxed wood on dry snow	0.1	0.04
Metal on metal (lubricated)	0.15	0.06
Teflon on Teflon	0.04	0.04
Ice on ice	0.1	0.03
Synovial joints in humans	0.01	0.003

Note: All values are approximate. In some cases, the coefficient of friction can exceed 1.0.

Table A21 Coefficients of friction for common objects on other objects

Material	Dielectric Constant κ	Dielectric Strength* (10^6V/m)
Air (dry)	1.000 59	3
Bakelite	4.9	24
Fused quartz	4.3	8
Mylar	3.2	7
Neoprene rubber	6.7	12
Nylon	3.4	14
Paper	3.7	16
Paraffin-impregnated paper	3.5	11
Polystyrene	2.56	24
Polyvinyl chloride	3.4	40
Porcelain	6	8

Table A22 Dielectric constants

Material	Dielectric Constant κ	Dielectric Strength* (10^6V/m)
Pyrex glass	5.6	14
Silicone oil	2.5	15
Strontium titanate	233	8
Teflon	2.1	60
Vacuum	1.000 00	∞
Water	80	3

Table A22 Dielectric constants

INDEX

S

T